Control Systems

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Third Year ECE

Unit-I

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UNIT- I

- Introduction to Control system
  - Control System – Definition and Practical Examples
  - Basic Components of a Control System
- Feedback Control Systems:
  - Feedback and its Effect
  - Types of Feedback Control Systems
  - Transfer Function
- Block Diagrams:
  - Representation and reduction
  - Signal Flow Graphs
- Modeling of Physical Systems:
  - Electrical Networks and Mechanical Systems
  - Force-Voltage Analogy
  - Force-Current Analogy

Effect of feedback on time constant of a control system:

Less the time constant, faster is the response.

Consider an open loop system with overall transfer function as $G(s) = \frac{k}{1 + sT}$.

Let the input be unit step, $r(t) = u(t)$ » $R(s) = \frac{1}{s}$

$$C(s) = \frac{K}{1 + sT} \cdot R(s) = \frac{k}{1 + sT} \cdot \frac{1}{s}$$

By using partial fraction method

$$C(s) = \frac{A}{S} + \frac{B}{1 + sT}$$

$$C(s) = \frac{A\,(1 + sT) + Bs}{s\,(1 + sT)}$$

Therefore, $k = A(1 + sT) + Bs$

For $s = 0$, $A = k$

For $s = -1/T$, $B = -Tk$

$$C(s) = \frac{k}{S} - \frac{kT}{1 + sT}$$

$$C(s) = \frac{k}{S} - \frac{k}{s + 1/T}$$

Taking Inverse Laplace transform

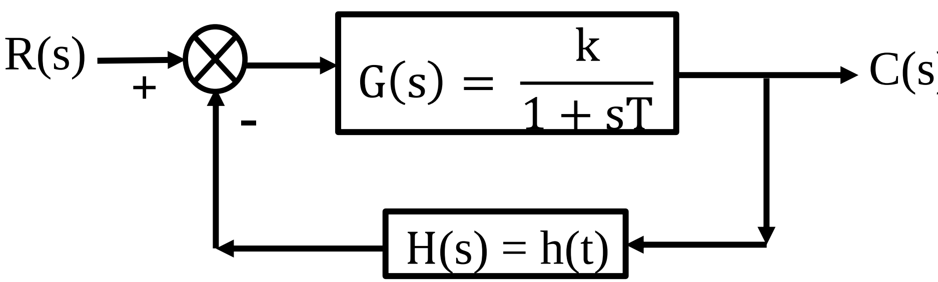
$$e(t) = k\,u(t) - k\,e^{-t/T}\,u(t)$$

$$e(t) = k\,(1 - e^{-t/T})\,u(t)$$

Where T = time constant ------------- (4)

Consider an open loop system

Here feedback $H(s) = h(t)$ is introduced in the system



$$T(s) = \frac{C(s)}{R(s)} = \frac{\frac{k}{1+sT}}{1 + \frac{k}{1+sT} \cdot H(s)}$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{k}{1 + sT + k\,H(s)}$$

Let us consider H(s) is a unity value function, i.e. H(s) = 1

$$T(s) = \frac{C(s)}{R(s)} = \frac{k}{sT + k + 1}$$

$$C(s) = \frac{k\left(\frac{1}{T}\right)}{s + \frac{k+1}{T}} \cdot R(s)$$

for unit step input, $r(t) = u(t) \gg R(s) = \frac{1}{s}$

$$C(s) = \frac{\frac{k}{T}}{s + \frac{k+1}{T}} \cdot \frac{1}{s}$$

taking inverse laplace transform for C(s)

$$c(t) = \frac{k}{T} L^{-1}\left(\frac{1}{s \cdot \left(s + \frac{k+1}{T}\right)}\right)$$

$$c(t) = \frac{k}{T} L^{-1}\left(\frac{A}{s} + \frac{B}{\left(s + \frac{k+1}{T}\right)}\right)$$

$$\frac{1}{s \cdot \left(s + \frac{k+1}{T}\right)} = \frac{A}{s} + \frac{B}{\left(s + \frac{k+1}{T}\right)}$$

For $s = 0$, $A = \frac{T}{k+1}$

For $s = -\frac{k+1}{T}$, $B = \frac{-T}{k+1}$

$$c(t) = \frac{k}{T} L^{-1}\left(\frac{\frac{T}{k+1}}{s} - \frac{\frac{T}{k+1}}{\left(s + \frac{k+1}{T}\right)}\right)$$

$$c(t) = \frac{k}{T}\left(\frac{T}{k+1} u(t) - \frac{T}{k+1} e^{-(k+1)t/T} u(t)\right)$$

$$c(t) = \frac{k}{k+1}\left(1 - e^{-(k+1)t/T}\right) u(t)$$

Where, time constant = $\frac{T}{k+1}$ --------- (5)

It can be observed that the new time constant due to unity feedback system is $\frac{T}{k+1}$. Thus for positive value of $k > 1$, the time constant $\frac{T}{k+1}$ is less than T.

Thus it can be concluded from equation (4) and (5) that the time constant of closed loop system is less than open loop system. We know that lesser the time constant faster is the response. Hence feedback improves the time constant of a system.

Effect of feedback on overall gain of a control system:

Consider an open loop system with overall transfer function = G(s) = overall gain of the system.

If the feedback H(s) is introduced in such a system, the overall gain becomes $\frac{G(s)}{1 \pm G(s)\,.\,H(s)}$.

The +ve and –ve sign in the denominator gets decide by the sign of feedback. For a –ve feedback the gain G(s) is reduced by $\frac{G(s)}{1 + G(s)\,.\,H(s)}$,

so due to –ve feedback overall gain of the system reduces.

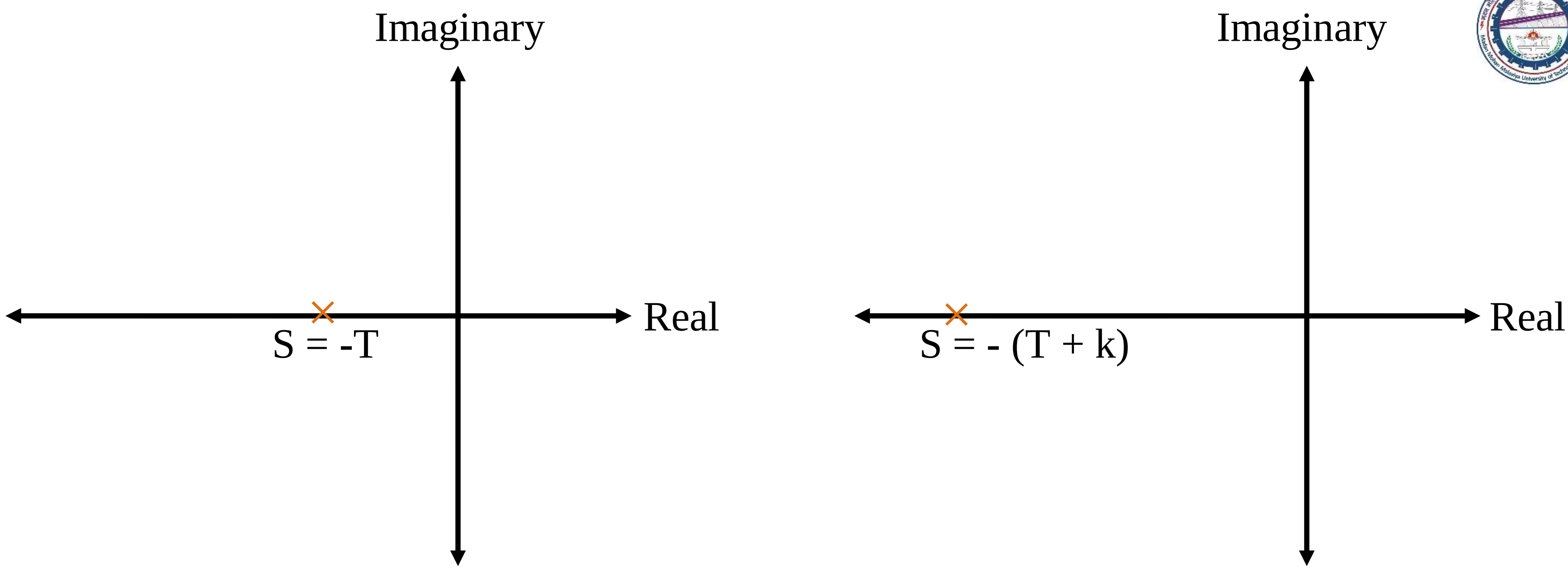
Effect of feedback on stability of a control system:

Consider an open loop system with overall transfer function = G(s) = $\frac{k}{s+T}$ then open loop pole is located at s = -T.

Now, let the unity feedback (H(s) = 1) is introduced in the system. The overall transfer function of closed loop becomes $\frac{k}{s+T+k}$

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,.\,H(s)} = \frac{\frac{k}{s+T}}{1 + \frac{k}{s+T}\,.\,1} = \frac{k}{s+T+k}$$

The closed loop poles is now located at s = - (T + k)



For open loop

For closed loop

The stability of the system depends on the location of poles in s- plane. Thus it can be concluded that the feedback effect the stability of the system.

Types of feedback control systems:

There are two main types of feedback control systems: negative feedback and positive feedback. In a positive feedback control system the setpoint and output values are added. In a negative feedback control the setpoint and output values are subtracted. As a rule negative feedback systems are more stable than positive feedback systems. Negative feedback also makes systems more immune to random variations in component values and inputs

Transfer Function:

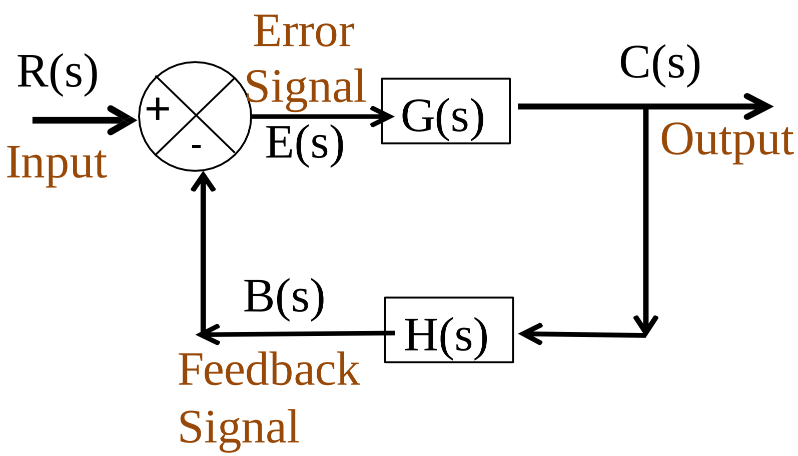
It is a mathematical expression relating output or response of the system to the input. It is denoted by $T(s) = \frac{C(s)}{R(s)}$

- **Open Loop control system:-**

$$C(s) = G(s)\,.\,R(s)$$

$$\frac{C(s)}{R(s)} = G(s) = T(s) \text{ OLTF}$$

- **Closed Loop control system:-**



Error signal is given by;

$$E(s) = R(s) - B(s) \text{ --------------------- } (1)$$

$$R(s) = E(s) + B(s)$$

Gain of feedback network is given by;

$$H(s) = \frac{B(s)}{C(s)}, \qquad B(s) = H(s)\,.\,C(s) \text{ -------- } (2)$$

Gain for CL system is given by;

$$G(s) = \frac{C(s)}{E(s)}$$

$$C(s) = G(s).E(s) \text{ ---------------(3)}$$

Substitute value of E(s) from eq. 1 to 3

$$C(s) = G(s)\,.\,(R(s) - B(s))$$

$$C(s) = G(s)\,.\,R(s) - G(s)\,.\,B(s) \text{ --------(4)}$$

Substitute value of B(s) from eq. 2 to 4

$$C(s) = G(s)R(s) - G(s).H(s).C(s)$$

$$G(s).R(s) = C(s) + G(s).H(s).C(s)$$

$$G(s).R(s) = C(s)(1 + G(s).H(s))$$

Transfer function is given by;

$$T.F. = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)} = T(s) \text{ CLTF}$$

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Block Diagram of a Control System:

- ✓ If the system is simple & has limited parameters then it is easy to analyze such systems using the methods discussed earlier i.e transfer function, if the system is complicated and also have number of parameters then it is very difficult to analyze it.
- ✓ To overcome this problem block diagram representation method is used.
- ✓ It is a simple way to represent any practically complicated system. In this each component of the system is represented by a separate block known as functional block.
- ✓ These blocks are interconnected in a proper sequence.
- ✓ The block diagram has following five basic elements associated with it
    a) Blocks
    b) T.F of elements shown inside the block
    c) Summing point
    d) Take off points
    e) Arrow
- ✓ For a closed loop systems, the functions of comparing the different signals is indicated by the summing point, while a point from which a signal is taken for feedback purpose is indicated by take off point in block diagram.
- ✓ The signal can travel along the arrow only.

Advantages of block diagram:

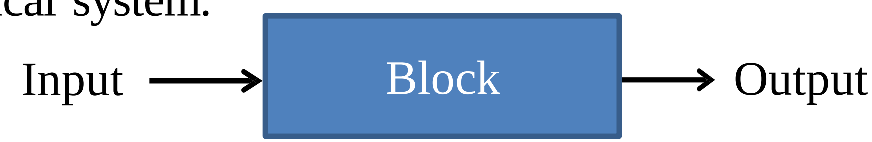
a) Very simple to construct the block diagram for complicated systems.
b) The function of individual element can be visualised from the block diagram.
c) Individual as well as the overall performance of the system can be studied by using transfer function.
d) Overall closed loop transfer function can be easily calculated using block diagram reduction rule.

Disadvantages of block diagram:

a) Block diagram does not includes about physical construction of the system.
b) Source of energy is generally not shown in the block diagram.

Basic elements of block diagram:

Blocks:- It is shorthand, pictorial representation of the cause and effect relationship between input and output of a physical system.

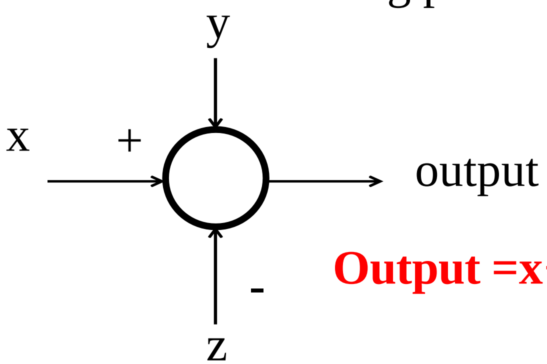


Output:- The value of the input is multiplied to the value of block gain to get the output.

Output, $Y(s) = 2s \cdot X(s)$

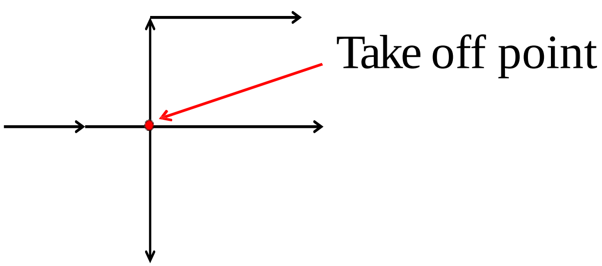
Basic elements of block diagram:

Summing Point:- Two or more signals can be added/ subtracted at summing point.



Output =x+y-z

Take off Point:- The output signal can be applied to two or more points from a take off point.



Forward Path:- The direction of flow of signal is from input to output.

Feedback Path:- The direction of flow of signal is from output to input.