Hydrograph Analysis

As Prepared By-

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Hydrograph

Record of River Discharge over a period of time ;

Q vs t

River Discharge : Q = A X v

= **cross sectional area** X **rivers mean (average) velocity**

(at a particular point in its course)

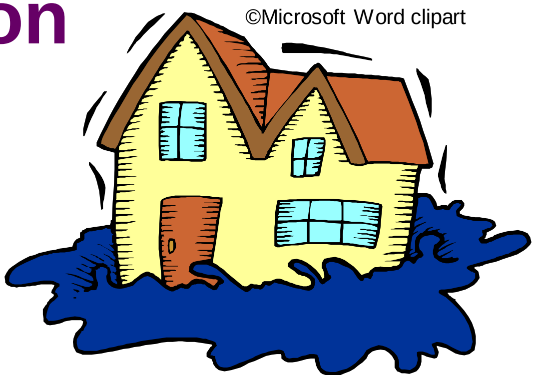
Storm Hydrographs

Show the change in discharge caused by a period of rainfall

Why Construct & Analyse Hydrographs？

- To find out discharge patterns of a particular drainage basin

- Help predict flooding events, therefore influence implementation of flood prevention measures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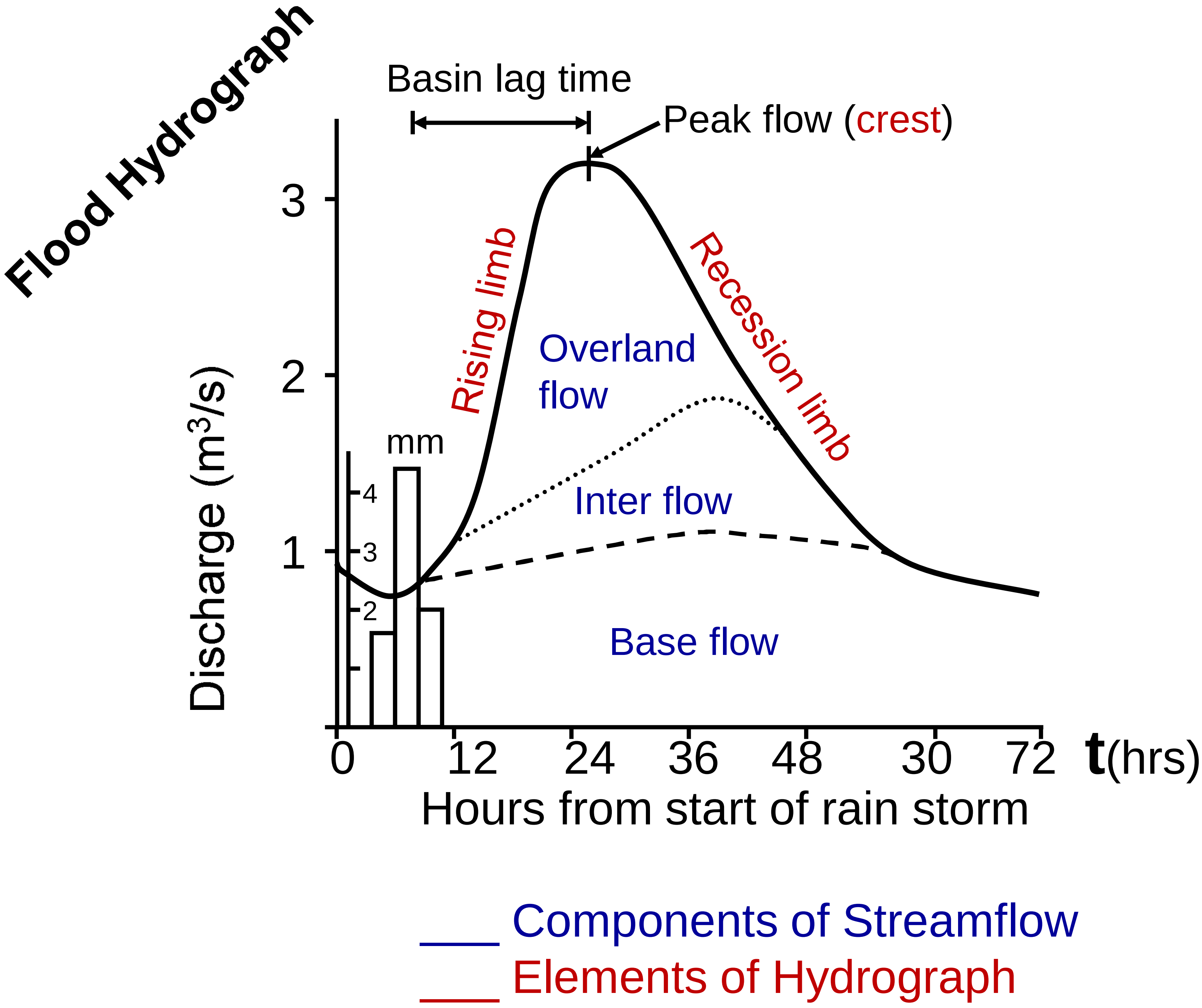
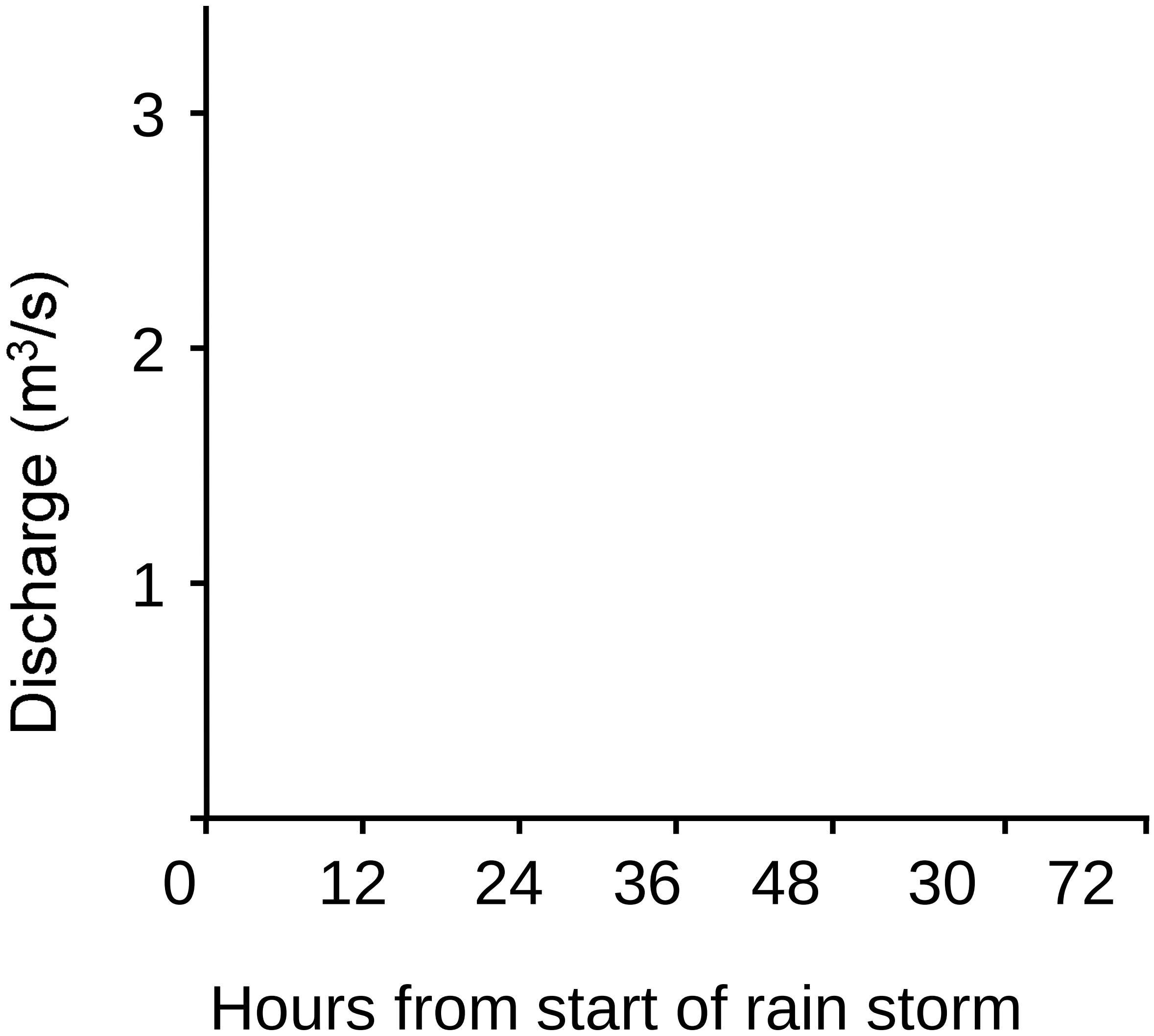
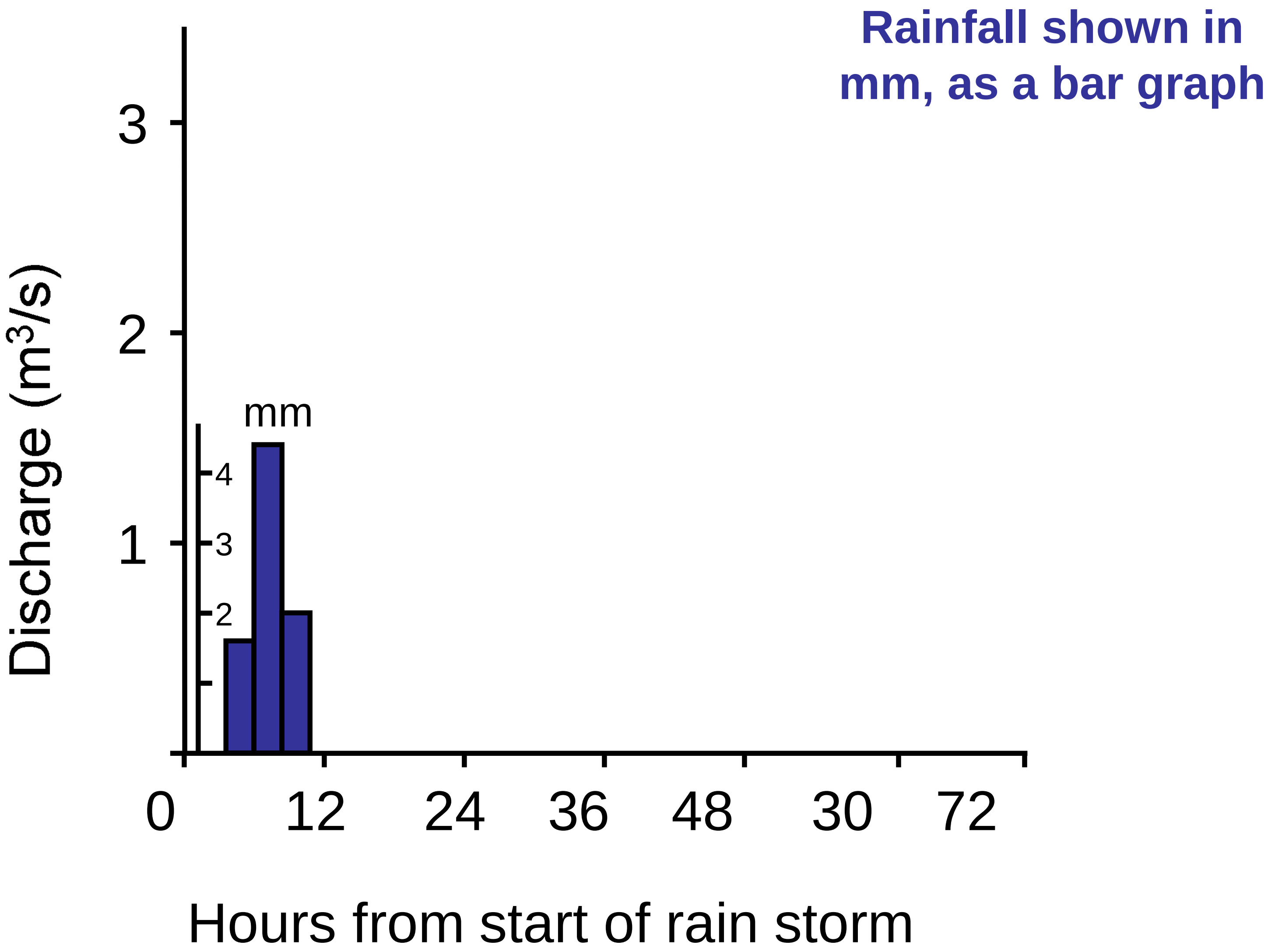
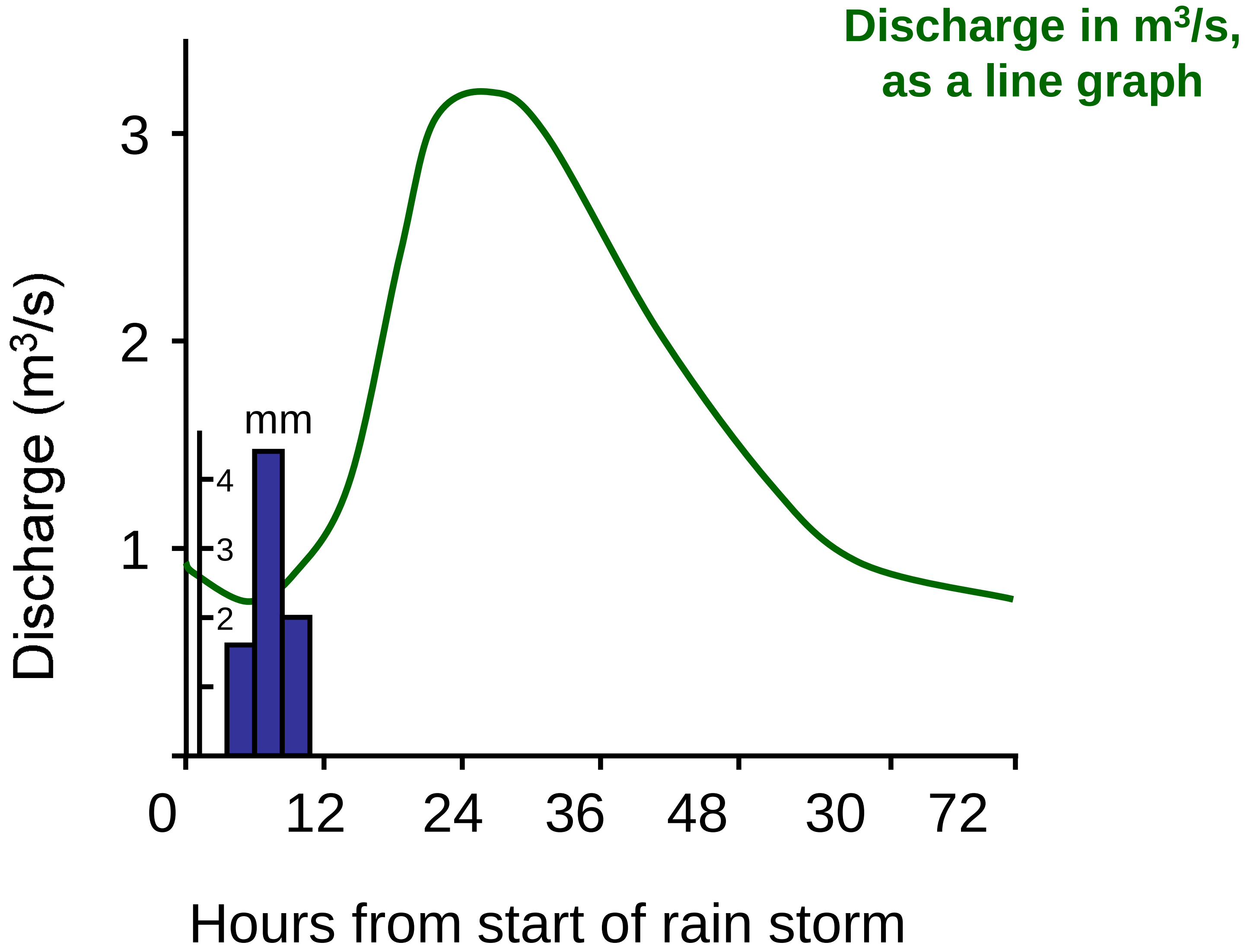


Flood Hydrograph
Basin lag time
Peak flow (crest)
Rising limb
Recession limb
Overland flow
Inter flow
Base flow
mm
4
3
2
3
2
1
Discharge (m3/s)
0
12
24
36
48
30
72
t(hrs)
Hours from start of rain storm
___ Components of Streamflow
___ Elements of Hydrograph

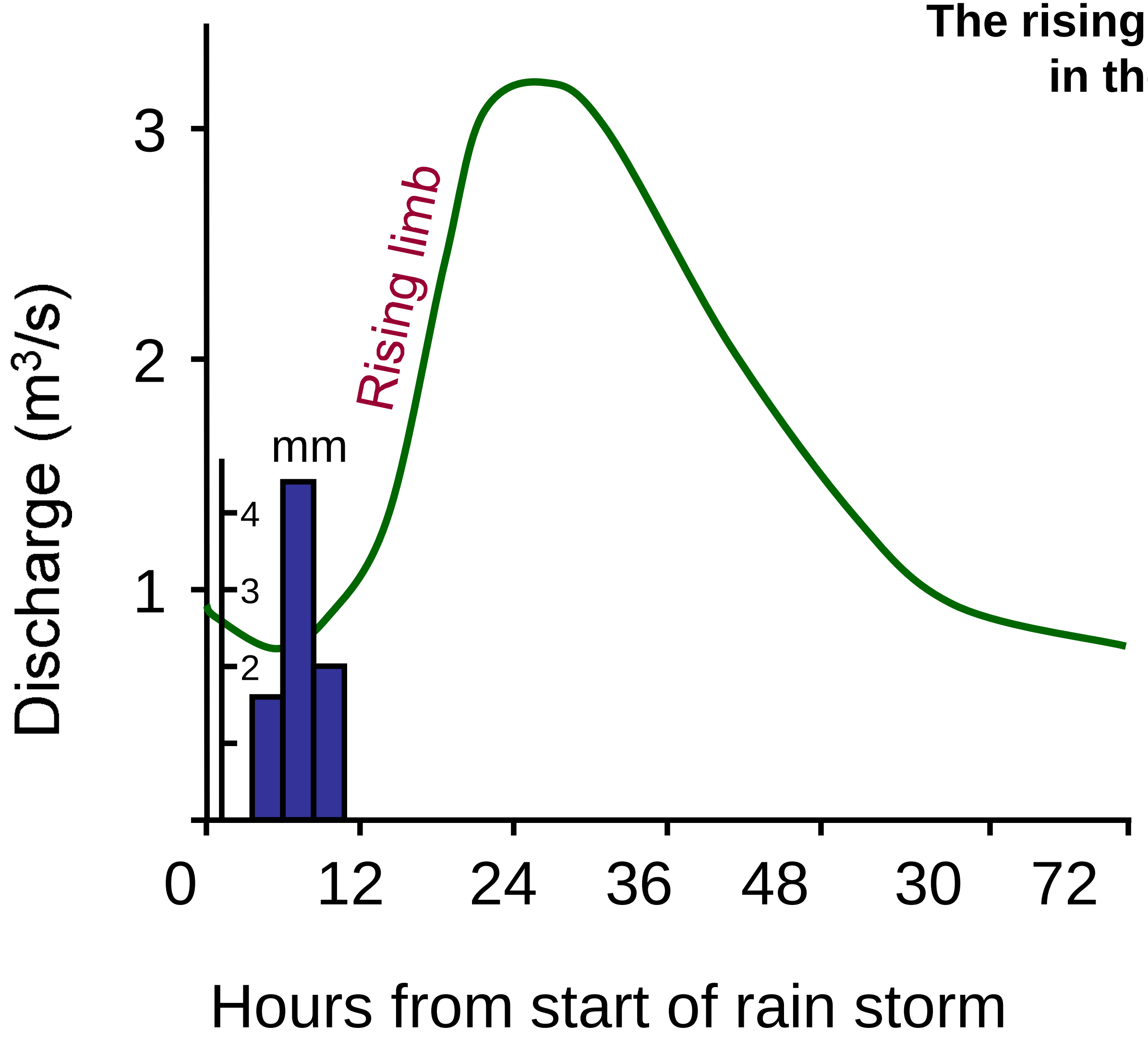

Discharge (m3/s)
3
2
1
0
12
24
36
48
30
72
Hours from start of rain storm


Rainfall shown in
mm, as a bar graph
Discharge (m^3/s)
3
2
1
mm
4
3
2
0
12
24
36
48
30
72
Hours from start of rain storm

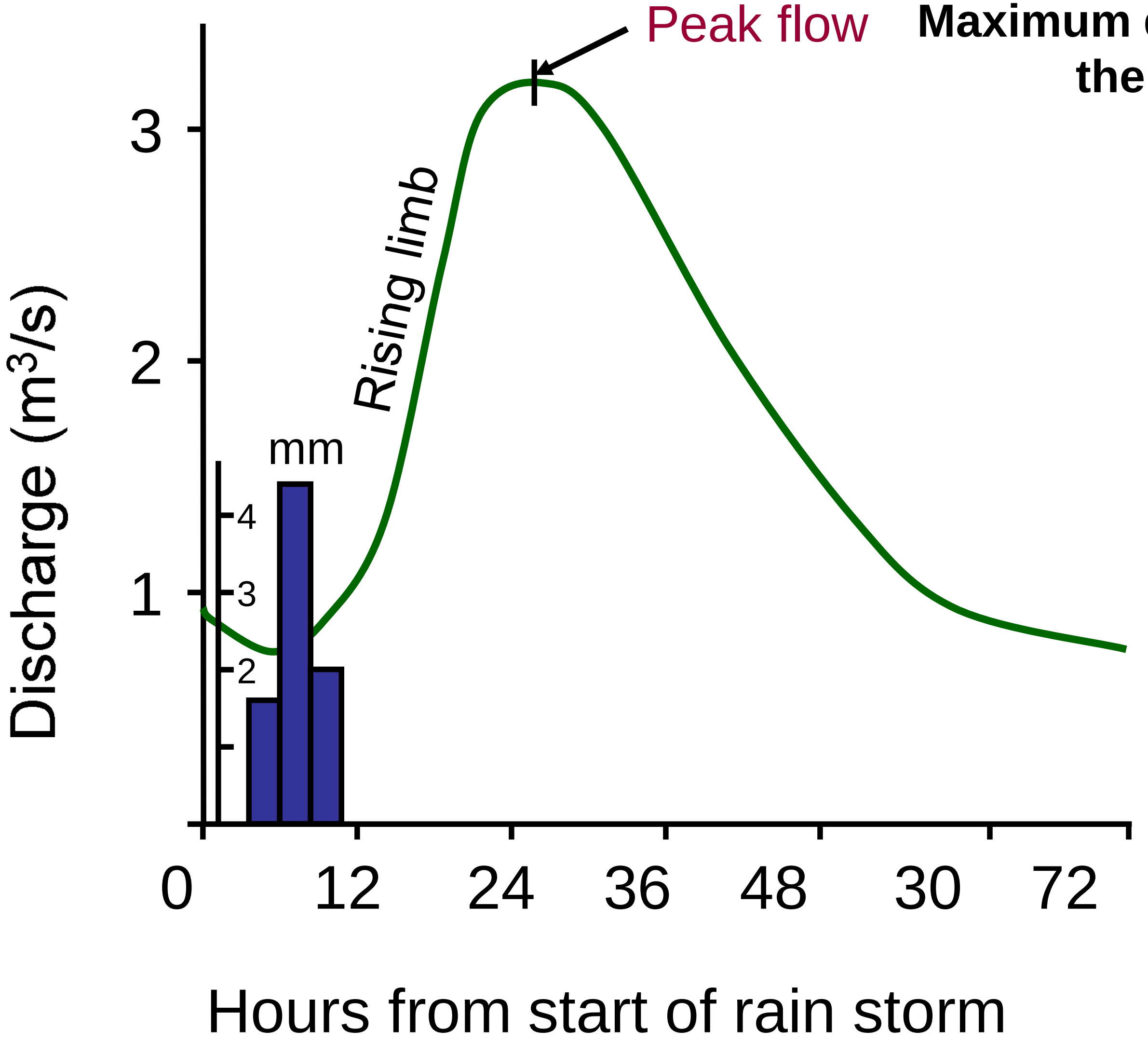

Discharge in m³/s,
as a line graph
Discharge (m³/s)
3
2
1
mm
4
3
2
0
12
24
36
48
30
72
Hours from start of rain storm

Rising limb

The rising flood water in the river



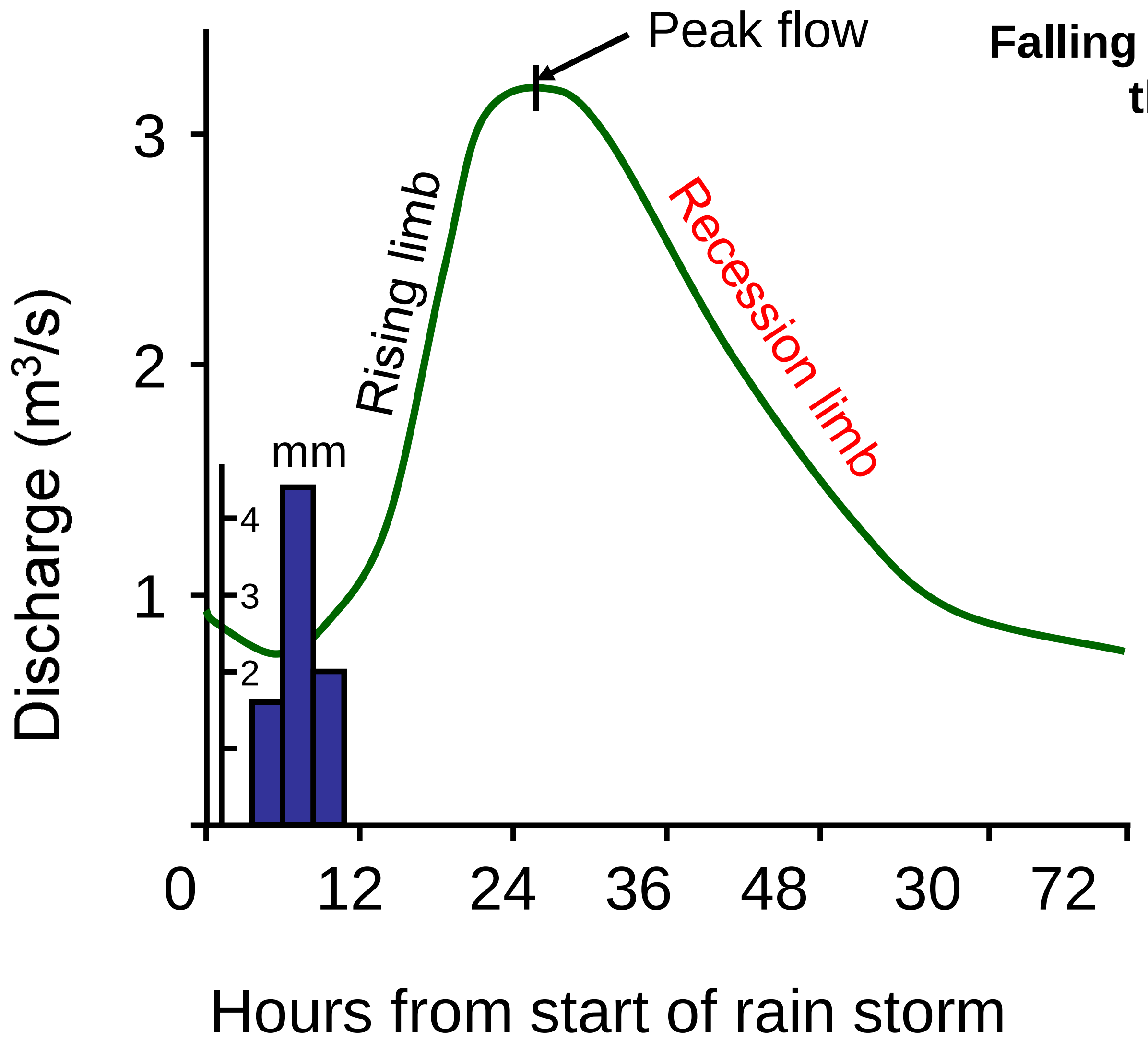
Peak flow



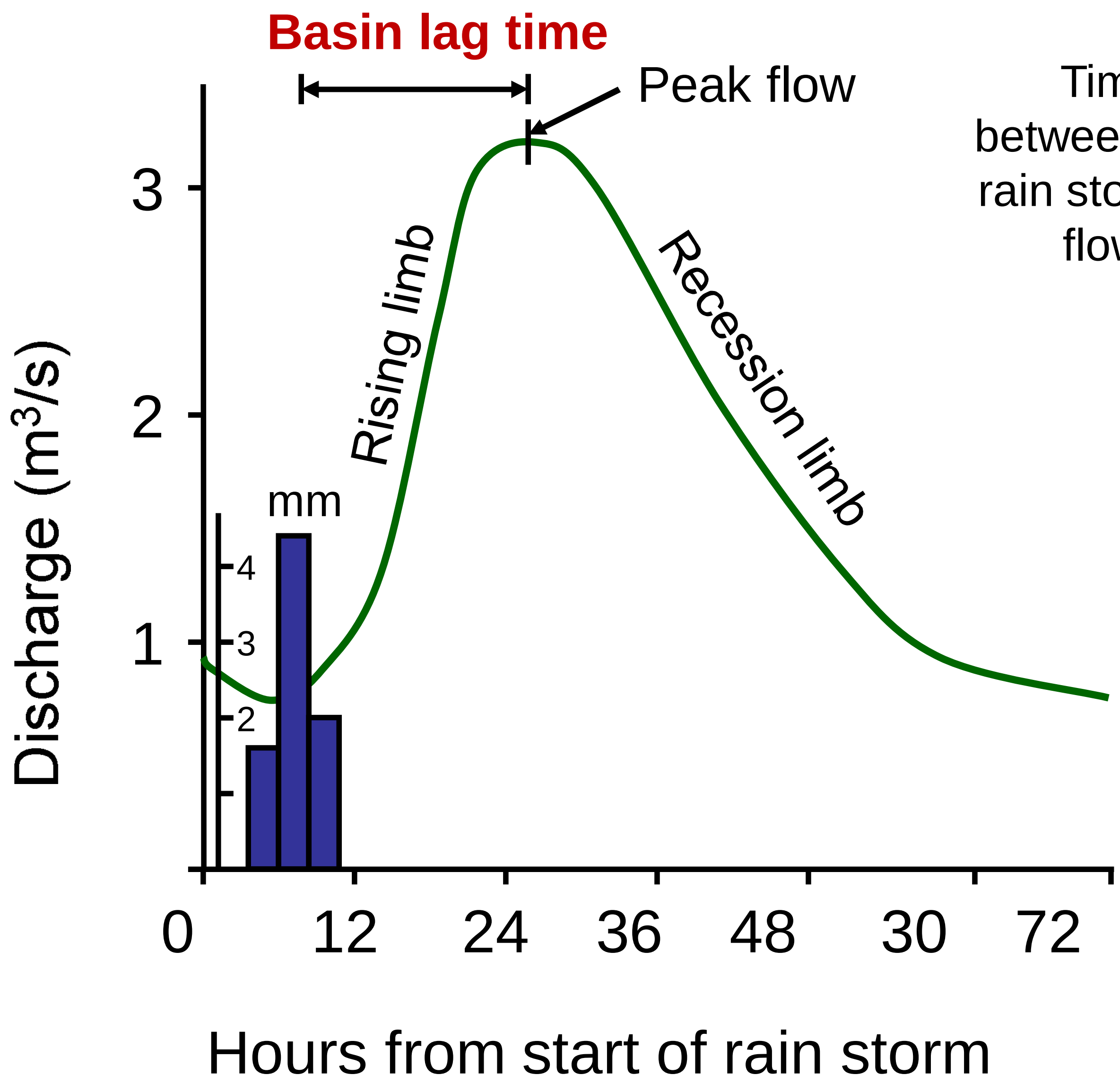
Maximum discharge in the river

Recession limb

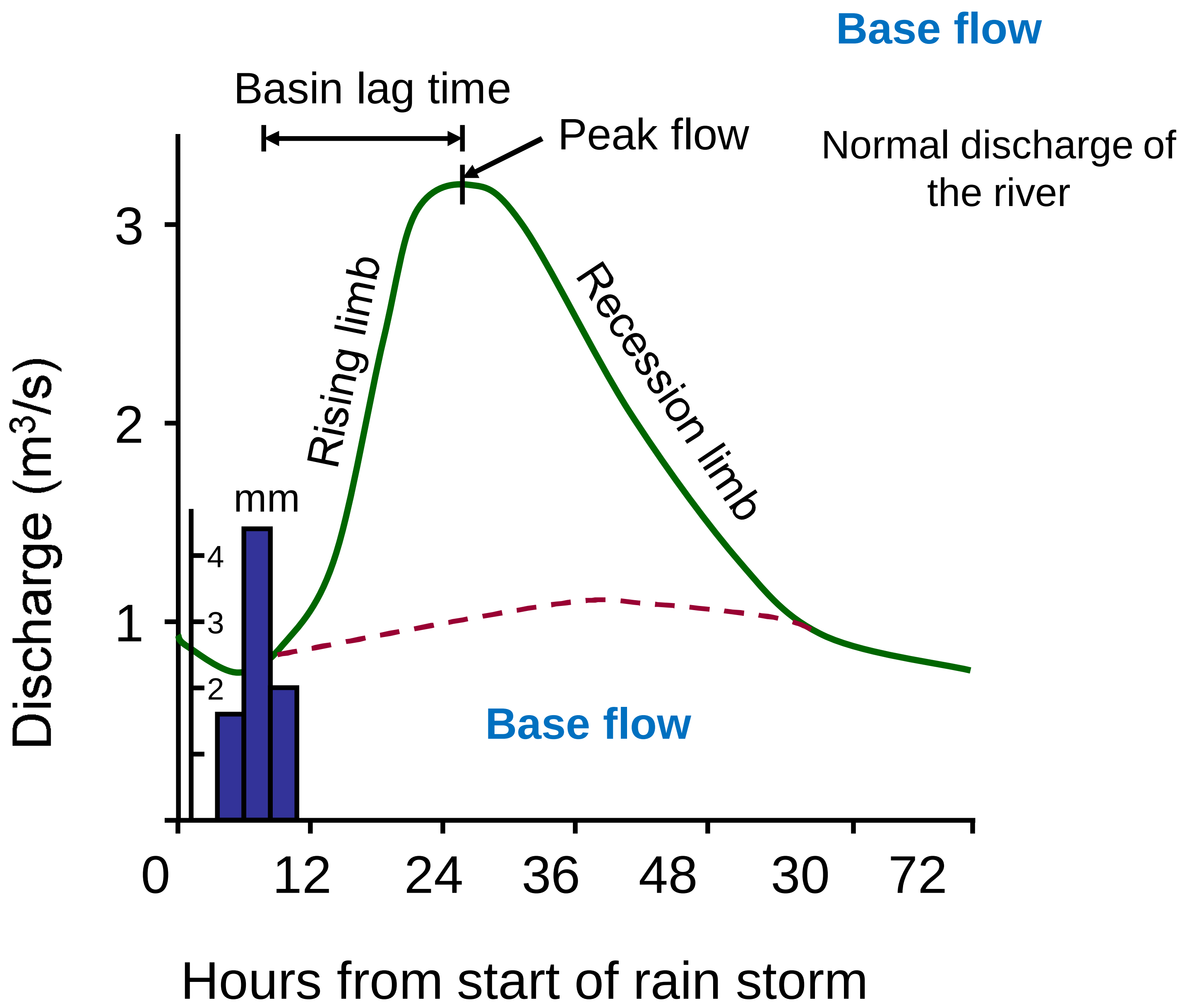
Falling flood water in the river

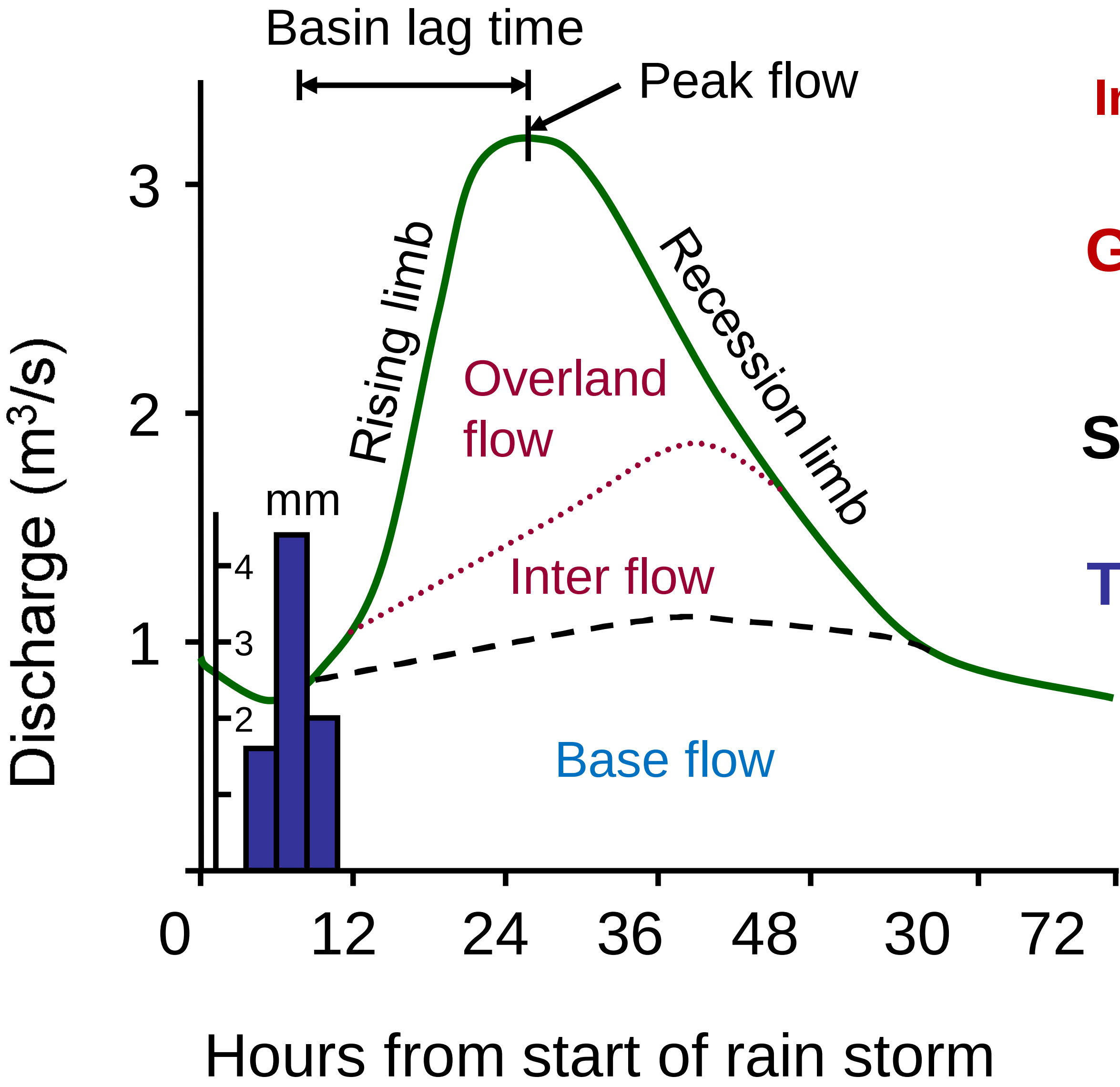


Basin lag time



Time difference between the peak of the rain storm and the peak flow of the river


Base flow
Basin lag time
Peak flow
Normal discharge of the river
Rising limb
Recession limb
mm
4
3
2
3
2
1
Discharge (m3/s)
Base flow
0
12
24
36
48
30
72
Hours from start of rain storm



Overland flow
+
Inter flow
+
GW (BF)
=
Stream Flow
Or
Total Runoff

Overland flow

Volume of water reaching the river from surface run off

Inter flow

Volume of water reaching the river through the soil and underlying rock layers

GW

The Base flow

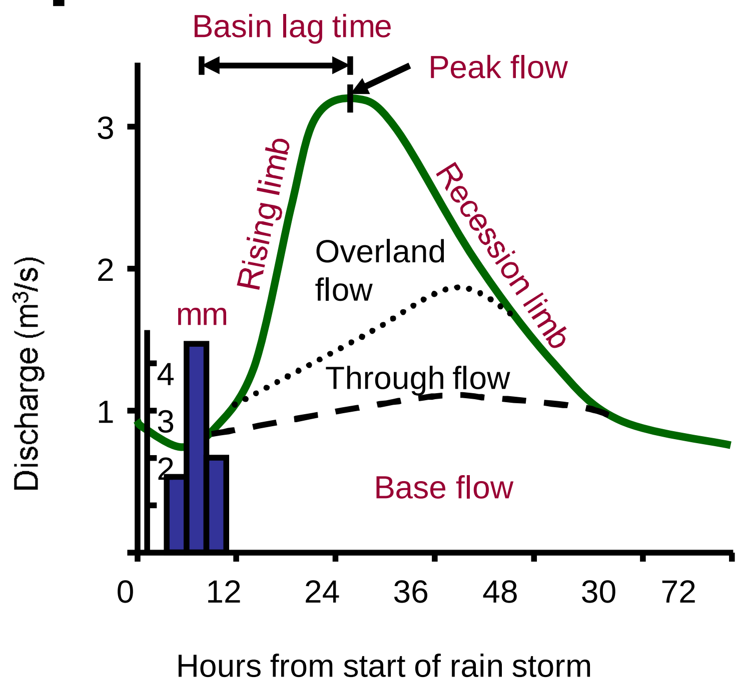
Factors influencing Storm Hydrographs

- Area
- Shape
- Slope
- Rock Type
- Soil
- Land Use
- Drainage Density
- Precipitation / Temp
- Tidal Conditions

Interpretation of Storm Hydrographs

You need to refer to:

- Rising Limb
- Recession Limb
- Lag time
- Rainfall Intensity



- Peak flow compared to Base flow
- Recovery rate, back to Base flow

Area

- Large basins receive more precipitation than small therefore have larger runoff

- Larger size means longer lag time as water has a longer distance to travel to reach the trunk river

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Shape

- Elongated basin will produce a lower peak flow and longer lag time than a circular one of the same size

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Slope

- Channel flow can be faster down a steep slope therefore steeper rising limb and shorter lag time

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Rock Type

- Permeable rocks mean rapid infiltration and little overland flow therefore shallow rising limb

Area	**Rock Type**	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Soil

- Infiltration is generally greater on thick soil, although less porous soils eg. clay act as impermeable layers

- The more infiltration occurs the longer the lag time and shallower the rising limb

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Land Use

- Urbanisation - concrete and tarmac form impermeable surfaces, creating a steep rising limb and shortening the time lag

- Afforestation - intercepts the precipitation, creating a shallow rising limb and lengthening the time lag

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Drainage Density

- A higher density will allow rapid overland flow

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Precipitation & Temperature

- Short intense rainstorms can produce rapid overland flow and steep rising limb

- If there have been extreme temperatures, the ground can be hard (either baked or frozen) causing rapid surface run off

- Snow on the ground can act as a store producing a long lag time and shallow rising limb. Once a thaw sets in the rising limb will become steep

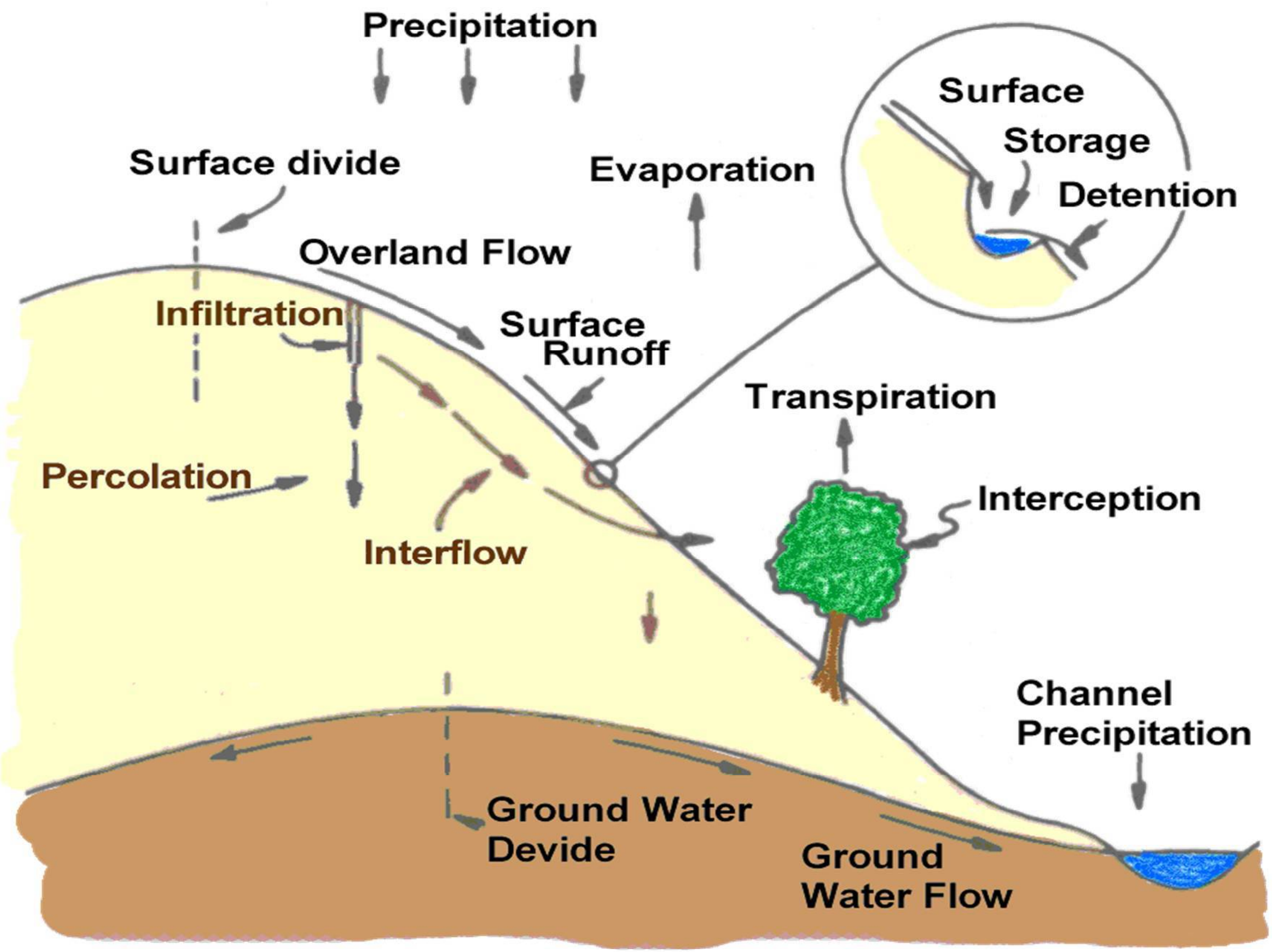
Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Tidal Conditions

- High spring tides can block the normal exit for the water, therefore extending the length of time the river basin takes to return to base flow

Area	Rock Type	Drainage Density
Shape	Soil	Precipitation / Temp
Slope	Land Use	Tidal Conditions

Hydrograph Analysis

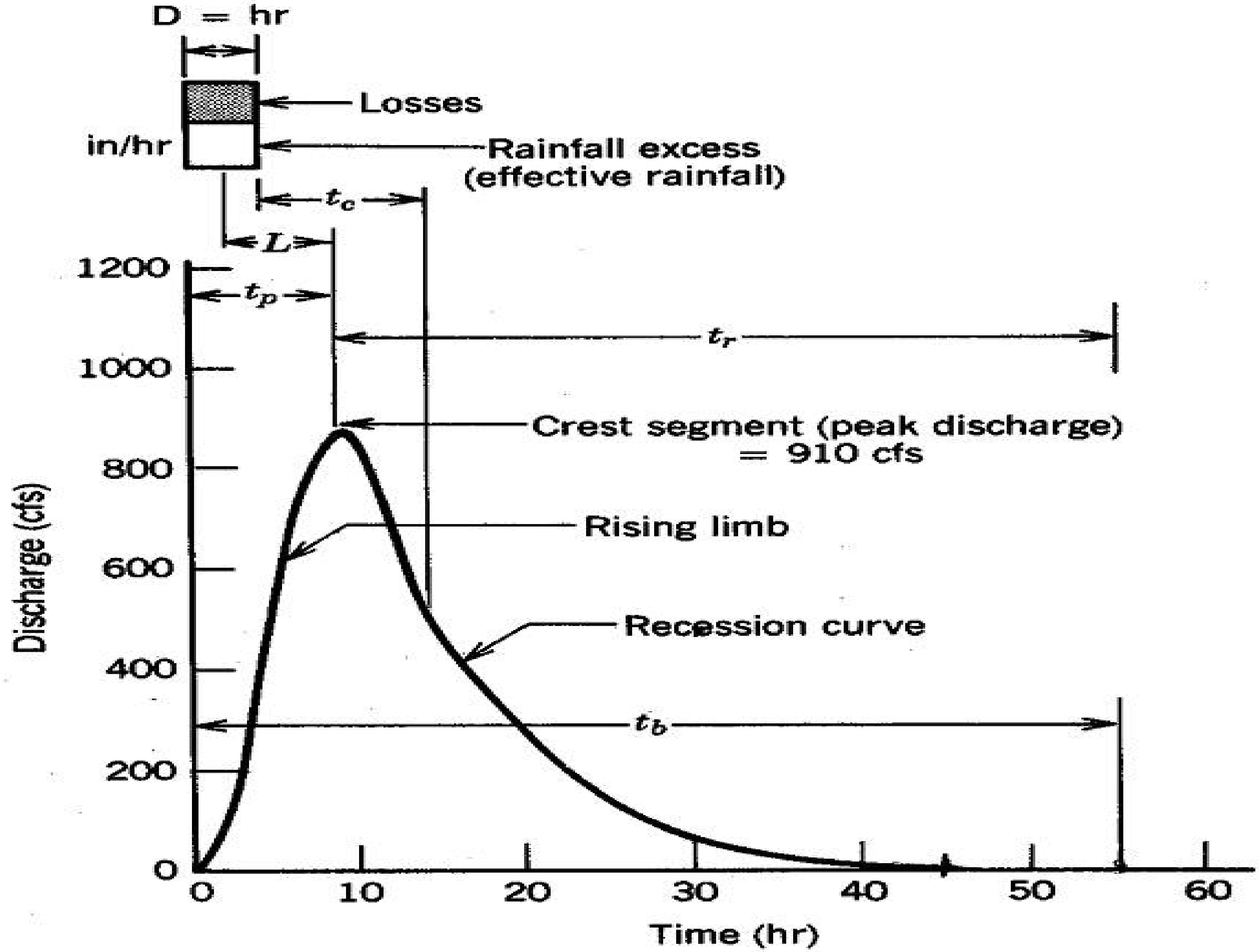


Hydrograph Analysis :

Hydrograph : Q vs t

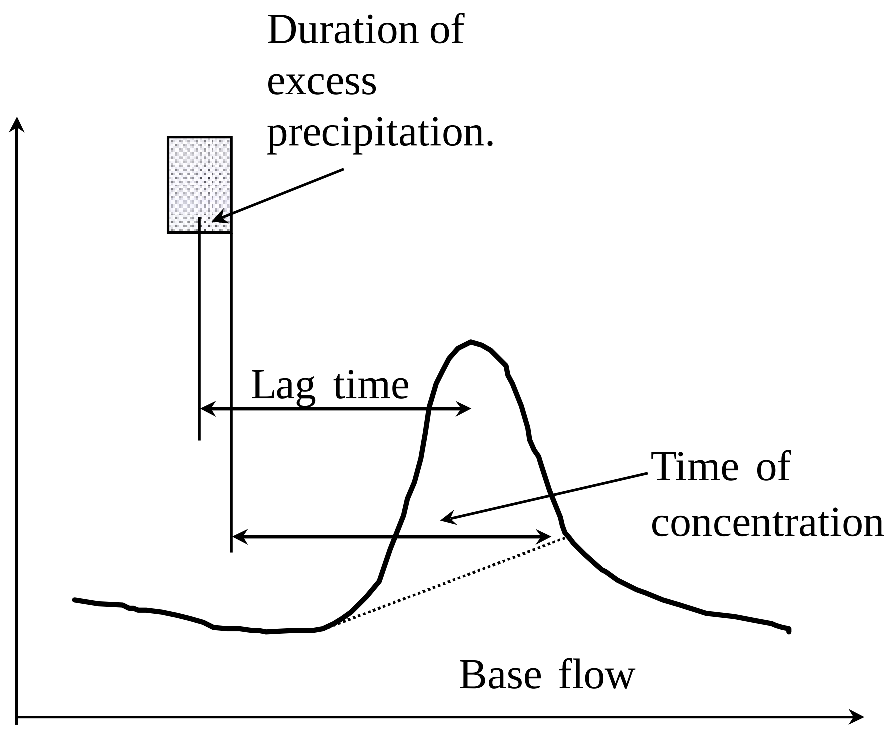
- **Duration , t**
- **Lag Time , t_L**
- **Time of Concentration , tc**
- **Rising Limb**
- **Recession Limb (falling limb)**
- **Peak Flow ,Q_p**
- **Time to Peak (rise time),t_p**
- **Recession Curve**
- **Base flow , BF**
- **Separation of BF from Runoff**

Hydrograph Components



Taken from Wanielista, M., R. Kersten, and R. Eaglin, *Hydrology: Water Quantity and Quality Control,* p. 184

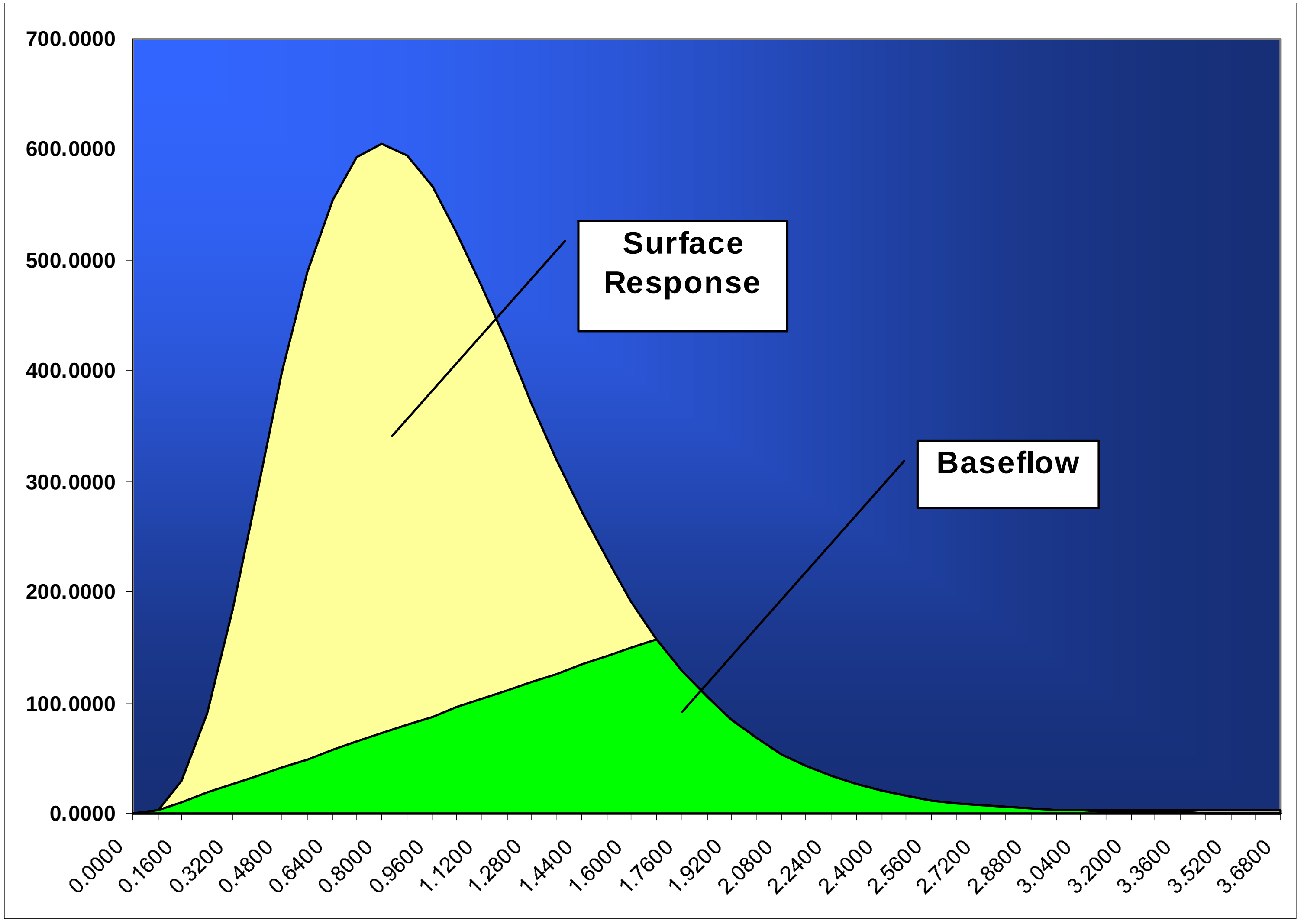
Graphical Representation



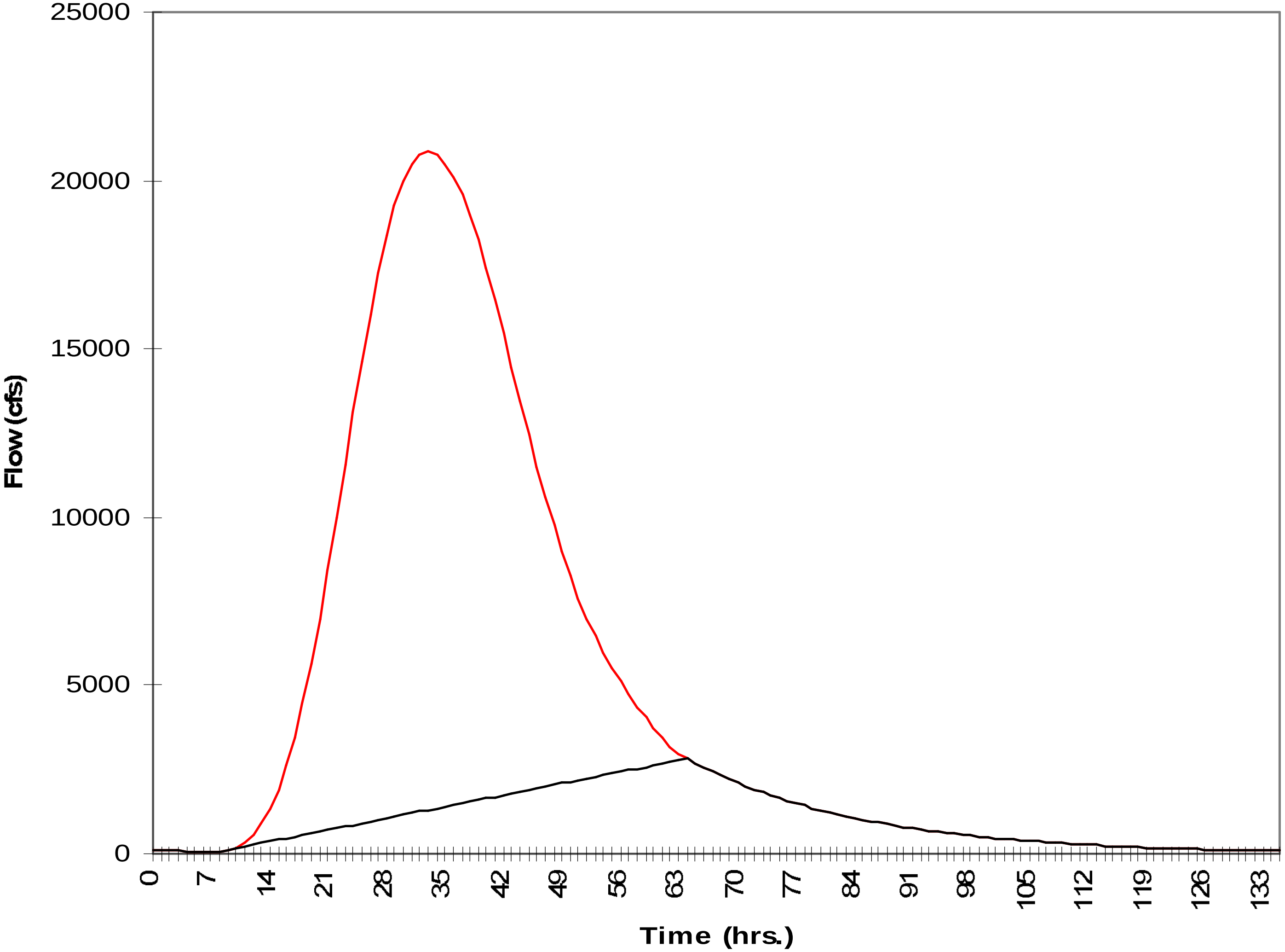
Separation of Baseflow

- ✓... generally accepted that the inflection point on the recession limb of a hydrograph is the result of a change in the controlling physical processes of the excess precipitation flowing to the basin outlet.
- ✓In this example, base flow is considered to be a straight line connecting that point at which the hydrograph begins to rise rapidly and the inflection point on the recession side of the hydrograph.
- ✓the inflection point may be found by plotting the hydrograph in semi-log fashion with flow being plotted on the log scale and noting the time at which the recession side fits a straight line.

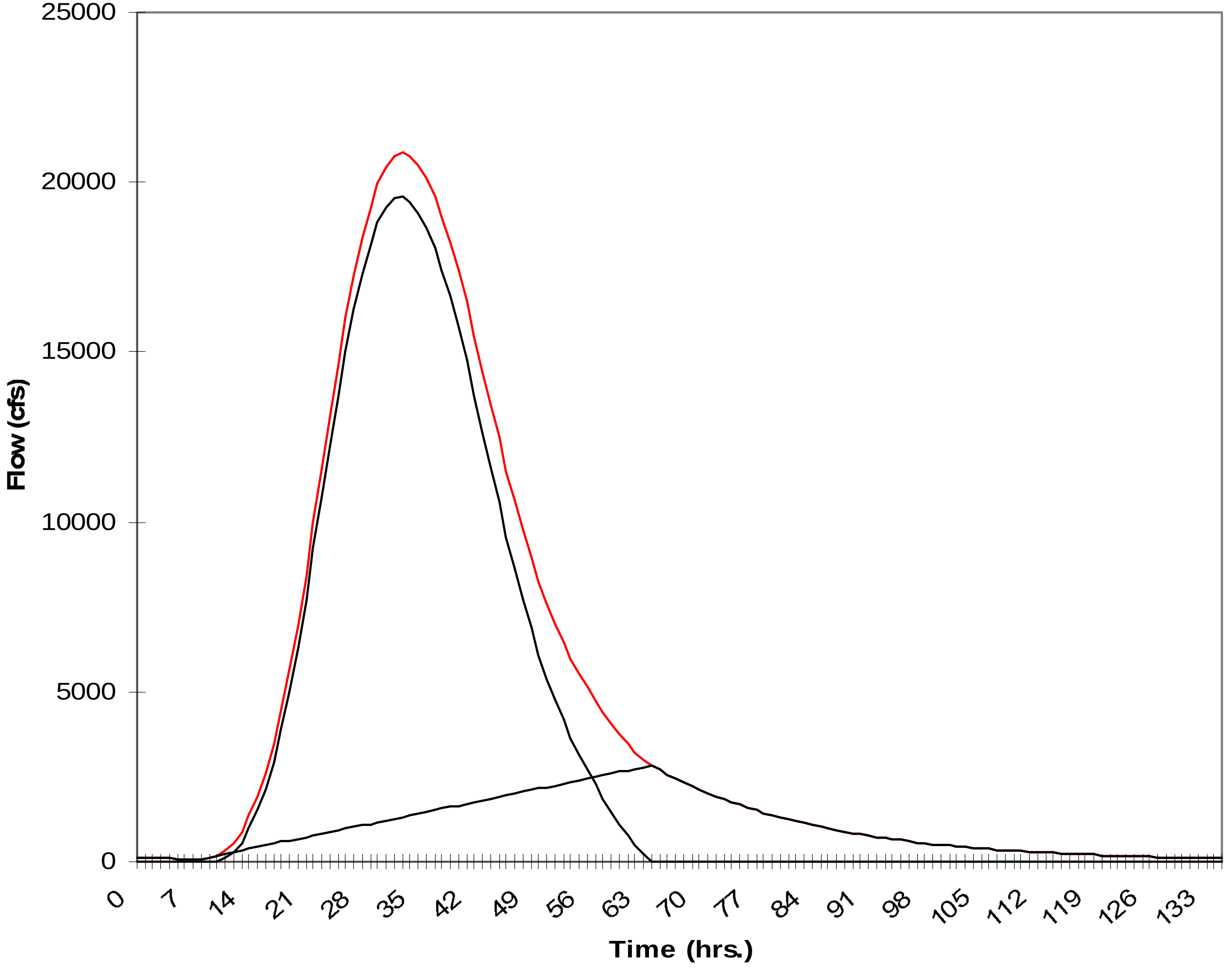
Separation of Baseflow



Hydrograph & Baseflow



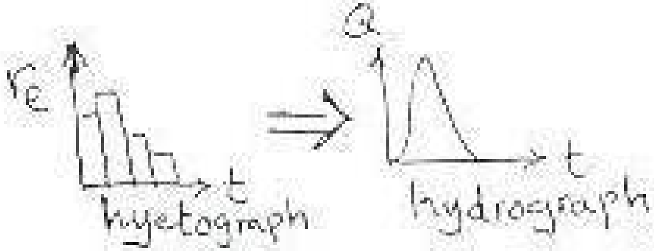
Separate Baseflow



$$Q = d * UH$$

Runoff = rainfall depth * Unit Hydrograph

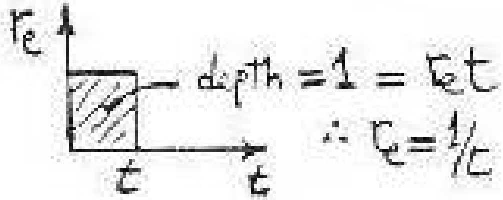
Unit Hydrograph(UH) Theory:



UH Theory

- Developed by Sherman (1932) to determine the direct runoff hydrograph from the effective rainfall r_e hyetograph.

- UH is defined as the direct runoff resulting from a storm having one unit (1 in. or 1 cm) of effective rain depth occurring uniformly over the watershed at a uniform rate during a specified period of time. The specified time could be any finite duration; i.e. 1-hr UH, 6-hr UH, 1-day UH).



Basic Assumptions

- The Basic assumptions are:

  - Constant intensity of r_e within its duration.
  - Uniformly distributed r_e over the watershed.
  - Constant basin characteristics with time (No topography change)
  - It involves the rules of linearity, proportionality and superpositioning.

Unit Hydrograph

- ***The hydrograph (direct runoff)resulting from 1-inch (or 1cm) of excess precipitation spread uniformly in space and time over a watershed for a given duration.***

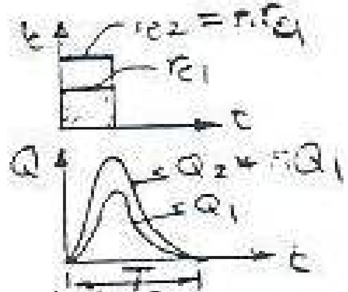
- **The key points :**

  ✓**1-inch (1cm) of EXCESS precipitation**
  Spread uniformly over space - evenly over the watershed

  ✓**Uniformly in time - the excess rate is constant over the time interval**

  ✓**There is a given duration. t-UH (ex. 2hr-UH)**

Propositions of the UH

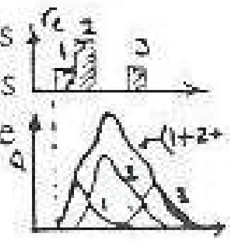


The following are the basic propositions of the unit hydrograph:

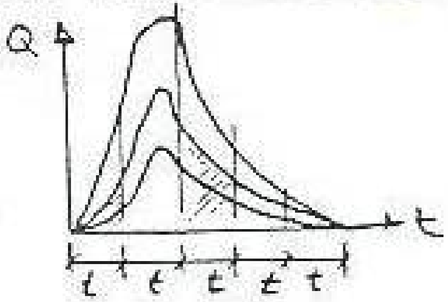
(i) Same runoff time base - For all (unit) storms of different intensities, the period of surface runoff (i.e. time base, base width or base period) is approximately the same, although they produce different runoff volumes.

(ii) Proportional ordinates - For (unit) storms of different intensities, the ordinates of the hydrograph at any given time, are in the same proportion as the rainfall intensities.

(iii) Principle of superposition - If there is a continuous storm and/or isolated storms of uniform intensity net rain, they may be divided into unit storms and hydrographs of runoff for each storm obtained and the ordinates added with the appropriate time lag to get the combined hydrograph.



(iv) Same distribution percentages - If the total period of surface runoff (i.e. time base or base width) is divided into equal time intervals the percentage of surface runoff that occurs during each of these periods will be same for all unit storms of different intensities.



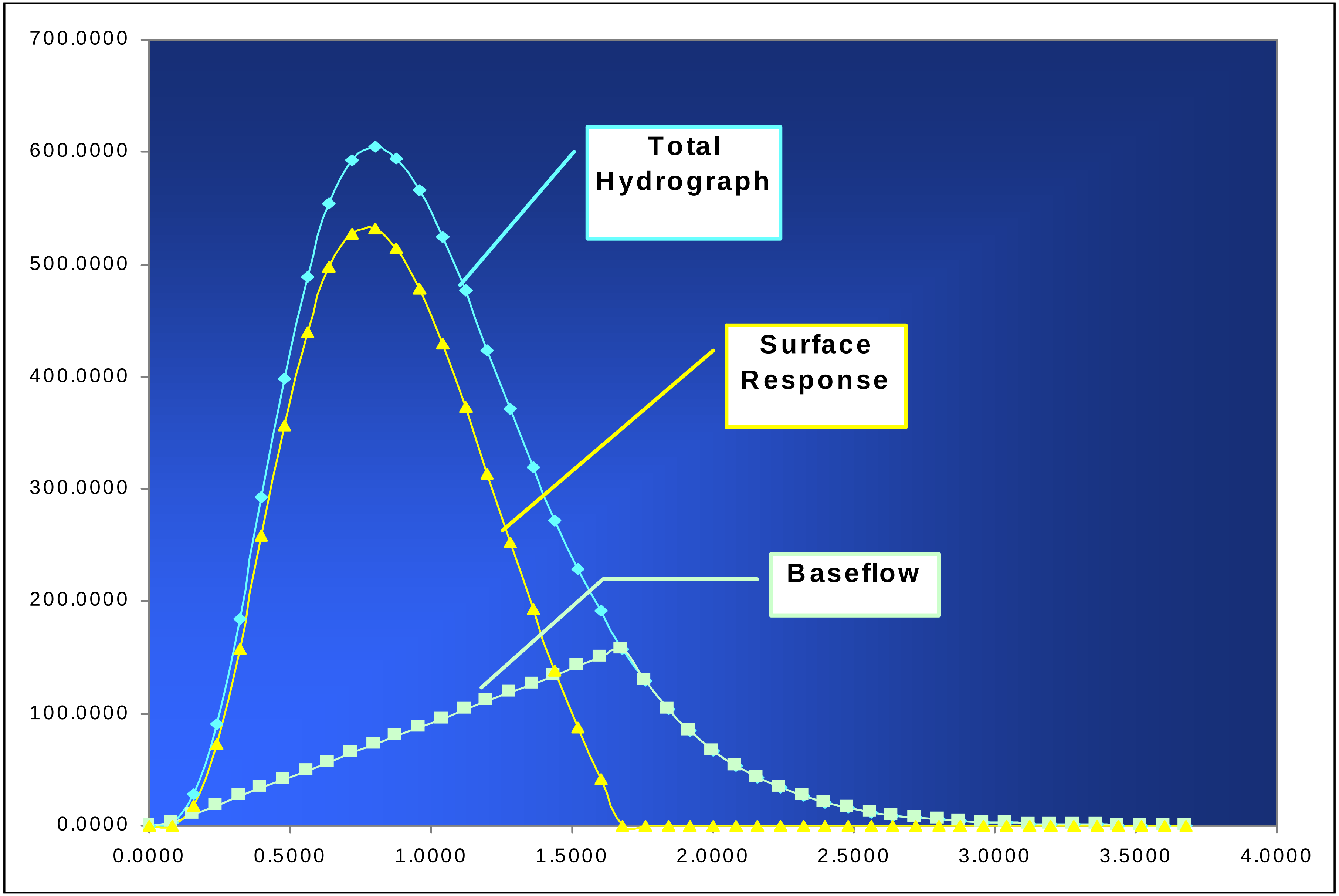
Methods of Developing UH's

- From Streamflow Data
- Synthetically
  - Snyder
  - SCS
  - Time-Area (Clark, 1945)
- "Fitted" Distributions
- Geomorphologic

Unit Hydrograph Derivation

- **i) Tabulate the total hydrograph with time distribution.**
- **ii) Tabulate the baseflow if given or separate with method of our choice.**
- **iii) Find the Direct Runoff Hydrograph(DRH) by subtracting the baseflow from the total hydrograph.(DRH = Q – BF)**
- **iv) Find the volume of water under the DRH**
- $$\text{Vol.} = \Sigma Q * \Delta t$$

Unit Hydrograph Derivation



Unit Hydrograph Derivation

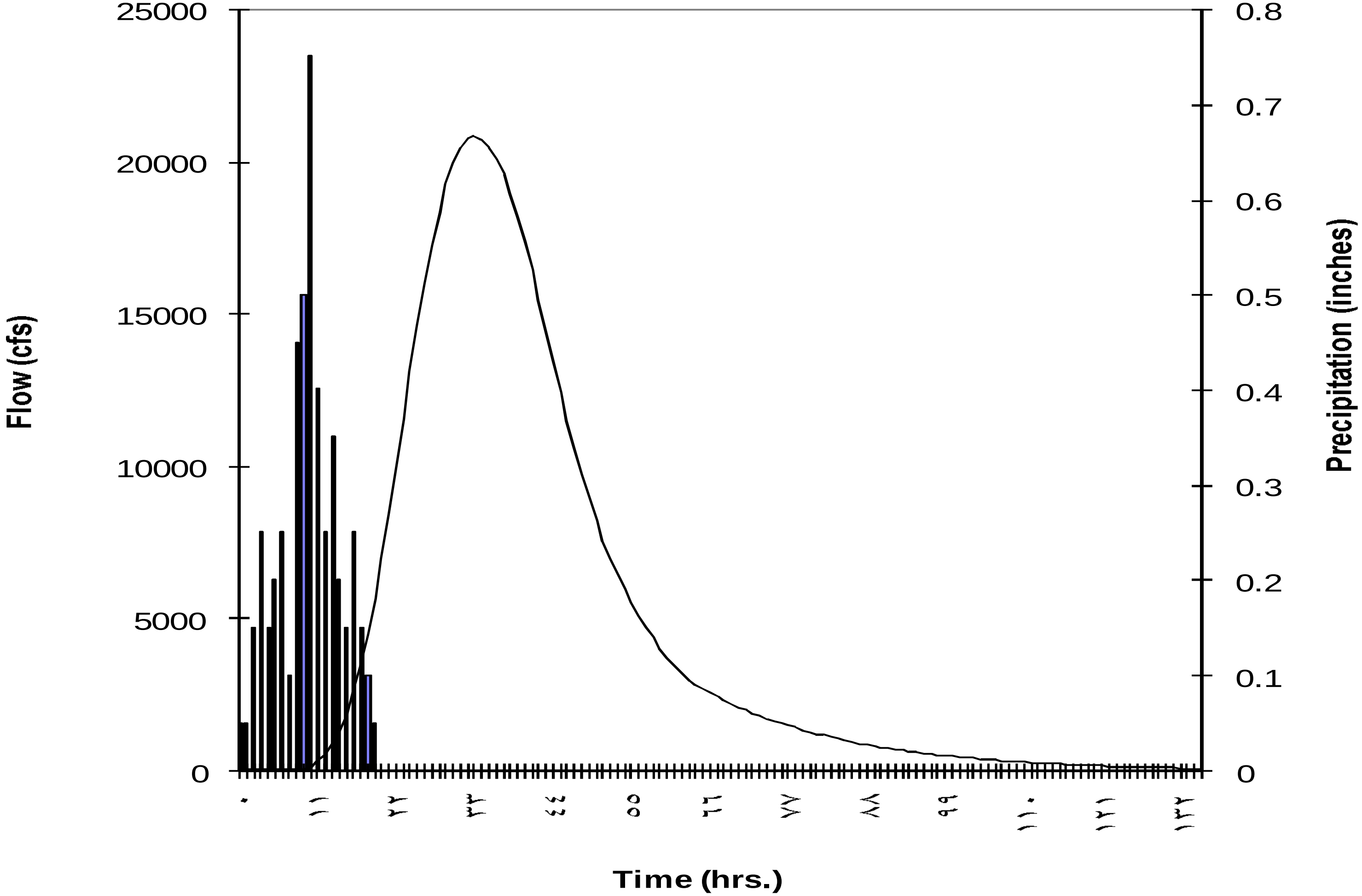
- **v) Divide the volume of water(step iv) by the drainage area(A) to get effective rainfall(d_e) (runoff) per unit area.**

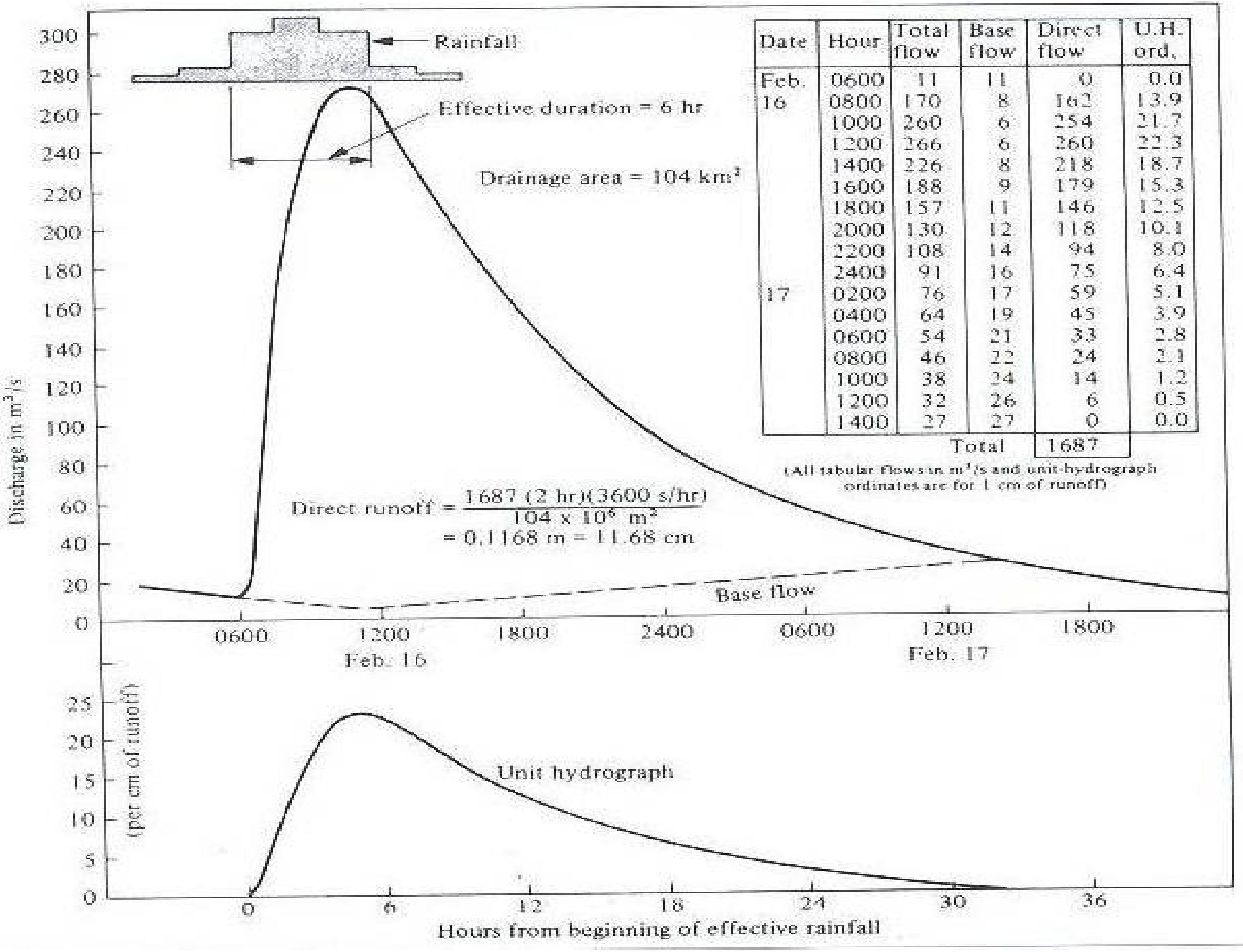
$$d_e = vol. / A$$

- **vi) Divide the ordinates of the DRH by the d_e of effective rainfall(step v).**

- **The result is a unit hydrograph(UH) for the duration of storm.**

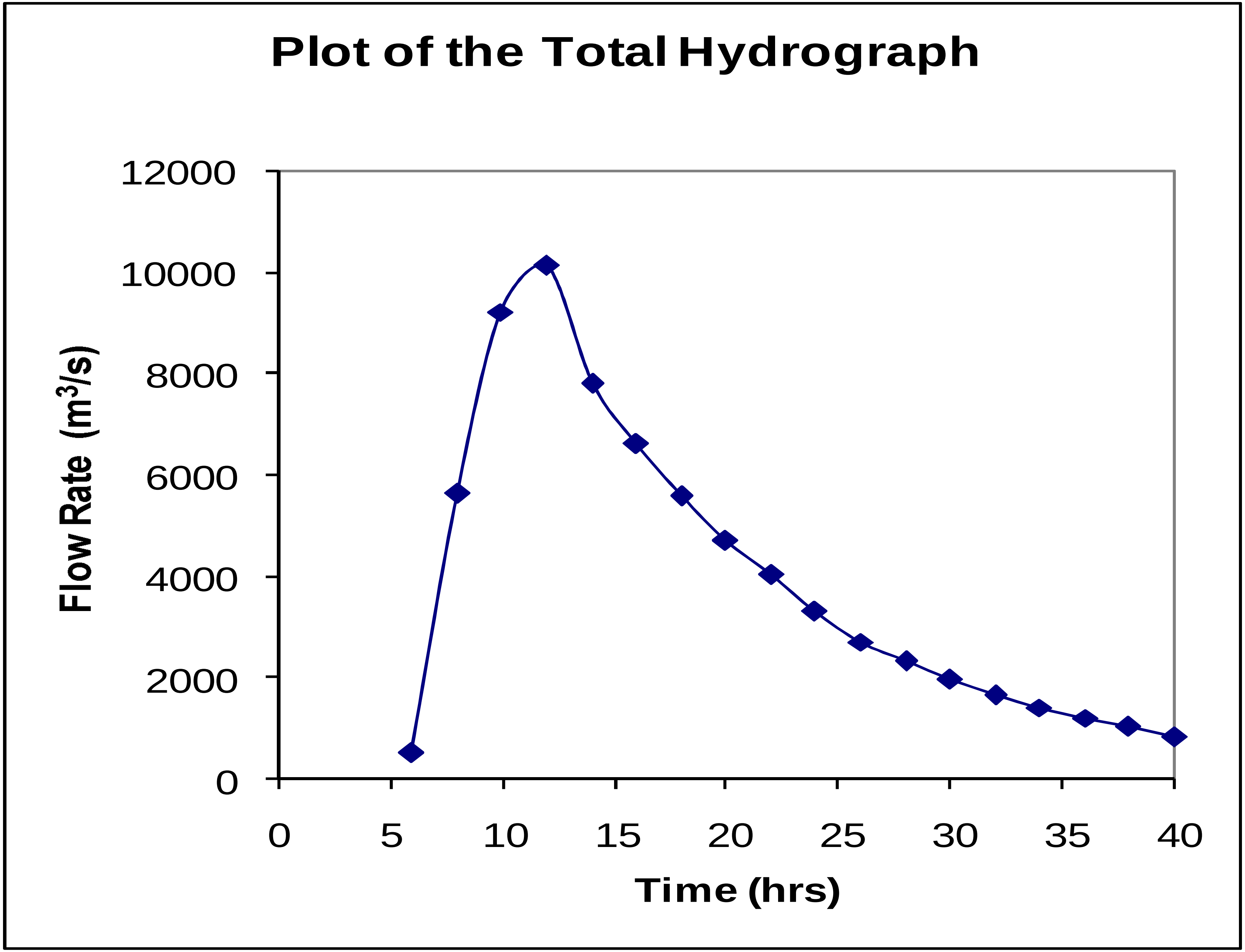
Unit Hydrograph Derivation





Date	Hour	Total flow	Base flow	Direct flow	U.H. ord.
Feb. 16	0600	11	11	0	0.0
	0800	170	8	162	13.9
	1000	260	6	254	21.7
	1200	266	6	260	22.3
	1400	226	8	218	18.7
	1600	188	9	179	15.3
	1800	157	11	146	12.5
	2000	130	12	118	10.1
	2200	108	14	94	8.0
	2400	91	16	75	6.4
17	0200	76	17	59	5.1
	0400	64	19	45	3.9
	0600	54	21	33	2.8
	0800	46	22	24	2.1
	1000	38	24	14	1.2
	1200	32	26	6	0.5
	1400	27	27	0	0.0
			Total	1687	

(All tabular flows in m³/s and unit-hydrograph ordinates are for 1 cm of runoff)



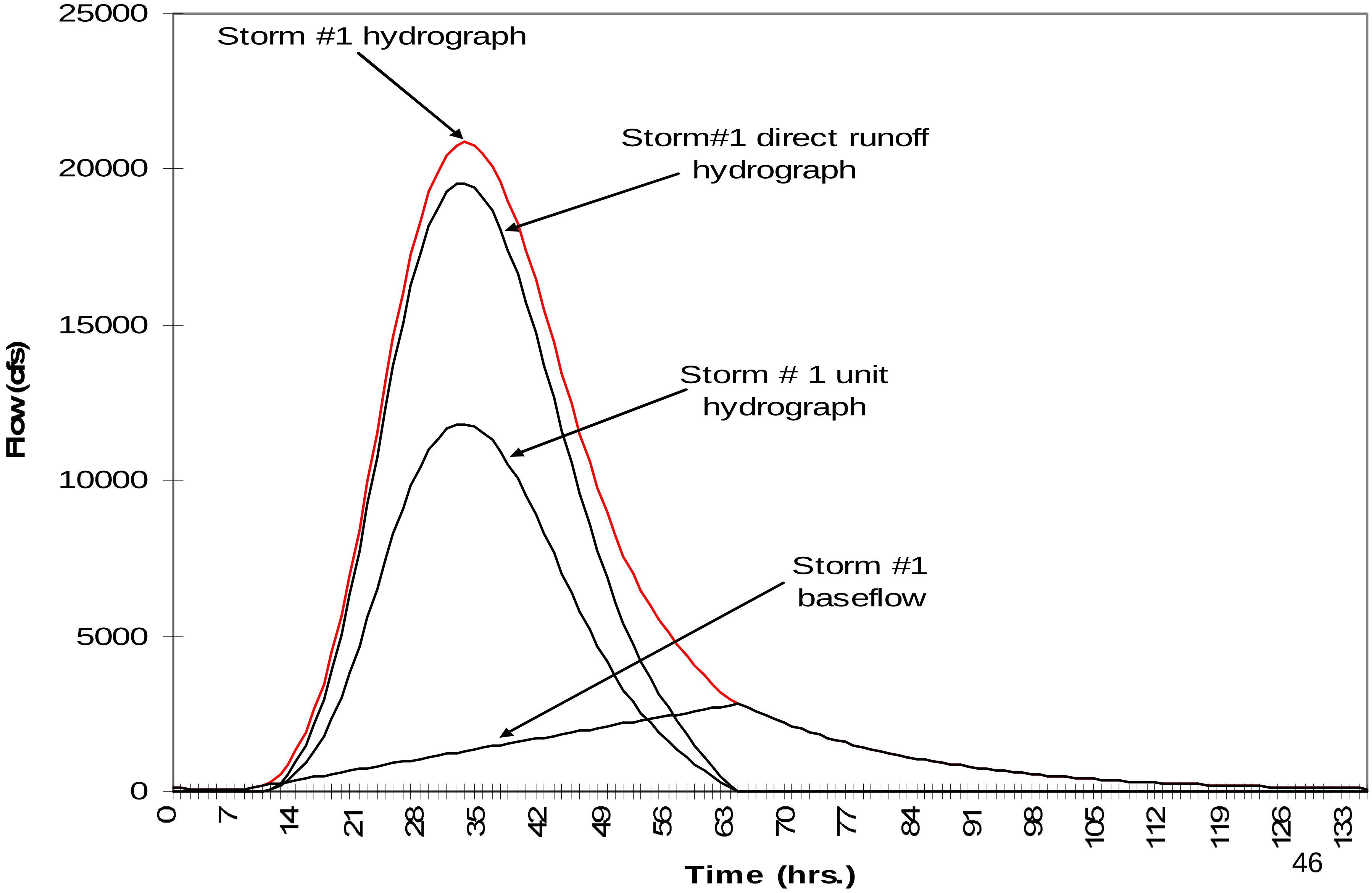
Plot of the Total Hydrograph
Flow Rate (m3/s)
12000
10000
8000
6000
4000
2000
0
0
5
10
15
20
25
30
35
40
Time (hrs)

Obtain UH Ordinates

- **The ordinates of the unit hydrograph are obtained by dividing each flow in the direct runoff hydrograph by the depth of excess precipitation.**

- **In this example, the units of the unit hydrograph would be cfs/inch (of excess precipitation).**

Final UH



25000
20000
15000
10000
5000
0
Flow (cfs)
Storm #1 hydrograph
Storm#1 direct runoff hydrograph
Storm # 1 unit hydrograph
Storm #1 baseflow
0
7
14
21
28
35
42
49
56
63
70
77
84
91
98
105
112
119
126
133
Time (hrs.)

Determine Duration of UH

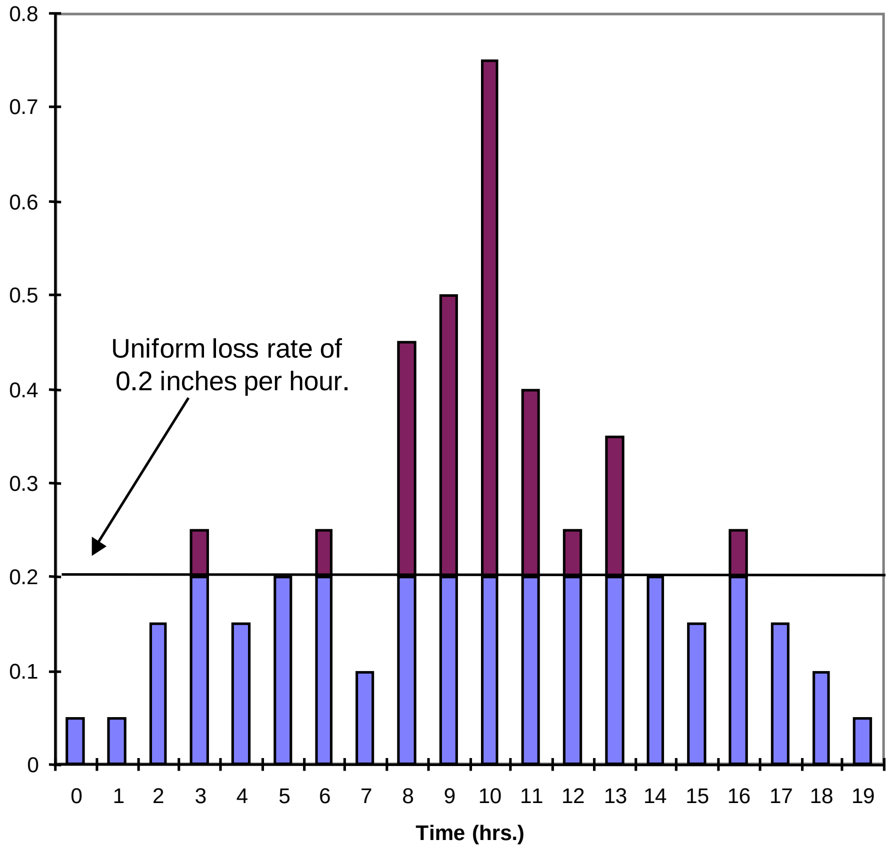
- The duration of the derived unit hydrograph is found by examining the precipitation for the event and determining that precipitation which is in excess.

- This is generally accomplished by plotting the precipitation in hyetograph form and drawing a horizontal line such that the precipitation above this line is equal to the depth of excess precipitation as previously determined.

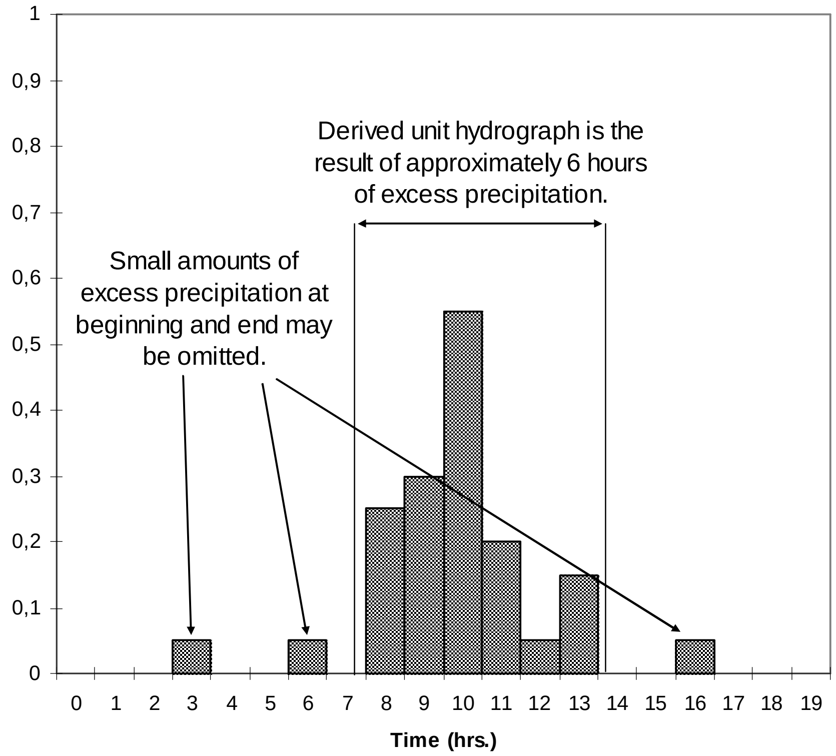
- This horizontal line is generally referred to as the **Φ-index** and is based on the assumption of a constant or uniform infiltration rate.

- The uniform infiltration necessary to cause 1.65 inches of excess precipitation was determined to be approximately 0.2 inches per hour.

Estimating Excess Precip.



Excess Precipitation



Changing the Duration

Once UH is obtained from storm of duration t_1 and excess rain of $(1/t_1)$ the UH of duration equal to any integral multiple (n) of t_1 can be obtained by using the principle of superposition. A simultaneous addition of ordinates using the principle of superposition will result in a runoff hydrograph having a duration of nt_1 and intensity of $1/t_1$. Dividing the ordinates of this hydrograph by the excess precip. depth ($n t_1 \, 1/t_1 = n$) will give rise to the $n t_1$ UH. (see figure (a) below) (1) Direct method

If UH is required for a duration other than an integral multiple of the given duration, the above method will not work. The solution can be obtained by using a

(2) S-curve hydrograph (SH).

S-curve hydrograph is the total hydrograph resulting from a series of continuous uniform-intensity storms of duration t_1 and intensity $1/t_1$. The method involves the following steps:

1. Generate the S-curve hydrograph by adding a series of UH of duration t_1 each lagged by t_1. ($i_e = 1/t_1$ and $t = t_1$).

2. Shift the S-curve in time by t_2 (new desired UH duration) and subtract the ordinates of the two S-curves. This results in hydrograph of intensity $1/t_1$ and duration t_2

3. Convert the hydrograph to UH by multiplying all the hydrograph ordinates by t_1 / t_2 ($i_e = 1/t_2$ and $t = t_2$) (see handout).

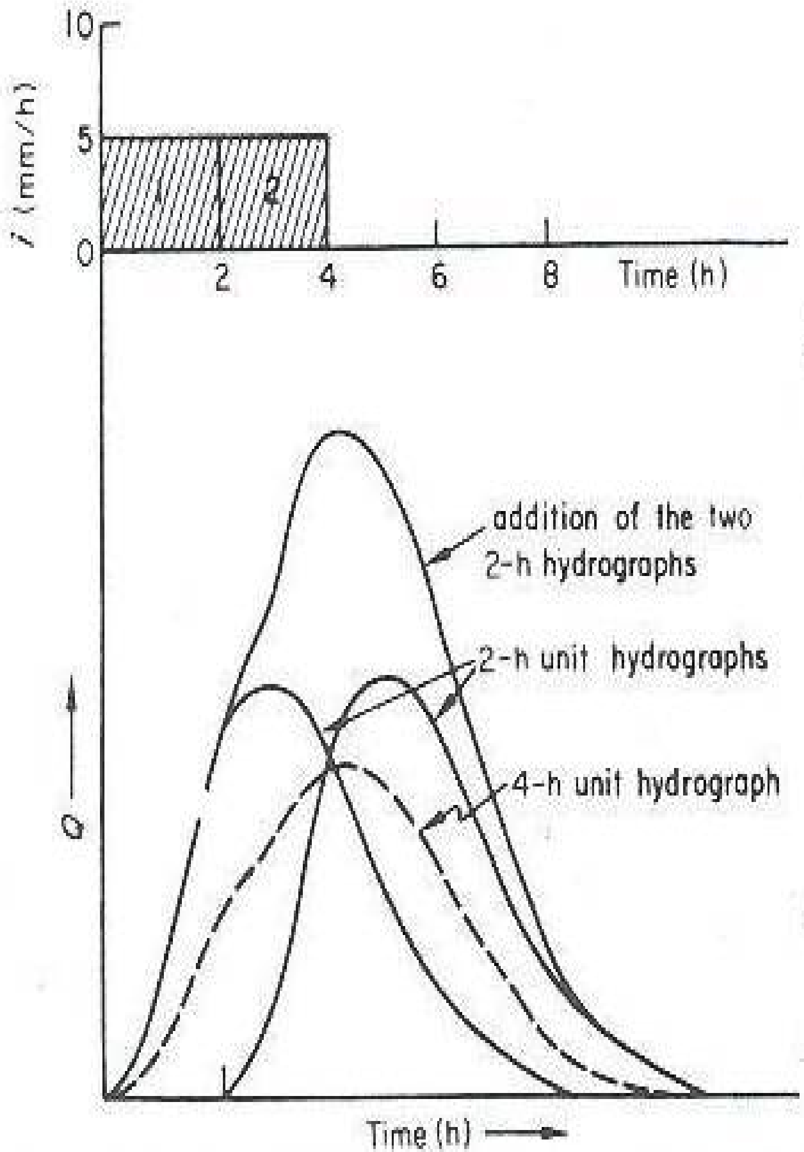


Fig.(a) Direct Method

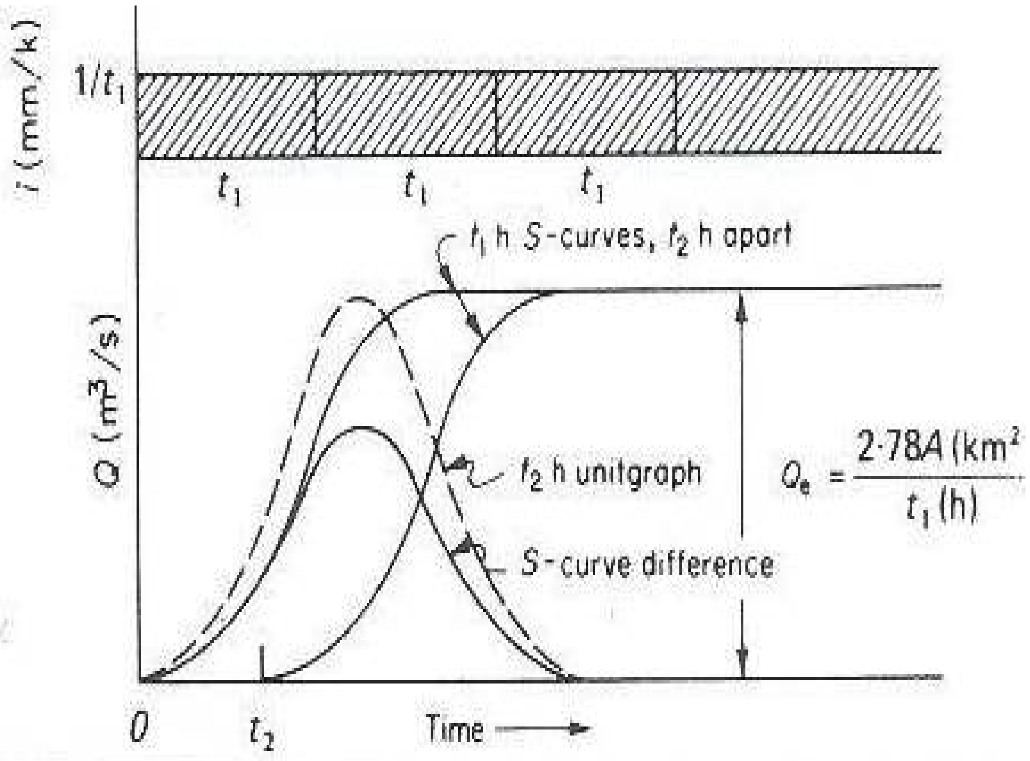


Fig (b) : S-curve method

S-curve method (all values except those in columns (1) are in units of m^3/s)

(1) Time (h)	(2) 4-h unitgraph	(3) S-curve additions	(4) S-curve columns (2) + (3)	(5) lagged S-curve (by 3hr)	(6) Column (4) minus column (5)	(7) Column 6 × (4/3) = 3-h unitgraph
0	0	—	0	—	0	0
1	6	—	6	—	6	8
2	36	—	36	—	36	48
3	66	—	66	0	66	88
4	91	0	91	6	85	113
5	106	6	112	36	76	101
6	93	36	129	66	63	84
7	79	66	145	91	54	72
8	68	91	159	112	47	63
9	58	112	170	129	41	55
10	49	129	178	145	33	44
11	41	145	186	159	27	36
12	34	159	193	170	23	31
13	27	170	197	178	19	25
14	23	178	201	186	15	20
15	17	186	203	193	10	13.5[a]
16	13	193	206	197	9	12[a]
17	9	197	206	201	5	6.5[a]
18	6	201	207	203	4	5.5[a]
19	3	203	206	206	0	0[a]
20	1.5	206	207	206	1	1.5[a]
21	0	206	206	207	−1	

Handwritten annotations: rows 0–3 "4hr period", subsequent groups marked ″; $\Sigma Q = 827$

[a] Slight adjustment is required to the tail of the 3-h unitgraph.

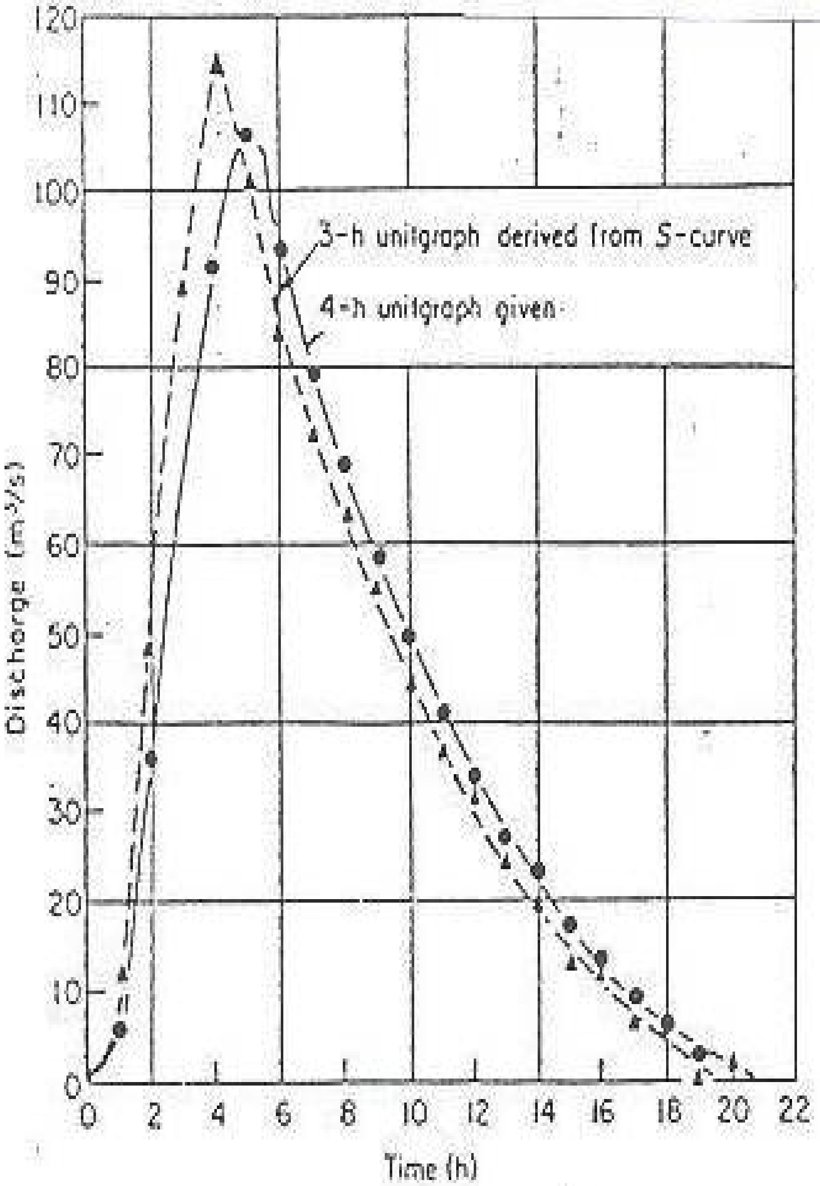


Figure Unitgraph derived by S-curve method

Check validity of 3hr UH:

$$\because d_e = \frac{\sum Q \cdot \Delta t}{A}$$

$$= \frac{827\,(1 * 3600)}{300 * 10^6}$$

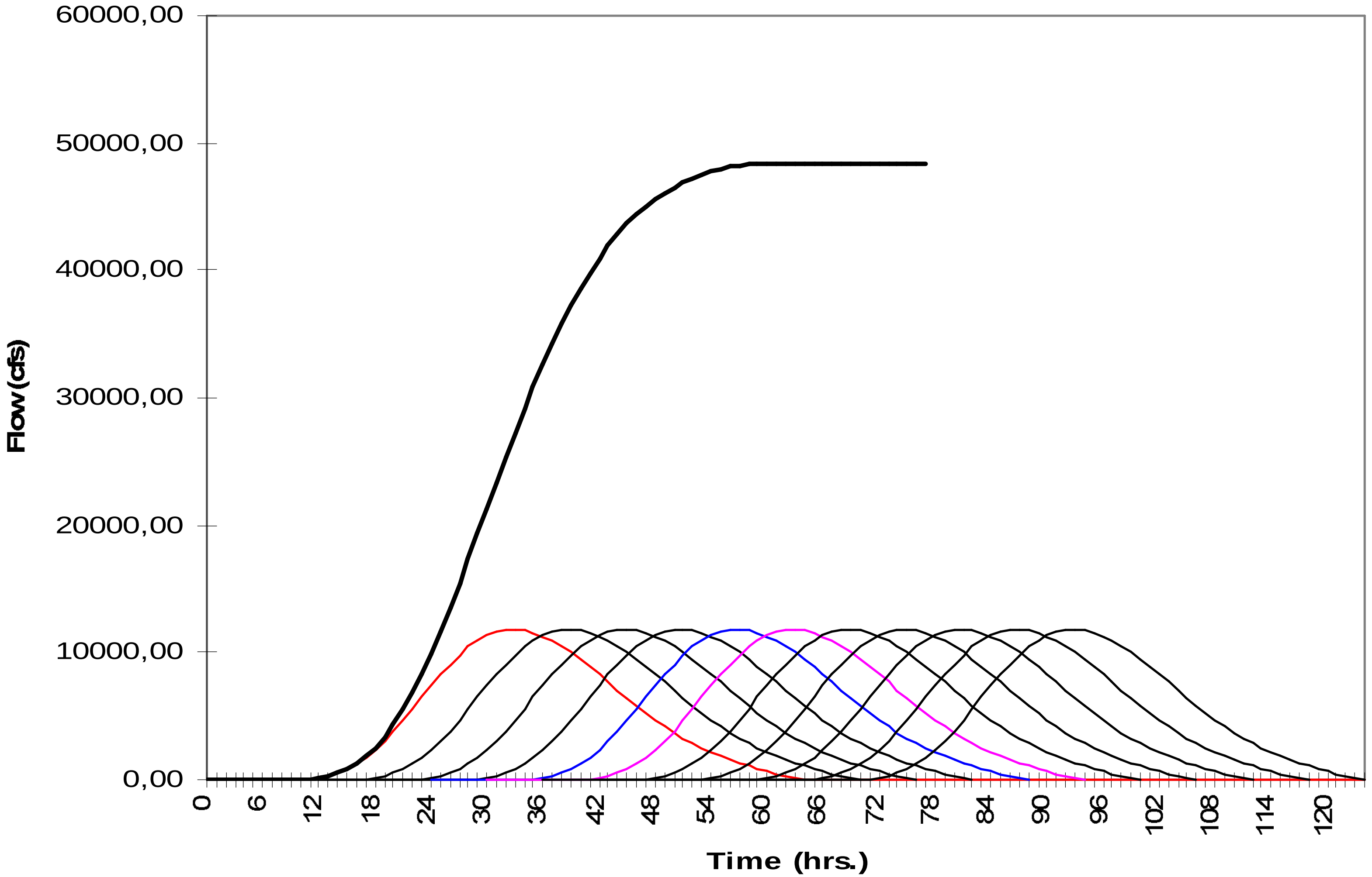
$$= 0.993 * 10^{-2}\ m$$

$$= 0.993\ cm$$

$$\simeq 1\ cm \Rightarrow \therefore O.K.$$

Valid UH

Develop S-Curve



Convert to 1-Hour Duration

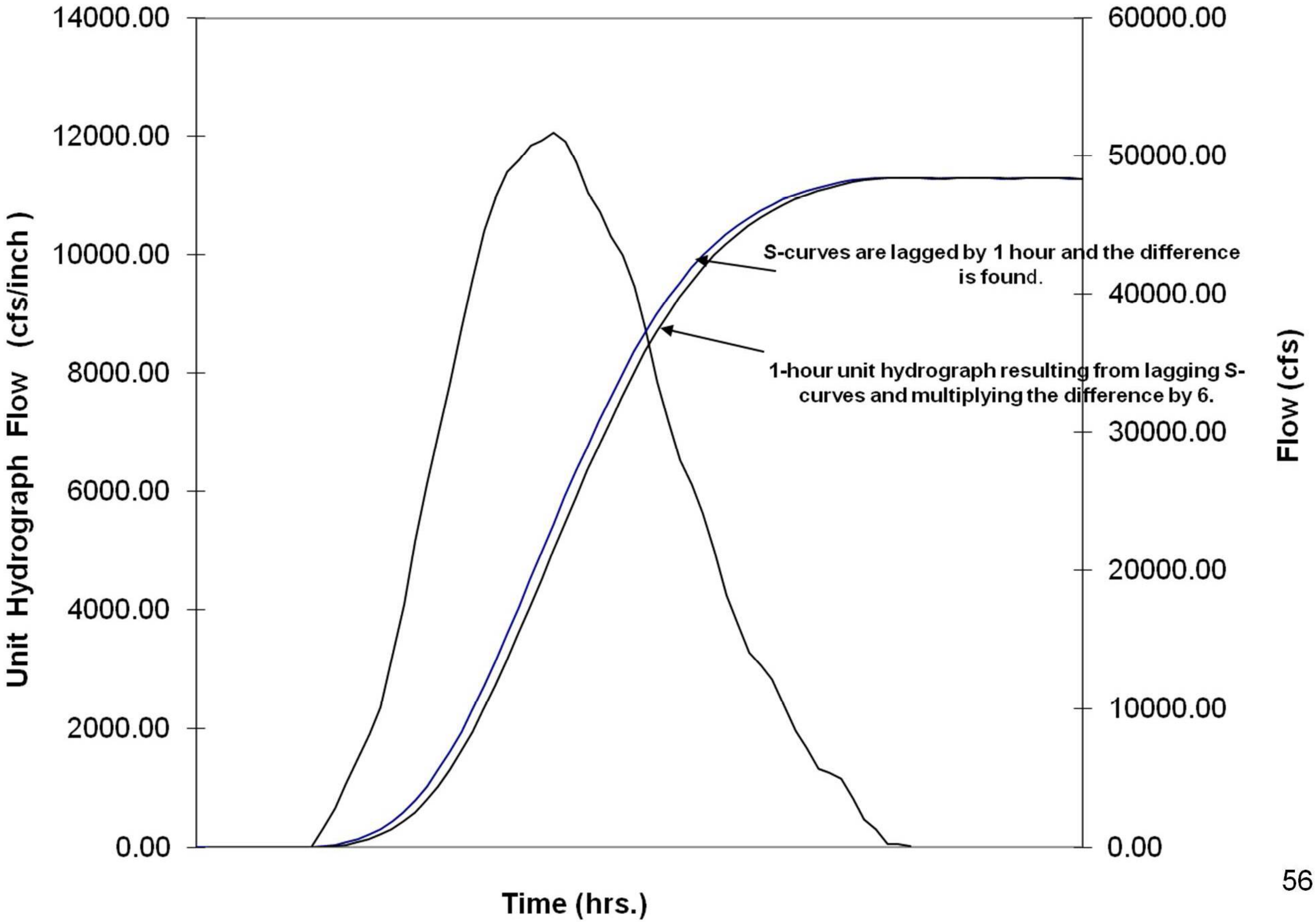
- To arrive at a 1-hour unit hydrograph, the S-curve is lagged by 1 hour and the difference between the two lagged S-curves is found to be a 1 hour unit hydrograph.

- However, because the S-curve was formulated from unit hydrographs having a 6 hour duration of uniformly distributed precipitation, the hydrograph resulting from the subtracting the two S-curves will be the result of 1/6 of an inch of precipitation.

- Thus the ordinates of the newly created 1-hour unit hydrograph must be multiplied by 6 in order to be a true unit hydrograph.

- The 1-hour unit hydrograph should have a higher peak which occurs earlier than the 6-hour unit hydrograph.

Final 1-hour UH



Shortcut Method

- **There does exist a shortcut method for changing the duration of the unit hydrograph if the two durations are multiples of one another.**

- **This is done by displacing the the unit hydrograph.**

- **For example, if you had a two hour unit hydrograph and you wanted to change it to a four hour unit hydrograph.**

Shortcut Method Example

- **First, a two hour unit hydrograph is given and a four hour unit hydrograph is needed.**
- **There are two possibilities, develop the S - curve or since they are multiples use the shortcut method.**

Time (hr)	Q
0	0
1	2
2	4
3	6
4	10
5	6
6	4
7	3
8	2
9	1
10	0

Shortcut Method Example

- The 2 hr-UH is then displaced by 2 hours.
- This is done because the 2 hr-UH will be used to represent a 4 hr-UH.

Time (hr)	Q	Displaced UHG
0	0	
1	2	
2	4	0
3	6	2
4	10	4
5	6	6
6	4	10
7	3	6
8	2	4
9	1	3
10	0	2
11		1
12		0

Shortcut Method Example

•**These two hydrographs are then summed.**

Time (hr)	Q	Displaced UHG	Sum
0	0		0
1	2		2
2	4	0	4
3	6	2	8
4	10	4	14
5	6	6	12
6	4	10	14
7	3	6	9
8	2	4	6
9	1	3	4
10	0	2	2
11		1	1
12		0	0

Shortcut Method Example

•**Finally the summed hydrograph is divided by two.**

•**This is done because when two unit hydrographs are added, the area under the curve is two units. This has to be reduced back to one unit of runoff.**

Time (hr)	Q	Displaced UHG	Sum	4 hour UHG
0	0		0	0
1	2		2	1
2	4	0	4	2
3	6	2	8	4
4	10	4	14	7
5	6	6	12	6
6	4	10	14	7
7	3	6	9	4.5
8	2	4	6	3
9	1	3	4	2
10	0	2	2	1
11		1	1	0.5
12		0	0	0

Average Several UH's

- **It is recommend that several unit hydrographs be derived and averaged.**
- **The unit hydrographs must be of the same duration in order to be properly averaged.**
- **It is often not sufficient to simply average the ordinates of the unit hydrographs in order to obtain the final unit hydrograph. A numerical average of several unit hydrographs which are different "shapes" may result in an "unrepresentative" unit hydrograph.**
- **It is often recommended to plot the unit hydrographs that are to be averaged. Then an average or representative unit hydrograph should be sketched or fitted to the plotted unit hydrographs.**
- **Finally, the average unit hydrograph must have a volume of 1 inch of runoff for the basin.**

Synthetic UHG's

- Snyder
- SCS
- Time-area
- IHABBS Implementation Plan :

NOHRSC Homepage

http://www.nohrsc.nws.gov/

http://www.nohrsc.nws.gov/98/html/uhg/index.html

Snyder

- **Since peak flow and time of peak flow are two of the most important parameters characterizing a unit hydrograph, the Snyder method employs factors defining these parameters, which are then used in the synthesis of the unit graph (Snyder, 1938).**
- **The parameters are C_p, the peak flow factor, and C_t, the lag factor.**
- **The basic assumption in this method is that basins which have similar physiographic characteristics are located in the same area will have similar values of C_t and C_p.**
- **<u>Therefore, for ungaged basins, it is preferred that the basin be near or similar to gaged basins for which these coefficients can be determined.</u>**

Basic Relationships

$$t_{LAG} = C_t (L \bullet L_{ca})^{0.3}$$

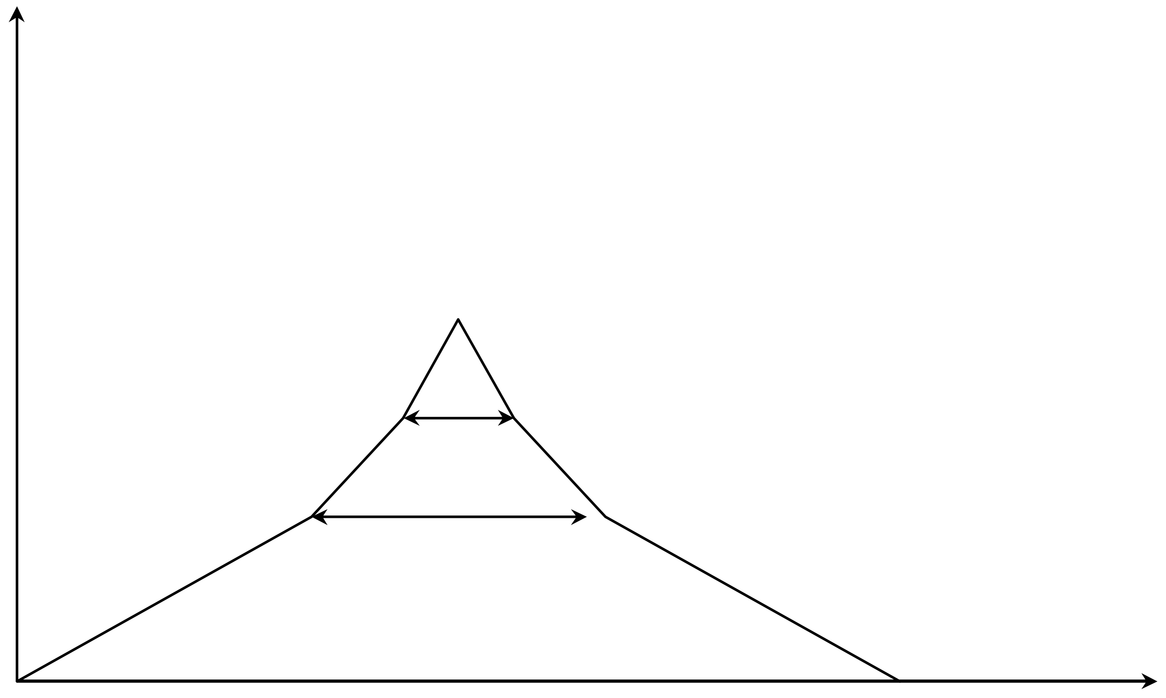
$$t_{duration} = t_{LAG} / 5.5$$

$$t_{base} = 3 + \frac{t_{LAG}}{8}$$

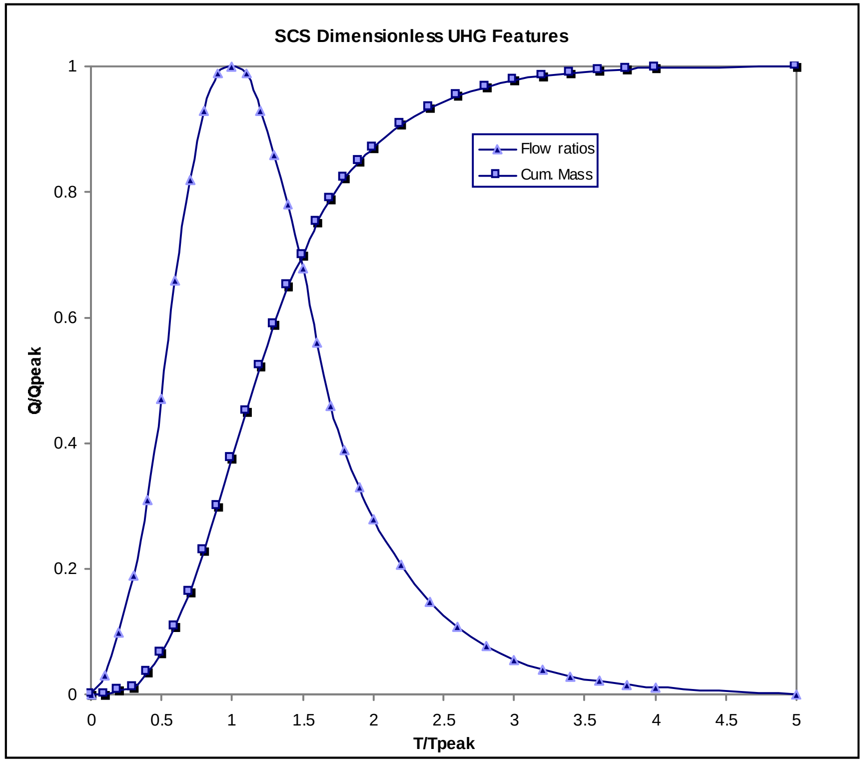
$$q_{peak} = \frac{640 \quad AC_p}{t_{LAG}}$$

Final Shape

The final shape of the Snyder unit hydrograph is controlled by the equations for width at 50% and 75% of the peak of the UH:



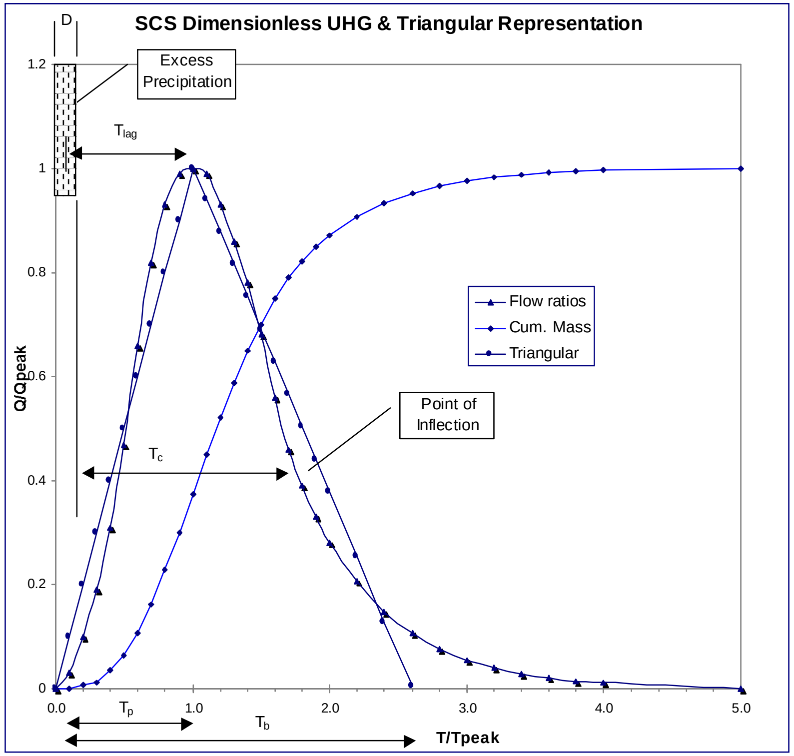
SCS


SCS Dimensionless UHG Features
Q/Qpeak
T/Tpeak
Flow ratios
Cum. Mass
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Dimensionless Ratios

Time Ratios (t/t_p)	Discharge Ratios (q/q_p)	Mass Curve Ratios (Q_a/Q)
0	.000	.000
.1	.030	.001
.2	.100	.006
.3	.190	.012
.4	.310	.035
.5	.470	.065
.6	.660	.107
.7	.820	.163
.8	.930	.228
.9	.990	.300
1.0	1.000	.375
1.1	.990	.450
1.2	.930	.522
1.3	.860	.589
1.4	.780	.650
1.5	.680	.700
1.6	.560	.751
1.7	.460	.790
1.8	.390	.822
1.9	.330	.849
2.0	.280	.871
2.2	.207	.908
2.4	.147	.934
2.6	.107	.953
2.8	.077	.967
3.0	.055	.977
3.2	.040	.984
3.4	.029	.989
3.6	.021	.993
3.8	.015	.995
4.0	.011	.997
4.5	.005	.999
5.0	.000	1.000

Triangular Representation



Triangular Representation

$$T_b = 2.67 \text{ x } T_p$$

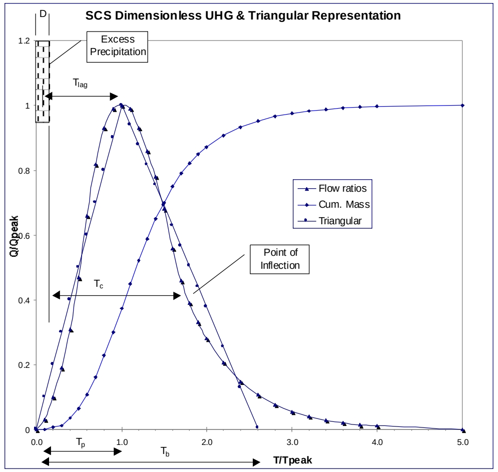
$$T_r = T_b - T_p = 1.67 \text{ x } T_p$$

$$Q = \frac{q_p T_p}{2} + \frac{q_p T_r}{2} = \frac{q_p}{2}(T_p + T_r)$$

$$q_p = \frac{2Q}{T_p + T_r}$$

$$q_p = \frac{654.33 \text{ x } 2 \text{ x } A \text{ x } Q}{T_p + T_r}$$

$$q_p = \frac{484\, A\, Q}{T_p}$$



The 645.33 is the conversion used for delivering 1-inch of runoff (the area under the unit hydrograph) from 1-square mile in 1-hour (3600 seconds).

484 ?

$$q_p = \frac{484\, A\, Q}{T_p}$$

Comes from the initial assumption that 3/8 of the volume under the UHG is under the rising limb and the remaining 5/8 is under the recession limb.

General Description	Peaking Factor	Limb Ratio (Recession to Rising)
Urban areas; steep slopes	575	1.25
Typical SCS	484	1.67
Mixed urban/rural	400	2.25
Rural, rolling hills	300	3.33
Rural, slight slopes	200	5.5
Rural, very flat	100	12.0

Duration & Timing?

Again from the triangle

$$T_p = \frac{D}{2} + L$$

L = Lag time

$$L = 0.6 * T_c$$

$$T_c + D = 1.7\,T_p$$

$$\frac{D}{2} + 0.6\,T_c = T_p$$

For estimation purposes : $D = 0.133\,T_c$

Time of Concentration

- Regression Eqs.
- Segmental Approach

A Regression Equation

$$T_{lag} = \frac{L^{0.8}(S+1)^{0.7}}{1900(\%Slope)^{0.5}}$$

where : T_{lag} = lag time in hours
L = Length of the longest drainage path in feet
S = (1000/CN) - 10 (CN=curve number)
%Slope = The average watershed slope in %

Segmental Approach

- More “hydraulic” in nature
- The parameter being estimated is essentially the time of concentration or longest travel time within the basin.
- In general, the longest travel time corresponds to the longest drainage path
- The flow path is broken into segments with the flow in each segment being represented by some type of flow regime.
- The most common flow representations are overland, sheet, rill and gully, and channel flow.

A Basic Approach $V = kS^{\frac{1}{2}}$

K	**Land Use / Flow Regime**
0.25	Forest with heavy ground litter, hay meadow (overland flow)
0.5	Trash fallow or minimum tillage cultivation; contour or strip cropped; woodland (overland flow)
0.7	Short grass pasture (overland flow)
0.9	Cultivated straight row (overland flow)
1.0	Nearly bare and untilled (overland flow); alluvial fans in western mountain regions
1.5	Grassed waterway
2.0	Paved area (sheet flow); small upland gullies

McCuen (1989) and SCS (1972) provide values of k for several flow situations (slope in %)

Flow Type	**K**
Small Tributary - Permanent or intermittent streams which appear as solid or dashed blue lines on USGS topographic maps.	2.1
Waterway - Any overland flow route which is a well defined swale by elevation contours, but is not a stream section as defined above.	1.2
Sheet Flow - Any other overland flow path which does not conform to the definition of a waterway.	0.48

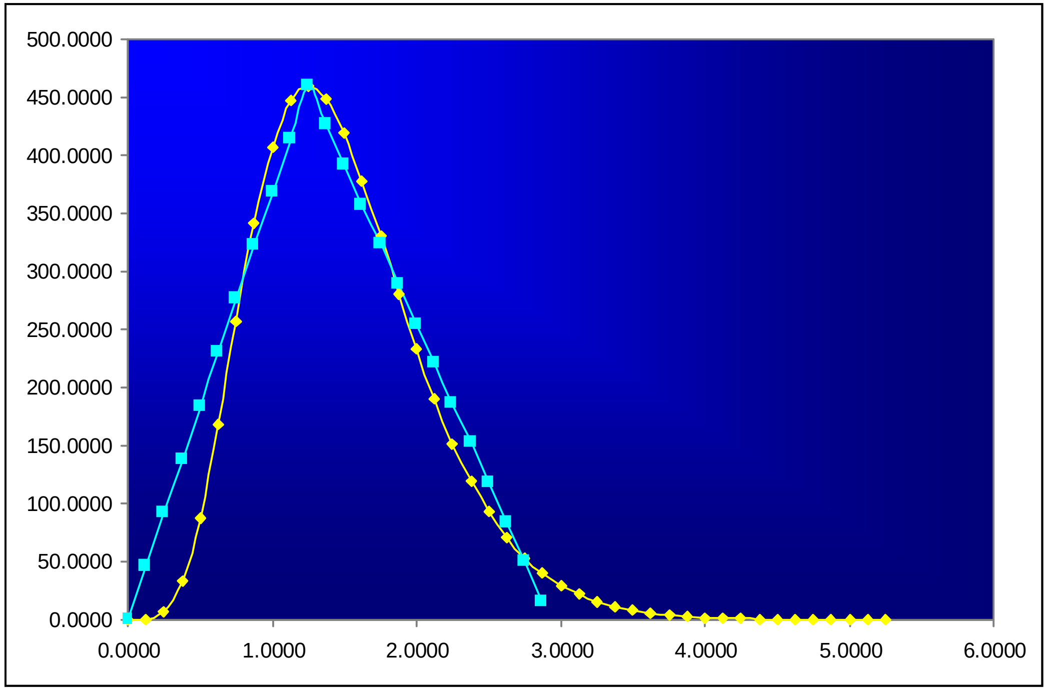
Sorell & Hamilton, 1991

Triangular Shape

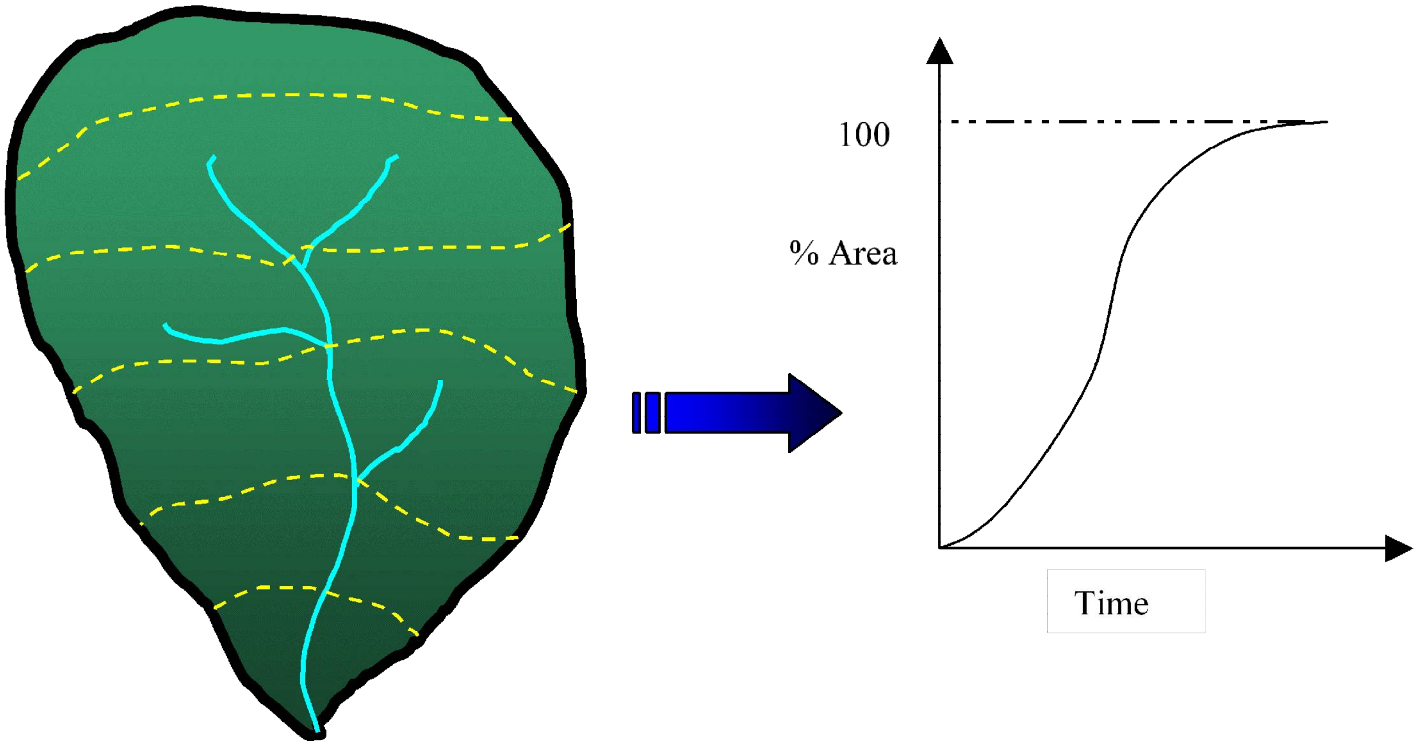
- In general, it can be said that the triangular version will not cause or introduce noticeable differences in the simulation of a storm event, particularly when one is concerned with the peak flow.
- For long term simulations, the triangular unit hydrograph does have a potential impact, due to the shape of the recession limb.
- The U.S. Army Corps of Engineers (HEC 1990) fits a Clark unit hydrograph to match the peak flows estimated by the Snyder unit hydrograph procedure.
- It is also possible to fit a synthetic or mathematical function to the peak flow and timing parameters of the desired unit hydrograph.
- Aron and White (1982) fitted a gamma probability distribution using peak flow and time to peak data.

Fitting a Gamma Distribution

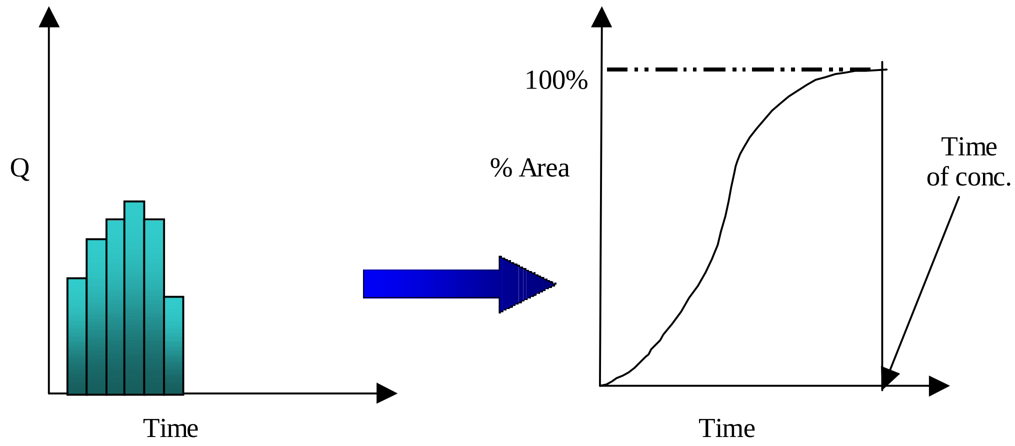
$$f(t;a,b) = \frac{t^a e^{-t/b}}{b^{a+1}\Gamma(a+1)}$$



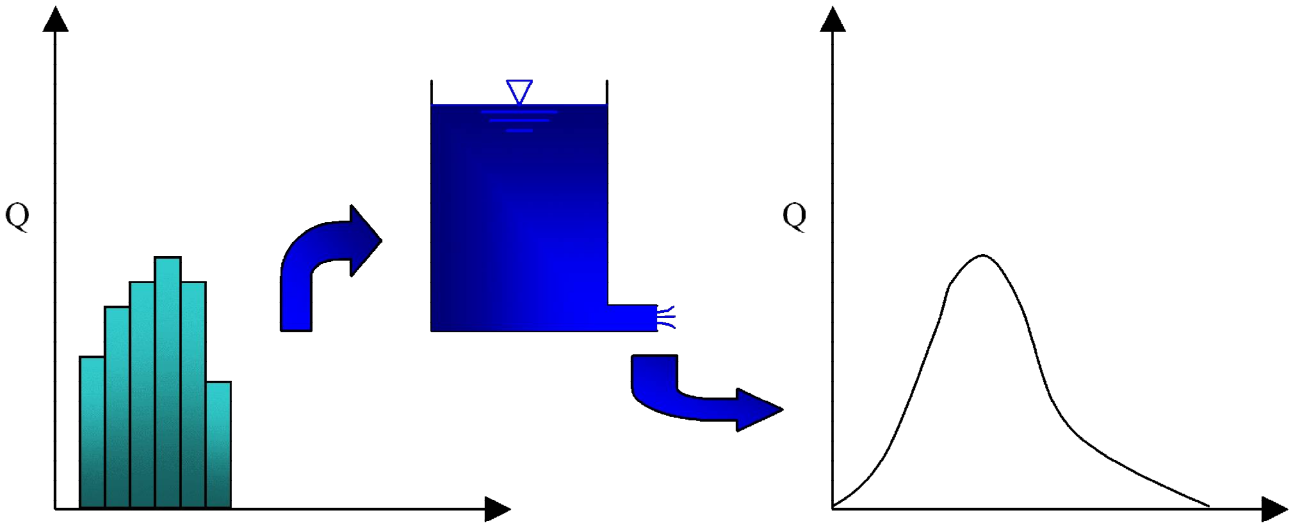
Time-Area



Time-Area

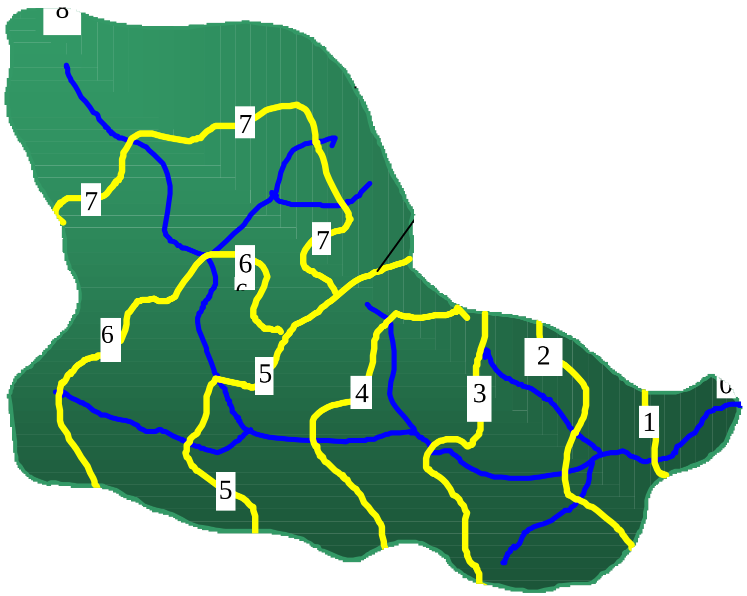


Time-Area



Hypothetical Example

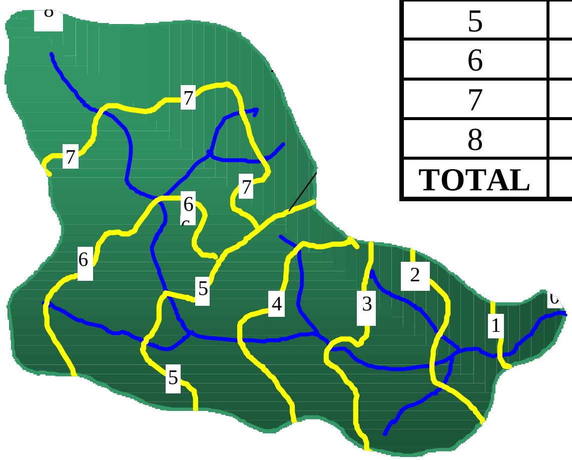
- A 190 mi^2 watershed is divided into 8 isochrones of travel time.
- The linear reservoir routing coefficient, R, estimated as 5.5 hours.
- A time interval of 2.0 hours will be used for the computations.



Rule of Thumb

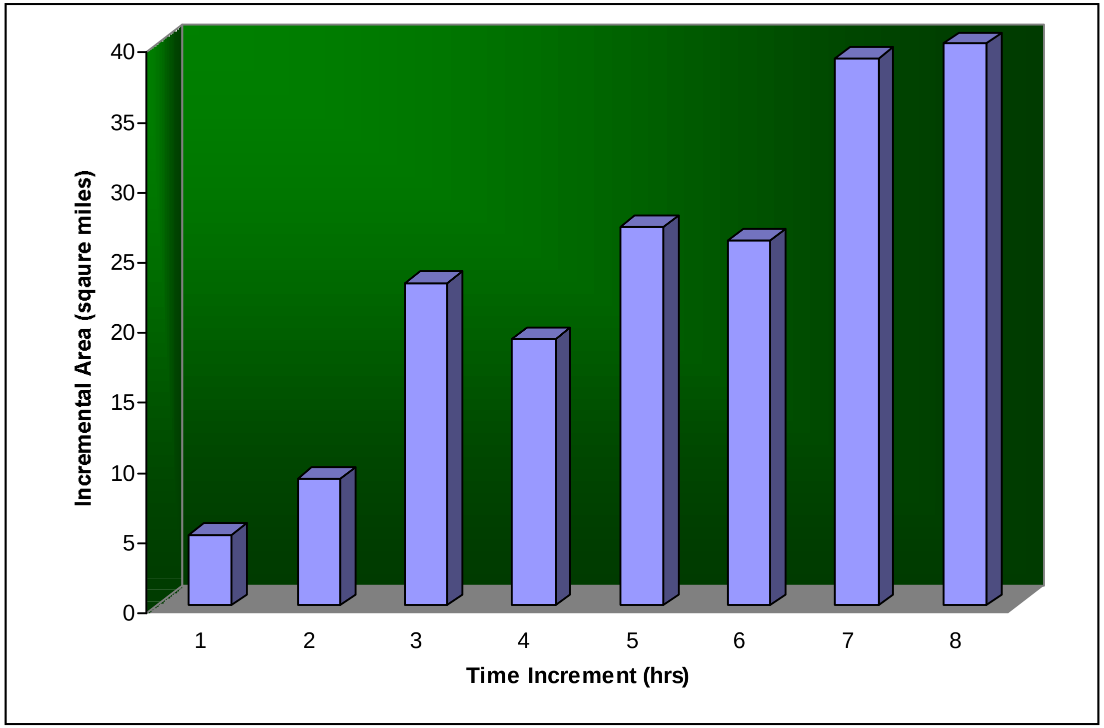
R - The linear reservoir routing coefficient can be estimated as approximately 0.75 times the time of concentration.

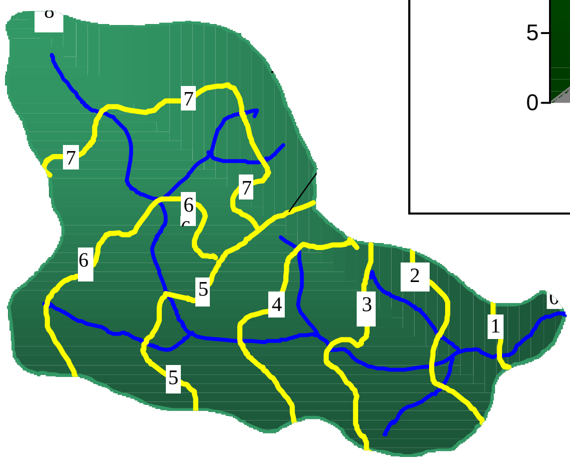
Basin Breakdown



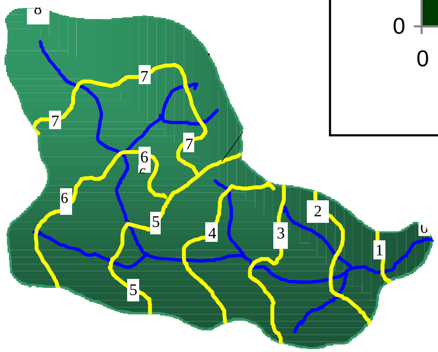
Map Area #	Bounding Isochrones	Area (mi^2)	Cumulative Area (mi^2)	Cumulative Time (hrs)
1	0-1	5	5	1.0
2	1-2	9	14	2.0
3	2-3	23	37	3.0
4	3-4	19	58	4.0
5	4-5	27	85	5.0
6	5-6	26	111	6.0
7	6-7	39	150	7.0
8	7-8	40	190	8.0
TOTAL		**190**	**190**	**8.0**

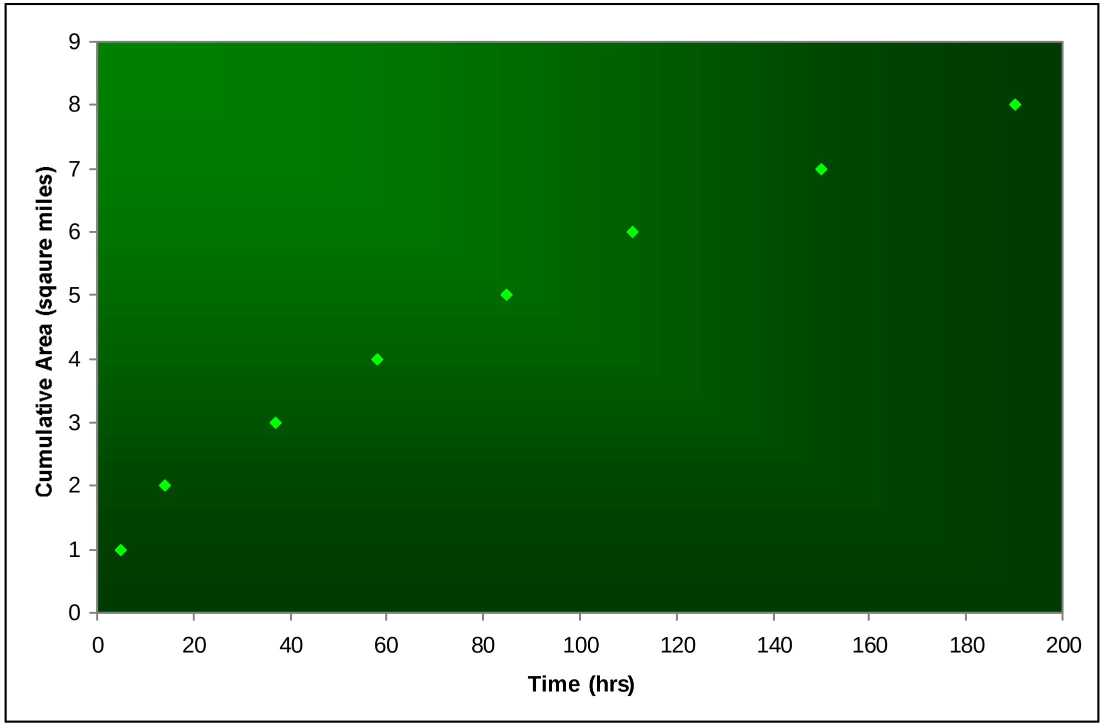
Incremental Area



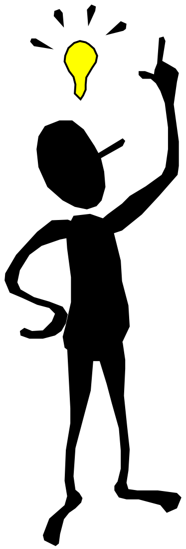


Cumulative Time-Area Curve





Trouble Getting a Time-Area Curve?



$$TA_i = 1.414 T_i^{1.5} \qquad \text{for } (0 \le \text{Ti} \le 0.5)$$

$$1 - TA_i = 1.414(1 - T_i)^{1.5} \qquad \text{for } (0.5 \le \text{Ti} \le 1.0)$$

Synthetic time-area curve - The U.S. Army Corps of Engineers (HEC 1990)

Instantaneous UHG

$$IUH_i = cI_i + (1-c)IUH_{(i-1)}$$

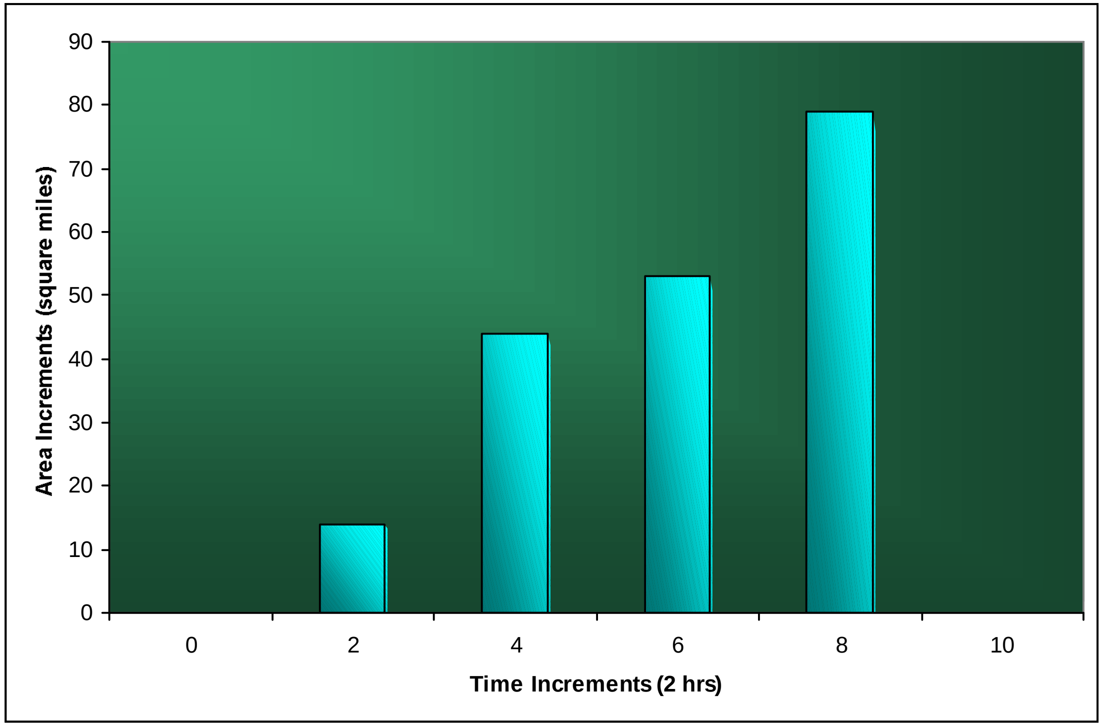
$$c = \frac{2\Delta t}{2R + \Delta t}$$

- ✓ **Δt = the time step used n the calculation of the translation unit hydrograph**
- ✓ **The final unit hydrograph may be found by averaging 2 instantaneous unit hydrographs that are a Δt time step apart.**

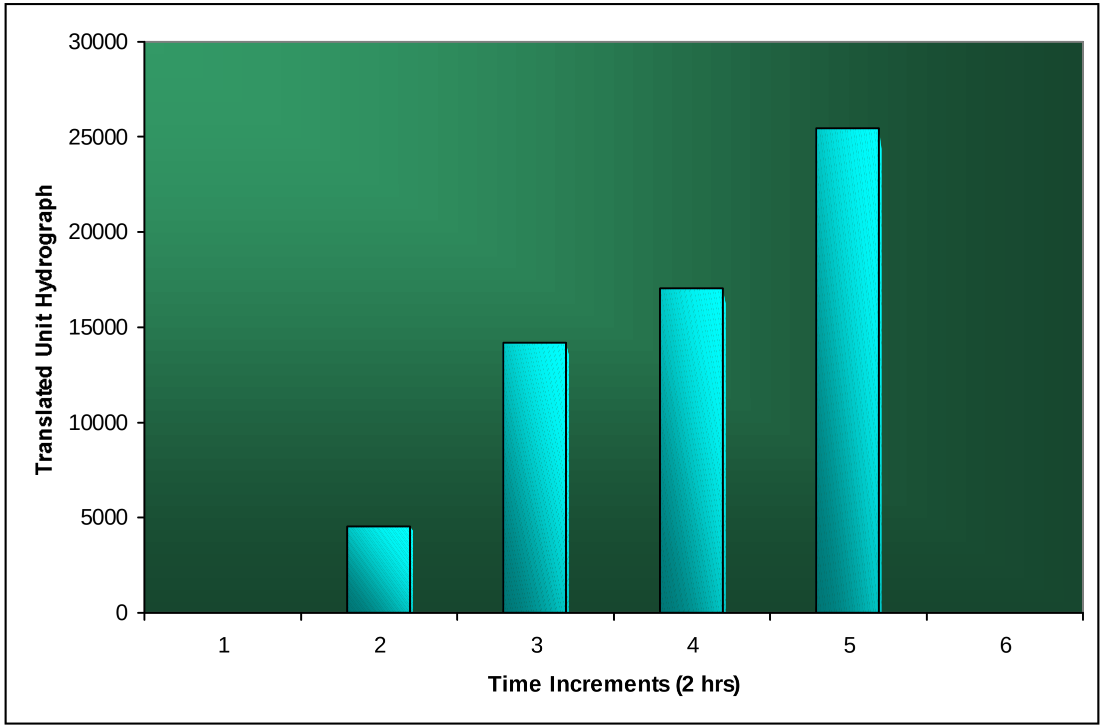
Computations

Time (hrs) (1)	Inc. Area (mi^2) (2)	Inc. Translated Flow (cfs) (3)	Inst. UHG (4)	IUHG Lagged 2 hours (5)	2-hr UHG (cfs) (6)
0	0	0	0		0
2	14	4,515	1391	0	700
4	44	14,190	5333	1,391	3,360
6	53	17,093	8955	5,333	7,150
8	79	25,478	14043	8,955	11,500
10	0	0	9717	14,043	11,880
12			6724	9,717	8,220
14			4653	6,724	5,690
16			3220	4,653	3,940
18			2228	3,220	2,720
20			1542	2,228	1,890
22			1067	1,542	1,300
24			738	1,067	900
26			510	738	630
28			352	510	430
30			242	352	300
32			168	242	200
34			116	168	140
36			81	116	100
38			55	81	70
40			39	55	50
42			26	39	30
44			19	26	20
46			13	19	20
48				13	

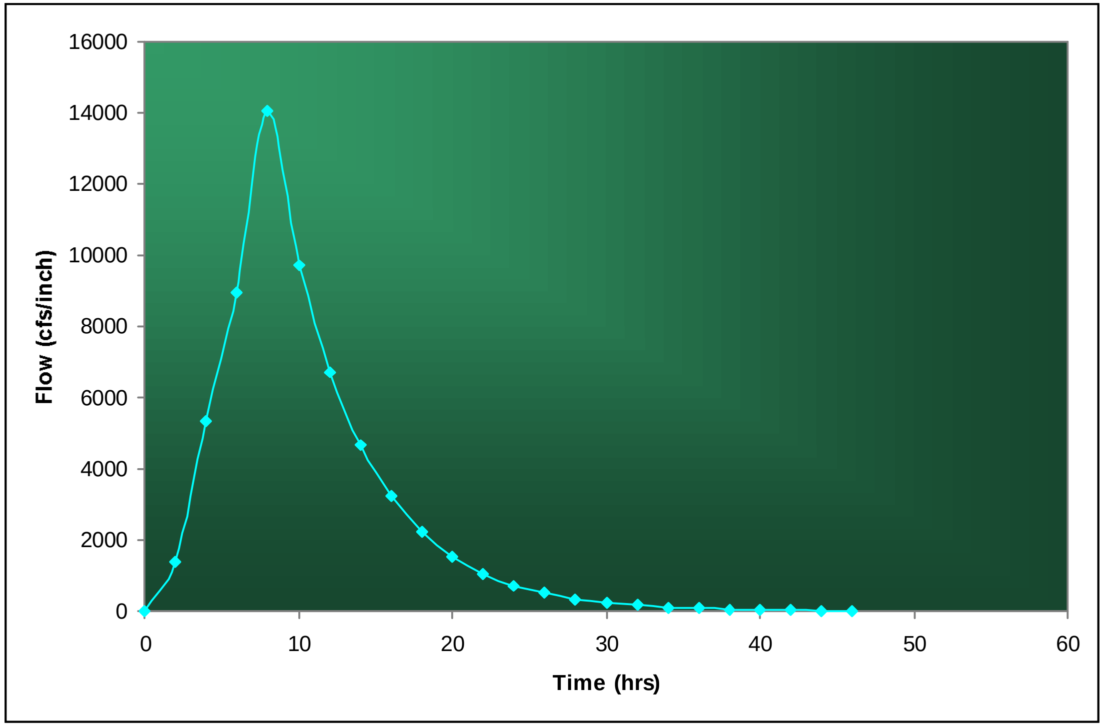
Incremental Areas



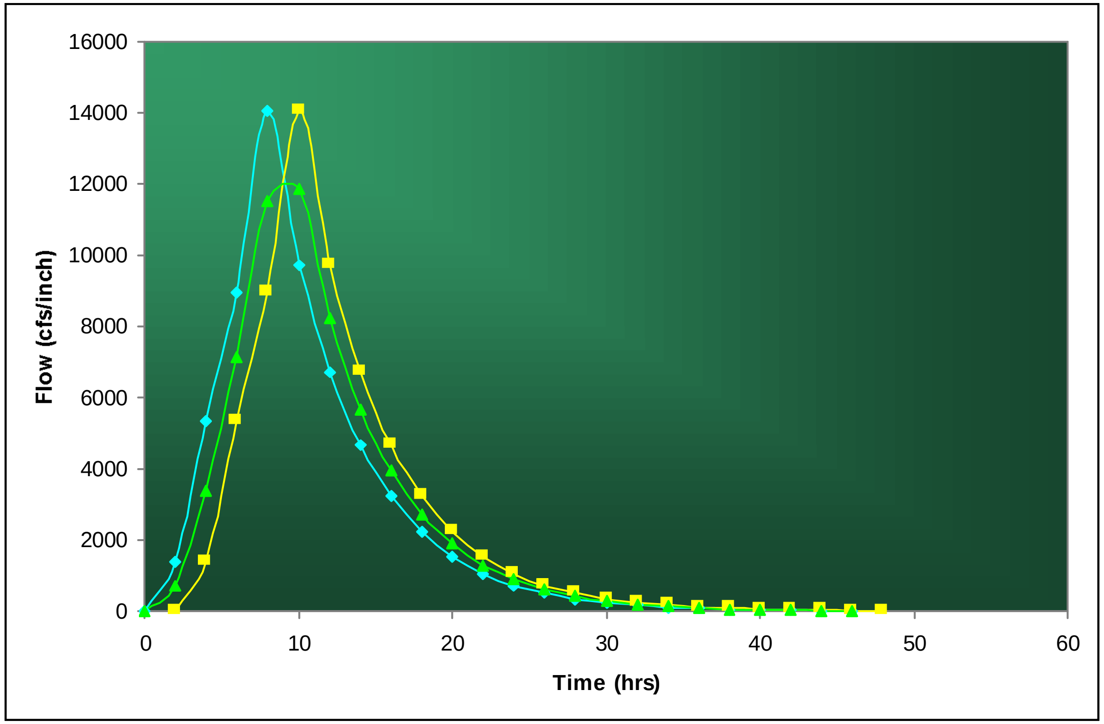
Incremental Flows



Instantaneous UHG



Lag & Average



16000
14000
12000
10000
8000
6000
4000
2000
0
Flow (cfs/inch)
0
10
20
30
40
50
60
Time (hrs)

Engineering Hydrology

BY-

Dr. Yunes Mogheir
Dr. Ramadan Al Khatib

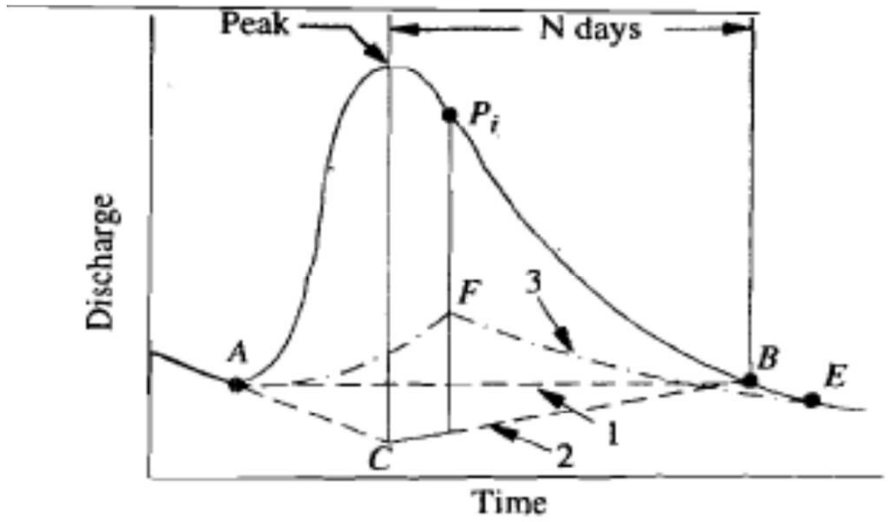


Fig. 6.5 Base flow seperation methods

- Previous Chapter estimation of long-term runoff was examined
- the present chapter examines in detail the short-term runoff phenomenon by the *storm hydrograph* or *flood hydrograph or simply Hydrograph*
- The runoff measured at the stream-gauging station will give a typical hydrograph as shown in Fig. 6.1

- The flood hydrograph is formed as a result of uniform rainfall of duration, T_r. over a catchment.

- The Hydrograph (Figure 6.1) has three characteristic regions:

  (i) the rising limb *AB*, joining point A, the starting point of the rising curve and point *B*, the point of inflection,

  (ii) the crest segment *BC* between the two points of inflection with a peak *P* in between,

  (iii) the falling limb or *depletion curve CD* starting from the second point of inflection C.

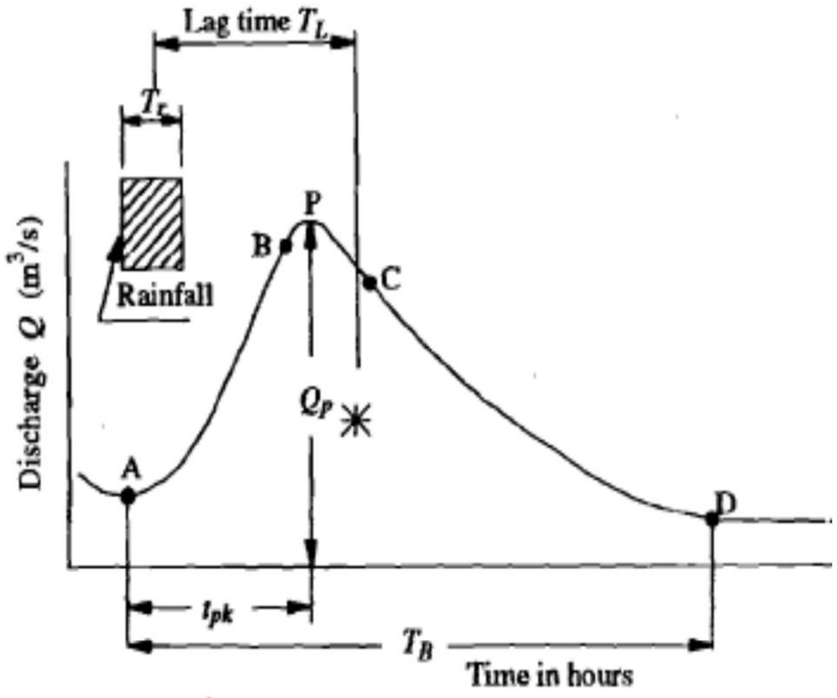


Fig. 6.1 Elements of a Flood Hydrograph

- Timing of the Hydrograph

1. t_{pk} : the time to peak (Q_p) from the starting point A,
2. *lag time* T_L : the time interval from the centre of mass of rainfall to the centre of mass of hydrograph,
3. T_B : the time base of the hydrograph

- Factor Influencing the Hydrograph

1. Watershed Characteristics such as

   size, shape, slope, storage

2. Infiltration Characteristics

   soil and land use and cover

3. Climactic Factors
   - rainfall intensity and pattern
   - aerial distribution
   - duration
   - type (rainfall vs snowmelt)

- Generally, the climatic factors control the rising limb
- catchment characteristics determine the recession limb

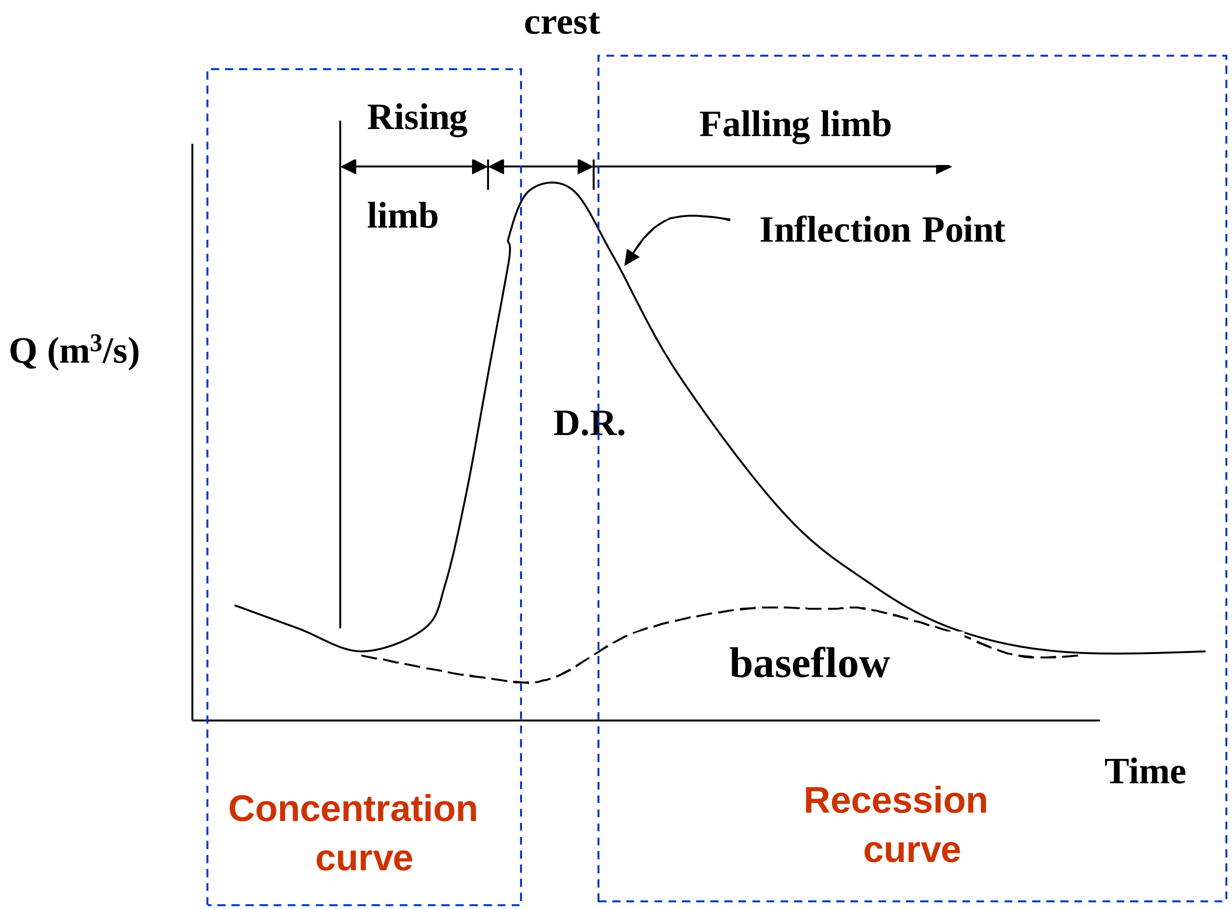
6.3 COMPONENTS OF A HYDROGRAPH

the essential components of a hydrograph are:

(i) the rising limb,

(ii) the crest segment, and

(iii) the recession limb.



Rising Limb

- The rising limb of a hydrograph (concentration curve) represents the increase in discharge due to the gradual building up of storage in channels and over the catchment surface.
- As the storm continues more and more flow from distant parts reach the basin outlet. At the same time the infiltration losses also decrease with time.

Crest

- The peak flow occurs when the runoff from various parts of the catchment at the same time contribute the maximum amount of flow at the basin outlet.
- Generally for large catchments, the peak flow occurs after the end of rainfall,
- the time interval from the centre of mass of rainfall to the peak being essentially controlled by basin and storm characteristics.

Recession Limb

- It extends from the point of inflection at the end of the crest segment to the start of the natural groundwater flow
- It represents the withdrawal of water from the storage built up in the basin during the earlier phases of the hydrograph.
- The starting point of the recession limb (the point of inflection) represents the condition of maximum storage.
- Since the depletion of storage takes place after the end of rainfall, the shape of this part of the hydrograph is independent of storm characteristics and depends entirely on the basin characteristics.
- The storage of water in the basin exists as

  - surface storage, which includes both surface detention and channel storage,

  -interflow storage, and

  -groundwater storage, i.e. base-flow storage.

- Barnes (1940) showed that the recession of a storage can be expressed as

$$Q_t = Q_0 K_r^t$$

which $Q_{0:}$ the initial discharge and

Q_t :are discharges at a time interval of *t* days;

K: is a recession constant of value less than unity.

- Previous Equation can also be expressed in an alternative form of the exponential decay as

$$Q_t = Q_0 e^{-at}$$

where $a = -\ln K$,

- The recession constant K; can be considered to be made up of three components to take care of the three types of storages as:

$K = K_{rs} \cdot K_{ri} \cdot K_{rb}$

where K_{rs} = recession constant for surface storage (0.05 to 0.20),

K_{ri}= recession constant for interflow (0.50 to 0.85) and

K_{rb} = recession constant for base flow (0.85 to 0.99)

Example 6.1

6.4 Base Flow Separation

- The surface hydrograph is obtained from the total storm hydrograph by separating the quick-response flow from the slow response runoff.

- The base flow is to be deducted from the total storm hydrograph to obtain the surface flow hydrograph in three methods

Method I: Straight line method

- Draw a horizontal line from start of runoff to intersection with recession limb (**Point A**).
- Extend from time of peak to intersect with recession limb using a lag time, N.

$$N = 0.83\, A^{0.2}$$

Where: A = the drainage area in Km^2 and N = days where **Point B** can be located and determine the end of the direct runoff .

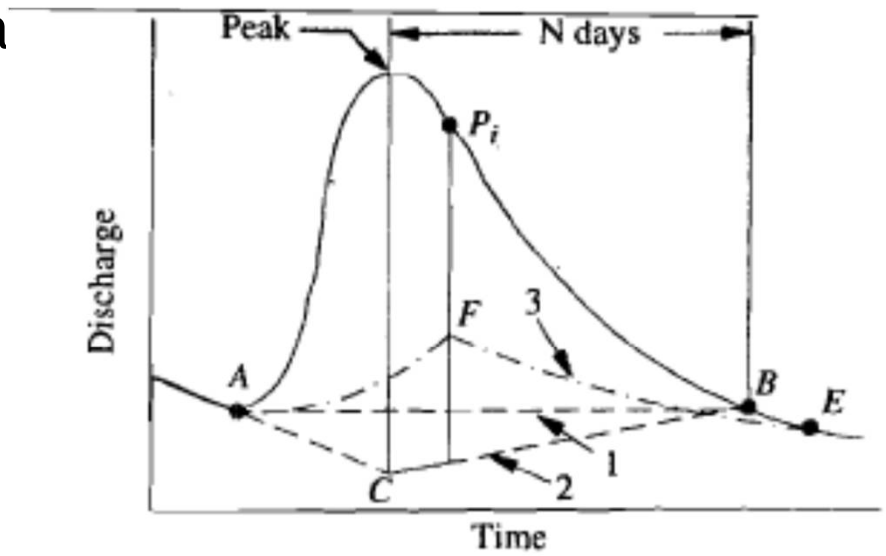


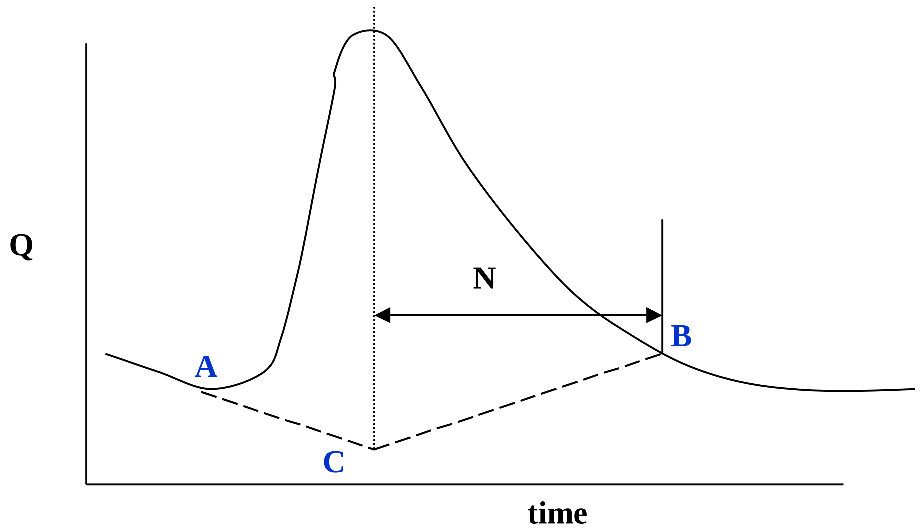
Fig. 6.5 Base flow seperation methods

Method II:

- In this method the base flow curve existing prior to the beginning of the surface runoff is extended till it intersects the ordinate drawn at the peak (point C in Fig, 6.5). This point is joined to point *B* by a straight line.

- Segment *AC* and *CB* separate the base flow and surface runoff.

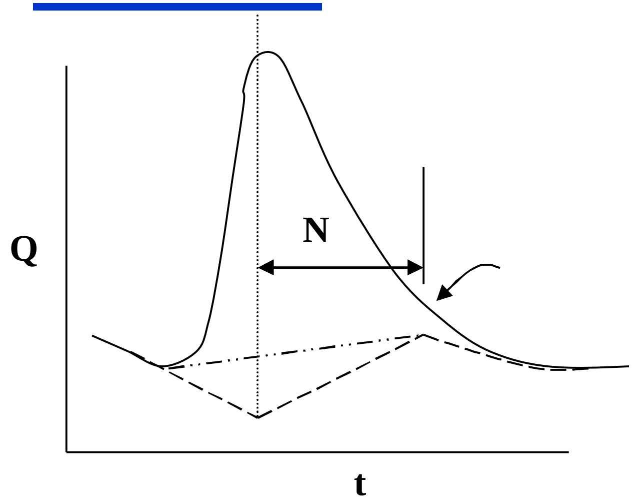
- This is probably the most widely used base-flow separation procedure.



Method III

- In this method the base flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line *EF* in Fig. 6.5), Points *A* and *F* are joined by an arbitrary smooth curve.
- This method of base-flow separation is realistic in situations where the groundwater contributions are significant and reach the stream quickly.
- The selection of anyone of the three methods depends upon the local practice and successful predictions achieved in the past.
- The surface runoff hydrograph obtained after the base-flow separation is also known as *direct runoff hydrograph* (DRH).

Method V



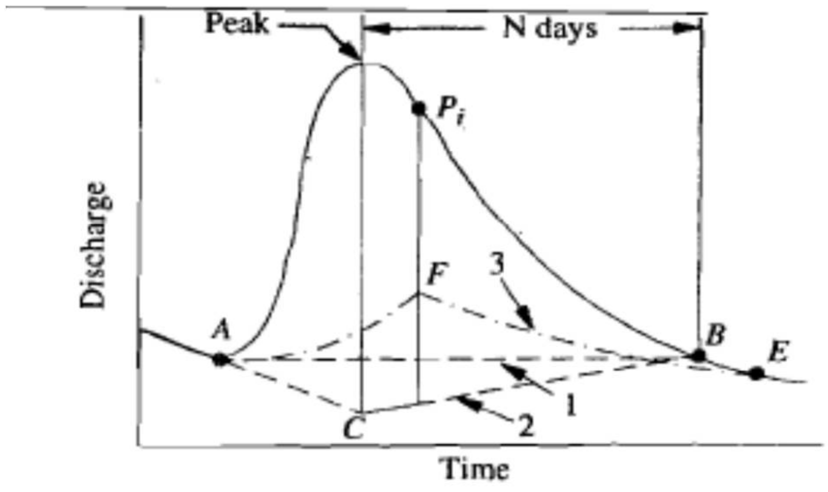


Fig. 6.5 Base flow seperation methods

6.5 **EFFECTIVE RAINFALL**

- Figure 6.6. show, the hyetograph of a storm. The initial loss and infiltration losses are subtracted from it. The resulting hyetograph is known' as *effective rainfall hyetograph* (ERH). It is also known as *hyetograph of rainfall excess* or *supra rainfall*.
- *Both DRH and ERH represent the same Total quantity but in different units*
- *ERH is usually in cm/h against time*
- *The area multiplied by the catchment Area gives the total volume of the direct runoff (total area of DRH)*

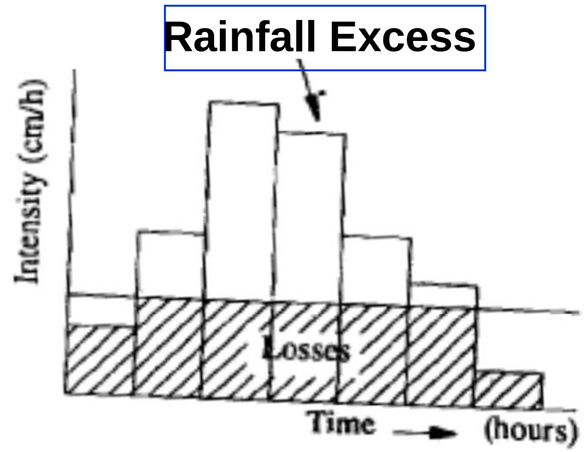


Fig. 6.6 Effective rainfall hyetograph (ERH)

EXAMPLE 6.2. *Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km^2 produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and φ index.*

Time from start of rainfall (h)	–6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m^3/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

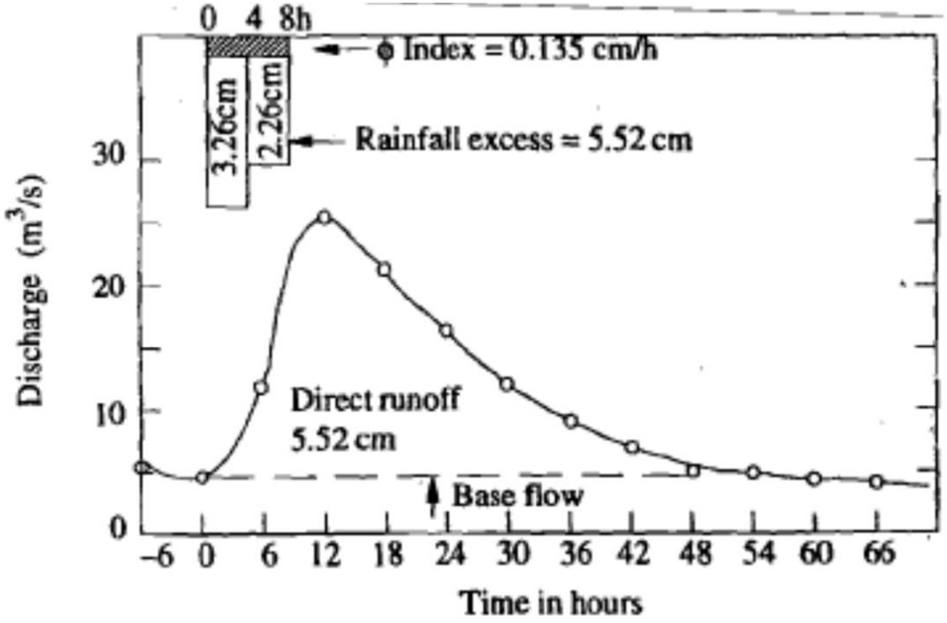


Fig. 6.7 Base flow seperation—Example 6.2

6.6 **UNIT HYDROGRAPH**

- A large number of methods are proposed to solve this problem and of them probably the most popular and widely used method is the *unit-hydrograph method.*

- A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (D hours).

- The term unit here refers to a unit depth of rainfall excess which is usually taken as 1 cm.
- The duration, being a very important characteristic, is used as indication to a specific unit hydro graph. Thus one has a 6-h unit hydrograph, 12-h unit hydrograph, etc. and in general a *D-h* unit hydrograph applicable to a given catchment.

The definition of a unit hydrograph implies the following:

- It relates only the direct runoff to the rainfall excess. Hence the volume of water contained in the unit hydrograph must be equal to the rainfall excess.

As 1 cm depth of rainfall excess is considered the area of the unit hydrograph is equal to a volume given by 1cm over the catchment.

- The rainfall is considered to have an average intensity of *excess rainfall* (ER) of *I/D* cm/h for the duration *D-h* of the storm.

- The distribution of the storm is considered to be uniform all over the catchment.

- Fig 6.9 shows a typical 6-h unit hydrograph. Here the duration of the rainfall excess is 6 h

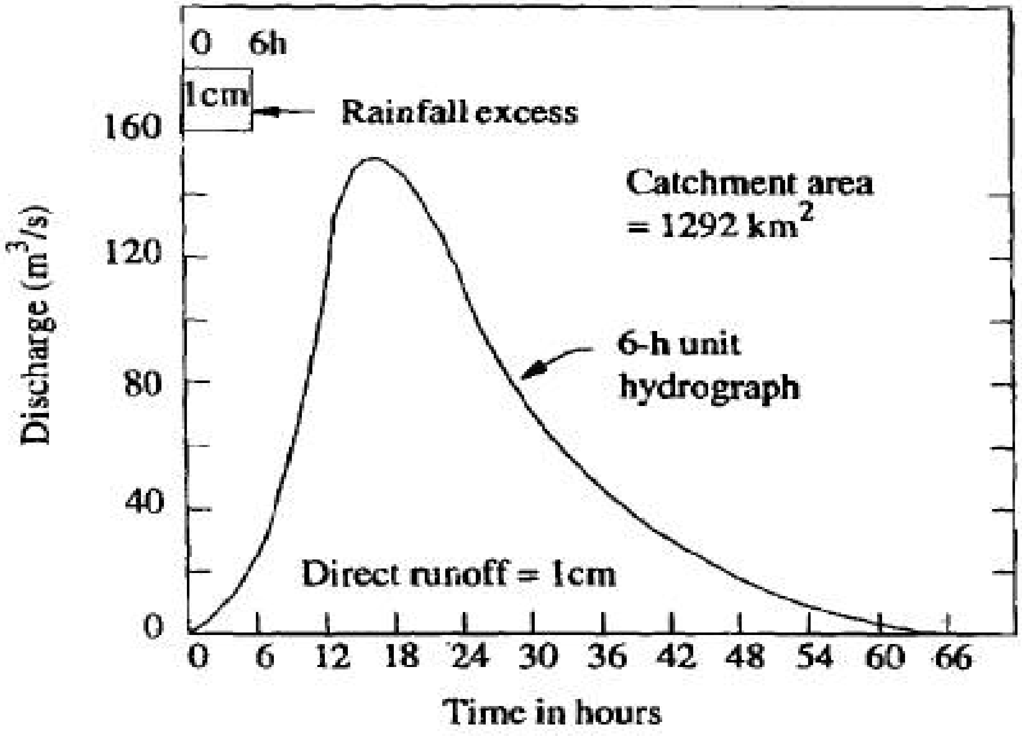


Fig. 6.9 Typical 6–h unit hydrograph

Area under the unit hydrograph = 12.92 X 10^6 m^3

Two basic assumptions constitute the foundations for the unit-hydrograph theory:

(i) the time invariance and

(ii) the linear response.

Time Invariance

This first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time-invariant. This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

Linear Response

- The direct-runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit-hydrograph theory.

- Linear response means that if an input $x_l\,(t)$ causes an output $y_l\,(t)$ and an input $.x_2\,(t)$ causes an output $y_2\,(t)$, then an input $x_l\,(t) + x_2\,(t)$ gives an output $y_1\,(t) + y_2(t)$.

- Consequently, if $x_2\,(t) = r\,X_l\,(t)$, then $y_z\,(t) = r\,y_l\,(t)$.

- Thus if the rainfall excess in a duration D is *r* times the unit depth, the resulting DRH will have ordinates bearing ratio *r* to those of the corresponding *D-h* unit hydrograph.

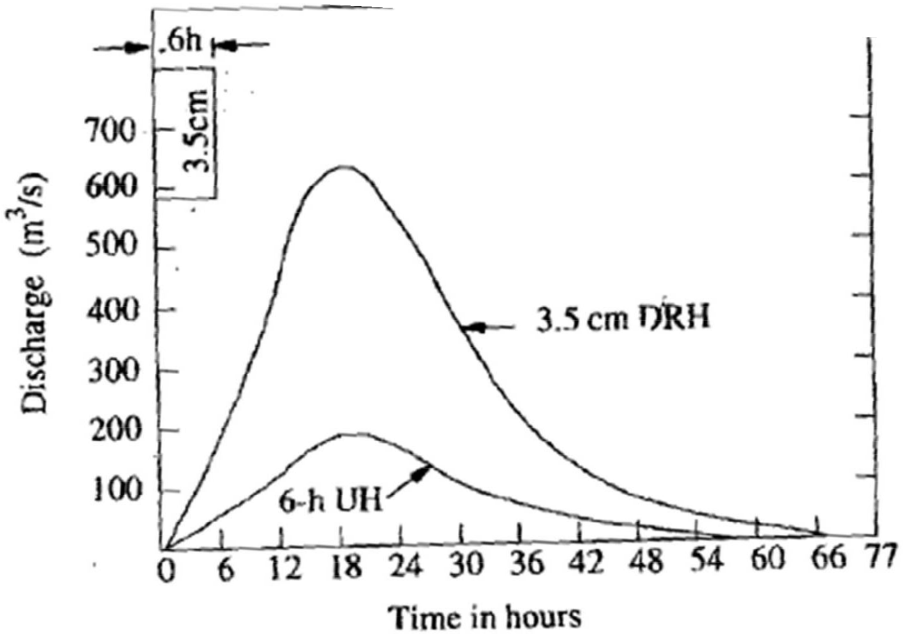
- Since the area of the resulting DRH should increase by the ratio *r,* the base of the DRH will be the same as that of the unit hydrograph.

- If two rainfall excess of *D-h* duration each occur consecutively, their combined effect is obtained by superposing the respective DRHs with due care being taken to account for the proper sequence of events. **(The method of superposition)**

EXAMPLE 6.4 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 hr.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m^3/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

- The desired ordinates of the DRH are obtained by multiplying the ordinates of the unit hydrograph by a factor of 3.5 as in Table 6.3.
- Note that the time base of DRH is not changed and remains the same as that of the unit hydrograph.



EXAMPLE 6.5 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.4. Calculate the resulting DRH.*

TABLE 6.4 CALCULATION OF DRH BY METHOD OF SUPERPOSITION — *EXAMPLE 6.5*

Time (h)	Ordinate of 6-h UH (m^3/s)	Ordinate of 3-cm DRH (col.2) × 3	Ordinate of 2-cm DRH (col.2 lagged by 6 h) × 2	Ordinate of 5-cm DRH (col. 3 + col.4) (m^3/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

Note : 1. The entries in col.4 are shifted by 6 h in time relative to col.2.

2. Due to unequal time interval of ordinates a few entries have to be interpolated to complete the table. These interpolated values are shown in parentheses.

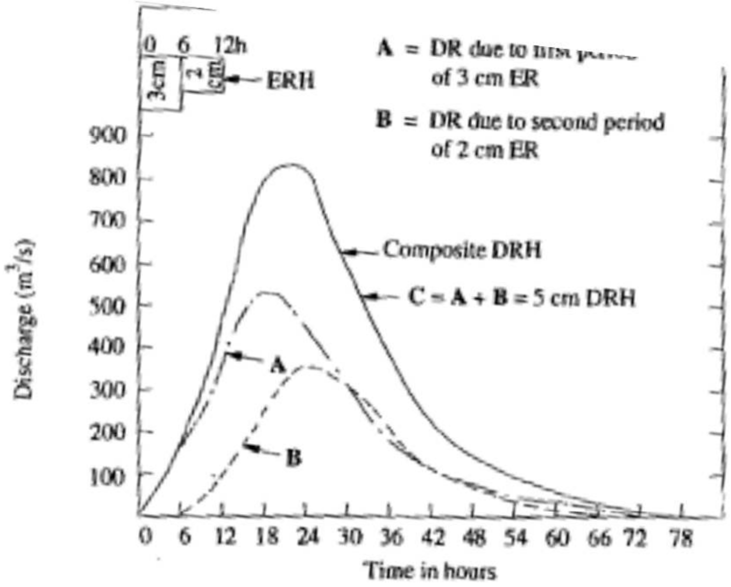


Fig. 6.10 (b) Principle of superposition—Example 6.5

Application of U-Hydrograph

- D-h U-hydrograph and storm hyetograph are available
- ERH is obtained by deducting the losses
- ERH is divided by M blocks of D-h duration
- Rainfall excesses is operated upon unit hydrograph successively to get different DHR curves

The direct runoff due to R_1 at time t is

$$Q_1 = R_1 . u[t]$$

The direct runoff due to R_2 at time $(t-D)$ is

$$Q_2 = R_2 . u[t-D]$$

Similarly, $Q_i = R_i . u[t-(i-1)D]$

and $Q_m = R_m . u[t-(M-1)D]$

Thus at any time t, the total direct runoff is

$$Q_t = \sum_{i=1}^{M} Q_i = \sum_{i=1}^{M} R_i . u[t-(i-1)D]$$

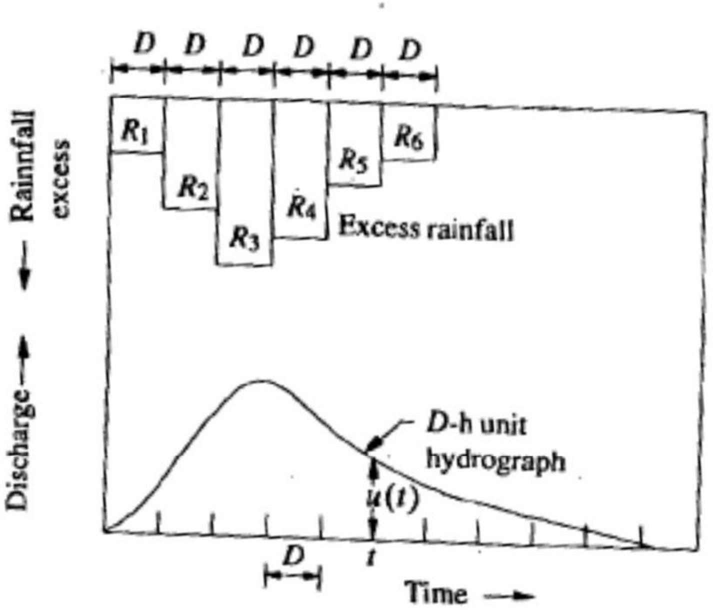


Fig. 6.11 DRH due to an ERH

EXAMPLE 6.6 *The ordinates of a 6- hour unit hydrograph of a catchment is given below.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48
Ordinate of 6-h UH	0	25	50	85	125	160	185	160	110	60	36	25

Time (h)	54	60	69
Ordinate of 6-h UH	16	8	0

Derive the flood hydrograph in the catchment due to the storm given below:

Time from start of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	3.5	11.0	16.5

The storm loss rate (ϕ – index) for the catchment is estimated as 0.25 cm/h. The base flow can be assumed to be 15 m^3/s *at the beginning and increasing by 2.0* m^3/s *for every 12 hours till the end of the direct–runoff hydrograph.*

Solution of Ex.6.6

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	(11.0-3.5) = 7.5	(16.5-11.0) = 5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

Time	Ordinaters of U.H.	DRH due to 2 cm ER Col. 2 × 2.0	DRH due to 2 cm ER × 6.0 (Advanced by 6 h)	DRH due to 4 cm ER Col.2 × 4.0 (Advanced by 12 h)	Ordinates of final DRH (Col 3+ 4+5)	Base flow (m^3/s)	Ordinates of flood hydrograph (m^3/s) (Col. 6 + 7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	960	340	1645	(17)	1662
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	640	1945	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	48	64	117	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

6.7 Derivation of Unit Hydrographs

- The area under each DRH is evaluated and the volume of the direct runoff obtained is divided by the catchment area to obtain the depth of ER.
- The ordinates of the various DHRs are divided by the respective ER values to obtain the ordinates of the unit hydrograph.

EXAMPLE 6.7 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km² due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	−6	0	6	12	18	24	30	36	42	48
Discharge (m^3/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5
Time from start of storm (h)	54	60	66	72	78	84	90	96	102	
Discharge (m^3/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0	

Solution of Example 6.7

SOLUTION: The storm hydrograph is plotted to scale (Fig. 6.13). Denoting the tim
beginning of storm as t, by inspection of Fig. 6.12,

A = beginning of DRH $t = 0$

B = end of DRH $t = 90$ h

P_m = peak $t = 20$ h

Hence

$$N = (90 - 20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\,(423)^{0.2} = 2.78 \text{ days},$$

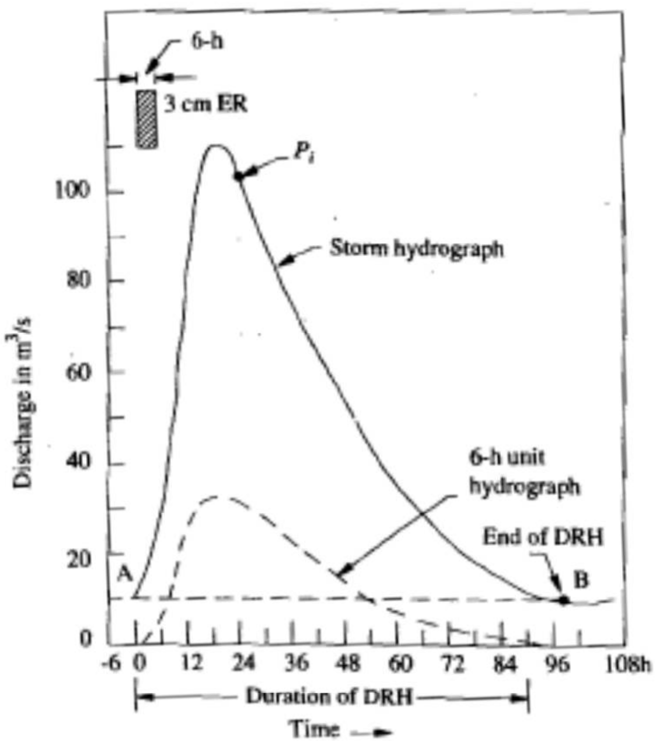


Fig. 6.13 Derivation of unit hydrograph from a storm hydrograph

- However, N =2.91 days is adopted for convenience.
- A straight line joining A and B is taken as the divide line for base-flow separation.
- The ordinates of DRH are obtained by Subtracting the base flow from the ordinates of the storm hydrograph.

$$\text{Volume of DRH} = 60 \times 60 \times 6 \times (\text{sum of DRH ordinates})$$

$$= 60 \times 60 \times 6 \times 587 = 12.68 \text{ Mm}^3$$

$$\text{Drainage area} = 423 \text{ km}^2 = 423 \text{ Mm}^2$$

$$\text{Runoff depth} = \text{ER depth} = \frac{12.68}{423} = 0.03 \text{ m} = 3 \text{ cm.}$$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph,, see Table 6.6

TABLE 6.6 CALCULATION OF THE ORDINATES OF A 6–h UNIT HYDROGRAPH—*EXAMPLE 6.7*

Time from beginning of storm (h)	Ordinate of storm hydrograph (m^3/s)	Base flow (m^3/s)	Ordinate of DRH (m^3/s)	Ordinate of 6-h unit hydrograph (Col. 4 ÷ 3)
1	2	3	4	5
–6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	101.0	33.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0