Complex frequency,

Complex frequency

Definition: A type of frequency that depends on two parameters ; one is the " σ" which controls the magnitude of the signal and the other is "w", which controls the rotation of the signal ; is known as "complex frequency".

A complex exponential signal is a signal of type

$$X(t) = X_m e^{St} \longrightarrow \quad (1)$$

where X_m and s are time independent complex parameter. and

$$S = \sigma + jw$$

where Xm is the magnitude of X(t)

sigma(σ) is the real part in S and is called neper frequency and is expressed in Np/s.
"w"is the radian frequency and is expressed in rad/sec. "S" is called complex frequency and is expressed in complex neper/sec.
Now put the value of S in equation (1), we get

$$X(t) = Xm\ e^{\sigma t+jwt}$$

$$X(t) = Xm\ e^{\sigma t}\, e^{jwt}$$

By using Euler's theorem. we have i.e $e^{i\theta} = \cos\theta + i\sin\theta$

$$X(t) = Xm\ e^{\sigma t}\,[\cos(wt) + j\sin(wt)]$$

The real part is

$$X(t) = Xm\ e^{\sigma t}\cos(wt)$$

and imaginary part is

$$X(t) = Xm\ e^{\sigma t}\sin(wt)$$

The physical interpretation of complex frequency appearing in the exponential form will be studied easily by considering a number of special cases for the different value of S.

Case no 1:

When w=0 and σ has certain value, then, the real part is

$$X(t) = Xm\ e^{\sigma t} \cos(wt)$$

$$X(t) = Xm\ e^{\sigma t}\ .\ 1 = Xm\ e^{\sigma t}$$

imaginary part is zero (0)

since

$$S = \sigma + jw$$

$$S = \sigma$$

as w = 0

Now there are also three cases in above case no 1

(i) If the neper frequency is positive i.e. $\sigma > 0$ the curve obtain is exponentially increasing curve as shown below.

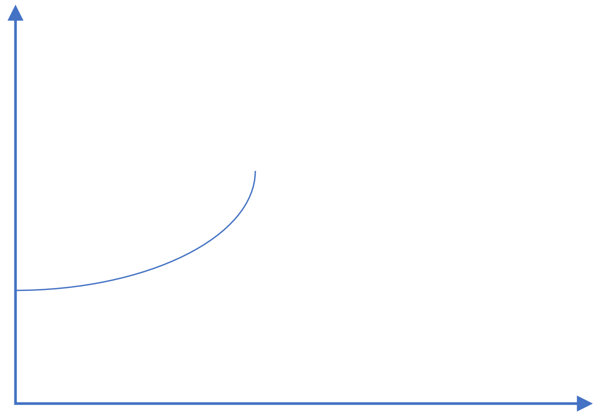


Fig:1

(ii) If $\sigma < 0$ then the curve obtain is exponentially decreasing curve as shown below

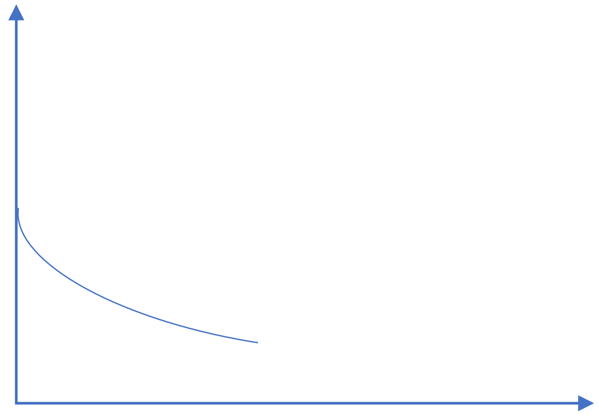


Fig:2

(iii) If $\sigma = 0$ then the curve obtain is the steady state d.c curve as shown below in fig:3

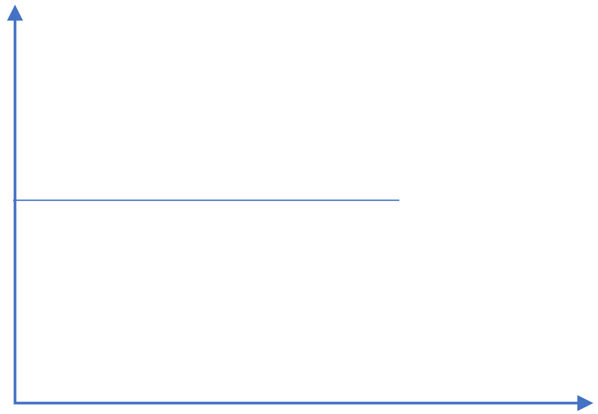
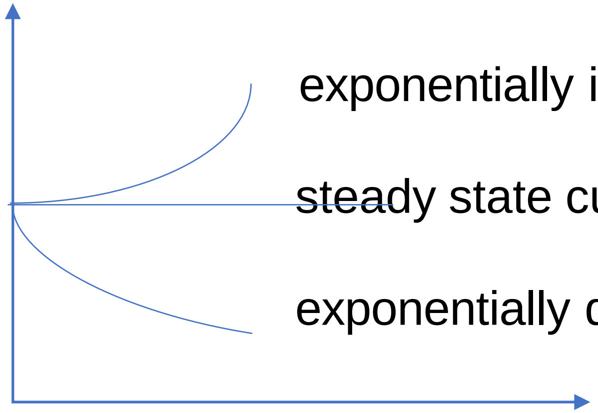


Fig:3

By combining all three curves we get



Case no 2

When σ = 0 and w has some value then, the real part is

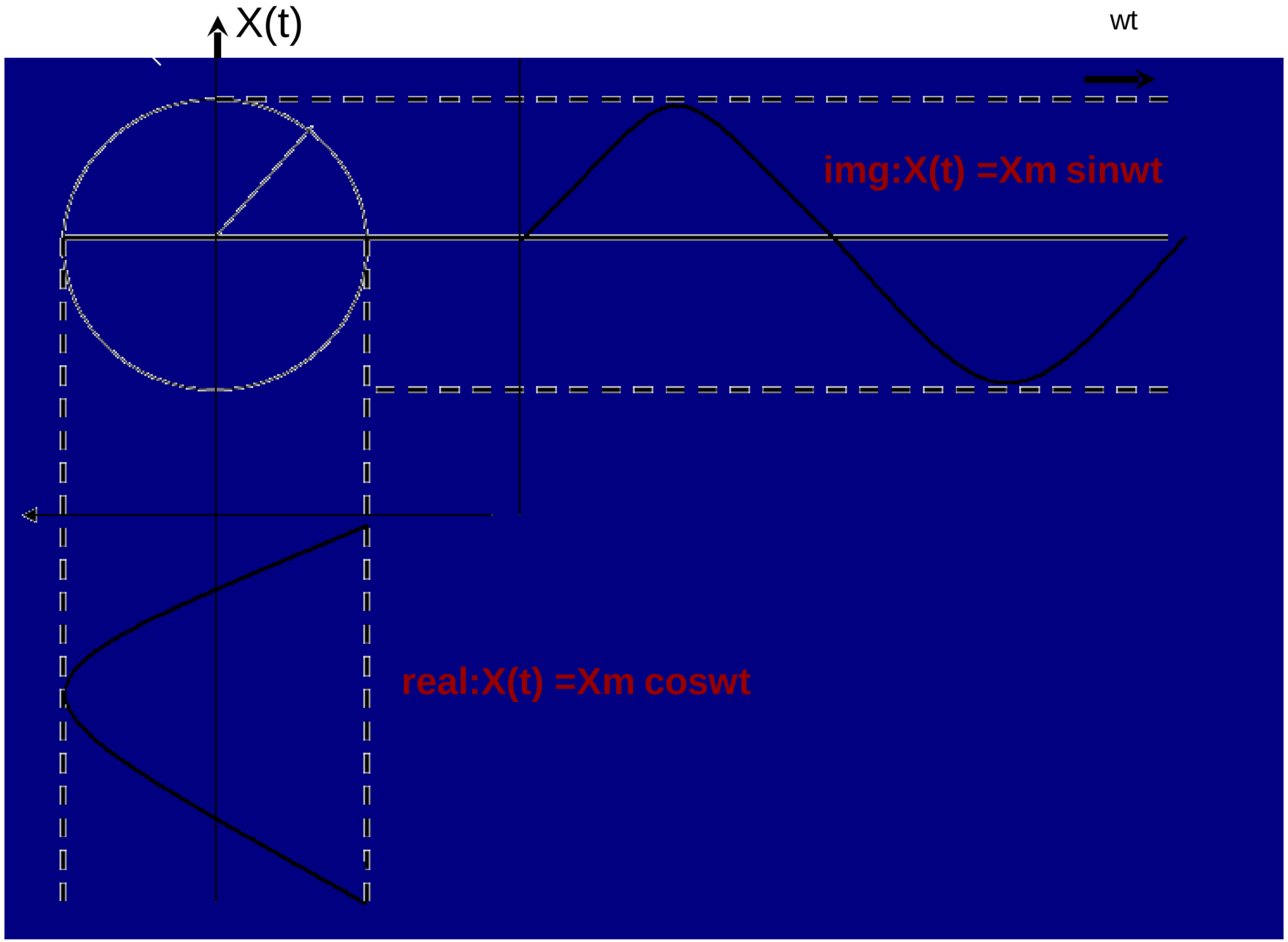
$$X(t) = Xm\, e^{0.t} \cos(wt)$$

$$X(t) = Xm \cos(wt)$$

and the imaginary part is

$$X(t) = Xm \sin(wt)$$

Hence the curve obtained is a sinusoidal steady state curve, as shown in the figure


X(t)
wt
img:X(t) =Xm sinwt
real:X(t) =Xm coswt

Case no 3:

When σ and w both have some value, then the real part is

$$X(t) = Xm\, e^{\sigma t} \cos(wt)$$

and the imaginary part is

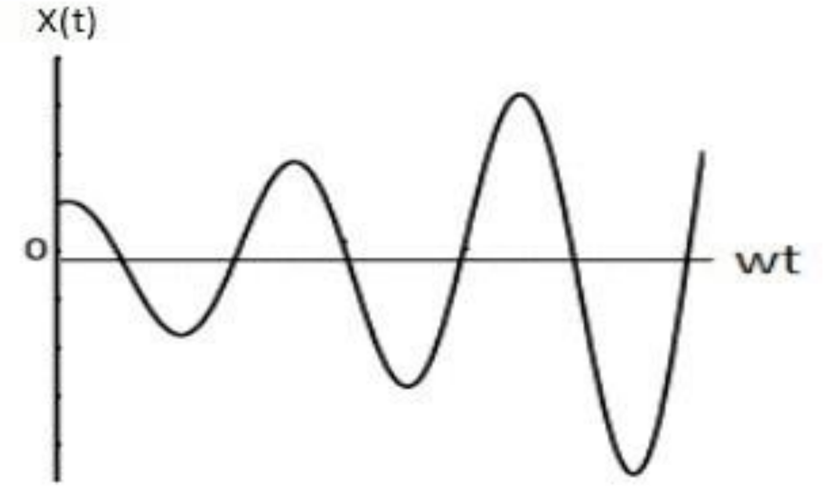
$$X(t) = Xm\, e^{\sigma t} \sin(wt)$$

So the curve obtained is time varying sinusoidal signal

These case no 3 is also has some two cases

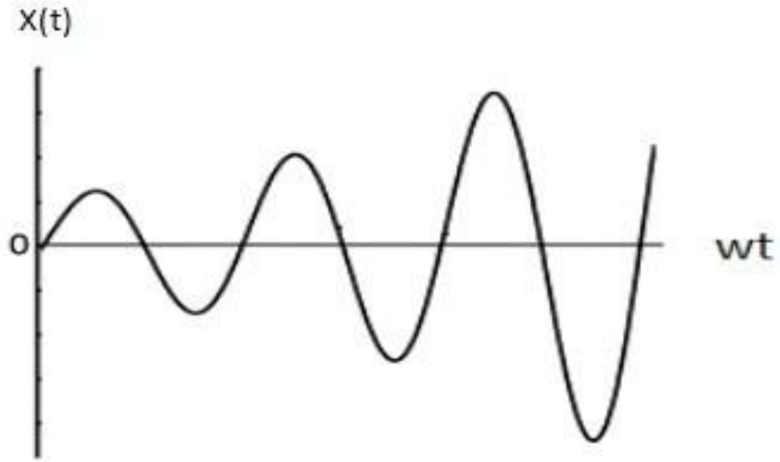
When σ > 0

REAL PART



$$X(t) = Xm\ e^{\sigma t} \cos(wt)$$

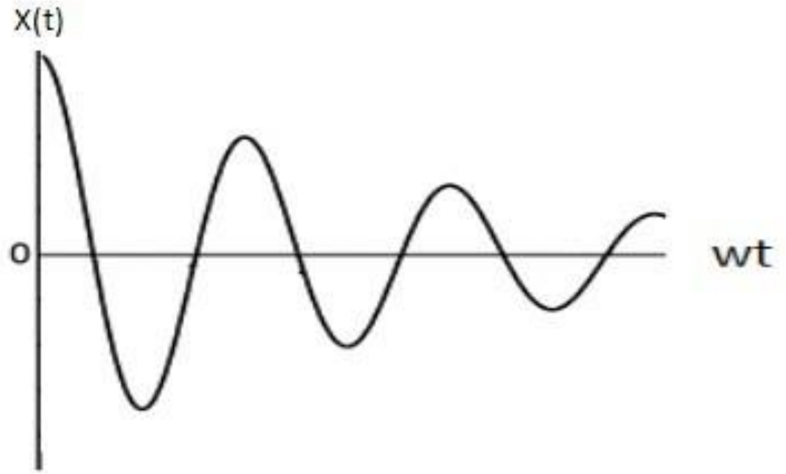
IMAGINARY PART



$$X(t) = Xm\ e^{\sigma t} \sin(wt)$$

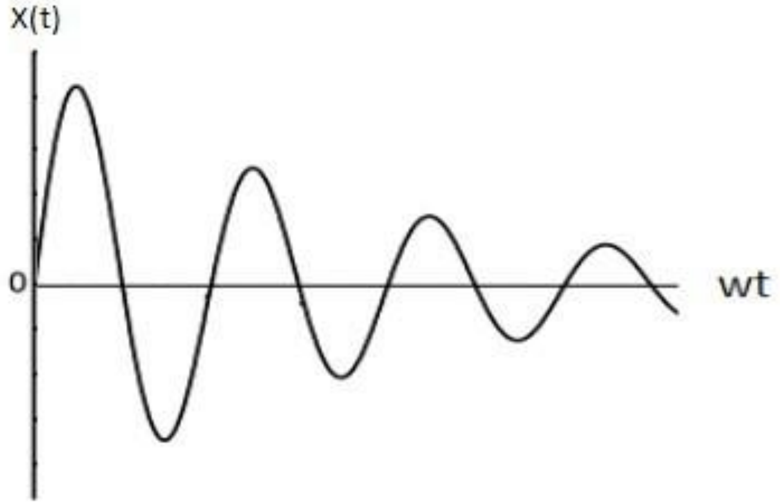
When σ < 0

REAL PART



$$X(t) = Xm\, e^{\sigma t} \cos(wt)$$

IMAGINARY PART



$$X(t) = Xm\, e^{\sigma t} \sin(wt)$$

Q : In the given circuit, Assume R_1 =1 ohm, R_2 =2 ohm and C =1F Find

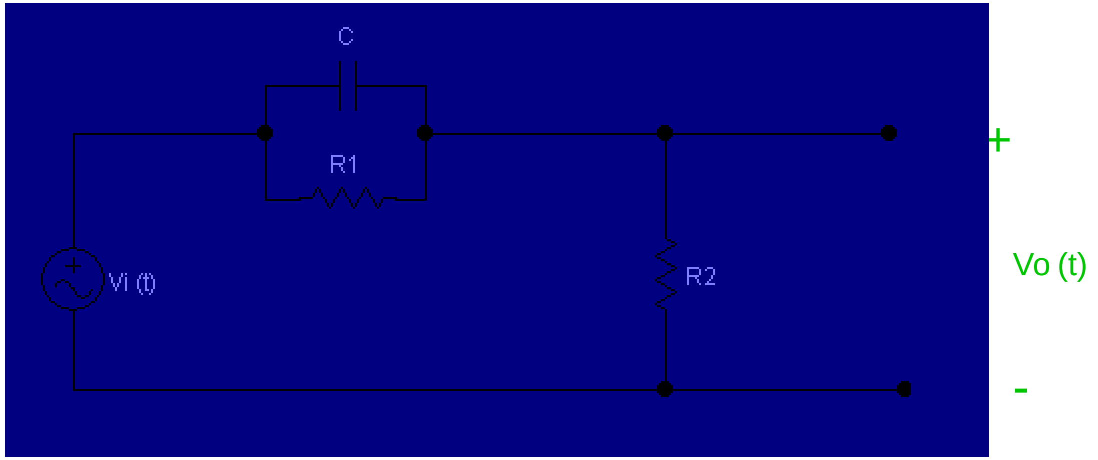
(i) Relation between $V_o(t)$ and $V_i(t)$

(ii) Find response to the following Input's

(a) $V_i(t)$= 12volt (b) $V_i(t)= 12e^{-3t}$ volt (c) $Vi(t)= 12e^{i2t}$ volt

(d) $Vi(t)=12e^{(-3+j2)t}$ volt (e) $Vi(t)= 10e^{-0.5t}\cos(1.5t)$v

Solution: The given circuit is



(i) By applying Kcl on above ckt, we have

$$\frac{V_i - V_o}{R_1} + \frac{V_i - V_o}{1/cs} = \frac{V_o}{R_2}$$

By putting values of R_1,R_2 and C, we get

$$\frac{V_i - V_o}{1} + \frac{V_i\text{-}V_o}{1/s} = \frac{V_o}{2}$$

$V_i - V_0 + sV_1 - sV_o = V_o/2$

$V_i(1+s) - s(1+V_o) = V_o/2$

$V_i(1+s) = V_o/2 + s(1+Vo)$

$V_i(1+s) = V_o(2s+3)/2$

$$V_i(2+2s)=V_0(\frac{2s+3}{2})$$

$$V_o(t)= \frac{(2s+2)}{(2s+3)}\, x V_i(t)$$

(ii) Response to the following Input is given below:

(a) $V_i(t) = 12$volts

$$V_o(t) = \left[\frac{2(0)+2}{2(0)+3}\right] x\, 12$$

as {e°=1 & s=0}

$$V_o(t) = 8 \text{ volt}$$

(b) $V_i(t) = 12e^{-3t}$

$$V_o(t) = \left[\frac{2(-3)+2}{2(-3)+3}\right] x\, 12e^{-3t}$$

$$V_o(t) = 16e^{-3t} \text{ volt}$$

(c) $V_i(t) = 12e^{j2t}$ volt

$$V_o(t) = \left[\frac{2(2j)+2}{2(2j)+3}\right] 12e^{j2t}$$

$$V_o(t)=\left[\frac{4j+2}{4j+3}\times\frac{4j-3}{4j-3}\right]\times 12e^{j2t}$$

$$V_o(t) = 10.3\ \angle 10.3^\circ \text{ volt}$$

(d) $V_i(t) = 12e^{(-3+j2)t}$ volt

$$V_o(t)=\left[\frac{2(-3+j2)+2}{2(-3+j2)+3}\right]\times 12e^{(-3+j2)t}$$

$$V_o(t)=13.57<8.130^o\ e^{(-3+j2)t} \text{ volt}$$

(e) $V_i(t) = 10e^{-0.5t}\cos(1.5t+30)$

Since the given response voltage is of only real part & we know that

$$V_i(t) = X_m e^{\sigma t}\cos(wt)$$

and we have given

$V_i = 10$, $\sigma = -0.5$, $w = 1.5$,

Since $S = \sigma + jw$

$S = -0.5+j1.5$

therefore

$$V_i(t) = 10\, e^{(-0.5+j1.5)t} \cos(1.5t)$$

$$V_0(t) = 10\, e^{(-0.5+j1.5)t} \left[\frac{2(-0.5+j1.5)+2}{2(-0.5+j1.5)+3} \right]$$

$$V_0(t) = 8.77\, e^{(-0.5+j1.5)t}$$

Finally

$$V_0(t) = 8.77\, e^{-0.5t}\,(\cos 1.5t + j \sin 1.5t) \text{ volt}$$


Thank You!