Principles of Communication (BEC-28)

Unit-4

Pulse Modulation and Digital Transmission of Analog Signal

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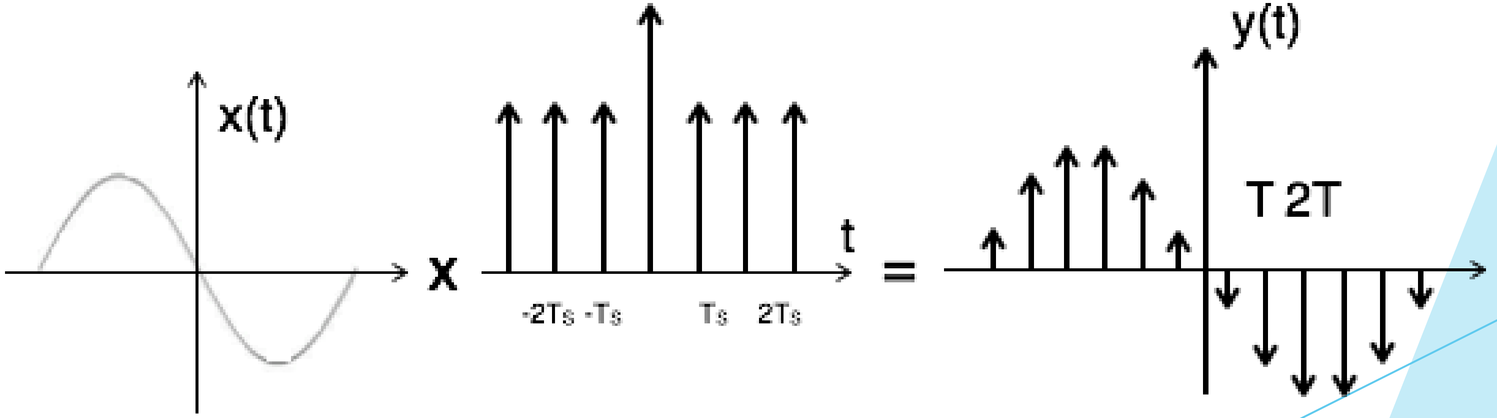
Content of Unit-IV

Pulse Modulation and Digital Transmission of Analog Signal: Sampling Theorem and its applications, Concept of Pulse Amplitude Modulation, Pulse width modulation and pulse position modulation, PCM, Pulse Time Modulation, TDM and FDM. Line Coding, Quantizer, Quantization Noise, Compounding multiplexer.

Three types of **sampling techniques**:

- **<u>Impulse sampling</u>:** Obtained by **multiplying input signal x(t) with impulse train of period 'T$_s$**.

Also called **ideal sampling**. Practically not used because pulse width cannot be zero and the generation of impulse train not possible.



Natural sampling

- This type of sampling similar to ideal sampling except for the fact that **instead of delta function**, now we use rectangular train of **period T_s**. i.e. multiply input signal x(t) to pulse train
- An **electronic switch** is used to periodically shift between the two contacts at a rate of **$f_s = (1/T_s)$ Hz**, staying on the input contact for C seconds and on the grounded contact for the remainder of each sampling
- The output $x_s(t)$ of the sampler consists of segments of x(t) and hence $X_s(t)$ can be considered as the product of x(t) and sampling function s(t).
- **$X_s(t) = x(t) \times s(t)$**

$$S(t) = \begin{cases} 1 & -\tau/2 < t < \tau/2 \\ 0 & \tau/2 < |t| < Ts/2 \end{cases} \quad \text{-------} (1)$$

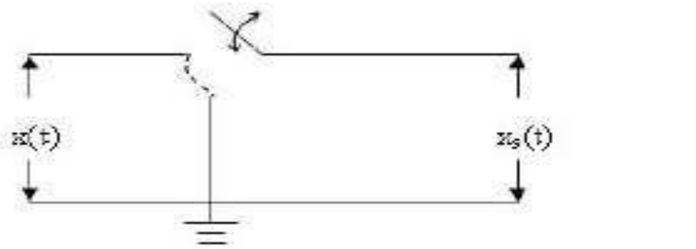


Fig: 2.11 Natural Sampling – Simple Circuit

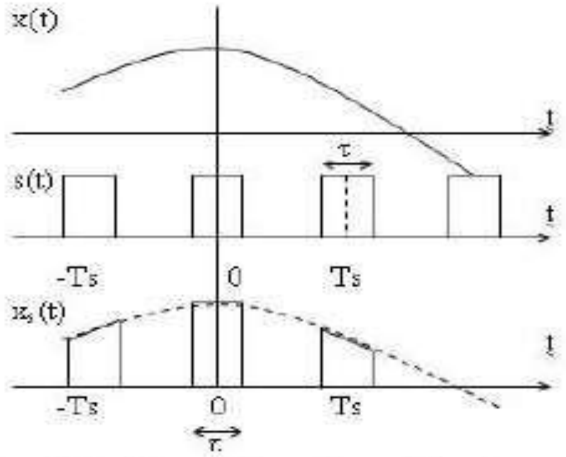
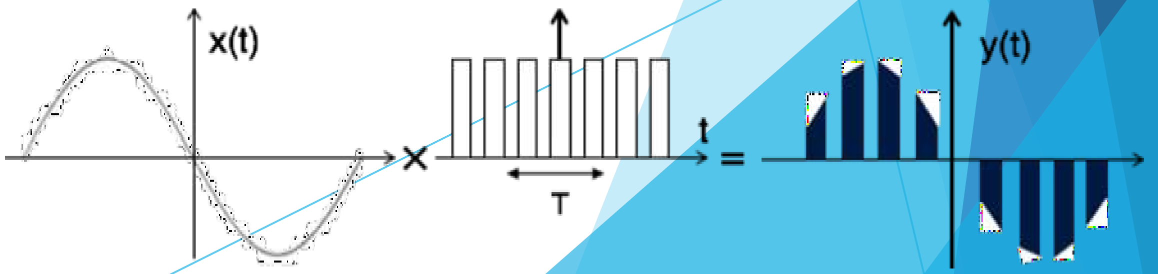


Fig: 2.12 Natural Sampling – Waveforms



Using Fourier series, we can rewrite the signal S(t) as:

$$S(t)= C_0+ \sum_{n=1}^{\infty} 2Cn\cos(n\omega_s t)$$

Where the Fourier coefficients $\mathbf{C_0=\frac{\tau}{T_s}}$ **and** $\mathbf{C_n=f_s\tau sinc(nf\tau_s)}$

Therefore: $x_s(t)=x(t)[C_0+ \sum_{n=1}^{\infty} 2Cn(\cos n\omega_s t)]$

$$\mathbf{x_s(t)=C_0x(t)+2C_1x(t)\cos(\omega_s t)+2C_2x(t)\cos(2\omega_s t)+\ldots\ldots}$$

Applying Fourier Transform for the above equation

Using $\mathbf{x(t)\leftrightarrow X(f)}$

$$\mathbf{x(t)\cos(2\pi f_0 t)\leftrightarrow \frac{1}{2}[X(f-f_0)+X(f+f_0)]}$$

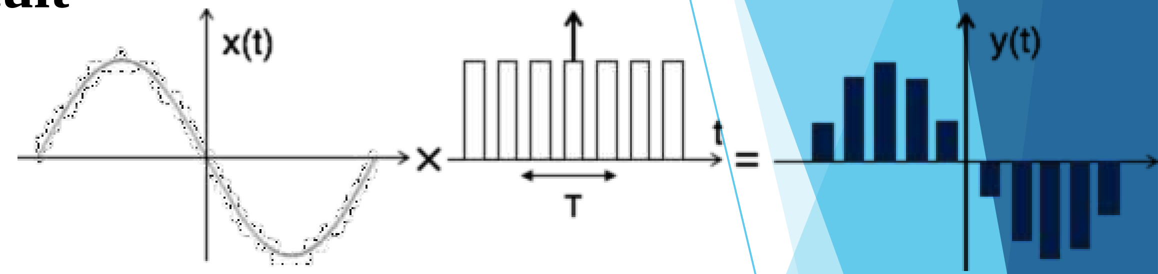
$$\mathbf{X_s(f)=C_0X(f)+C_1[X(f-f_0)+X(f+f_0)]+C_2[X(f-f_0)+X(f+f_0)]+\ldots\ldots\ldots\ldots}$$

$$\mathbf{X_s(f)= C_0X(f)+ \sum_{n=-\infty}^{\infty} C_n X(f-nfs)}$$

$$\mathbf{X_s(f) = A\tau/ Ts\,.[\,\Sigma\, sin\,c(n\,fs.\tau)\,X(f-n\,fs)]}$$

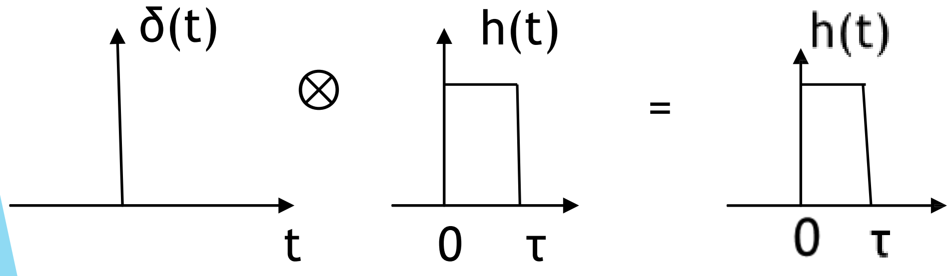
The signal $X_s(t)$ has the spectrum which consists of message spectrum and repetition of message spectrum periodically in the frequency domain with a period of fs. But **the message term is scaled by ‘Co”**(**sinc function**) which is **not the case in instantaneous sampling.**

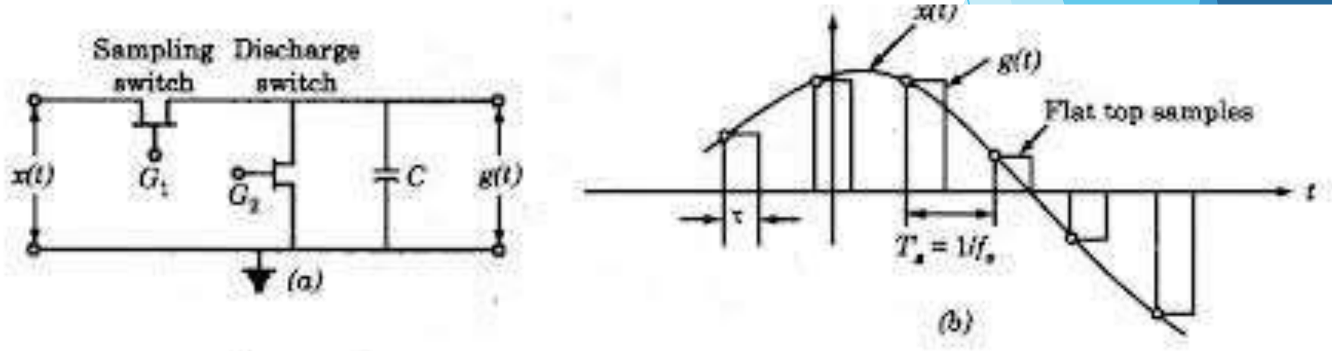
- **<u>Flat Top sampling</u>:** During transmission, noise is introduced at top of the transmission pulse which can be easily removed if the **pulse is in the form of flat top**.
- Here, the top of the samples are flat i.e. they have constant amplitude and is equal to the instantaneous value of the baseband signal x(t) at the start of sampling. Hence, it is called as **<u>flat top sampling or practical sampling</u>**.
- Flat top sampling makes use of **sample and hold circuit**



- Theoretically, **the sampled signal can be obtained by convolution of rectangular pulse h(t) with ideally sampled signal ,$s_\delta(t)$**

$g(t) = s(t) \otimes h(t)$





$f(t) \otimes \delta(t) = f(t)$; property of delta function
Applying a modified form; **s(t) in place of δ(t)**

On convolution of s(t) and h(t), we get a pulse whose duration is equal to h(t) only but amplitude defined by s(t).

Train of impulses given by:

$$\boldsymbol{\delta_{Ts}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nTs)}$$

Signal s(t) obtained by multiplication of message signal x(t) and $\delta_{Ts}(t)$

Thus, $\mathbf{s(t) = x(t).\ \delta_{Ts}(t)}$

$$\boldsymbol{s(t) = \sum_{n=-\infty}^{\infty} x(nTs)\delta(t - nT_s)}$$

Now sampled signal g(t) given as:

$$g(t) = s(t) \otimes h(t)$$

$$= \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

$$G(f) = S(f)\, H(f)$$

$$S(f) = f_s \sum X(f - nfs)$$

$$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nTs)\delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$\boldsymbol{g(t) = \sum_{-\infty}^{\infty} x(nTs) \int_{-\infty}^{\infty} \delta(\tau - nT_s)\, h(t-\tau)\, d\tau}$$

Using shifting property of delta function: $\int_{-\infty}^{\infty} f(t)\delta(t - to) = f(t_0)$

$$\boldsymbol{g(t) = \sum_{-\infty}^{\infty} x(nT_s) h(t - nTs)}$$

$$\boldsymbol{G(f) = f_s \sum_{-\infty}^{\infty} X(f - nfs) H(f)}$$ **Spectrum of flat top samples**

Example 1: *A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?*

Solution

The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). The sampling rate is therefore 400,000 samples per second.

Example 2: *A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?*

Solution

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Example 3: ***We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?***

Solution

The human voice normally contains frequencies from 0 to 4000 Hz. So the sampling rate and bit rate are calculated as follows:

$$\text{Sampling rate} = 4000 \times 2 = 8000 \text{ samples/s}$$
$$\text{Bit rate} = 8000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

Aperture Effect: Spectrum of flat topped sample is given by;

$$G(f)=f_s \sum [X(f-nf_s)H(f)] , \qquad \text{where } H(f)= \tau.\text{sin c}(f_s.t)e^{(-j\pi f\tau)}$$

This equation shows that signal g(t) is obtained by passing the **signal s(t) through a filter having transfer function H(f).**

Figure(a) shows **one pulse of rectangular pulse train** and **each sample of x(t) i.e. s(t)** is **convolved** with **this pulse**

Figure (b) shows the **spectrum of this pulse**. Thus, flat top sampling introduces an **amplitude distortion** in **reconstructed signal x(t) from g(t**). There is a **high frequency roll off** making **H(f) act like a LPF**, thus **attenuating the upper portion of message signal spectrum**. This is known as **aperture effect**

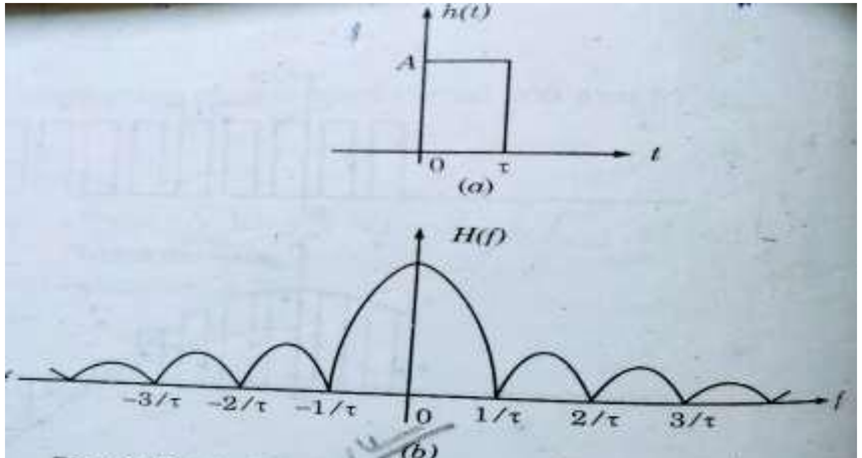
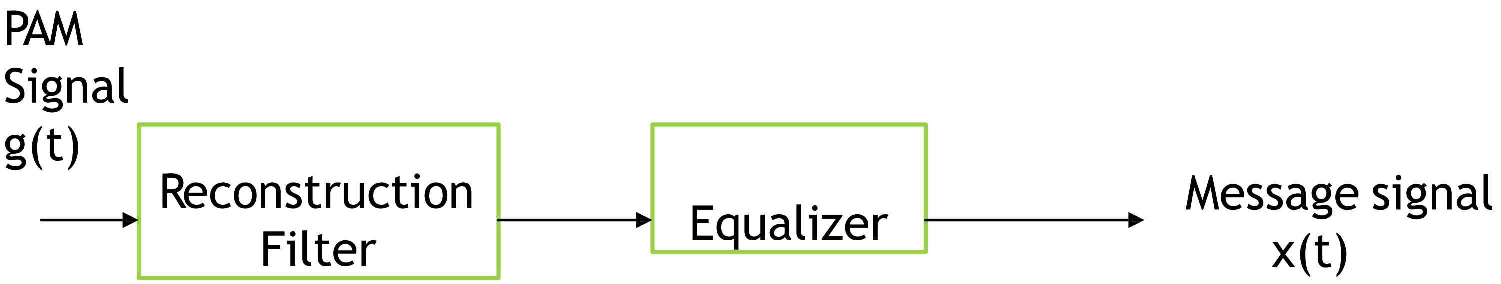


Fig. 3.18 *(a) One pulse of rectangular pulse train (b) spectrum of the pulse shown in figure (a).*

<u>How to minimize aperture effect</u>?? An **equalizer** at the receiver end is needed to compensate aperture effect. The receiver contains low pass reconstruction Filter with cut off slightly higher than f_m Hz.



Equalizer in cascade with reconstruction filter has the effect of decreasing the in band loss of reconstruction filter, **frequency increases in such away so as to compensate aperture effect.**

$$\mathbf{H_{eq}(f)} = \frac{K.e^{-j2\pi f td}}{H(f)},$$

where t_d is time delay introduced by LPF being equal to $\tau/2$

$$\mathbf{H_{eq}(f)} = \frac{K}{\tau sin\, c(f\tau)}$$


Thank You