Control Systems

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Unit-III

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Lecture 4

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Type of System

The open loop transfer function of unity feedback system can be written in two standard forms: the time constant form and the pole-zero form.

$$G(s) = \frac{K(s+z1)(s+z2)..................}{s^n(s+p1)(s+p2)................}$$ (Pole-zero form)

$$G(s) = \frac{K(1+Tz1s)(1+Tz2s)..................}{s^n(1+Tp1s)(1+Tp2s)................}$$ (Time constant form)

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Type-0 (Zero) System

Definition: A control system with no integration in the open loop transfer function and no pole of transfer function G(s) at the origin of s-plane is designated as "**Type-0** " system.

$$G(s) = \frac{K(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots\ldots\ldots\ldots}{(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots\ldots\ldots} \quad \text{(Standard form)}$$

An amplifier type control system is a practical example of Type-0 system

Type-1 (One) System

Definition: A control system with one integration in the open loop transfer function and one pole of transfer function G(s) at the origin of s-plane is designated as "**Type-1** " system.

$$G(s) = \frac{K(1+T\,z1s)(1+T\,z\,2s)\ldots\ldots\ldots\ldots\ldots\ldots}{s(1+T\,p1s)(1+T\,p\,2s)\ldots\ldots\ldots\ldots\ldots} \quad \text{(Standard form)}$$

An pneumatic type control system is a practical example of Type-1 system

Type-2 (Two) System

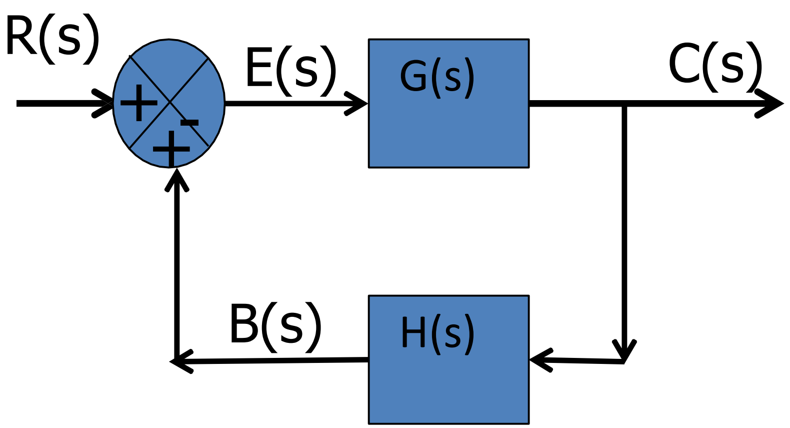
Definition: A control system with two integration in the open loop transfer function and two pole of transfer function G(s) at the origin of s-plane is designated as "**Type-2** " system.

$$G(s) = \frac{K(1+Tz1s)(1+Tz2s)\ldots\ldots\ldots\ldots\ldots\ldots}{s^2(1+Tp1s)(1+Tp2s)\ldots\ldots\ldots\ldots\ldots} \qquad \text{(Standard form)}$$

A mechanical displacement system is a practical example of Type-2 system

Derivation of Steady State Error

The steady state response is important to judge the accuracy of the output. The difference between the steady state response and desired reference gives the steady state error.



For given figure,

$$E(s) = R(s) - B(s)$$

But

$$B(s) = C(s).H(s)$$

$$E(s) = R(s) - C(s).H(s)$$

But

$$C(s) = G(s).E(s)$$

$$E(s) = R(s) - G(s).E(s).H(s)$$

$$R(s) = E(s) + G(s).E(s).H(s)$$

$$R(s) = E(s)\{1 + G(s).H(s)\}$$

$$E(s) = \frac{R(s)}{1 \pm G(s).H(s)}$$

Derivation of Steady State Error

In time domain,

$$e(t) = L^{-1} E(s)$$

and is the expression of error valid for all time. Steady state error is defined as,

$$e_{ss}(t) = \lim_{t \to \infty} e(t)$$

From the final value theorem in Laplace transform,

$$e_{ss}(t) = \lim_{s \to 0} sE(s)$$

Steady state error,

$$e_{ss}(t) = \lim_{s \to 0} sE(s)$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

Steady state error for step input:

A step input of magnitude A is applied,

$$r(t) = A.\,u(t) \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{A\} = \frac{A}{s}$$

Steady state error,

$$e_{ss}(t) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)\,H(s)}$$

Steady state error for step input:

$$e_{ss}(t) = \lim_{s \to 0} \frac{s\,\frac{A}{s}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{A}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \frac{A}{1 + \lim_{s \to 0} G(s)\,H(s)}$$

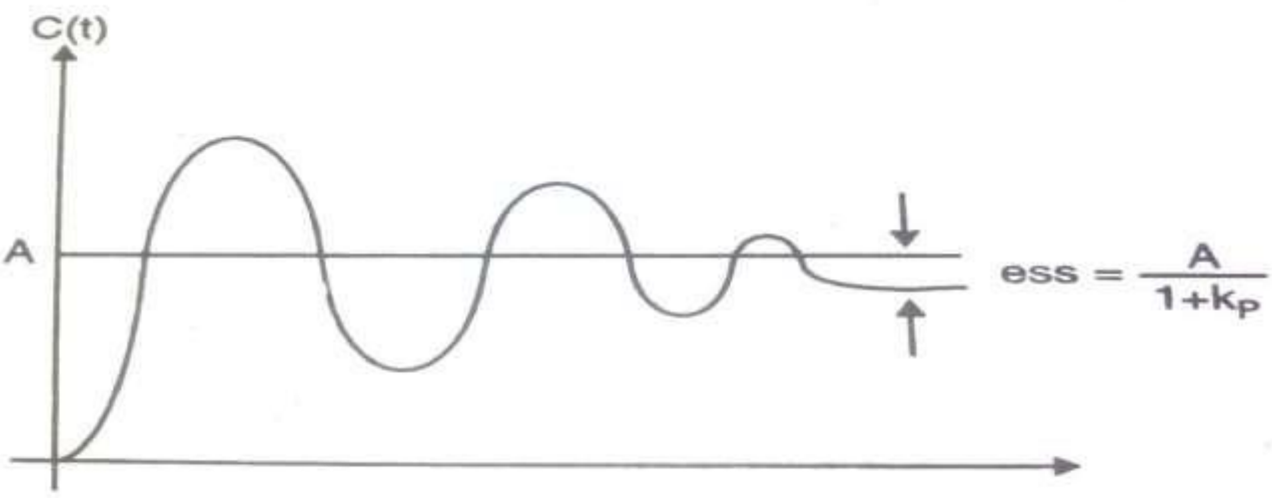
$$e_{ss}(t) = \frac{A}{1 + K_p}$$

Steady state error for step input:

The position error constant Kp of a system is defined as,

$$K_p = \lim_{s \to 0} G(s).H(s)$$

When a step input of magnitude A is given, in response to this gives $e_{ss}(t) = \dfrac{A}{1+K_p}$ steady state error



*Kp depends on type of system

Steady state error for ramp input:

A ramp input of slope A is applied,

$$r(t) = A.t \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{At\} = \frac{A}{s^2}$$

Steady state error,

$$e_{ss}(t) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

Steady state error for ramp input:

$$e_{ss}(t) = \lim_{s \to 0} \frac{s \frac{A}{s^2}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{\frac{A}{s}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{A}{s + sG(s)\,H(s)}$$

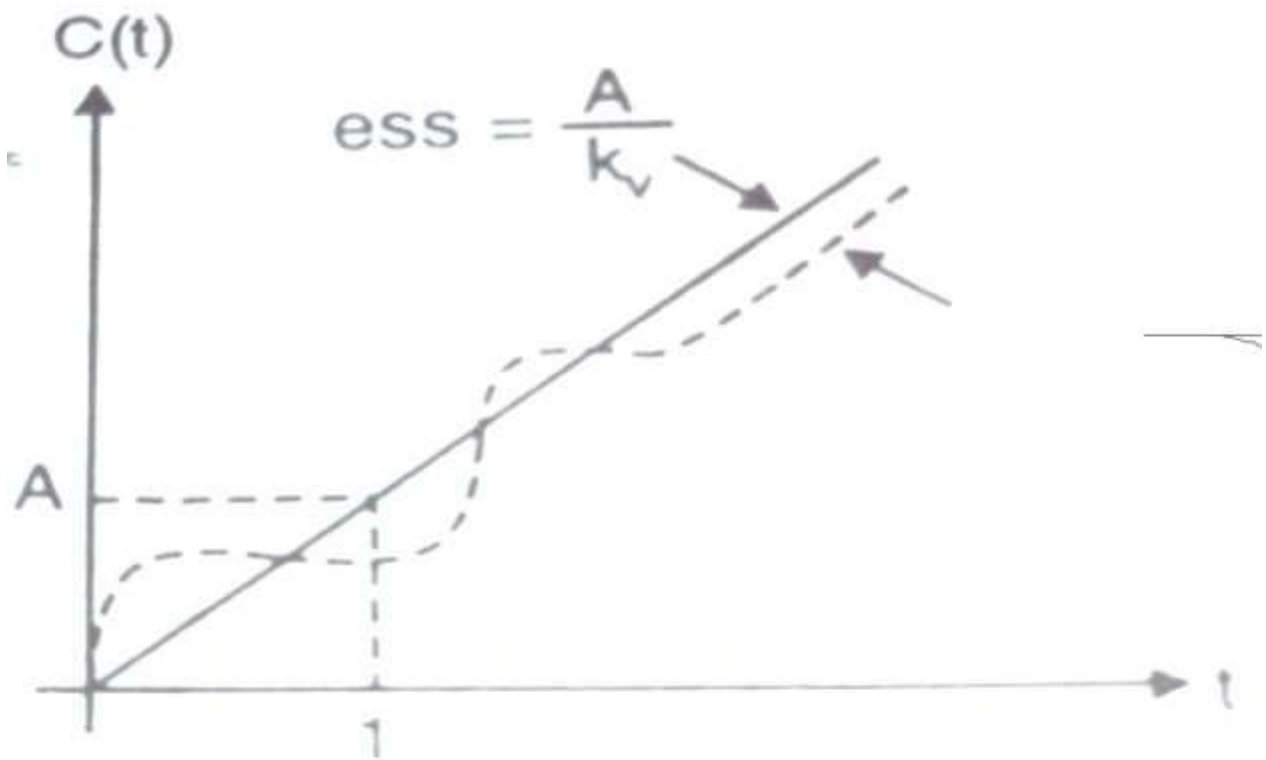
$$e_{ss}(t) = \frac{A}{0 + \lim_{s \to 0} sG(s)\,H(s)}$$

$$e_{ss}(t) = \frac{A}{K_v}$$

Steady state error for ramp input:

The velocity error constant Kv of a system is defined as,

$$K_v = \lim_{s \to 0} sG(s).H(s)$$



Steady state error for parabolic input:

A parabolic input of slope coefficient A/2 is applied,

$$r(t) = \frac{At^2}{2} \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform,

$$R(s) = L\{r(t)\} = L\{\frac{A}{2}t^2\} = \frac{A}{s^3}$$

Steady state error,

$$e_{ss}(t) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)\,H(s)}$$

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Steady state error for parabolic input:

$$e_{ss}(t) = \lim_{s \to 0} \frac{s \dfrac{A}{s^3}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{\dfrac{A}{s^2}}{1 + G(s)\,H(s)}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{A}{s^2 + s^2 G(s)\,H(s)}$$

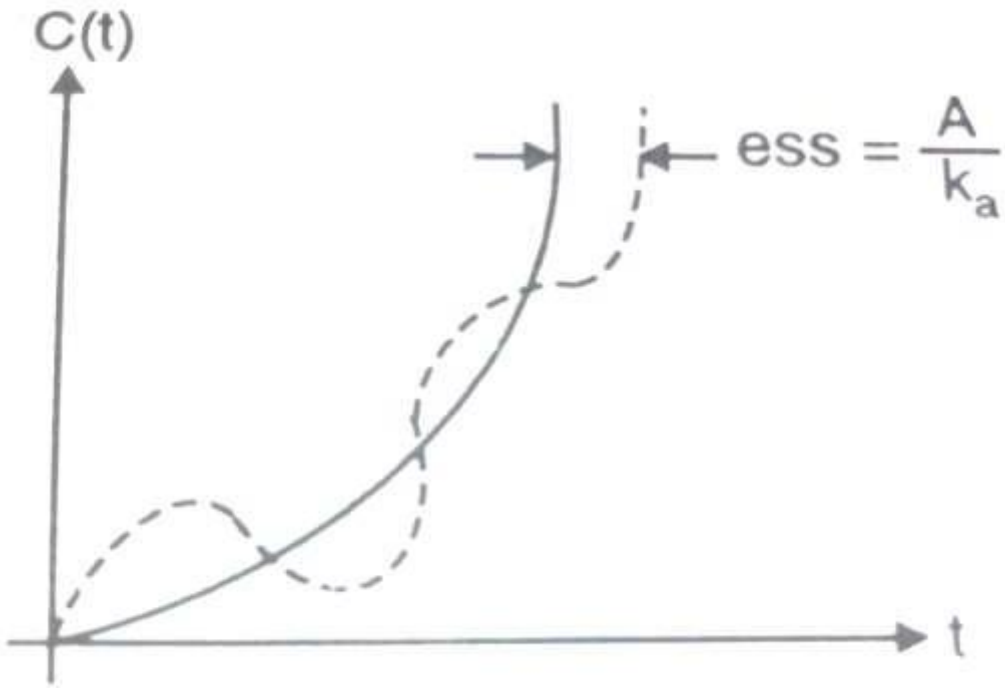
$$e_{ss}(t) = \frac{A}{0 + \lim_{s \to 0} s^2 G(s)\,H(s)}$$

$$e_{ss}(t) = \frac{A}{K_a}$$

Steady state error for parabolic input:

The acceleration error constant Ka of a system is defined as,

$$K_a = \lim_{s \to 0} s^2 G(s).H(s)$$



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Steady state error and Standard Signals

Summary:

Sr. No.	Input Signal	Steady State Error	Constant	Constant Expression
1	Step Input	$e_{ss}(t) = \frac{A}{1+K_p}$	Position Error Constant	$K_p = \lim_{s\to 0} G(s).H(s)$
2	Ramp Input	$e_{ss}(t) = \frac{A}{K_v}$	Velocity Error Constant	$K_v = \lim_{s\to 0} sG(s).H(s)$
3	Parabolic Input	$e_{ss}(t) = \frac{A}{K_a}$	Acceleration Error Constant	$K_a = \lim_{s\to 0} s^2 G(s).H(s)$