Control Systems

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Unit-I

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Lecture 7

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Mason's Rule (Mason, 1953)

- The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function.
- On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.
- The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

Mason's Gain Formula:

- The transfer function, ***C(s)/R(s)***, of a system represented by a signal-flow graph is;

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

Where,

n = number of forward paths.
P_i = the i^{th} forward-path gain.
Δ = Determinant of the system
Δ_i = Determinant of the i^{th} forward path

- Δ is called the signal flow graph determinant or characteristic function. Since $\Delta=0$ is the system characteristic equation.

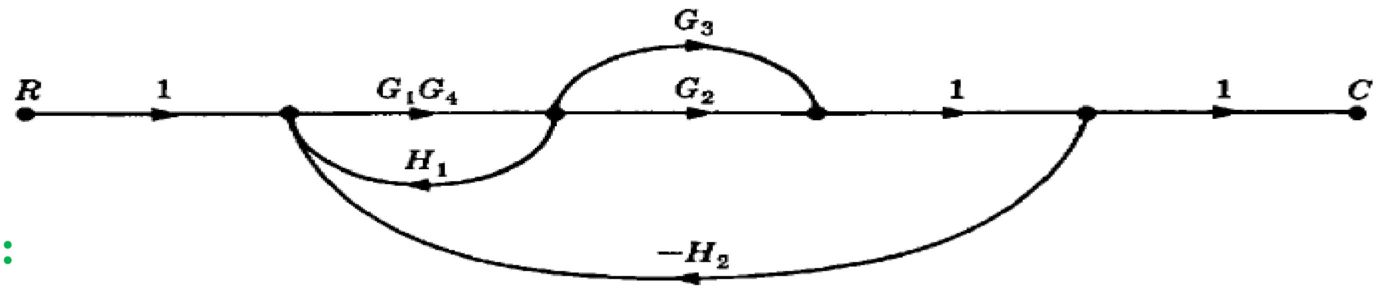
$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

- Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains.
- Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

Systematic approach for problems:

1. Calculate forward path gain P_i for each forward path *i*.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. etc
6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path *i*

Example#1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Solution:

There are two forward paths,

$P_1 = G_1G_2G_4$ $\qquad$ $P_2 = G_1G_3G_4$

Therefore, $\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$

Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+...

$\Delta_1 = 1$

Δ_2 = 1- (sum of all individual loop gains)+...

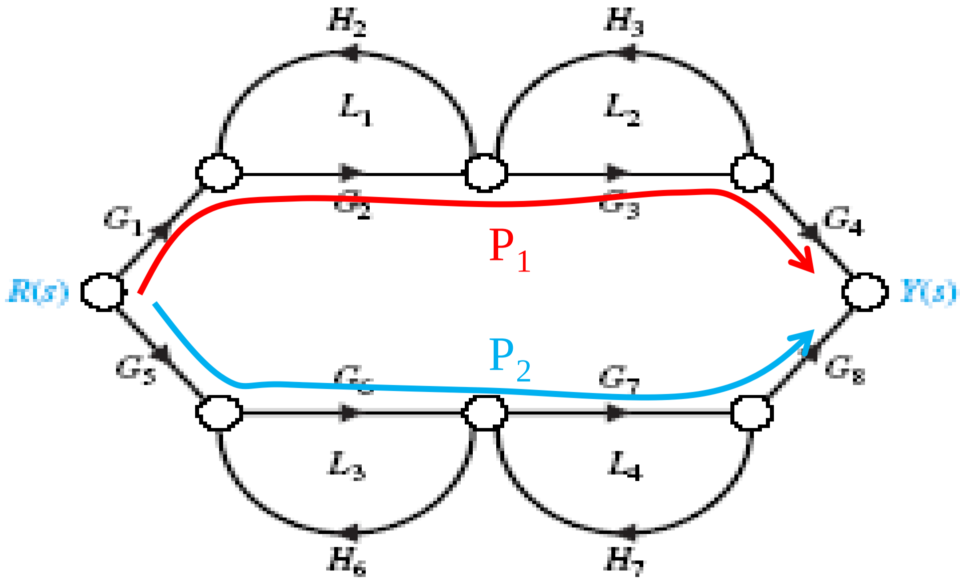
$\Delta_2 = 1$

Example#1: Continue

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

$$= \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Example#2: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



Solution:

1. Calculate forward path gains for each forward path.

   $P_1 = G_1G_2G_3G_4$ (path 1) and $P_2 = G_5G_6G_7G_8$ (path 2)

2. Calculate all loop gains. $L_1 = G_2H_2$, $L_2 = H_3G_3$, $L_3 = G_6H_6$, $L_4 = G_7H_7$

Example#2: Continue

3. Consider two non-touching loops.

L_1L_3 L_1L_4

L_2L_4 L_2L_3

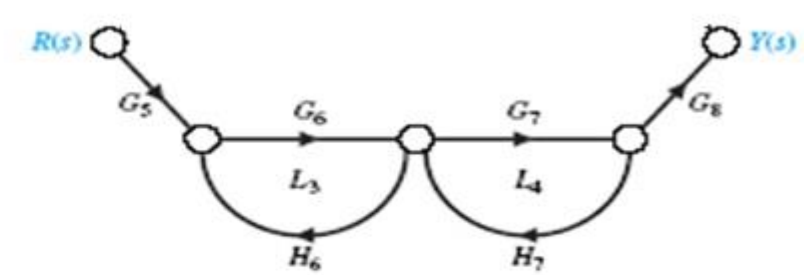
4. Consider three non-touching loops: None.

5. Calculate Δ from steps 2,3,4

$$\Delta = 1-(L_1+L_2+L_3+L_4)+(L_1L_3+L_1L_4+L_2L_3+L_2L_4)$$

$$\Delta = 1-(G_2H_2+H_3G_3+G_6H_6+G_7H_7)+ (G_2H_2G_6H_6+G_2H_2G_7H_7+H_3G_3G_6H_6+H_3G_3G_7H_7)$$

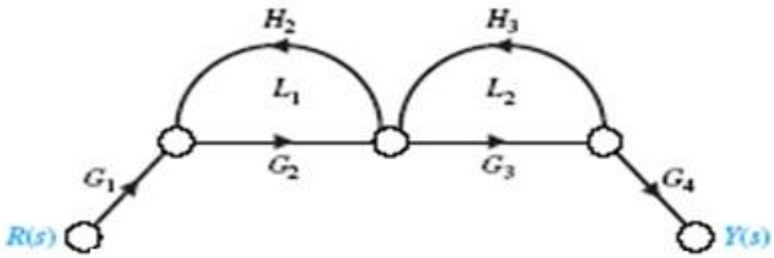
Eliminate forward path-1



$$\Delta_1 = 1-(L_3+L_4)$$

$$\Delta_1 = 1-(G_6H_6+G_7H_7)$$

Eliminate forward path-2



$$\Delta_2 = 1-(L_1+L_2)$$

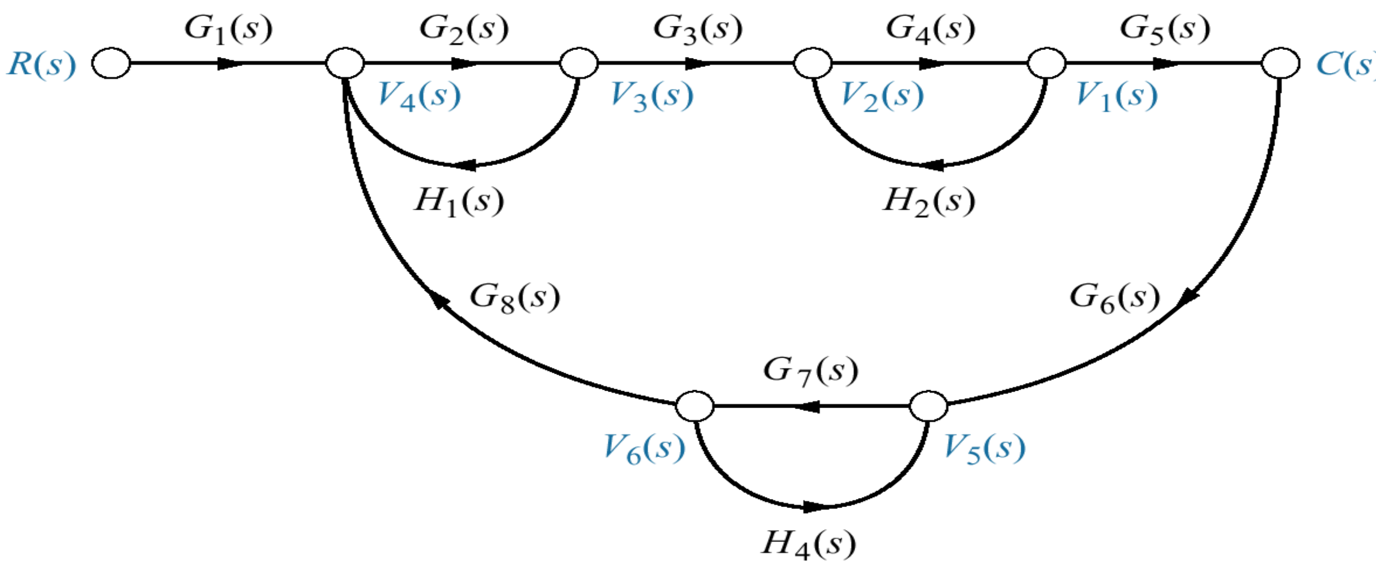
$$\Delta_2 = 1-(G_2H_2+G_3H_3)$$

Example#2: Continue

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4\left[1-\left(G_6H_6 + G_7H_7\right)\right] + G_5G_6G_7G_8\left[1-\left(G_2H_2 + G_3H_3\right)\right]}{1-\left(G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7\right) + \left(G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7\right)}$$

Example#3: Find the transfer function, C(s)/R(s), for the signal-flow graph in figure below.



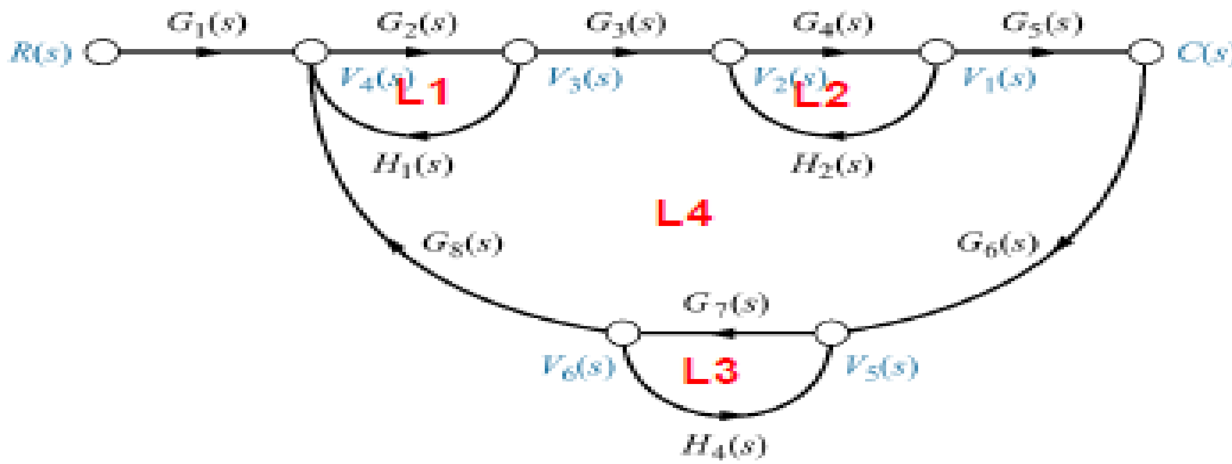
Solution:

- There is only one forward Path

$$P_1 = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$$

Example#3: Continue

- There are four feedback loops.



L1. $G_2(s)H_1(s)$ L3. $G_7(s)H_4(s)$

L2. $G_4(s)H_2(s)$ L4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$

- Non-touching loops taken two at a time.

L1 and L2: $G_2(s)H_1(s)G_4(s)H_2(s)$ L2 and L3: $G_4(s)H_2(s)G_7(s)H_4(s)$

L1 and L3: $G_2(s)H_1(s)G_7(s)H_4(s)$

- Non-touching loops taken three at a time.

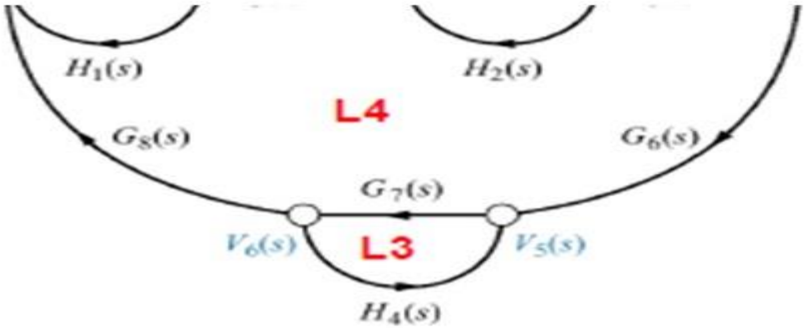
L1, L2, L3: $G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$

Example#3: Continue

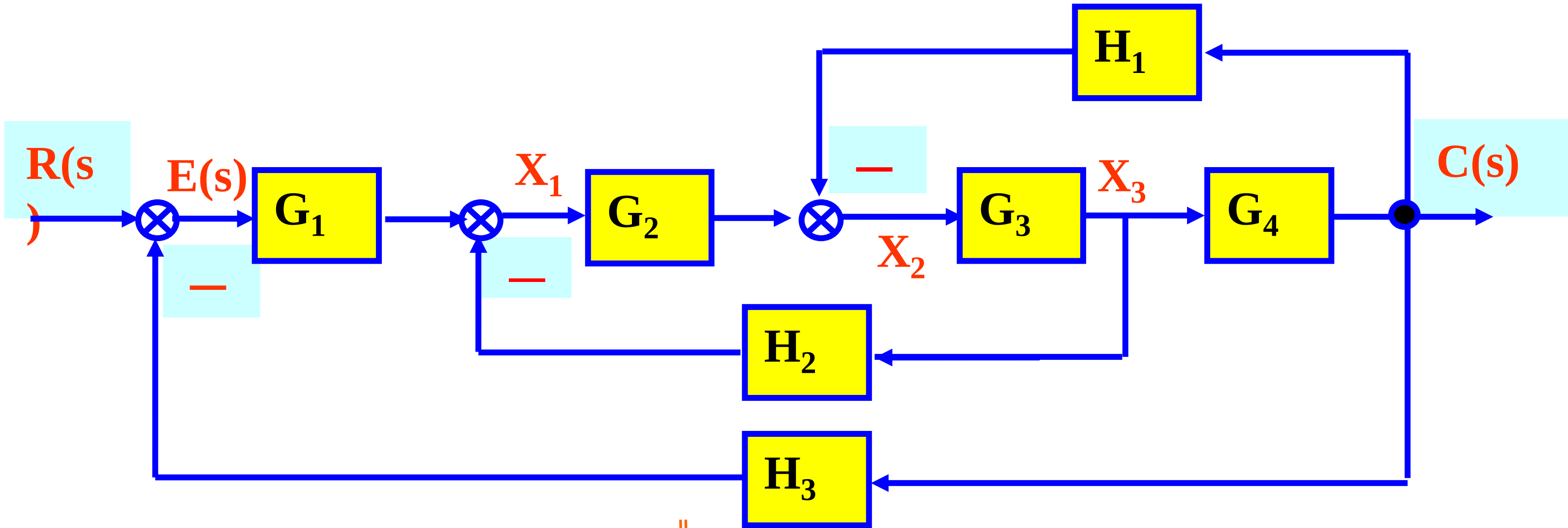
$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] + [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) + G_4(s)H_2(s)G_7(s)H_4(s)] - [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

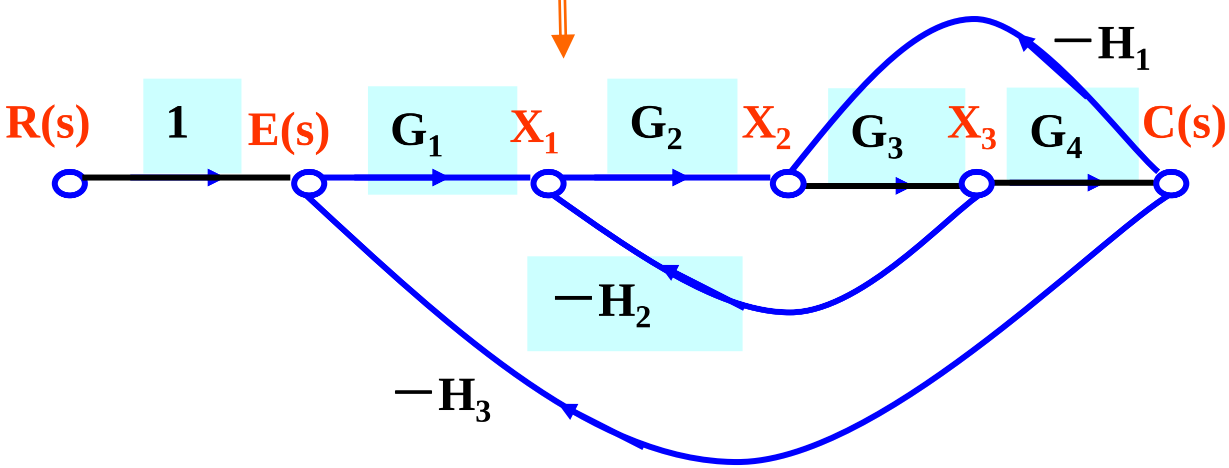
- Eliminate forward path-1

$$\Delta_1 = 1 - G_7(s)H_4(s)$$



Example#5: From Block Diagram to Signal-Flow Graph Models



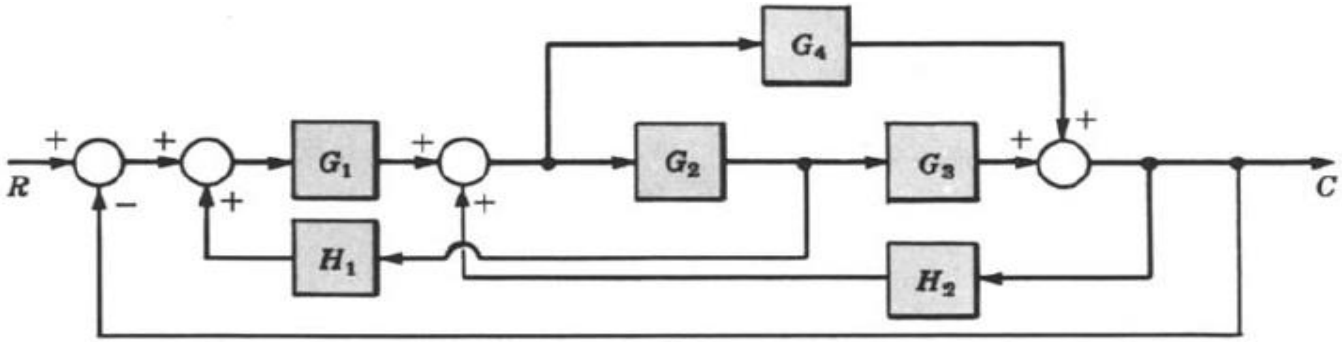


$$\Delta = 1 + (G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1)$$

$$P_1 = G_1G_2G_3G_4; \qquad \Delta_1 = 1$$

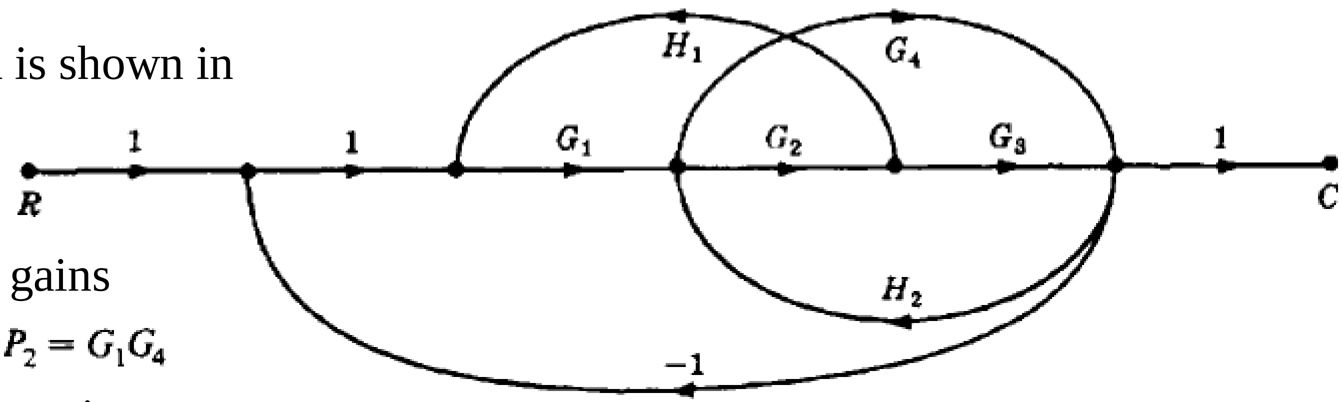
$$\Longrightarrow \quad G = \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2G_3G_4H_3 + G_2G_3H_2 + G_3G_4H_1}$$

Example#4: Find the control ratio C/R for the system given below.



Solution:

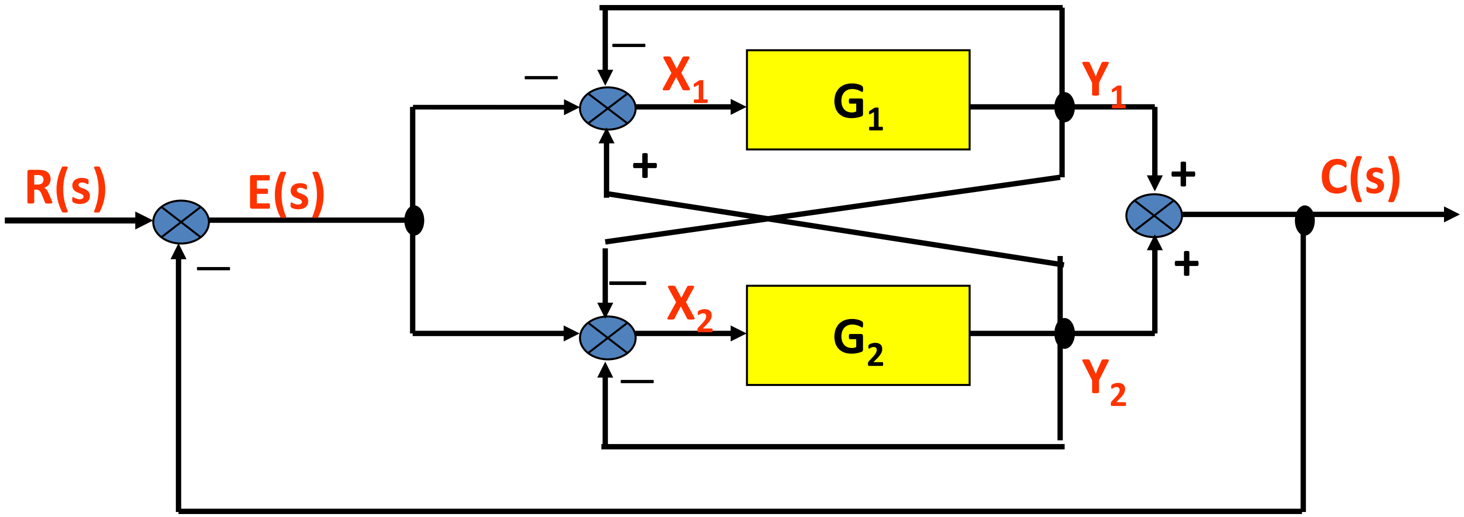
- The signal flow graph is shown in the figure.



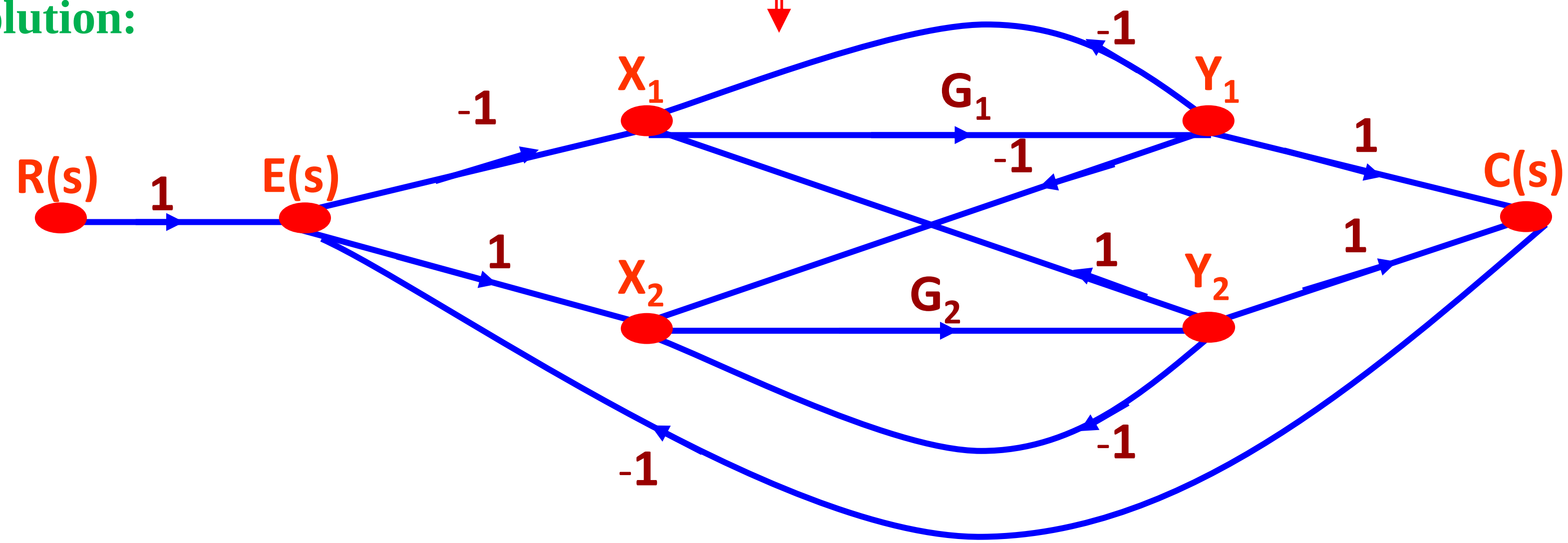
- The two forward path gains are $P_1 = G_1G_2G_3$ and $P_2 = G_1G_4$
- The five feedback loop gains are $P_{11} = G_1G_2H_1,\ P_{21} = G_2G_3H_2,\ P_{31} = -G_1G_2G_3,$ $P_{41} = G_4H_2,$ and $P_{51} = -G_1G_4.$
- All feedback loops touches the two forward paths, hence $\Delta_1 = \Delta_2 = 1$
- There are no non-touching loops, hence
$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51})$$
$$= 1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4$$
- Hence the control ratio

$$T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$$

Example#6: Find the control ratio C/R for the system given below.



Solution:



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Example#6: Continue

7 loops:

$[G_1 \cdot (-1)]$; $[G_2 \cdot (-1)]$; $[G_1 \cdot (-1) \cdot G_2 \cdot 1]$; $[(-1) \cdot G_1 \cdot 1 \cdot (-1)]$;

$[(-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1 \cdot (-1)]$; $[1 \cdot G_2 \cdot 1 \cdot (-1)]$; $[1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \cdot (-1)]$.

3 '2 non-touching loops' :

$[G_1 \cdot (-1)] \cdot [G_2 \cdot (-1)]$; $[(-1) \cdot G_1 \cdot 1 \cdot (-1)] \cdot [G_2 \cdot (-1)]$;

$[1 \cdot G_2 \cdot 1 \cdot (-1)] \cdot [G_1 \cdot (-1)]$.

Then: $\Delta = 1 + 2G_2 + 4G_1G_2$

4 forward paths: $p_1 = (-1) \cdot G_1 \cdot 1 \qquad \Delta_1 = 1 + G_2$

$$p_2 = (-1) \cdot G_1 \cdot (-1) \cdot G_2 \cdot 1 \qquad \Delta_2 = 1$$

$$p_3 = 1 \cdot G_2 \cdot 1 \qquad \Delta_3 = 1 + G_1$$

$$p_4 = 1 \cdot G_2 \cdot 1 \cdot G_1 \cdot 1 \qquad \Delta_4 = 1$$

We have

$$\frac{C(s)}{R(s)} = \frac{\sum p_k \Delta_k}{\Delta}$$

$$= \frac{G_2 - G_1 + 2G_1G_2}{1 + 2G_2 + 4G_1G_2}$$