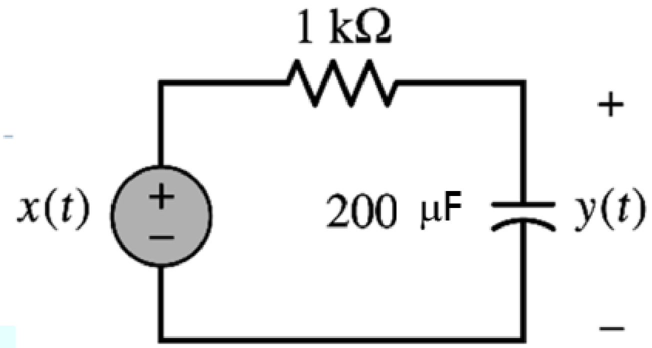
Example 6.4 RC Circuit



Find the Laplace transform of the output of the RC circuit for the input $x(t) = te^{2t}u(t)$.

<Sol.>

1. The impulse response of the RC circuit is $h(t)=\frac{1}{RC}e^{-t/(RC)}u(t)$

Then $H(s)=\frac{1}{RC}\frac{1}{s-\frac{-1}{RC}}=\frac{1}{1+sRC}$ Using RC = 0.2 s $\Rightarrow$ $H(s)=\frac{5}{s+5}$

2. Next, we use the s-domain differentiation property $-tx(t) \xleftrightarrow{\mathcal{L}_u} \frac{d}{ds}X(s)$

Then

$$X(s)=-\frac{d}{ds}\left(\frac{1}{s-2}\right)=\frac{1}{(s-2)^2}$$

3. We apply the convolution property to obtain the LT of the output $y(t)$:

$$Y(s)=\frac{5}{(s-2)^2(s+5)}$$

Inverse Laplace transform

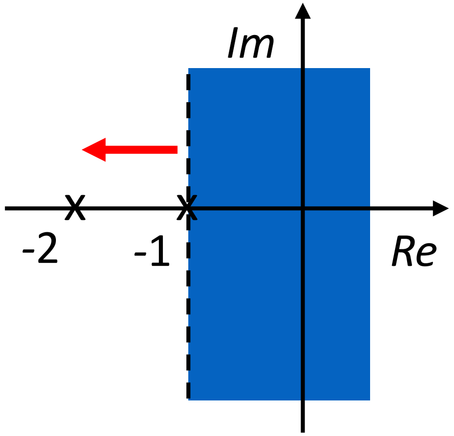
- **Example:1**
- Consider when

$$X(s) = \frac{1}{(s+1)(s+2)} \qquad \Re(s) > -1$$

- Like the inverse Fourier transform, expand as partial fractions

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} = \frac{1}{(s+1)} - \frac{1}{(s+2)}$$

- Pole-zero plots and ROC for combined & individual terms



$$e^{-t}u(t) \overset{L}{\leftrightarrow} \frac{1}{s+1}, \qquad \mathrm{Re}\{s\} > -1$$

$$e^{-2t}u(t) \overset{L}{\leftrightarrow} \frac{1}{s+2}, \qquad \mathrm{Re}\{s\} > -2$$

$$x(t) = (e^{-t} - e^{-2t})u(t) \overset{L}{\leftrightarrow} \frac{1}{(s+1)(s+2)}, \qquad \mathrm{Re}\{s\} > -1$$

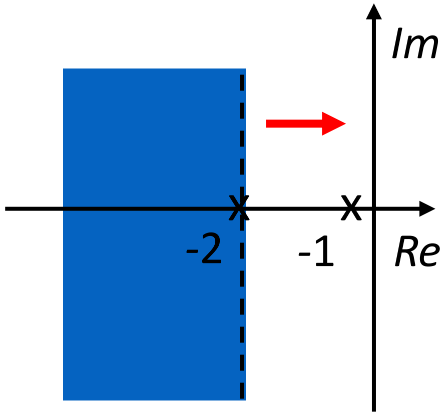
- **Example:2**

$$X(s) = \frac{1}{(s+1)(s+2)} \qquad \mathrm{Re}\{s\} < -2$$

- Consider when

$$X(s) = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)} = \frac{1}{(s+1)} - \frac{1}{(s+2)}$$

- Like the inverse Fourier transform, expand as partial fractions
- Pole-zero plots and ROC for combined & individual terms



$$-e^{-t}u(-t) \overset{L}{\leftrightarrow} \frac{1}{s+1}, \qquad \mathrm{Re}\{s\} < -1$$

$$-e^{-2t}u(-t) \overset{L}{\leftrightarrow} \frac{1}{s+2}, \qquad \mathrm{Re}\{s\} < -2$$

$$x(t) = (-e^{-t} + e^{-2t})u(-t) \overset{L}{\leftrightarrow} \frac{1}{(s+1)(s+2)}, \qquad \mathrm{Re}\{s\} < -2$$

- **Example:3**

Example 6.17

Find the inverse bilateral Laplace transform of $X(s)=\dfrac{-5s-7}{(s+1)(s-1)(s+2)}$

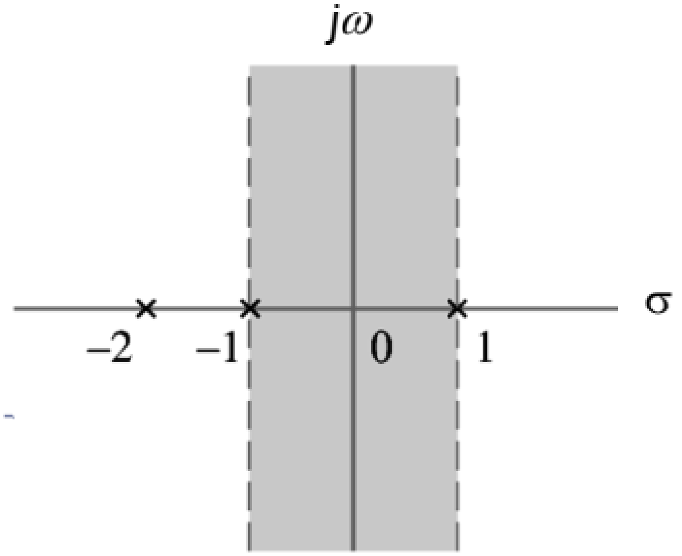
with ROC $-1<\mathrm{Re}(s)<1$

<Sol.>

Use the partial-fraction expansion $X(s)=\dfrac{1}{s+1}-\dfrac{2}{s-1}+\dfrac{1}{s+2}$

right-sided

left-sided



$$\Rightarrow \quad x(t)=e^{-t}u(t)+2e^{t}u(-t)+e^{-2t}u(t)$$

Initial Value theorem

- The value of function $f(t)$ at t→0 is called as the initial value.
- As per the initial value theorem, the initial value is given as:

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$

where $F(s)$ is the L.T. of $f(t)$

Limitations:

- In $F(s)$, the degree of denominator polynomial must be greater than degree of numerator polynomial.
- Example:

Find the initial value of $i(t) = (10 - 8e^{-2000t})u(t)$

$$(10 - 8e^{-2000t})u(t) \leftrightarrow \frac{10}{s} + \frac{-8}{s+2000}$$

$$\lim_{t\to 0^+} f(t) = \lim_{s\to\infty} s\left(\frac{10}{s} + \frac{-8}{s+2000}\right) = 2$$

Final Value theorem

- The value of function $f(t)$ at t→∞ is called as the final value.
- As per the final value theorem, the final value is given as:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

where $F(s)$ is the L.T. of $f(t)$

Limitations:

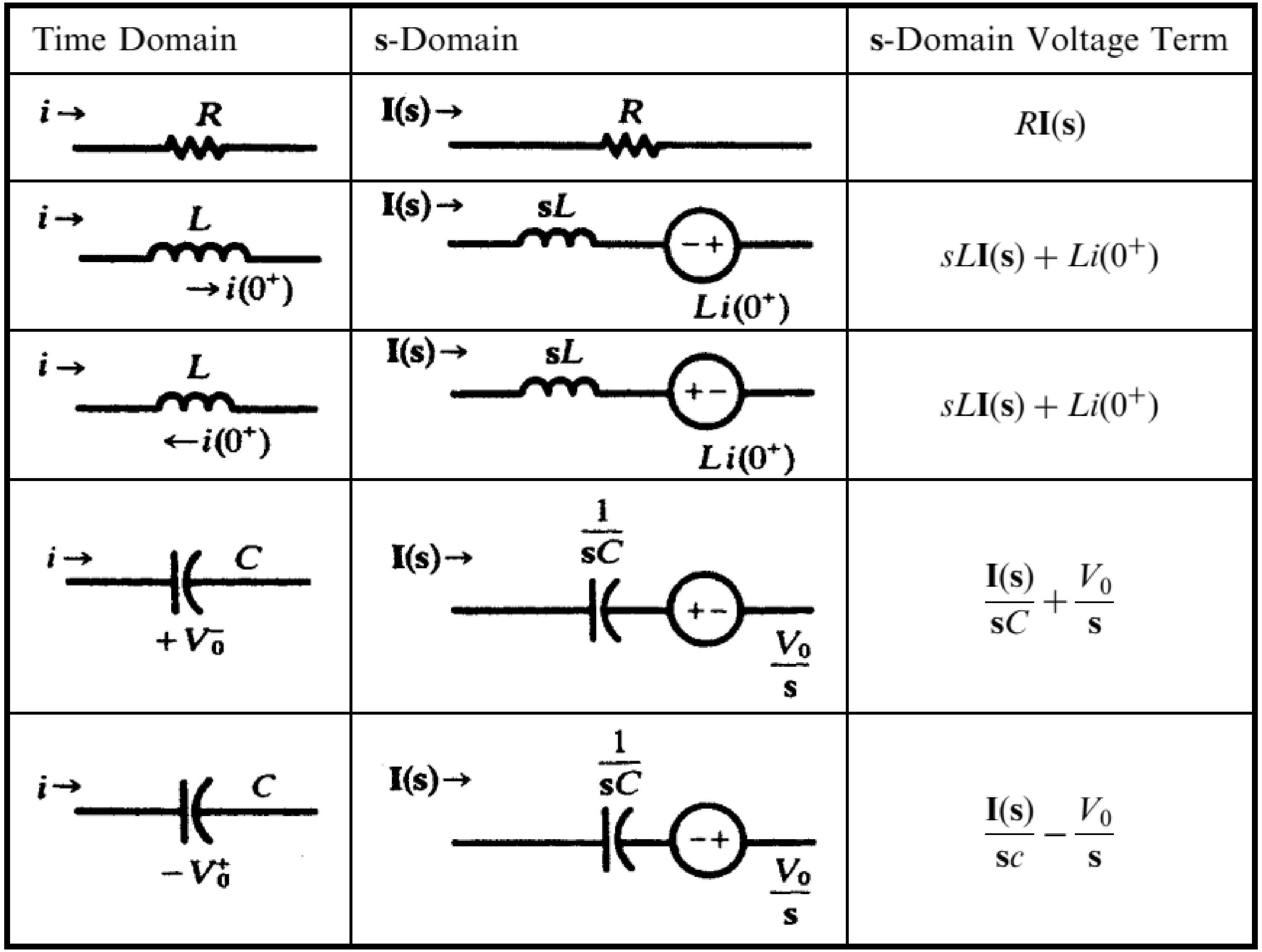
- $F(s)$ must not have poles in the right hand side of s-plane.
- Example:

Find the final value of $F(s) = \frac{1}{s(s+4)}$

$$\lim_{s\to 0} s\frac{1}{s(s+4)} = \frac{1}{4}$$

Transformed Circuits

Table 16-2

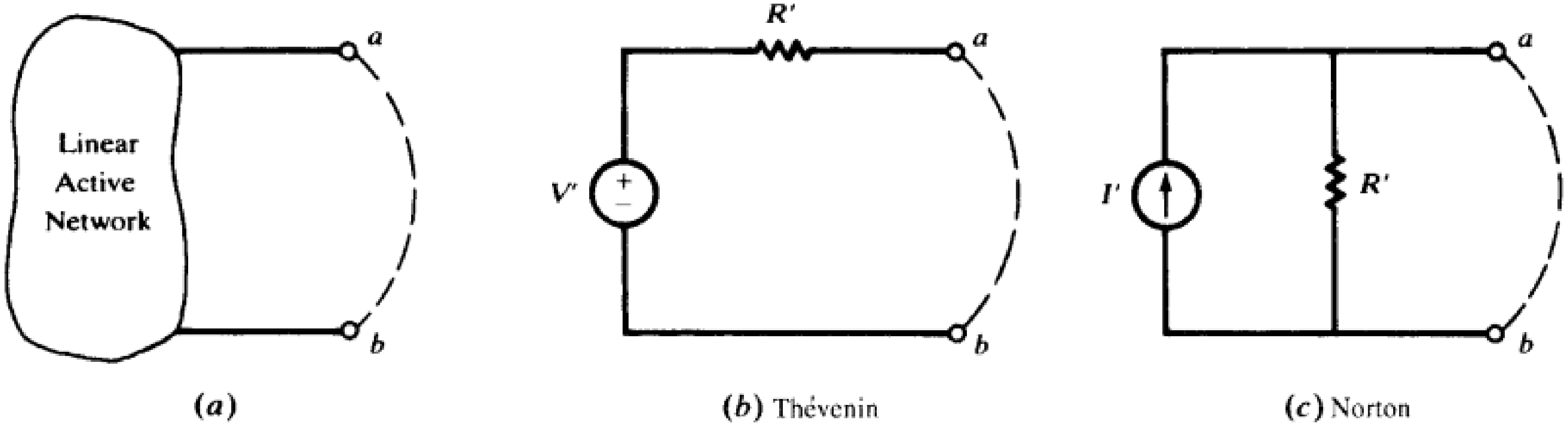


Time Domain	s-Domain	s-Domain Voltage Term
$i \rightarrow$ R	$\mathbf{I}(s) \rightarrow$ R	$R\mathbf{I}(s)$
$i \rightarrow$ L $\rightarrow i(0^+)$	$\mathbf{I}(s) \rightarrow$ sL $-+$ $Li(0^+)$	$sL\mathbf{I}(s) + Li(0^+)$
$i \rightarrow$ L $\leftarrow i(0^+)$	$\mathbf{I}(s) \rightarrow$ sL $+-$ $Li(0^+)$	$sL\mathbf{I}(s) + Li(0^+)$
$i \rightarrow$ C $+V_0^-$	$\mathbf{I}(s) \rightarrow$ $\frac{1}{sC}$ $+-$ $\frac{V_0}{s}$	$\frac{\mathbf{I}(s)}{sC} + \frac{V_0}{s}$
$i \rightarrow$ C $-V_0^+$	$\mathbf{I}(s) \rightarrow$ $\frac{1}{sC}$ $-+$ $\frac{V_0}{s}$	$\frac{\mathbf{I}(s)}{sc} - \frac{V_0}{s}$

Thevenin & Norton Theorem

4.9 THÉVENIN'S AND NORTON'S THEOREMS

A linear, active, resistive network which contains one or more voltage or current sources can be replaced by a single voltage source and a series resistance (*Thévenin's theorem*), or by a single current source and a parallel resistance (*Norton's theorem*). The voltage is called the *Thévenin equivalent voltage, V'*, and the current the *Norton equivalent current*, I'. The two resistances are the same, R'. When terminals *ab* in Fig. 4-12(*a*) are *open-circuited*, a voltage will appear between them.



(*a*)

(*b*) Thévenin

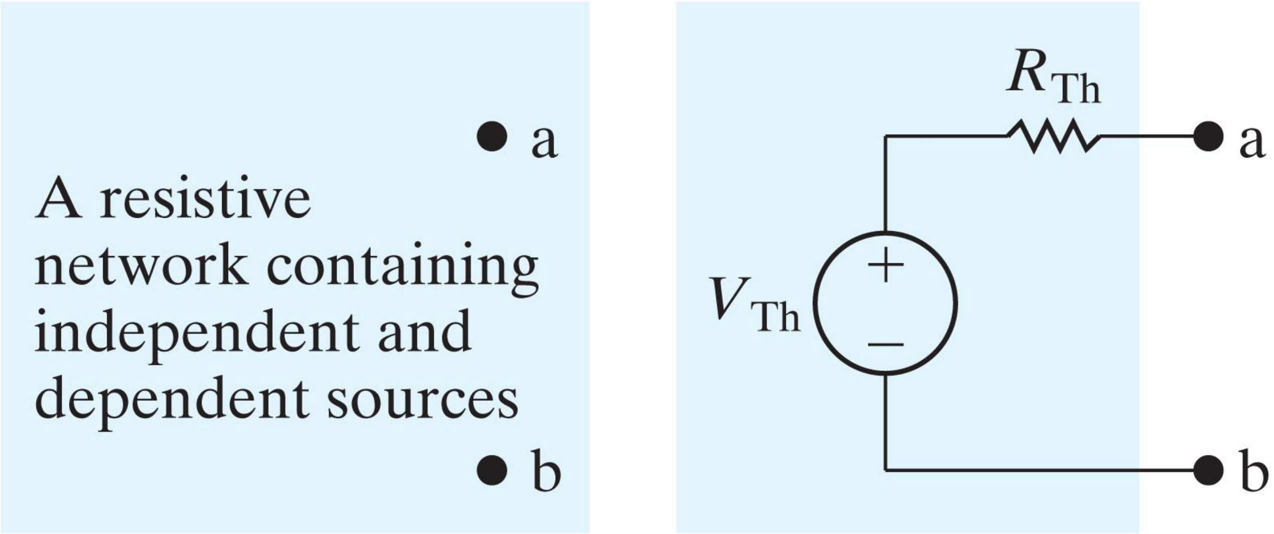
(*c*) Norton

Fig. 4-12

From Fig. 4-12(*b*) it is evident that this must be the voltage V' of the Thévenin equivalent circuit. If a *short circuit* is applied to the terminals, as suggested by the dashed line in Fig. 4-12(*a*), a current will result. From Fig. 4-12(*c*) it is evident that this current must be I' of the Norton equivalent circuit. Now, if the circuits in (*b*) and (*c*) are equivalents of the same active network, they are equivalent to each other. It follows that $I' = V'/R'$. If both V' and I' have been determined from the active network, then $R' = V'/I'$.

Why do we need them?

- Circuit “simplification”



Reduce the complicated circuit on the left to a voltage source in series with a resistor.