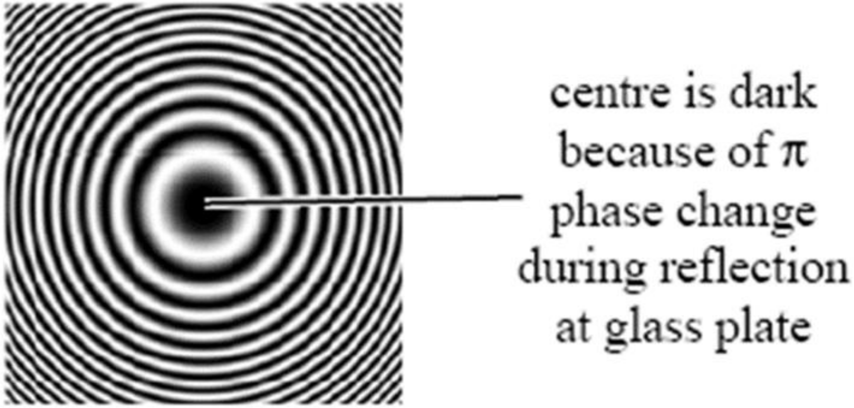
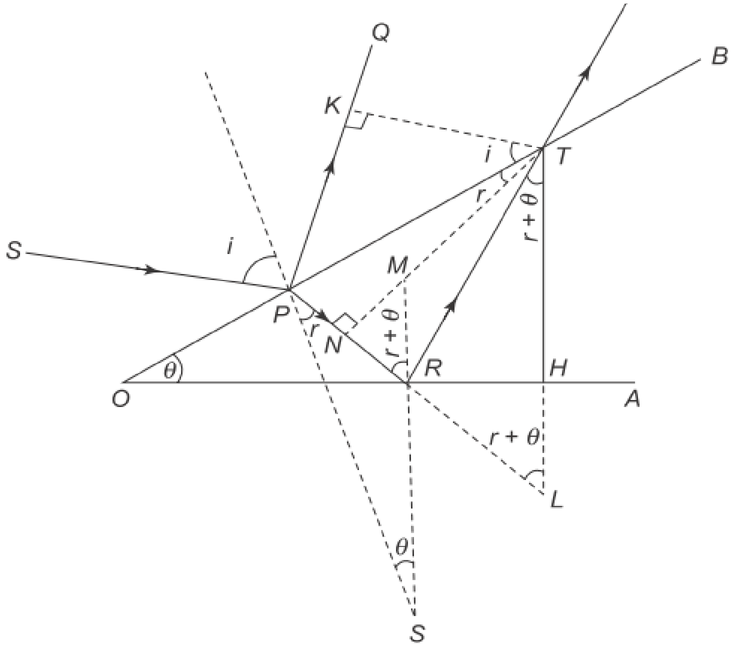
Interference

UNIT III
Optics
Lecture-3



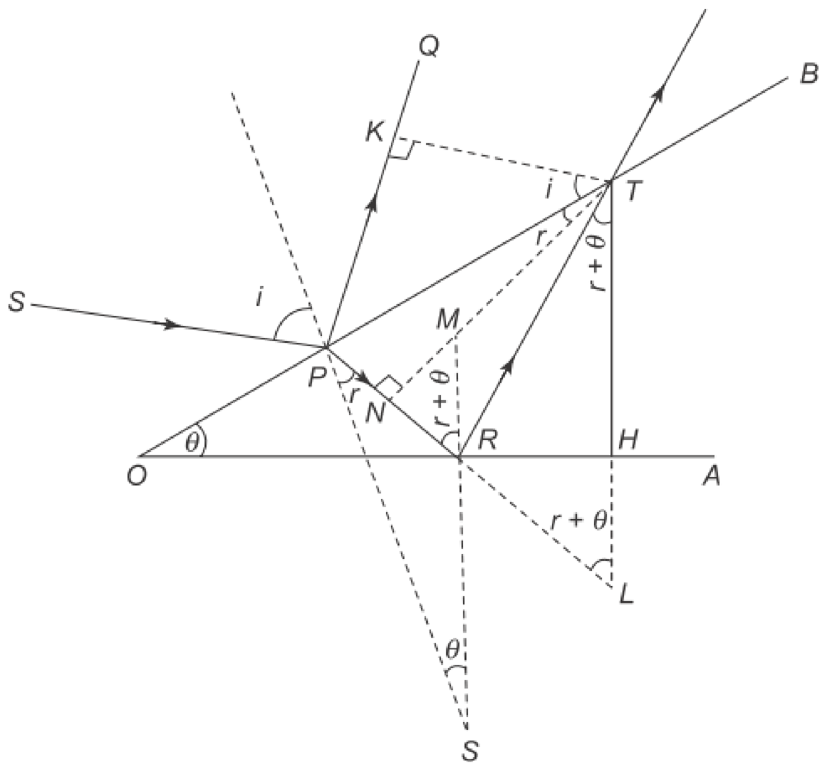


Content of Lecture

- INTERFERENCE DUE TO WEDGE-SHAPED THIN FILM
- FORMATION OF NEWTON’S RINGS
- EXPERIMENTAL METHOD FOR CALCULATION OF WAVELENGTH
- DETERMINATION OF REFRACTIVE INDEX OF A LIQUID BY NEWTON’S RING
- NEWTON’S RINGS WITH BOTH CURVED SURFACES

INTERFERENCE DUE TO WEDGE-SHAPED THIN FILM

Let us consider a wedge-shaped film of refractive index μ bounded by two plane surfaces OA and OB at angle θ. The thickness of the film is increasing from O to A.



INTERFERENCE DUE TO WEDGE-SHAPED THIN FILM

The optical path difference between reflected rays

p = μ(PR + RT) – PK

= μ(PN + NR + RT) – PK

From ΔPKT, sin i = $\frac{PK}{PT}$, and from Δ PNT, sin r = $\frac{PN}{PT}$

$$\mu = \frac{\sin i}{\sin r} = \frac{PK/PT}{PN/PT} = \frac{PK}{PN}$$

$$PK = \mu PN$$

$$p = \mu(PN + NR + RT) - \mu PN$$

$$p = \mu(NR + RT)$$

Now, ΔRTH and ΔRHL are congruent. Hence, $RT = RL$

$$\therefore \quad p = \mu(NR + RL)$$

$$= \mu NL$$

INTERFERENCE DUE TO WEDGE-SHAPED THIN FILM

Now from ΔTNL

$$\frac{NL}{TL} = \cos(r+\theta)$$

$$NL = TL\cos(r+\theta)$$

$$= (TH + HL)\cos(r+\theta)$$

$$= 2t\cos(r+\theta) \qquad (\because TH = HL)$$

$$\therefore \qquad p = 2\mu t\cos(r+\theta)$$

Now path difference as per Stokes treatment will become

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2}$$

(i) For maximum

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

or $$2\mu t\cos(r+\theta) = (2n-1)\frac{\lambda}{2}, n = 1, 2, 3,\ldots$$

(ii) For minimum

$$2\mu t\cos(r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

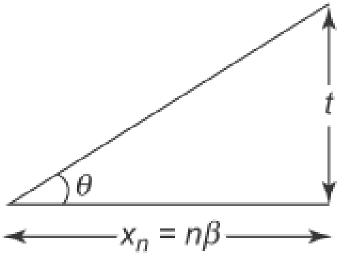
$$2\mu t\cos(r+\theta) = n\lambda$$

Fringe Width

$$\frac{t}{x_n} = \tan\theta$$

or $$t = x_n \tan\theta$$

Here, $x_n = n\beta$, β = fringe width and n = number of fringes.



∴ From Eq. (12.27), for a dark fringe,

$$2\mu x_n \tan\theta \cos(r+\theta) = n\lambda$$

or $$x_n = \frac{n\lambda}{2\mu\tan\theta\cos(r+\theta)}$$

For normal incidence and thin film $r = 0$, $\tan\theta = \theta$, and $\cos\theta = 1$. Therefore,

$$x_n = \frac{n\lambda}{2\mu\theta}$$

Similarly for $(n+1)$th fringe,

$$x_{n+1} = (n+1)\frac{\lambda}{2\mu\theta}$$

Therefore, distance or spacing between successive fringes, i.e., fringe width

$$x_{n+1} - x_n = (n+1)\frac{\lambda}{2\mu\theta} - n\frac{\lambda}{2\mu\theta}$$

or $$\beta = \frac{\lambda}{2\mu\theta}$$

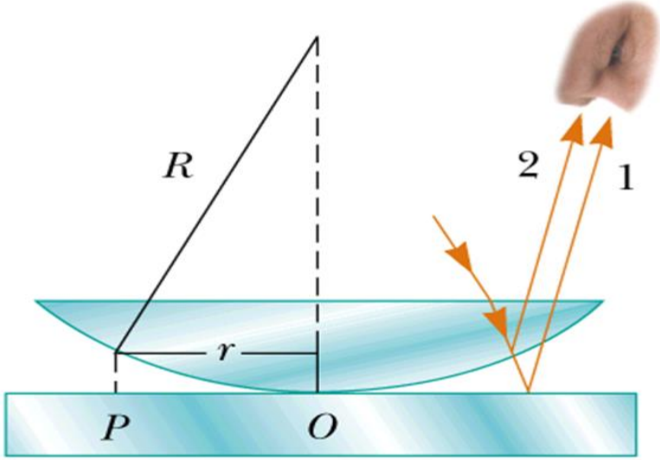
where θ is measured in radian.

FORMATION OF NEWTON'S RINGS

- When a planoconvex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film is formed between the lower surface of the lens and the upper surface of the plate.

- The thickness of the film gradually increases from the point of contact outwards.

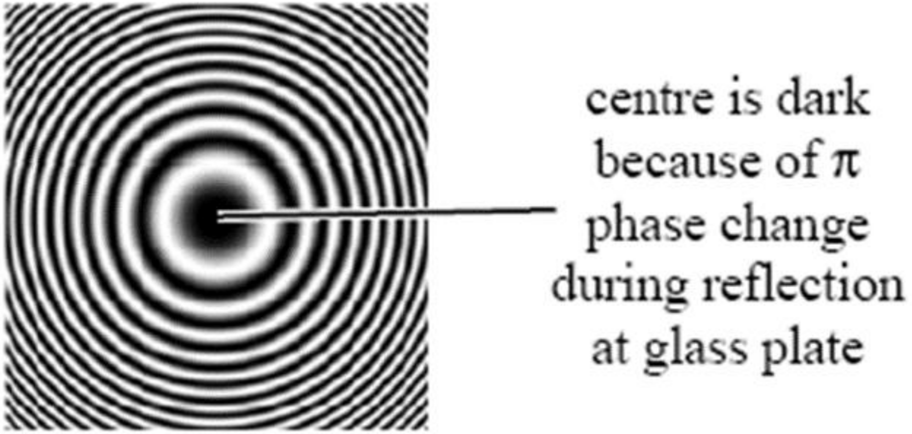
- If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings with their centre dark is formed in the air film.



FORMATION OF NEWTON'S RINGS

- These are called Newton's rings and can be seen through a low-power microscope focussed on the film. Newton's rings are formed as a result of interference between the light waves reflected from the upper and lower surfaces of the air film.

- Newton's rings are concentric and circular because of points equal to the thickness of film lie on circles with the point of the contact of the lens and the plate as centre.



Theory

In reflected light, the effective path difference between interfering rays will be

$$p = 2\mu t \cos r + \frac{\lambda}{2} *$$

Now, for air film, $\mu = 1$ and $r = 0$ for normal incidence. Therefore,

$$p = 2t + \frac{\lambda}{2}$$

- At the point of contact of the lens, the plate thickness t = 0; hence, p $= \frac{\lambda}{2}$. This is the condition for minimum intensity. Hence, the central spot is dark.

The condition for maximum intensity (bright fringe) is

$$p = n\lambda$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{(maxima)}$$

The condition for minimum intensity (dark fringe) is

$$p = (2n+1)\frac{\lambda}{2}$$

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{(minima)}$$

It is clear that a bright or dark fringe of any particular order n will occur for a constant value of thickness t.

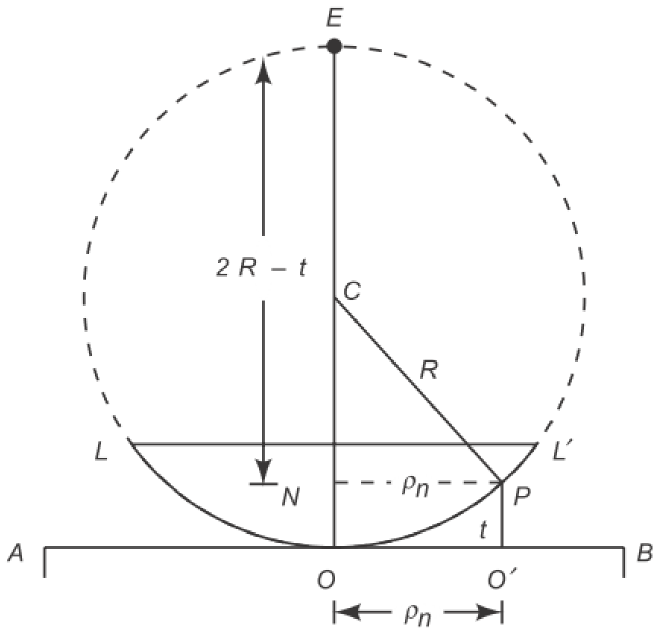
Diameters of Rings

From the property of a circle,

$$PN^2 = ON \times NE$$

$$\rho_n^{\,2} = t \times (2R - t)$$

$$= 2Rt - t^2$$



Since t is very small as compared to R, we can neglect t^2. Hence,

$$\rho_n^2 = 2Rt$$

or $$2t = \frac{\rho_n^2}{R}$$

Now the condition for a bright ring is given by

$$2t = (2n - 1)\frac{\lambda}{2}$$

$$\frac{\rho_n^2}{R} = (2n - 1)\frac{\lambda}{2}$$

$$\rho_n^2 = (2n - 1)\frac{\lambda R}{2}$$

Diameters of Rings

If D_n is the diameter of nth dark ring, then $D_n = 2\rho_n$. Therefore,

$$D_n^2 = 2(2n-1)\lambda R$$

$$D_n = \sqrt{2\lambda R}\sqrt{2n-1}$$

$$D_n \propto \sqrt{2n-1}$$

- Thus, the diameters of bright rings are proportional to the square roots of the odd natural numbers.

- The condition for a dark ring is given by

$$2t = n\lambda$$

$$\frac{\rho_n^2}{R} = n\lambda$$

If D_n is the diameter of nth dark ring, then $\rho_n = D_n/2$. Therefore,

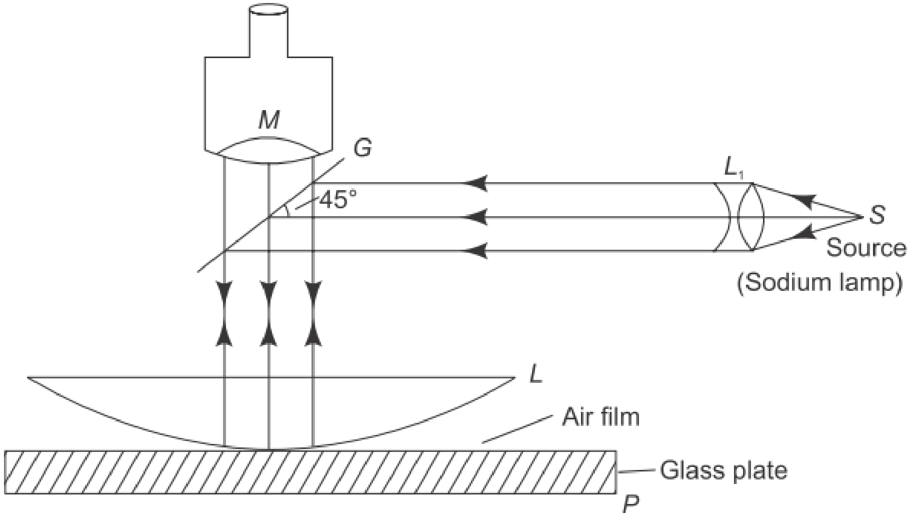
$$\frac{D_n^2}{4R} = n\lambda$$

$$D_n^2 = 4n\lambda R$$

$$D_n = \sqrt{4R\lambda}\sqrt{n}$$

$$D_n \propto \sqrt{n}$$

Experimental Method for Calculation of Wavelength



We know that for dark rings,

$$D_n^2 = 4n\,\lambda R$$

For $(n+p)$ dark ring,

$$D_{n+p}^2 = 4\,(n+p)\,\lambda R$$

Both will give

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

or

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

- The liquid whose refractive index is to be determined is placed between the lens and the glass plate P in the Newton's ring arrangement.
- The effective path difference between interfering rays is 2μt cos r + λ/2 for normal incidence r = 0, which gives path difference as 2μt + λ/2 .

The condition for nth bright fringe is

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \quad 2t = (2n-1)\frac{\lambda}{2\mu}$$

We know that if D_n is the diameter of nth bright rin

$$2t = \frac{D_n^2}{4R}$$

$$\therefore \quad \frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2\mu}$$

$$\therefore \quad D_n^2 = 2(2n-1)\frac{\lambda R}{\mu}$$

If D_{n+p} is the diameter of $(n+p)$th ring, then

$$D_{n+p}^2 = 2[2(n+p)-1]\frac{\lambda R}{\mu}$$

$$\therefore \quad [D_{n+p}^2 - D_n^2]_{\text{liquid}} = \frac{4p\lambda R}{\mu}$$

For air, $\mu = 1$,

$$\Rightarrow \quad [D_{n+p}^2 - D_n^2]_{\text{air}} = 4p\lambda R$$

$$\therefore \quad \mu = \frac{[D_{n+p}^2 - D_n^2]_{\text{air}}}{[D_{n+p}^2 - D_n^2]_{\text{liquid}}}$$

Newton's Rings with Both Curved Surfaces

Case 1: Lower surface is convex [Fig. 12.23(a)]. In this case, the thickness of the air film

$$t = t_1 + t_2 \tag{12.39}$$

If R_1 and R_2 are the radii of curvature of upper and lower curved surfaces, respectively, we can have

$$t = \frac{\rho_n^2}{2R_1} + \frac{\rho_n^2}{2R_2}$$

$$2t = \frac{\rho_n^2}{R_1} + \frac{\rho_n^2}{R_2} \tag{12.40}$$

where ρ_n be the radius of nth ring at the thickness t. The condition for dark ring at normal incidence is

$$2t = n\lambda \tag{12.41}$$

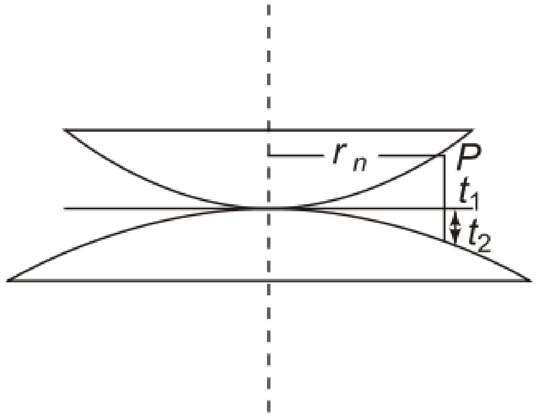
From Eqs. (12.40) and (12.41),

$$\rho_n^2\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = n\lambda \tag{12.42}$$

If D_n is the diameter of nth ring, $\rho_n = D_n/2$

$$\frac{D_n^2}{2}\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = n\lambda$$

$$D_n^2 = \frac{4n\lambda}{1/R_1 + 1/R_2} \tag{12.43}$$



(a)

Newton's Rings with Both Curved Surfaces

Case 2: Lower surface is concave [Fig. 12.23(b)]. Now,

$$t = t_1 - t_2$$

or
$$2t = \rho_n^2 \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

From Eqs. (12.41) and (12.43),

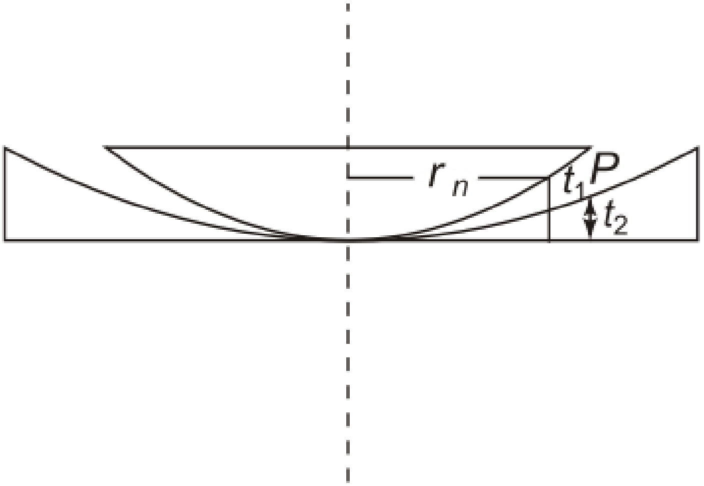
$$\rho_n^2 \left[\frac{1}{R_1} - \frac{1}{R_2}\right] = n\lambda$$

$$\frac{Dn^2}{4} \left[\frac{1}{R_1} - \frac{1}{R_2}\right] = n\lambda$$

$$\Rightarrow \quad D_n^2 = \frac{4n\lambda}{\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

Thus, combining Eqs. (12.43) and (12.45), we get

$$D_n^2 = \frac{4n\lambda}{\left[\frac{1}{R_1} \pm \frac{1}{R_2}\right]}$$



(b)

Example:1 In Newton's ring experiment, the diameter of 4th and 12th dark rings are 0.4 and 0.7 cm, respectively. Find the diameter of 20th dark ring.

Solution

The difference of $(n+p)$th and nth dark ring is given by

$$D^2_{n+p} - D^2_n = 4pR\lambda$$

Here $(n+p) = 12$ and $n = 4$, $D_{12} = 0.7$ cm, and $D_4 = 0.4$ cm.

$$\therefore \quad D^2_{12} - D^2_4 = 4 \times 8 \times R\lambda$$

$$D^2_{20} - D^2_4 = 4 \times 16 \times R\lambda$$

Dividing Eq. (3) by Eq. (2), we get

$$\frac{D_{20}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4\times16}{4\times8} = 2$$

$$D^2_{20} - D^2_4 = 2(D^2_{12} - D^2_4)$$

$$D^2_{20} = 2D^2_{12} - D^2_4$$

$$= 2 \times (0.7)^2 - (0.4)^2$$

$$D^2_{20} = 0.98 - 0.16 = 0.82$$

$$D_{20} = \sqrt{0.82} = 0.9 \text{ cm}$$

Example:2 The angle of the wedge is 0.3° and the wavelength of sodium D lines are 5890 Å and 5896 Å. Find the distance from the apex of the wedge at which the maximum due to the two wavelengths first coincide.

Solution

The condition for maxima for normal incidence in air film is given by

$$2t = (2n + 1)\frac{\lambda}{2} \qquad (1)$$

Let nth order maximum due to λ_1 coincides with $(n + 1)$th order maximum due to λ_2. Then

$$2t = (2n + 1) \cdot \frac{\lambda_1}{2} = (2n + 3) \cdot \frac{\lambda_2}{2} \qquad (2)$$

$$\therefore \qquad n = \frac{3\lambda_2 - \lambda_1}{2(\lambda_1 - \lambda_2)}$$

Substituting the value of n in Eq. (2), we get

$$2t = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}$$

If θ is the angle of wedge and x is the distance at which the maxima due to two wavelengths coincide $(\lambda_1 > \lambda_2)$, then

$$\tan\theta = \frac{t}{x}$$

$$\Rightarrow \qquad \theta = \frac{t}{x} \qquad (\tan\theta \approx \theta)$$

$$\Rightarrow \qquad t = \theta x$$

Substituting the value of t in Eq. (2), we get

$$2x\,\theta = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Here $\lambda_1 = 5896 \times 10^{-8}$ cm

$\lambda_2 = 5890 \times 10^{-8}$ cm

$$\theta = 0.3^\circ = \frac{0.3\times\pi}{180}$$

$$x = \frac{5896\times10^{-8}\times5890\times10^{-8}\times180}{2\left(5896\times10^{-8} - 5890\times10^{-8}\right)\times0.3\times3.14}$$

$$= 5.56 \text{ cm}$$

Assignment Based on this Lecture

- Discuss the phenomena interference due to wedge-shaped thin film and obtain the expression of fringe width.
- Explain the formation of newton's rings why its width decreases when we move from center to outward.
- Discuss the experimental method for calculation of wavelength of light using Newton's ring.
- Determination of refractive index of a liquid by newton's ring
- Explain the Newton's rings formed with both curved surfaces