EARTHQUAKE RESISTANT DESIGN

(BCE-42)

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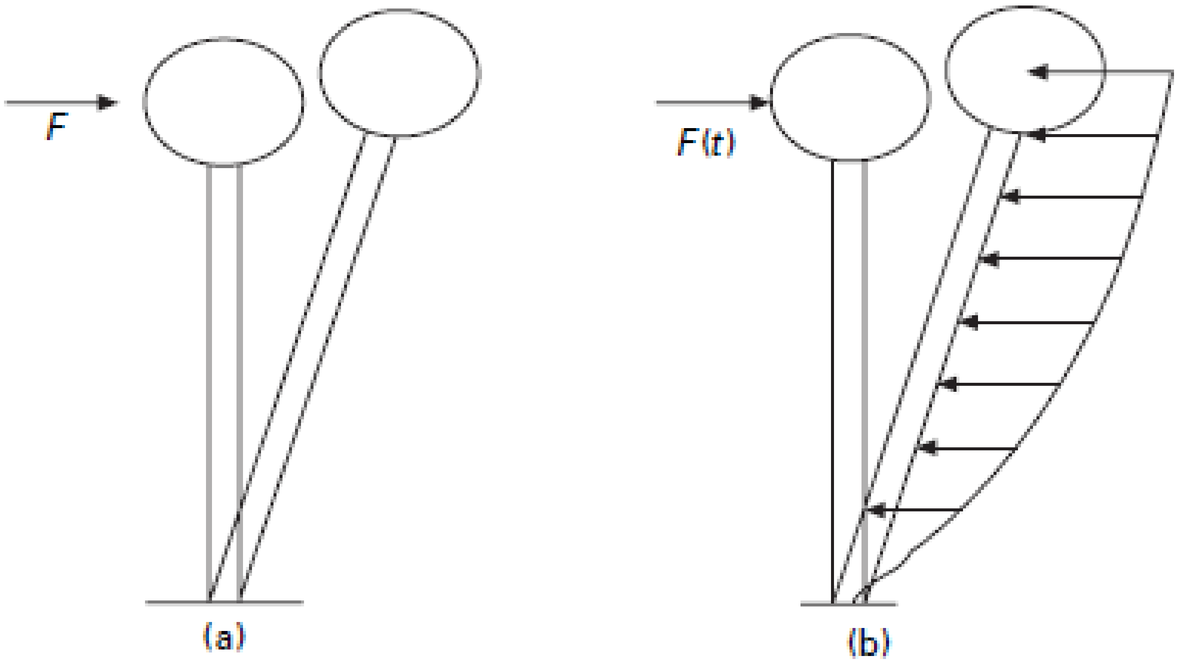
Topics Covered

UNIT- 2

- Definitions of basic problems in dynamics, static versus dynamic loads, different types of dynamic loads, un-damped and damped vibration of SDOF system, natural frequency, and periods of vibration, damping in structure, response to periodic loads, response to general dynamic load, response of structure subject to gravitational motion, lumped SDOF elastic systems, translational excitation.

Static Loads versus Dynamic Loads

- In static problem, load is constant with respect to time whereas in dynamic problem time is varying in nature because load and its response is vary with time.
- Static problem has only response i.e. *Displacement* where as Dynamic has three response i.e. *Displacement*, *velocity and acceleration.*
- Static problem has only one solution whereas dynamic problem has infinite number of solution which are time dependent in nature.
- In static problem the response calculated using static equilibrium whereas in dynamic problem it is also depend on the inertial forces.
- Static problem can be solve directly whereas dynamic problem is complex in nature due time dependency.



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Water tank subjected to static and dynamic loads: (a) static load; (b) dynamic load.

Dynamic load

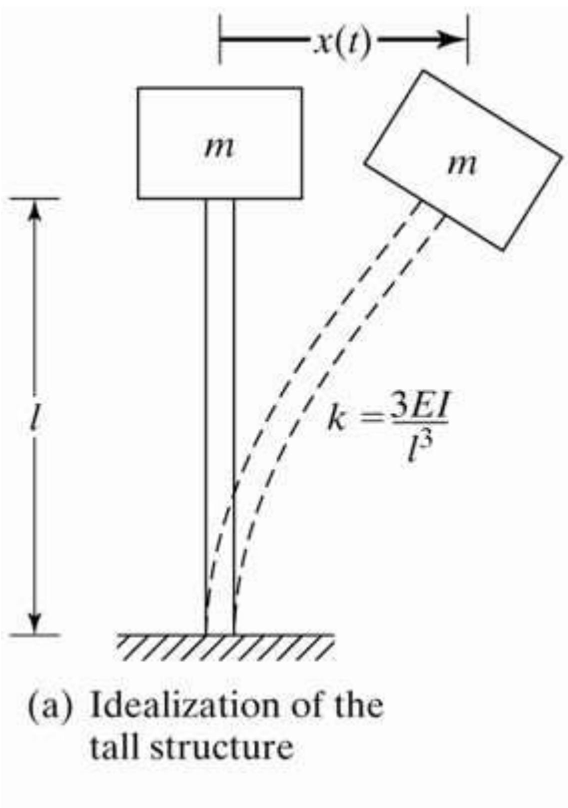
Dynamic load is any load whose magnitude, position and direction vary with time hence structural response the dynamic load is also vary with time.

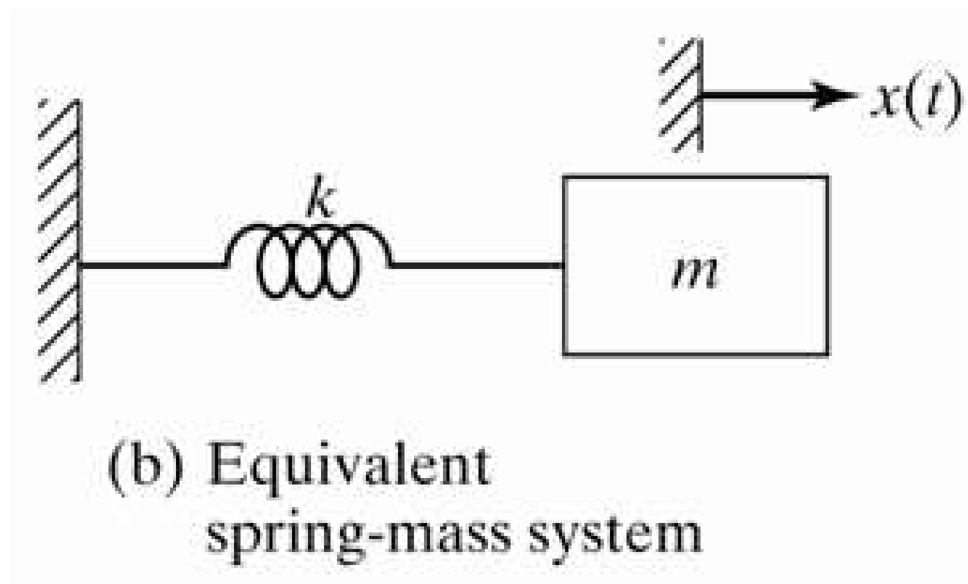
Types of dynamic loads.

1. Deterministic load: If the magnitude, point of application of the load and the variation of the load with respect to time are known, the loading is said to be deterministic and the analysis of a system to such loads is defined as deterministic analysis.
2. Non-deterministic load : if the variation of load with respect to time is not known, the loading is referred to as random or *stochastic* loading and the corresponding analysis is termed as non-deterministic analysis

Degree of freedom

1. Single Degree of Freedom: If a single coordinate is sufficient to define the position or geometry of the mass of the system at any instant of time is called single or one degree of freedom system.





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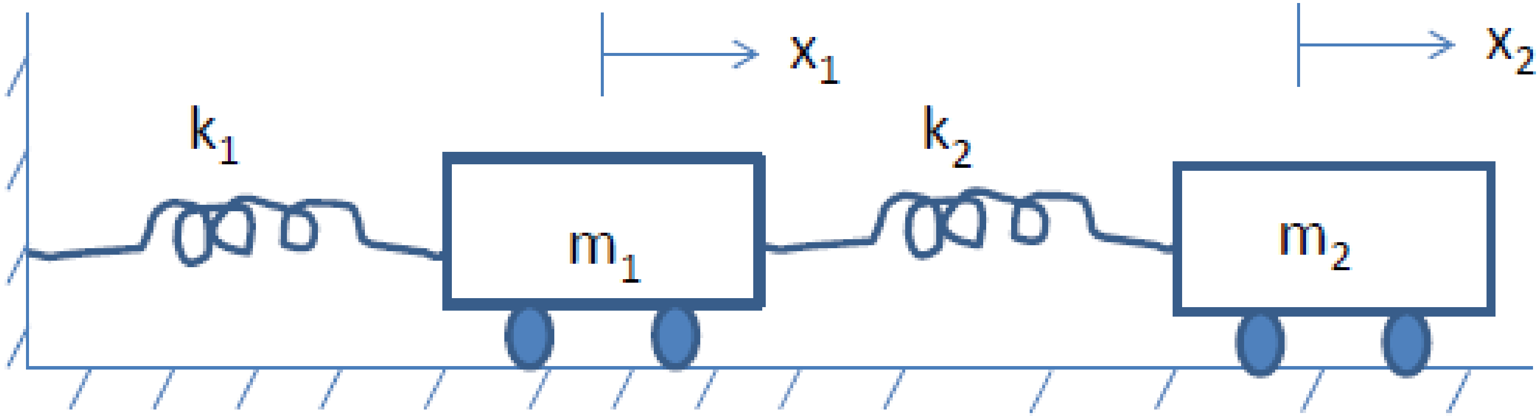
2. Multiple degree of freedom (MDoF)

If more than one independent coordinate is required to completely specify the position or geometry of different masses of the system at any instant of time, is called multiple degrees of freedom system.



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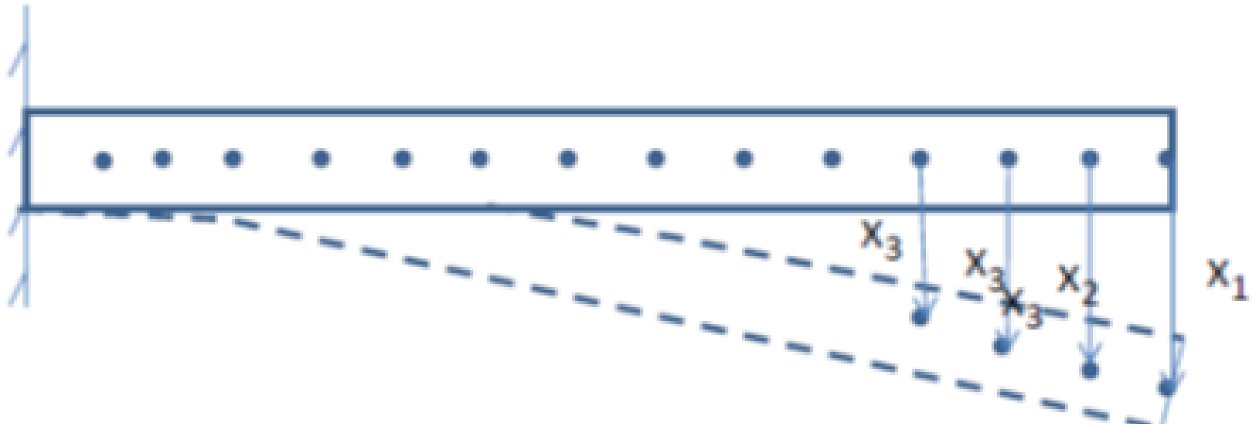
Example for MDOF system

3. Continuous system

If the mass of a system may be considered to be distributed over its entire length as shown in figure, in which the mass is considered to have infinite degrees of freedom, it is referred to as a continuous system. It is also known as distributed system.



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Cantilever beam with infinite number of degree of freedom

Free Vibration of Undamped - SDOF System

$$\alpha = 0; \beta = \omega;$$

$$@\, t = 0; x_0 = A;$$

$$\dot{x}(t) = -\omega A \sin \omega t + \omega B \cos \omega t;$$

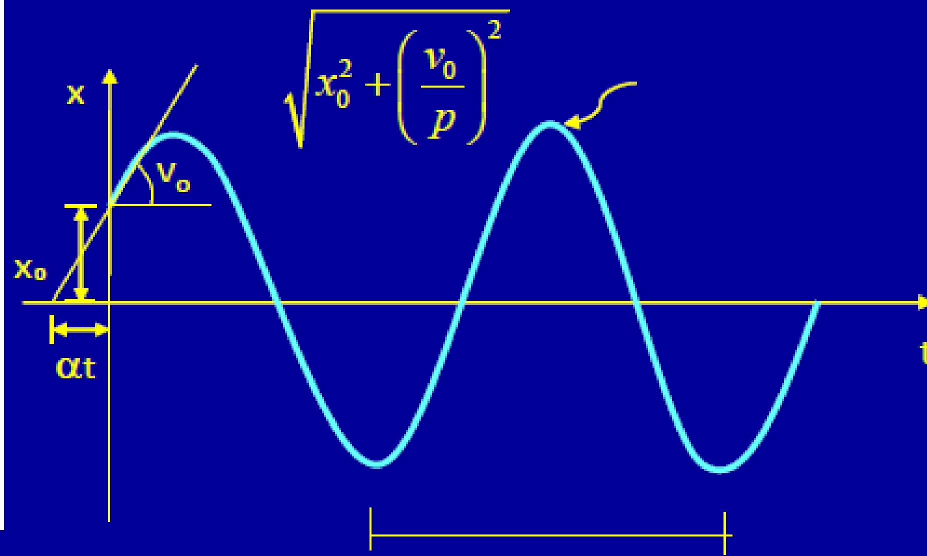
$$\dot{x}(t) = \omega B ; B = \frac{\dot{x}(0)}{\omega};$$

$$x(t) = x_0 \cos \omega t + \frac{\dot{x}(0)}{\omega} \sin \omega t;$$

$$where\ \omega = \sqrt{k/m}\ rad/\sec;$$

$$m = kg; k = N/m;$$

Amplitude of motion



$$T = \frac{2\pi}{p}$$

$$x(t) = x_0 \cos\ pt + \frac{v_0}{p} \sin\ pt$$

$V_o = \dot{X}_o$ & $P = \omega$

X_0=initial displacement

V_0=initial velocity

or

$$x(t) = \sqrt{x_0^2 + \left(\frac{v_0}{p}\right)^2}\ \sin(pt + \alpha)$$

where, $\tan\ \alpha = \dfrac{x_0}{v_0/p}$

ω=p is called circular frequency or angular frequency of vibration (Rad/s)

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Free Vibration of damped SDOF systems

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\ddot{x} + c\frac{\dot{x}}{m} + \frac{k}{m} = 0$$

$$\ddot{x} + 2\xi px + p^2x = 0$$

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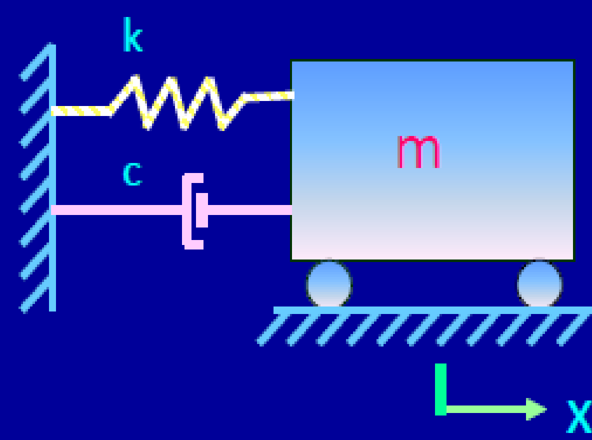

where,

$$p = \sqrt{\frac{k}{m}}$$

ρ is called circular frequency or angular frequency of vibration (Rad/s)

$$\zeta = \frac{c}{2mp} = \frac{c}{2\sqrt{km}}$$

(Dimensionless parameter) - A

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Solution of Eq.(A) may be obtained by a function in the form $x = e^{rt}$ where r is a constant to be determined. Substituting this into (A) we obtain,

$$e^{rt}\left(r^2 + 2\zeta pr + p^2\right) = 0$$

In order for this equation to be valid for all values of t,

$$r^2 + 2\zeta pr + p^2 = 0$$

or $$r_{1,2} = p\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

Thus $e^{r_1 t}$ and $e^{r_2 t}$ are solutions and, provided r_1 and r_2 are different from one another, the complete solution is

$$x = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

The constants of integration c_1 and c_2 must be evaluated from the initial conditions of the motion.

Note that for $\zeta > 1$, r_1 and r_2 are real and negative

for $\zeta < 1$, r_1 and r_2 are imaginary and

for $\zeta = 1$, $r_1 = r_2 = -p$

Solution depends on whether ζ is smaller than, greater than, or equal to one.

For $\zeta < 1$ (Light Damping) :

(B)

$$x(t) = e^{-\zeta pt}\left[A\cos p_d t + B\sin p_d t\right]$$

where, $$p_d = p\sqrt{1-\zeta^2}$$

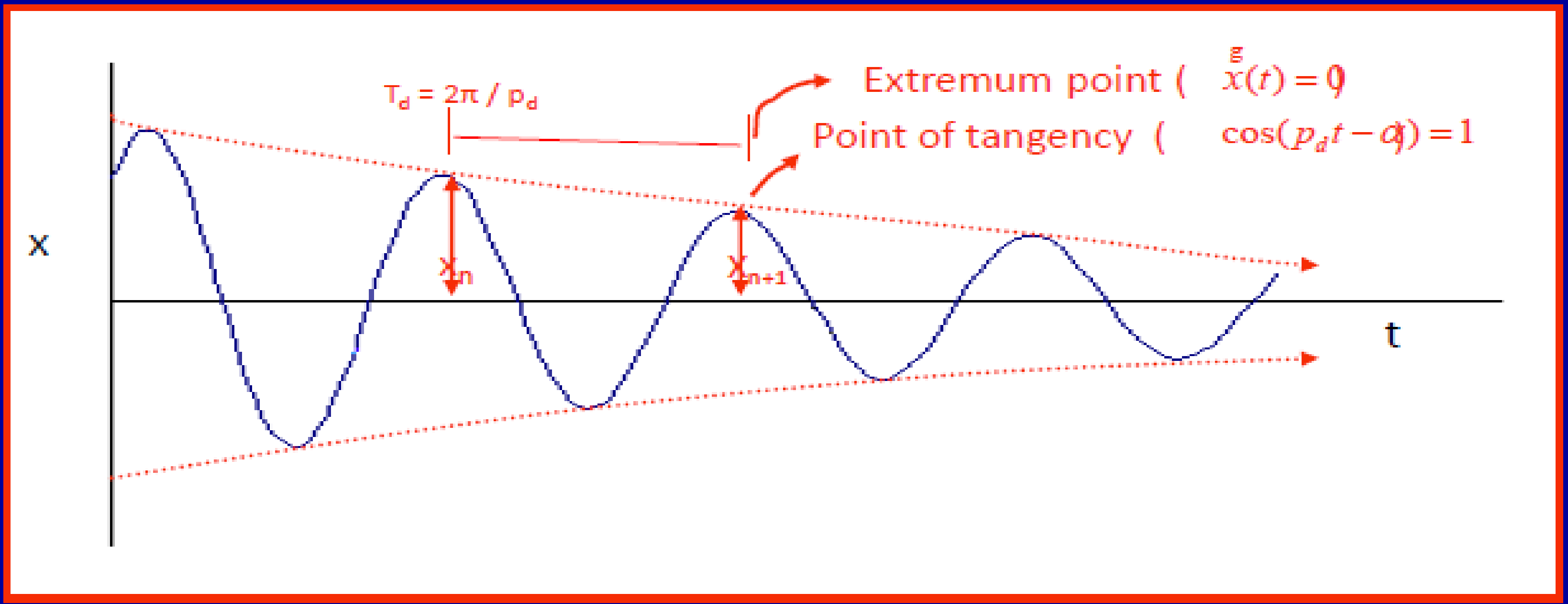
'A' and 'B' are related to the initial conditions as follows

$$A = x_0$$

$$B = \frac{v_0}{p_d} + \frac{\zeta}{\sqrt{1-\zeta^2}}x_0$$

In other words, Eqn.B can also be written as,

$$x(t) = e^{-\zeta pt}\left[x_o\cos p_d t + \left(\frac{v_o}{p_d} + \frac{\zeta}{\sqrt{1-\zeta^2}}x_o\right)\sin p_d t\right]$$



$$T_d = \frac{2\pi}{p_d} = \text{Damped natural period}$$

$$p_d = p\sqrt{1-\zeta^2} = \text{Damped circular natural frequency}$$

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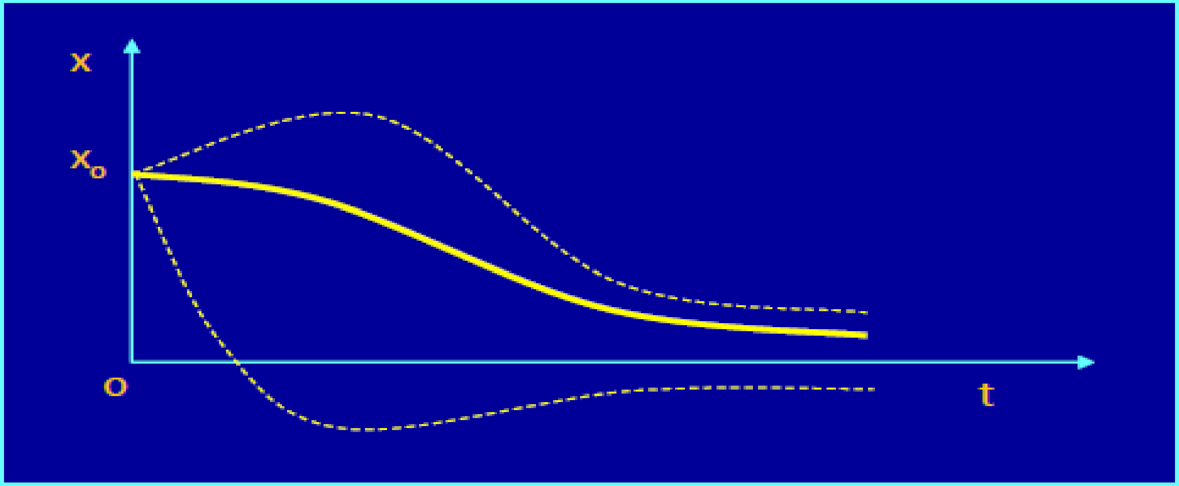
For $\zeta > 1$ (Heavy Damping)

Such system is said to be over damped or super critically damped.

$$x(t) = C_1 e^{(-)t} + C_2 e^{(-)t}$$

i.e., the response equation will be sum of two exponentially decaying curve

In this case r1 and r2 are real negative roots.



For $\zeta = 1$

Such system is said to be critically damped.

$$x(t) = C_1 e^{-pt} + C_2 t e^{-pt}$$

With initial conditions,

$$x(t) = \left[x_0 \left(1 + pt \right) + v_0 t \right] e^{-pt}$$

The value of 'c' for which $\zeta = 1$ Is known as the critical coefficient of damping

$$C_{cr} = 2mp = 2\sqrt{km}$$

Therefore, $\zeta = \dfrac{C}{C_{cr}}$

Period of Vibration and Natural Frequency

Period of Vibration (Time Period): For SHM, time period is the time interval in which the phase of vibrating particle changes by 2π and it is denoted by T.

$$T = \frac{2\pi}{\omega_n}$$

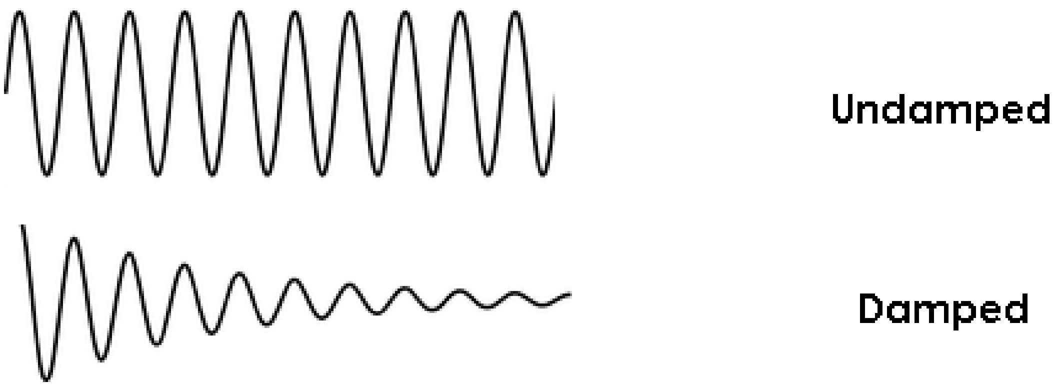
Natural Frequency: It is the frequency at which a system tends to oscillate in the absence of any driving or damping force. Free vibrations of an elastic body are called natural vibrations and occur at a frequency called the natural frequency. Denoted by ω.

In a mass spring system, with mass *m* and spring stiffness *k*, the natural frequency can be calculated as:

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Damping in structures

Damping is phenomenon in which that makes any vibrating body/structures to decay in amplitude of motion gradually by means of energy dissipation through various mechanism.



Some modern damping devices:

1. Metallic Yielding dampers
2. Friction dampers
3. Visco elastic dampers
4. Tuned Mas dampers
5. Viscous Fluid dampers
6. Tuned Liquid dampers
7. Active Tuned Mas dampers

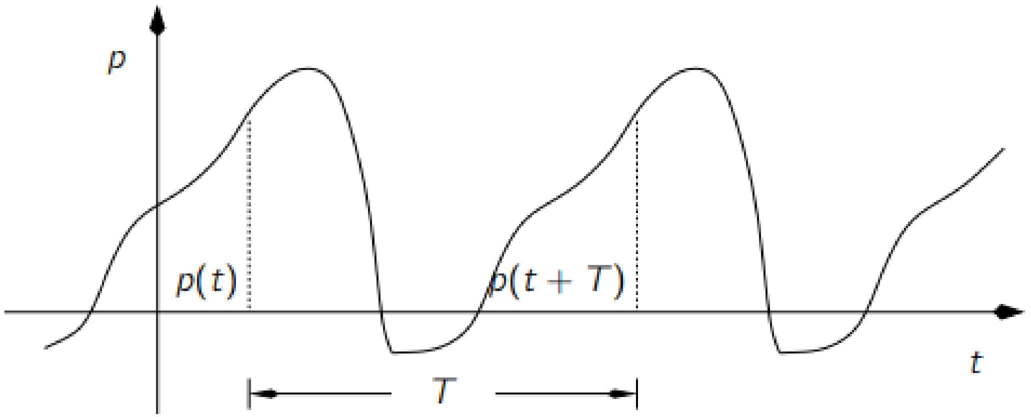
Response To Periodic Loading

Periodic loading is the loading which repeats its after some definite periods.

A periodic loading is characterized by the identity

$$p(t) = p(t + T)$$

Where T is period of loading .



Let us consider a SDOF system is subjected to periodic loading, that means a forcing function repeat itself after time interval T.

A Fourier series is an expansion of a periodic function in terms of an infinite sum of sine's and cosines in series of harmonic loading terms. For periodic loading of period T, shown in above fig. Fourier series can be written as

$$F(T) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots\ldots\ldots + a_n \cos n\omega t + b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots\ldots\ldots + b_n \sin n\omega t$$

Or it can be written as

$$F(T) = a_0 + \sum_{m=1}^{\infty} a_m \cos\omega_m t + \sum_{m=1}^{\infty} m_m \sin\omega_m t$$

where

$$a_0 = \frac{1}{T}\int_0^T F(t)dt \qquad a_m = \frac{2}{T}\int_0^T F(t)\cos\omega_m t dt \qquad b_m = \frac{2}{T}\int_0^T F(t)\sin\omega_m t dt$$

m = 1, 2, 3, 4……

a_0 a_m b_m are Fourier coefficients

The response obtained from SDOF system

$$x(t) = \frac{F}{k(1-\beta^2)}\left[\sin\omega t + \beta\sin\omega_n t\right]$$

Textbooks

1. Earthquake Resistant Design of Structures - P. Agarwal & M. Shrikhande

2. Dynamics of Structures Theory and Applications to Earthquake Engineering - Anil K. Chopra

3. Dynamics of Structures - R.W. Clough & J. Penjien.

4. I.S. Codes No. 1893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