MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-4

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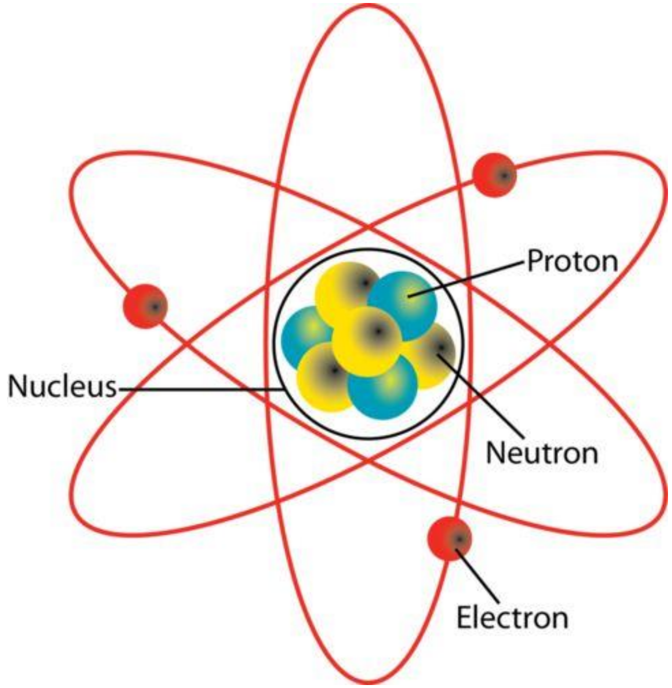
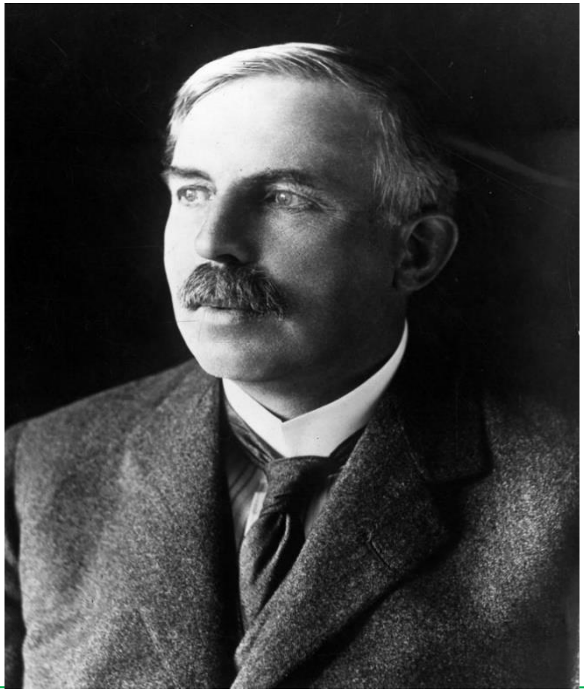




Size of the Nucleus

Nucleus

the massive, positively charged central part of an atom, made up of protons and neutrons

- In his experiment Rutheford observed that the light nuclei the distance of closest approach is of the order of 3x10^{-15} m.

- The distance of closest approach, at which scattering begins to take place was identified as the measure of nuclear size.

- Nuclear size is defined by nuclear radius, also called rms charge radius. It can be measured by the scattering of electrons by the nucleus.
- The problem of defining a radius for the atomic nucleus is similar to the problem of atomic radius, in that neither atoms nor their nuclei have definite boundaries.

Nuclear Size

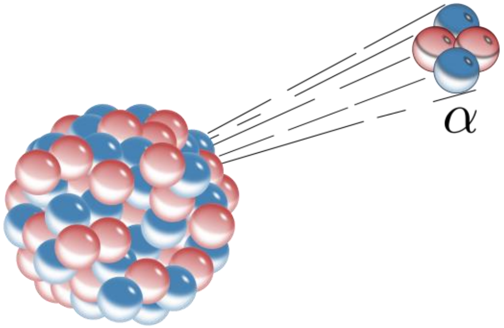
- However, the nucleus can be modelled as a sphere of positive charge for the interpretation of electron scattering experiments: because there is no definite boundary to the nucleus, the electrons “see” a range of cross-sections, for which a mean can be taken.
- The qualification of “rms” (for “root mean square”) arises because it is the nuclear cross-section, proportional to the square of the radius, which is determining for electron scattering.
- The first estimate of a nuclear charge radius was made by Hans Geiger and Ernest Marsden in 1909, under the direction of Ernest Rutherford.
- The famous Rutherford gold foil experiment involved the scattering of α-particles by gold foil, with some of the particles being scattered through angles of more than 90°, that is coming back to the same side of the foil as the α-source.
- Rutherford was able to put an upper limit on the radius of the gold nucleus of 34 femtometers (fm).

Nuclear Mass Radius

- If we ignore small asymmetries of some nuclei from spherical shape and assuming uniform distribution of nucleons inside the nucleus, the nuclear mass density ρ_m remains constant over most of the nuclear volume and decreases rapidly to zero.

- As nuclear mass is nearly proportional to the mass number A.

- Mass density $\rho_m = \frac{Mass\ Number}{Volume} = \frac{A}{V}$ = constant



Size of Nucleus

- Thus the Nuclear volume is nearly propotional to the mass number A of the nucleus i. e. V $\propto A$

- Or $\frac{4}{3}\pi R^3 \propto A$

- Hence the Radius of the nucleus R$\propto A^{\frac{1}{3}}$ or R = $R_0\ A^{\frac{1}{3}}$

- Where R_0 is the constant and called the nuclear radius parameter

- The Nuclear radius is defined is called the nuclear mass radius.

Nuclear Charge Radius

- The Nuclear charge (+Ze) is uniformly distributed throughout the nuclear volume, therefore the nuclear charge density ρ_e is nearly same throughout the nuclear volume.

- Thus the distribution nuclear charge follows the same pattern as nuclear mass distribution.

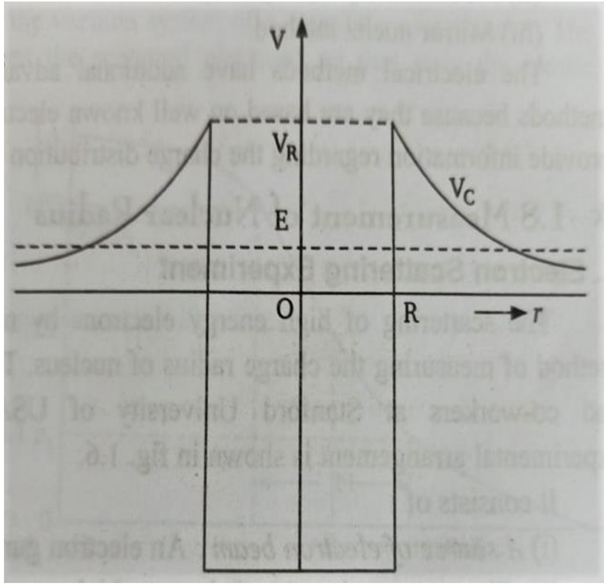
- The radius measured on the basis of the nuclear charge is called the charge radius.

- Clearly the mass radius and charge radius of the nucleus are very nearly same

Potential Radius

- Suppose a light positively charged particle of charge ne (e. g., a proton, n =1 or and α- particle, n=2) is under the electrostatic repulsive force of nucleus of charge (+Ze).

- When probing charge particle is outside the nucleus (r > R), the potential energy V is negative due to strong electrostatic repulsive force, but when it is inside the nucleus (r< R) the potential energy V is negative due to strong nuclear short range – attractive force, so that the potential energy diagram of the probing particle is shown in the figure.



- The nucleus is surrounded by Coulomb potential barrier,

$$V_c = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(ne)}{r}$$, Where r>R

- At nuclear surface r = R, The barrier height is given by $V_R = \frac{1}{4\pi\epsilon_0}\frac{(Ze)(ne)}{R}$

Methods for determination of Nuclear Size

- The methods for determining the nuclear size are classified into two main categories

1. Nuclear force Methods : These Methods are based on the range of nuclear forces. In all such methods the nucleus is probed by nucleon or light nucleus. These Methods include

- (a) α- particle Scattering
- (b) Neutron Scattering
- (C) Isotope Shift in line spectra
- (d) α- decay

Methods for determination of Nuclear Size

2. Electrical Methods : These Methods are based on the Electric field and charge distribution of nucleus. In all such methods the nucleus is probed by either electron or muon. These Methods include

- (a) $Electron\ Scattering$ Scattering
- (b) Muonic X-Ray Method and
- (C) Mirror Nuclei Method

➢ The electrical methods have additional advantage over the nuclear force methods because they are based on well known electrical interactions and also they provide information regarding the range distribution within the nucleus.

Mirror Nuclei Method

- A nucleus consists of protons and neutron
- The common term used for proton or neutron is nucleon.

- Two nuclei having same number of nucleons (Protons+ neutrons) but the number of protons in one of them being equal to the number of neutrons in the other are called the mirror nuclei.

- The example of mirror nuclei are $6^{C^{11}}$ and $5^{B^{11}}$. $6^{C^{11}}$ consists of 6 protons and 5 neutrons while $5^{B^{11}}$ contains 5 protons and 6 neutrons, thus satisfying the condition of mirror nuclei.

Mirror Nuclei Method

- The existence of protons in the nuclei gives rise to repulsive Coulomb forces and hence nuclear Coulomb energy E_e.

- In 1938 Bethe suggested that the difference of coulomb energies between two neighbouring mirror nuclei (like $6^{C^{11}}$ and $5^{B^{11}}$) can be used to calculate nuclear radii.

- The nuclear Coulomb energy may be calculated by classical and quantum mechanical methods.

Mirror Nuclei Method

- According to classical method a nucleus is analogous to a uniformly charged sphere of radius R.

- If Z is atomic number of nucleus, then charge on nucleus, q = Ze volume of nucleus, V= $\frac{4}{3}\pi R^3$.

- Charge density, ρ= $\frac{Charge}{Volume}$ = $\frac{Ze}{\frac{4}{3}\pi R^3}$

- The electrostatic energy of nucleus is the work done in assembling the nuclear charge assuming initially at infinite separation. Suppose nucleus is formed by assembling infinity thin spherical shells.

Mirror Nuclei Method

- Suppose at any instant we have a spherical core of radius x. Charge on this spherical core
- $q_1 = \frac{4}{3}\pi x^3 \rho$
- Now we bring a thin spherical of radius x and thickness dx from infinity in the process of assembling the charge.
- The charge on this shell $q_2 = (4\pi x^2 dx)\rho$.
- The electrostatic potential energy of this spherical core and thin spherical shell
- $dE_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{x}$

Mirror Nuclei Method

- $= \frac{1}{4\pi\epsilon_0} \frac{\left(\frac{4}{3}\pi x^3 \rho\right)((4\pi x^2 dx)\rho}{x} = \frac{1}{4\pi\epsilon_0}\left(\frac{16}{3}\pi^2\rho^2 x^4 dx\right)$
- Total electrostatic potential energy of nucleus is obtained by integrating this expression between the limits 0 and R i. e. Total electrostatic potential energy
- $E_e = \int_0^R \frac{1}{4\pi\epsilon_0}\left(\frac{16}{3}\pi^2\rho^2 x^4 dx\right)$
- $= \frac{1}{4\pi\epsilon_0}\left(\frac{16}{3}\pi^2\rho^2\right)\left[\frac{x^5}{5}\right]_0^R = \frac{1}{4\pi\epsilon_0}\frac{16}{3}\pi^2\rho^2\left[\frac{R^5}{5}\right]$
- Using the value of ρ in above equation we get
- $E_e = \frac{1}{4\pi\epsilon_0}\frac{16}{3}\pi^2\left[\frac{Ze}{\frac{4}{3}\pi R^3}\right]^2 \frac{R^5}{5}$

Mirror Nuclei Method

- $E_e = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R}$
- The difference in Coulomb energies between two neighbouring nuclei of charges (Ze) and (Z+1)e having the same radius R is
- $\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{(Z+1)^2e^2}{R}$ - $\frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R}$
- $= \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2}{R}[(Z+1)^2 - Z^2]$
- i.e. $\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2}{R}$ (2Z+1)
- In the Mirror Nuclei (2Z+1)= A, where A is mass number or the nucleon number
- $\Delta E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{e^2}{R}$ A

Mirror Nuclei Method

- Taking experimental observations of mirror nuclei ($_{14}Si^{29}$, $_{15}P^{29}$) the difference in the coulomb energies comes out to be 4.96 Mev. Substituting this value, R= 4.96 F.
- From the relation R= $R_0A^{1/3}$, we obtain the radial parameter R_0= 1.5 F.
- This simple consideration provides only the approximate value of the nuclear radius. The consideration of detailed nuclear structure reduces to the value of R.
- A quantum mechanical treatment which takes into account the nature of potential is tedious and gives the nuclear radius a few percent smaller.
- By introducing the correction to this model Kfoed- Hansen estimated the nuclear radius parameter to be
- R_0 = (1.28$\pm$0.5) F

Nuclear Spin and Angular Momentum

- A nucleus like an electron possesses an intrinsic angular momentum, commonly known as spin.

- We can picture a nucleus consisting of nucleons moving in certain orbits about the centre of mass of nucleus.

- This is in accordance with the independent particle model.

- Each nucleon (e.g. proton or neutron) has intrinsic spin quantum number ½ exactly equal to that of an electron and integral orbital quantum number just as in atomic physics.

- The resultant total angular momentum quantum number (denoted by I) is an integral for nuclei with even A and half for nuclei with odd A.

Nuclear Spin and Angular Momentum Contd…

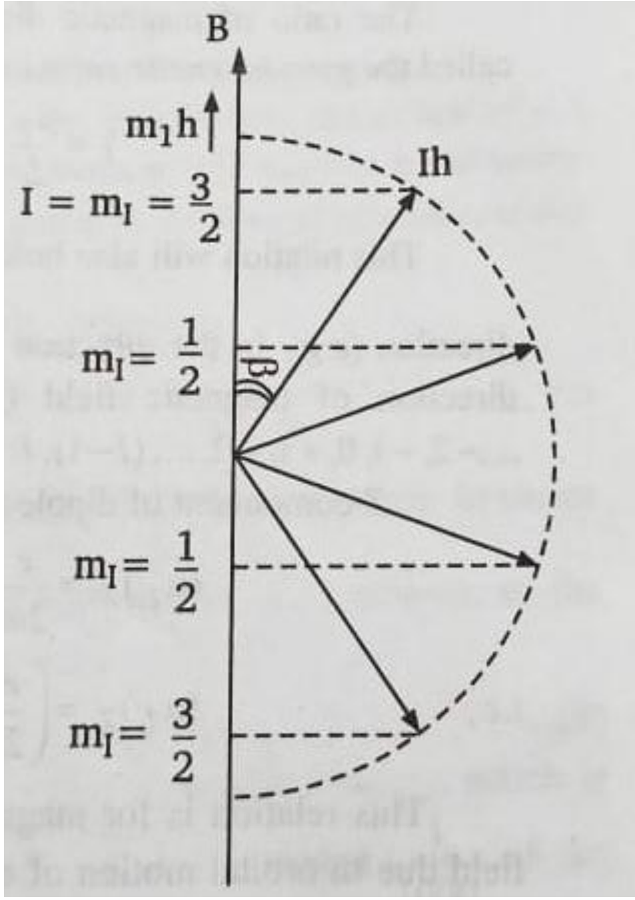
- As in atomic physics the total angular momentum of nucleus is given by
- $J_N = \sqrt{I(I+1)}\ \hbar = I^* \hbar$
- If an external magnetic field is applied then the component of angular momentum along the direction of the magnetic field (say Z axis) will be $(J_N)_Z = m_I \hbar$. Where m_I is the nuclear magnetic quantum number.
- Nuclear magnetic quantum number takes the values –I, -I+1, ----I-1, I.
- Thus the nuclear spins are quantized in the space in the presence of magnetic field and have (2I+1) different orientations.
- The Largest value of m_I is I and the largest observable component of the total nuclear angular moment is I $\hbar$.

Nuclear Spin and Angular Momentum Contd…

- The space quantization of nuclear angular momentum may be represented by a vector diagram shown in figure.

- If β is the angle between the directions of magnetic field $\vec{B}$ and the nuclear angular momentum I^* ħ, then

- Cos $\beta = \frac{m_I\ ħ}{I^*\ ħ} = \frac{m_I}{I^*} = \frac{m_I}{\sqrt{I(I+1)}}$
- As m_I = –I, -I+1, ----I-1, I
- When I= $\frac{3}{2}$, m_I = - $\frac{3}{2}$, - $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{2}$
- and I(I+1) = $\frac{3}{2}$x $\frac{5}{2}$



Nuclear Spin and Angular Momentum Contd…

- Number of Orientations
- (2I+I) = 2x$\frac{3}{2}$+1 =4
- Allowed values of cosβ
- Cos β = $\frac{-3/2}{\sqrt{\frac{3}{2}\times\frac{5}{2}}}$, $\frac{-1/2}{\sqrt{\frac{3}{2}\times\frac{5}{2}}}$, $\frac{1/2}{\sqrt{\frac{3}{2}\times\frac{5}{2}}}$, $\frac{3/2}{\sqrt{\frac{3}{2}\times\frac{5}{2}}}$
- $\frac{-3}{\sqrt{15}}, \frac{-1}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{3}{\sqrt{15}}$
- Thus for I = $\frac{3}{2}$, there are four possible orientations of nuclear angular momentum in the presence of external magnetic field.

Nuclear Spin and Angular Momentum Contd…

- The spin of a nucleus can be experimentally determined by several methods. The results obtained give an important connection between the spin and mass number:
- (i) A nucleus with an odd mass number has a half integral spin and that with an even mass number has an integral spin.
- (ii) For the ground state of even Z(number of Proton) and even (number of neutrons), it has been found without exception I=0