Initial value theorem

- In mathematical analysis, the initial value theorem is a theorem used to relate frequency domain expressions to the time domain behaviour as time approaches zero.
- Condition of existence of IVT:
  - The Laplace transform of x(t) is X(s) and If time t approaches to (0^+) then the function x(t) should exists.

- Mathematical expression of IVT

$$x(t = 0) = \lim_{s \to \infty} sF(s)$$

Question

- Find the initial value of the following functions

$$F(s) = \frac{(s+1)(9s+4)}{s(2s+3)(6s+7)}$$

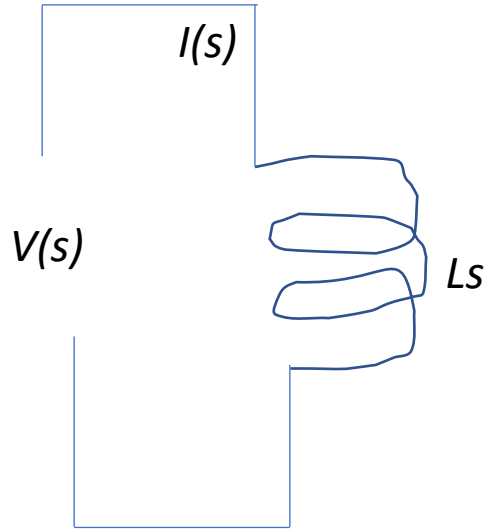
$$f(t) = 3 + \delta(t)$$

Solution

- $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} s \times \frac{(s+1)(9s+4)}{s(2s+3)(6s+7)} = \frac{9}{12}$
- NOT POSSIBLE TP APPLY IVT (why??)

Question

- Current flowing through 4H inductor is given by $I(s) = \frac{10}{s(s+2)}$find the initial voltage of the inductor.



Ans : 40 volt

Final value theorem

- Final value theorems relates frequency domain expressions to the time domain behaviour as time approaches infinity
- Condition of existence of FVT :
  - The Laplace transform of x(t) is X(s) and sX(s) has no pole on imaginary axis and in the R.H.P. (Right half Plane).

- Mathematical expression of FVT

$$x(t \to \infty) = \lim_{s \to 0} sF(s)$$

Question

- Find the final value of the following function

$$f(t) = 3 + e^{2t}$$

$$F(s) = \frac{10}{s} - \frac{40}{s(5 + 8s)}$$

Solution

- FVT not applicable
- $f(t \to \infty) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} s \times \left[\frac{10}{s} - \frac{40}{s(5+8s)}\right] = 2$


Thank You!