Control Systems

Subject Code: BEC-26

Third Year ECE

Unit-I

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Lecture 6

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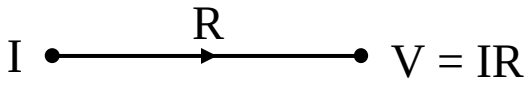
UNIT- I

- Introduction to Control system
  - Control System – Definition and Practical Examples
  - Basic Components of a Control System
- Feedback Control Systems:
  - Feedback and its Effect
  - Types of Feedback Control Systems
- Block Diagrams:
  - Representation and reduction
  - Signal Flow Graphs
- Modeling of Physical Systems:
  - Electrical Networks and Mechanical Systems
  - Force-Voltage Analogy
  - Force-Current Analogy

Signal Flow Graph (SFG) Representation:

1. SFG is a graphical representation of variables of a set of linear algebraic equations representing the system.
2. Variables are represented by small circles called nodes.
3. The line joining the nodes are called branches which is associated with a T.F and an arrow.
4. Eg: V = IR
5. Let V & I are the variables in which I is the input & V is the output



Properties of SFG:

1. SFG is only applicable to LTI system.
2. The signal gets multiplied by branch gain when it travels along it.
3. The value of variable is represented by any node is the algebraic sum of the signals entering at the node.

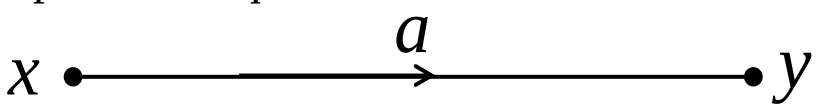
4. The no. of branches leaving a node doesn't affect the value of variable represented by that node.

Introduction

- ✓ SFG is a graphical representation of variables of a set of linear algebraic equations representing the system.
- ✓ Variables are represented by small circles called nodes.
- ✓ The directed lines joining the nodes are called branches which is associated with a T.F and an arrow.
- ✓ Alternative method to block diagram representation, developed by Samuel Jefferson Mason.
- ✓ Advantage: the availability of a flow graph gain formula, also called Mason's gain formula.
- ✓ It depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

Fundamentals of Signal Flow Graphs

- Consider a simple equation below and draw its signal flow graph: $y = ax$
- The signal flow graph of the equation is shown below;

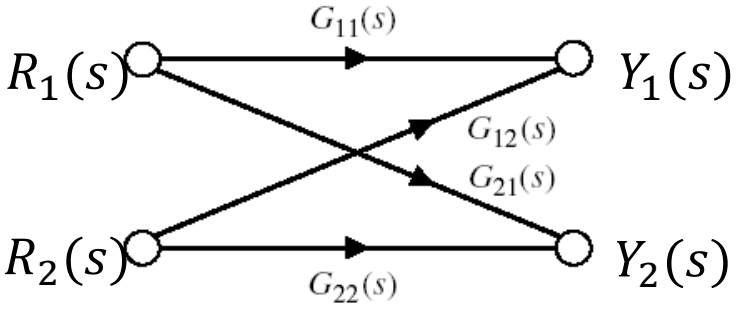


- Every variable in a signal flow graph is represented by a **Node**.
- Every transmission function in a signal flow graph is represented by a **Branch**.
- Branches are always **unidirectional**.
- The arrow in the branch denotes the **direction** of the signal flow.

Signal-Flow Graph Model 1:

$$Y_1(s) = G_{11}(s)\cdot R_1(s) + G_{12}(s)\cdot R_2(s)$$

$$Y_2(s) = G_{21}(s)\cdot R_1(s) + G_{22}(s)\cdot R_2(s)$$

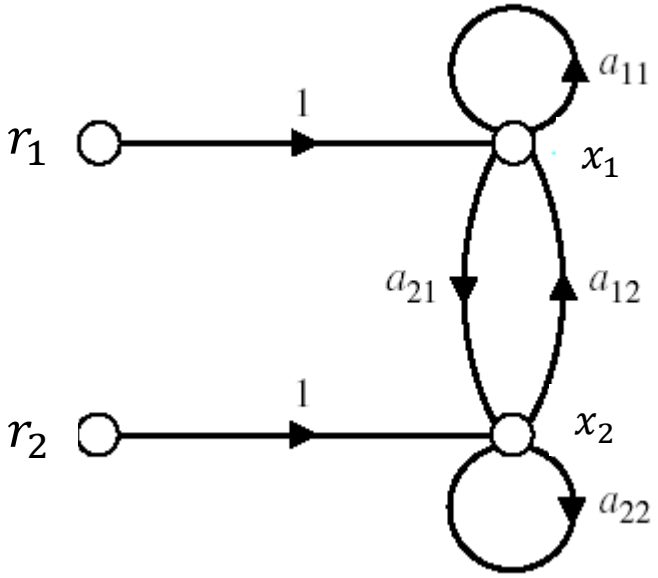


Signal-Flow Graph Model 2:

r_1 and r_2 are inputs and *x_1* and *x_2* are outputs

$$a_{11}\cdot x_1 + a_{12}\cdot x_2 + r_1 = x_1$$

$$a_{21}\cdot x_1 + a_{22}\cdot x_2 + r_2 = x_2$$



Signal-Flow Graph Model 3:

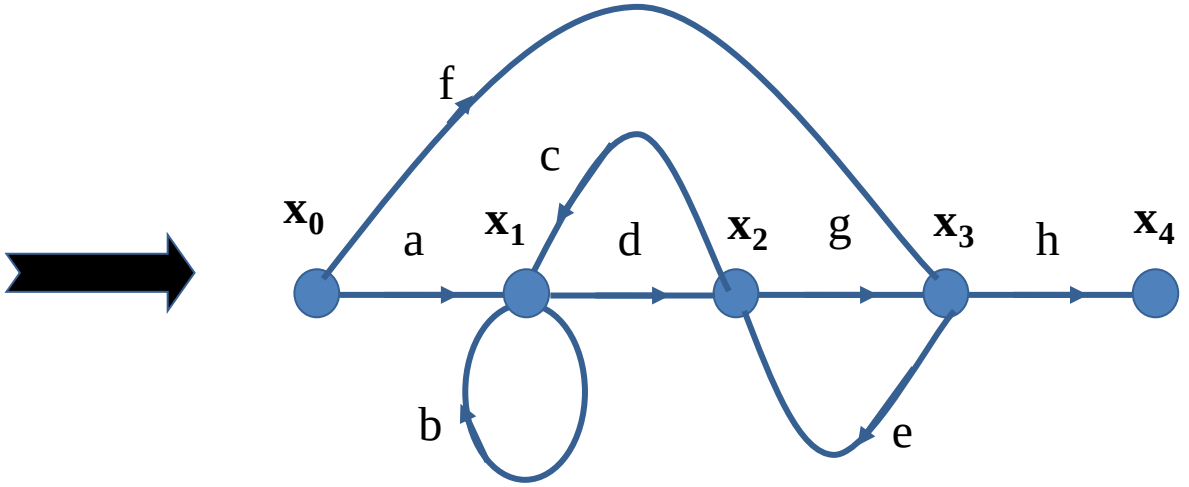
x_o is input and x_4 is output

$$x_1 = ax_0 + bx_1 + cx_2$$

$$x_2 = dx_1 + ex_3$$

$$x_3 = fx_0 + gx_2$$

$$x_4 = hx_3$$



Terminologies of SFG:

- An **input node** or source contain only the outgoing branches. i.e., $\mathbf{X_1}$
- An **output node** or sink contain only the incoming branches. i.e., $\mathbf{X_4}$
- A **path** is a continuous, unidirectional succession of branches along which no node is passed more than ones. i.e.,

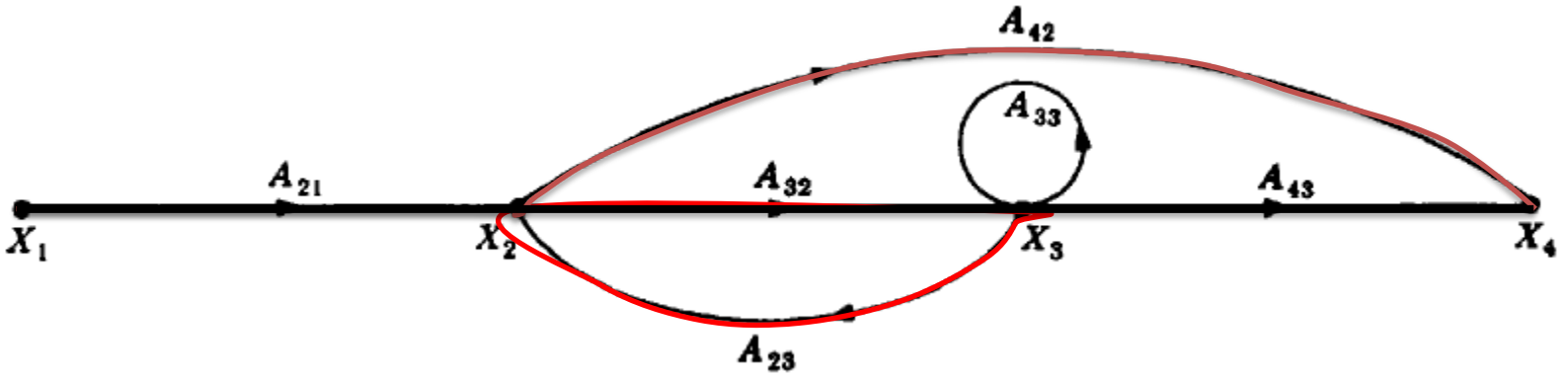
  X1 to X2 to X3 to X4 **X1 to X2 to X4** **X2 to X3 to X4**

- A **forward path** is a path from the input node to the output node. i.e.,

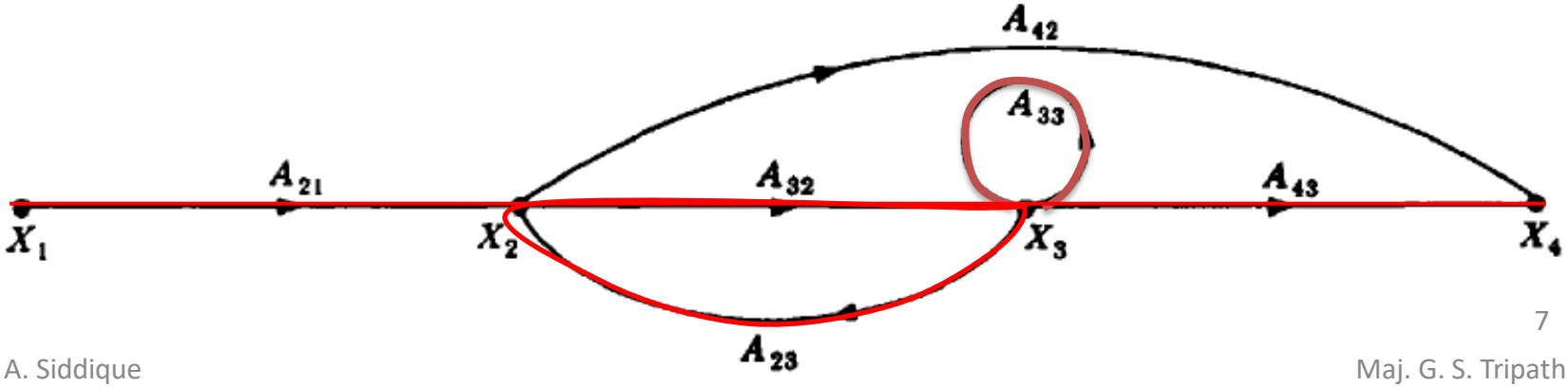
  $\mathbf{X_1}$ **to** $\mathbf{X_2}$ **to** $\mathbf{X_3}$ **to** $\mathbf{X_4}$, and $\mathbf{X_1}$ **to** $\mathbf{X_2}$ **to** $\mathbf{X_4}$, are forward paths.

Cntd…

- A **feedback path** or feedback loop is a path which originates and terminates on the same node. i.e.; **X_2 to X_3 and back to X_2** is a feedback path.



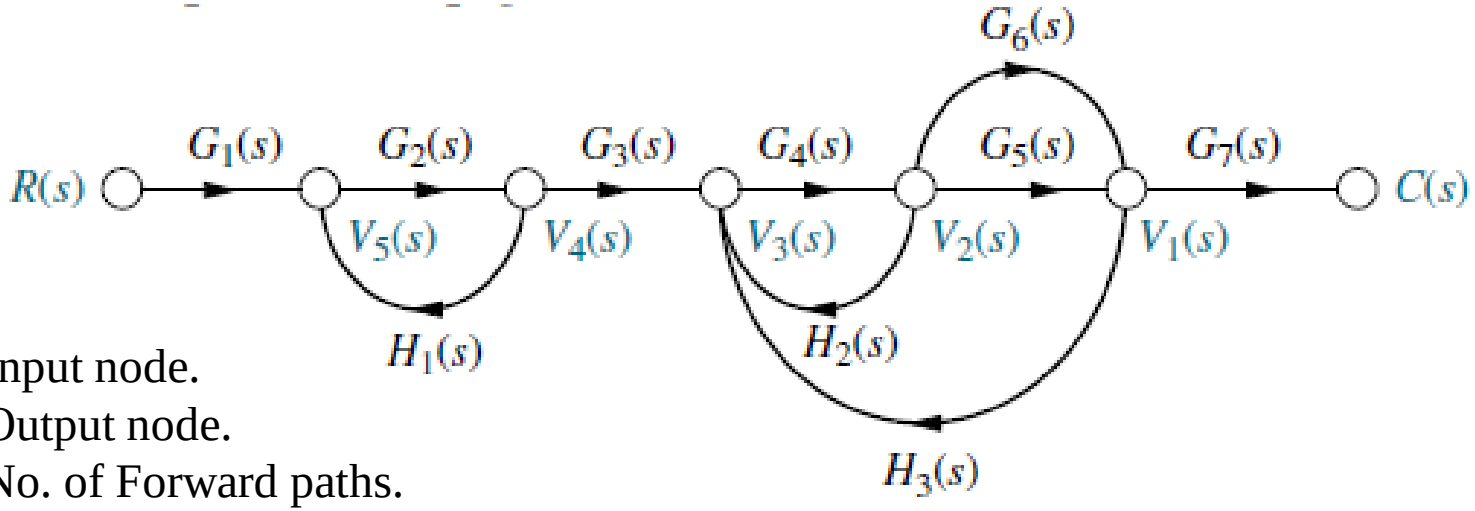
- A **self-loop** is a feedback loop consisting of a single branch. i.e.; **A_{33}** is a self loop.
- The **gain** of a branch is the transmission function of that branch.
- The **path gain** is the product of branch gains encountered in traversing a path. i.e. the gain of forwards path **X_1 to X_2 to X_3 to X_4** is **$A_{21}A_{32}A_{43}$**



Cntd…

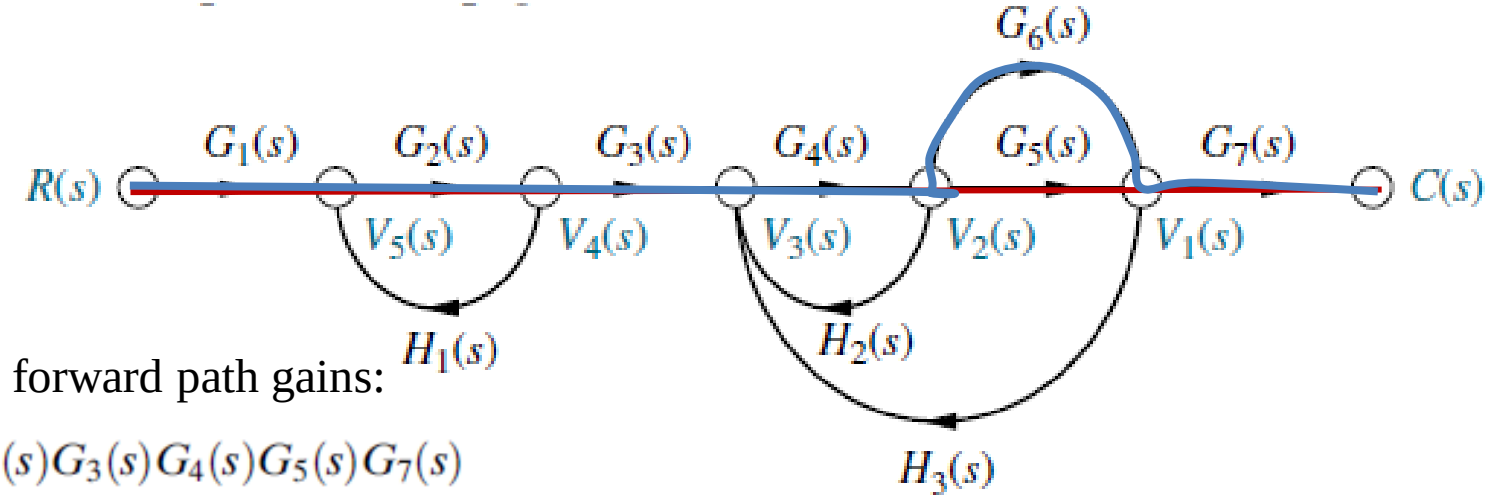
- The **loop gain** is the product of the branch gains of the loop. i.e., the loop gain of the feedback loop from X_2 to X_3 and back to X_2 is $A_{32}A_{23}$.
- Two loops, paths, or loop and a path are said to be **non-touching** if they have no nodes in common.

Problem 1:Consider the signal flow graph below and identify the following:

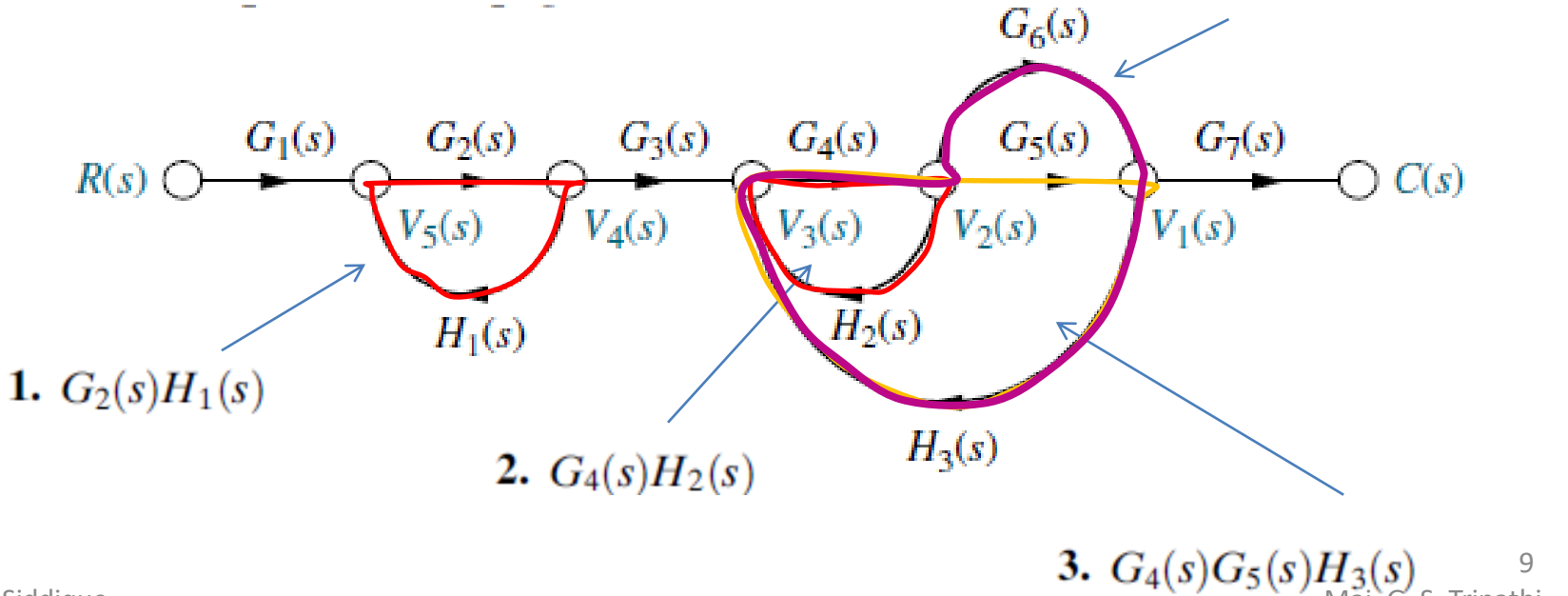


a) Input node.
b) Output node.
c) No. of Forward paths.
d) No. of Feedback paths (loops).
e) Determine the loop gains of the feedback loops.
f) Determine the path gains of the forward paths.
g) Non-touching loops

Problem 1: Contd..



- There are two forward path gains:
  1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$
  2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$
- There are four loops:



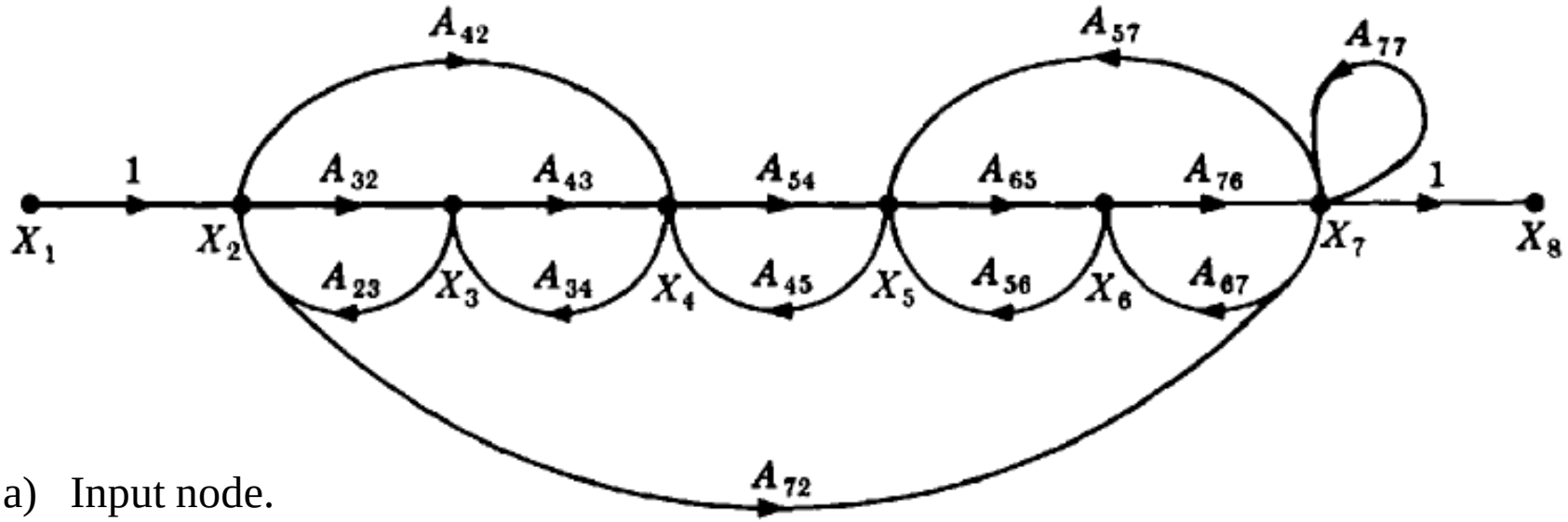
Problem 1: Contd..

- Nontouching loop gains:

1. $[G_2(s)H_1(s)][G_4(s)H_2(s)]$
2. $[G_2(s)H_1(s)][G_4(s)G_5(s)H_3(s)]$
3. $[G_2(s)H_1(s)][G_4(s)G_6(s)H_3(s)]$

a) Input node: R(s)
b) Output node: C(s)
c) No of Forward paths: two
d) No. of Feedback paths (loops): four
e) Determine the loop gains of the feedback loops:
   1. G2(s)H1(s)
   2. G4(s)H2(s)
   3. G4(s)G5(s)H3(s)
   4. G4(s)G6(s)H3(s)
f) Determine the path gains of the forward paths:
   1. G1(s) G2(s) G3(s) G4(s) G5(s) G7(s)
   2. G1(s) G2(s) G3(s) G4(s) G6(s) G7(s)
g) Non-touching loops:
   1. [G2(s)H1(s)][G4(s)H2(s)]
   2. [G2(s)H1(s)][G4(s)G5(s)H3(s)]
   3. [G2(s)H1(s)][G4(s)G6(s)H3(s)]

Problem 2: Consider the signal flow graph below and identify the following:



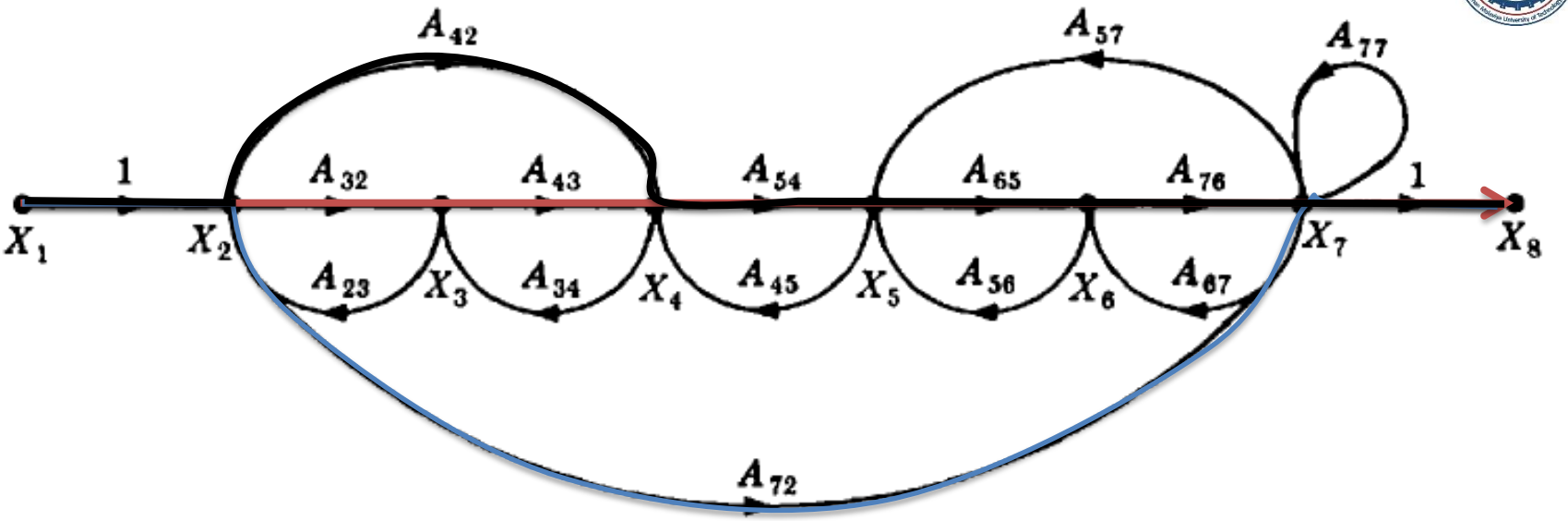
a) Input node.
b) Output node.
c) Forward paths.
d) Feedback paths.
e) Self loop.
f) Determine the loop gains of the feedback loops.
g) Determine the path gains of the forward paths.

- Input and output Nodes

a) Input node X_1

a) Output node X_8

(c) Forward Paths

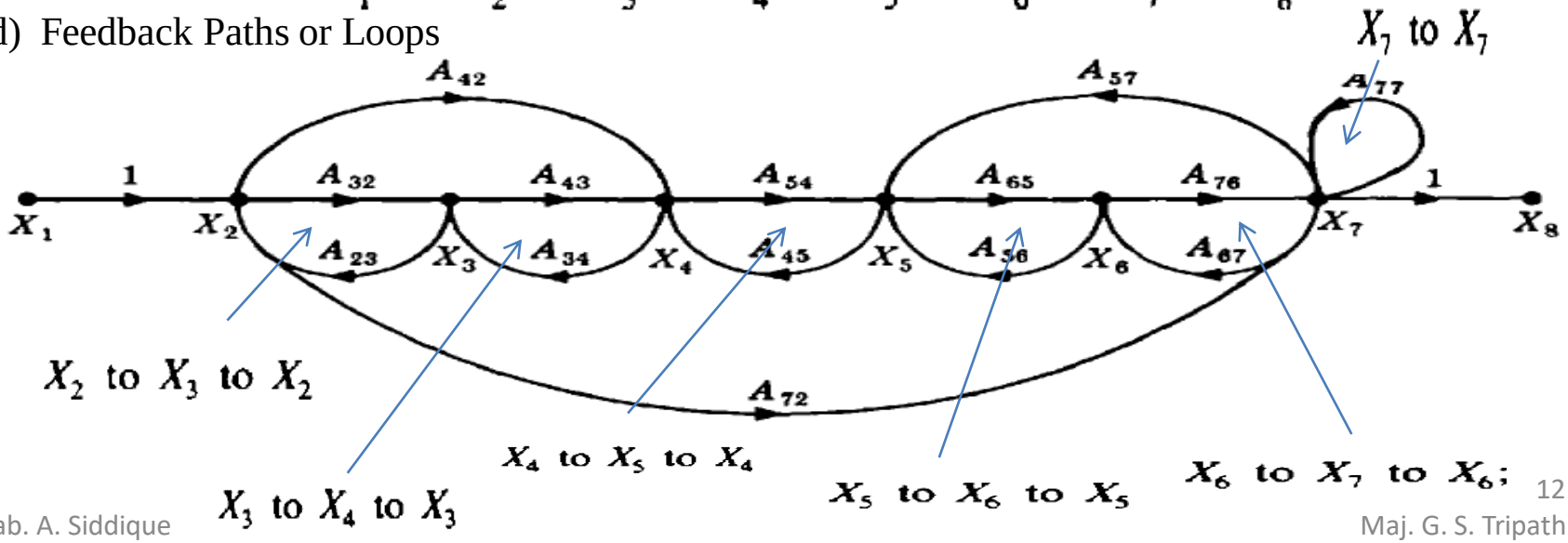


X_1 to X_2 to X_7 to X_8

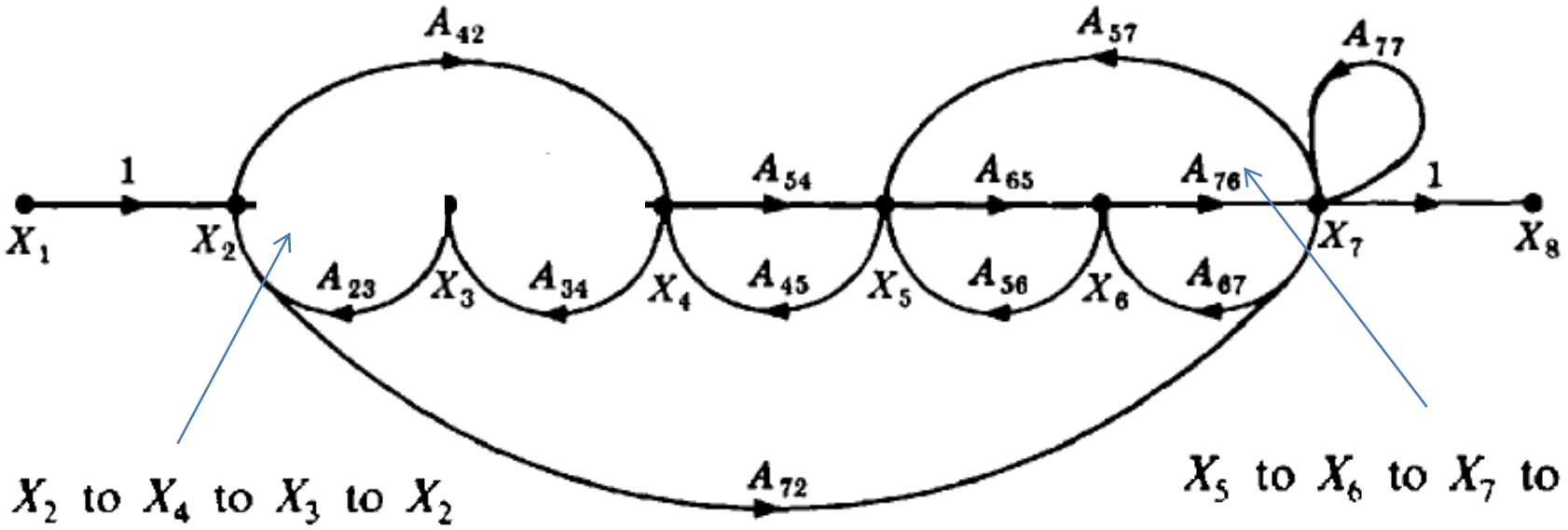
X_1 to X_2 to X_4 to X_5 to X_6 to X_7 to X_8

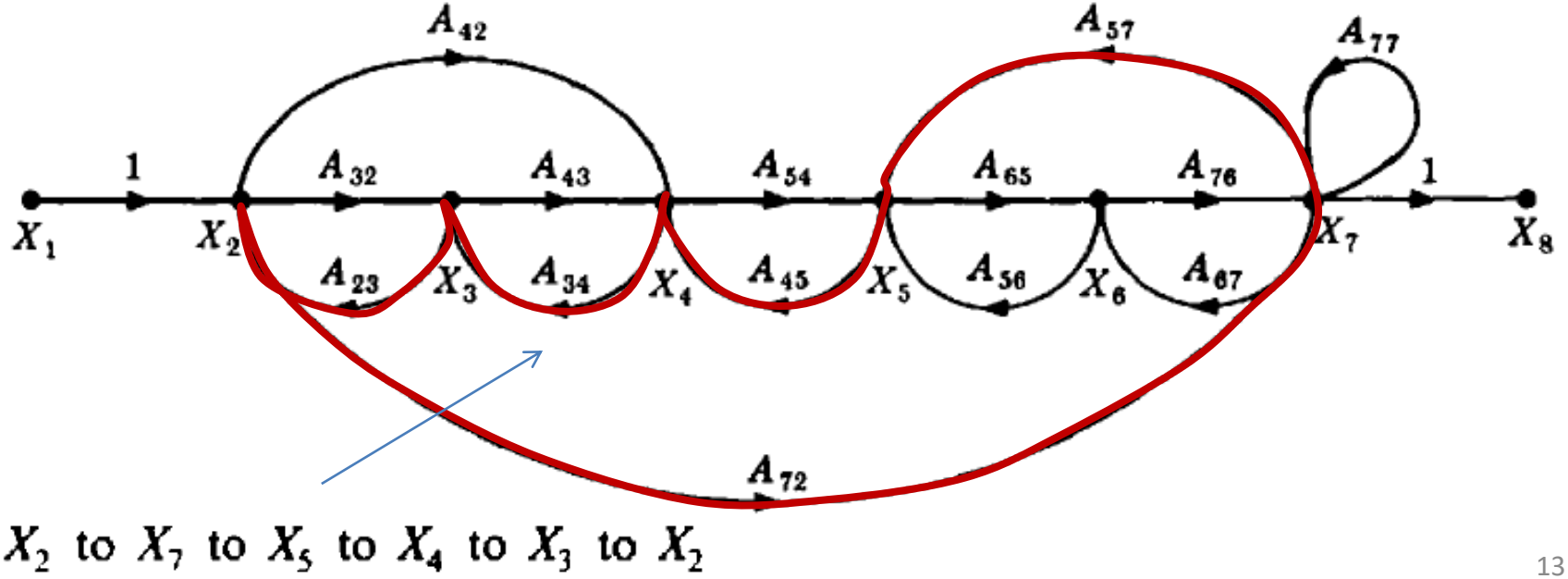
X_1 to X_2 to X_3 to X_4 to X_5 to X_6 to X_7 to X_8

(d) Feedback Paths or Loops

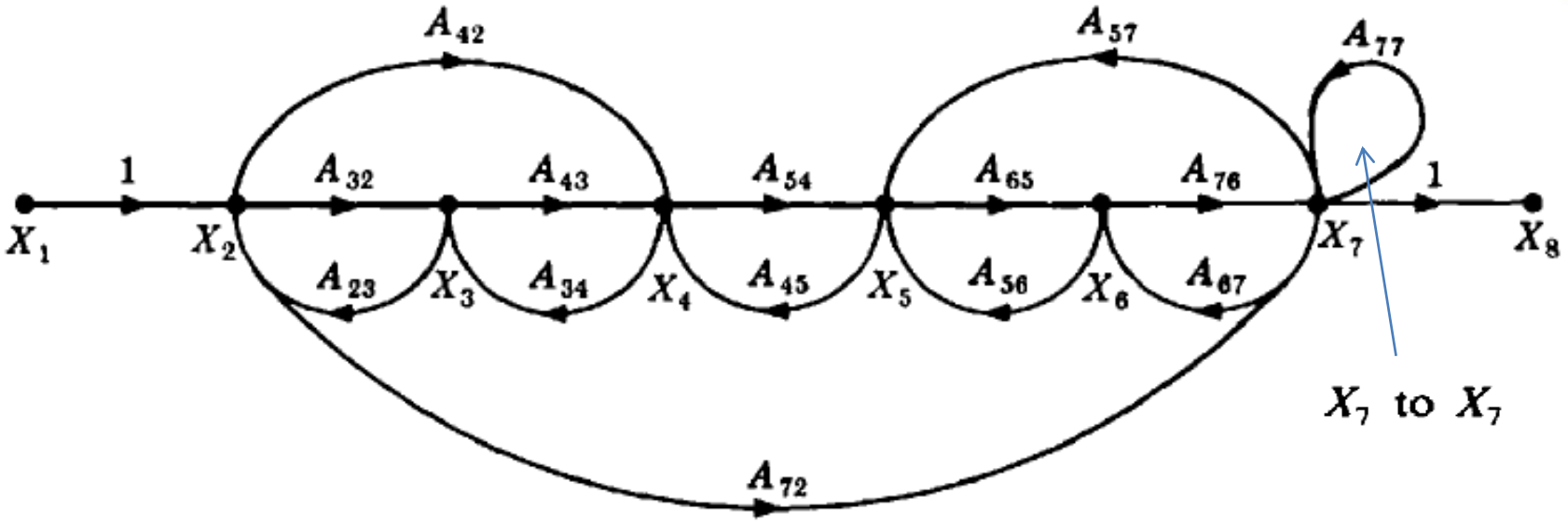


(d) Feedback Paths or Loops





(e) Self Loop(s)



(f) Loop Gains of the Feedback Loops

$A_{32}A_{23}$ $A_{76}A_{67}$; $A_{42}A_{34}A_{23}$ $A_{72}A_{57}A_{45}A_{34}A_{23}$;

$A_{43}A_{34}$; A_{77} $A_{65}A_{76}A_{57}$; $A_{72}A_{67}A_{56}A_{45}A_{34}A_{23}$

$A_{54}A_{45}$; $A_{65}A_{56}$

(g) Path Gains of the Forward Paths

A_{72} $A_{32}A_{43}A_{54}A_{65}A_{76}$ $A_{42}A_{54}A_{65}A_{76}$