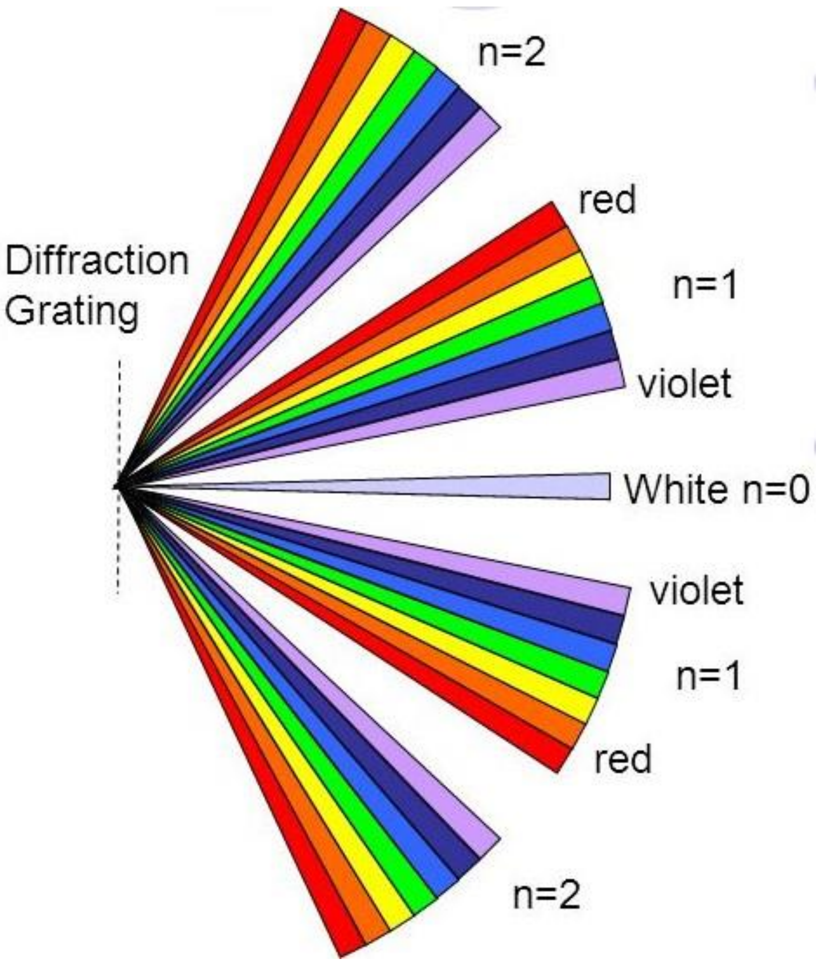
Diffraction



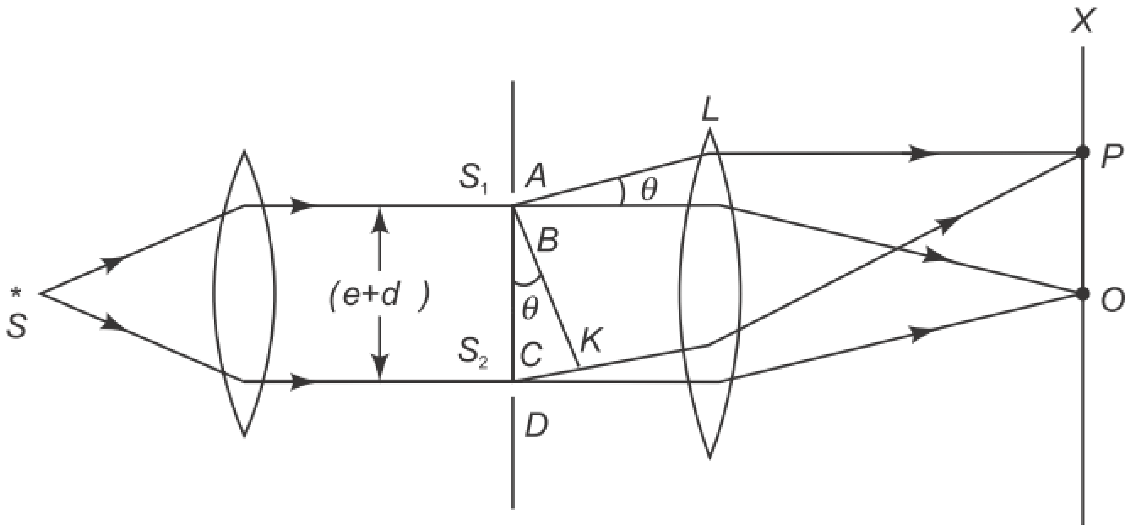
UNIT III
Optics
Lecture-5

Content of Lecture

- FRAUNHOFER DIFFRACTION DUE TO DOUBLE SLIT
- PLANE DIFFRACTION GRATING
- MISSING ORDERS OR ABSENT SPECTRA
- DETERMINATION OF WAVELENGTH OF LIGHT BY GRATING

DIFFRACTION DUE TO DOUBLE SLIT

- Let a parallel beam of monochromatic light be incident normally upon two parallel slits AB and CD, each of width e, separated by opaque space of width d.
- The distance between the corresponding points of the two slits is (e + d) as is shown in Fig.
- The pattern obtained on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superposed.



DIFFRACTION DUE TO DOUBLE SLIT CONTINUD…..

From the theory of diffraction at a single slit, the resultant amplitude due to wavelets diffracted from each slit in a direction θ is

$$R = \frac{A \sin \alpha}{\alpha}$$

where A is a constant and $\alpha = (\pi e \sin \theta)/\lambda$.

Let the two slits S1 and S2 be the two coherent sources, each sending wavelets of amplitude $\frac{A \sin \alpha}{\alpha}$ in a direction θ.

Consequently, the resultant amplitude at point P on the screen will be the result of interference between two waves of same amplitude having a phase difference d.

DIFFRACTION DUE TO DOUBLE SLIT CONTINUD…..

- The path difference between the wavelets from S1 and S2 in the direction θ is

$$S_2K = (e + d) \sin \theta$$

- Hence, the corresponding phase difference

$$\delta = \frac{2\pi}{\lambda}(e + d) \sin \theta$$

The resultant amplitude at P can be determined by

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 + \left(\frac{A \sin \alpha}{\alpha}\right)^2 + 2\left(\frac{A \sin \alpha}{\alpha}\right)\left(\frac{A \sin \alpha}{\alpha}\right) \cos \delta$$

$$= 4 A^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \frac{\delta}{2}$$

$$= 4 A^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

where $\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e + d) \sin \theta.$

Therefore, the resultant intensity at P is

$$I = R^2 = 4A^2 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

DIFFRACTION DUE TO DOUBLE SLIT CONTINUD.....

Thus, the resultant intensity depends on two factors:

- ❑ $\frac{Sin^2\,\alpha}{\alpha^2}$ gives the diffraction pattern due to each individual slit.
- ❑ $Cos^2\beta$ gives the interference pattern due to light waves from the two slits.

➢The diffraction term $\frac{Sin^2\,\alpha}{\alpha^2}$ gives a central maximum in the direction θ = 0, having alternately minima and subsidiary maxima of decreasing intensity on either side

The minima are obtained in the directions given by

$$\sin\alpha = 0$$

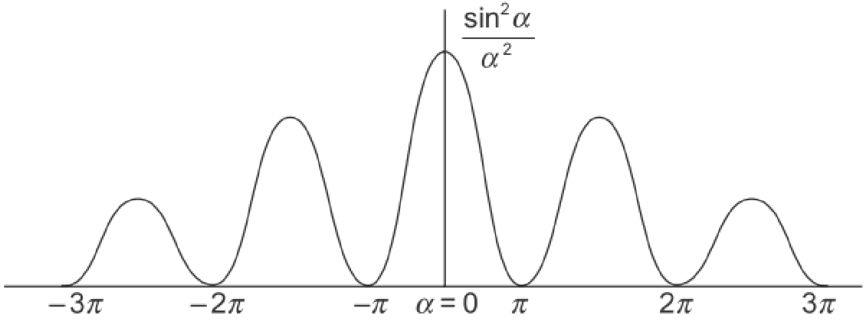
or $\alpha = \pm m\pi$

$$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

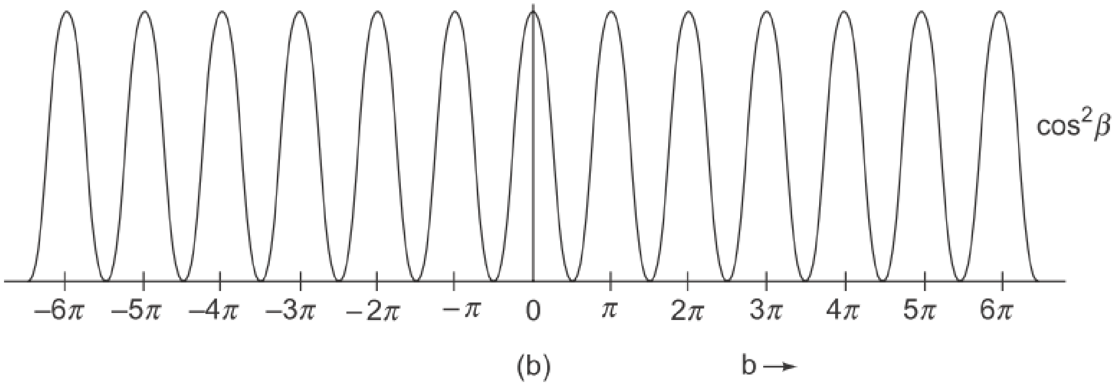
$$e\sin\theta = \pm m\lambda$$

where $m = 1, 2, 3, \ldots$ (but not zero).

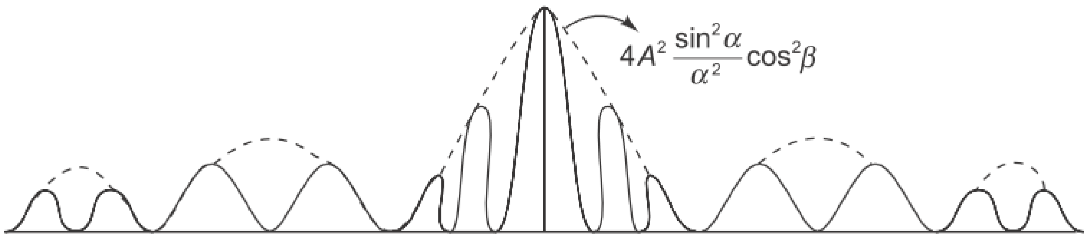
Depiction of diffraction pattern of double slit



(a)



(b)



(c)

Maxima due to interference

- The interference term $Cos^2\beta$ gives a set of equidistant dark and bright fringes.

- The bright fringes (maxima) are obtained in the directions given by

$$\cos^2\beta = 1$$

or

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

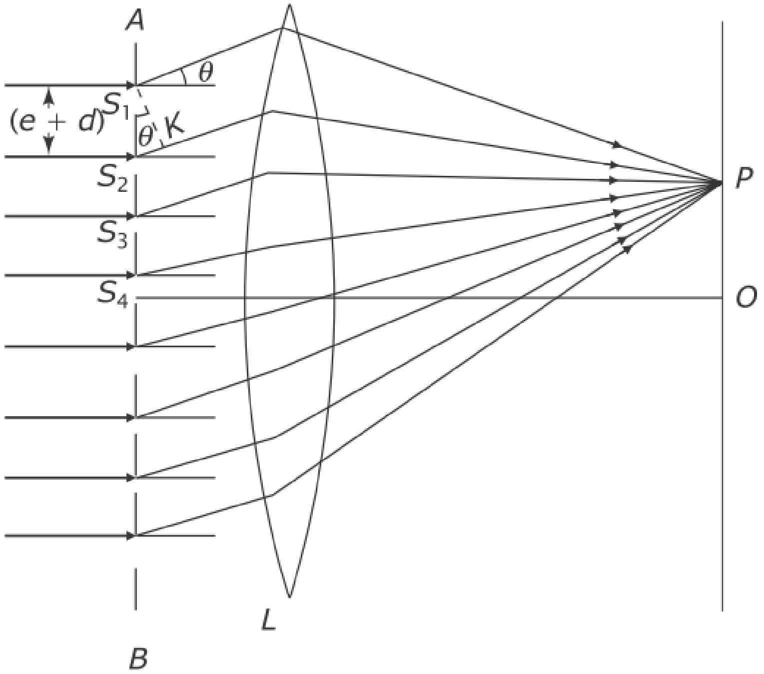
$$(e+d)\sin\theta = \pm n\lambda$$

- where n = 0, 1, 2,…. The various maxima corresponding to n = 0, 1, 2,… are zero-order, first-order, second-order, … maxima, respectively

PLANE DIFFRACTION GRATING

- Let AB be the section of a grating having width of each slit as e, and d the width of each opaque space between the slits.

- The quantity (e + d) is called grating element, and two consecutive slits separated by the distance (e + d) are called corresponding points.



Analysis

Path difference between the rays from the slits S_1 and S_2 is

$$S_2K = S_1S_2 \sin\theta = (e+d)\sin\theta$$

The corresponding phase difference $= \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$ (say)

Hence, the resultant amplitude in the direction θ is obtained from

$$R = \frac{A' \sin(nd/2)}{\sin(d/2)}$$

where A' = resultant amplitude from each slit

$$R = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta} \qquad I = R^2 = \frac{A^2\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}$$

For n = 0, we get the zero-order maximum. For n = ±1, ±2, ±3,..., we obtain the first-order, second-order, third-order, … principal maxima, respectively. The ± sign shows that there are two principal maxima for each order lying on either side of the zero-order maximum.

Principal Maxima

When $\sin\beta = 0$, i.e., $\beta = \pm n\pi$, where $n = 0, 1, 2, 3, \ldots$, we have $\sin N\beta = 0$ and hence $\sin(N\beta)/\sin\beta = (0/0)$ which is an indeterminate quantity.

Therefore, taking limits and applying L′Hospital's rule of mathematics

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

The intensity is then

$$I = R^2 = \frac{A^2\sin^2\alpha}{\alpha^2} N^2$$

which is maximum. These maxima are most intense and are called principal maxima. They are obtained in the directions given by

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = \pm n\lambda^*$$

Minima

When $\sin N\beta = 0$ but $\sin\beta \neq 0$, then

$$\frac{\sin N\beta}{\sin\beta} = 0$$

which gives $I = 0$ is a minimum. These minima are obtained in the directions given by

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$N\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

$$N(e+d)\sin\theta = \pm m\lambda$$

- where m takes all integral values except O, N, 2N,……….. nN, because these values of m give sinβ = 0, which gives the principal maxima.

- It is also clear from above that there are (N – 1) minima between two successive principal maxima.

MISSING ORDERS OR ABSENT SPECTRA

The principal maxima in the grating spectrum are obtained in the directions given by

$$(e + d) \sin \theta = n\lambda \qquad (n = 0, 1, 2, 3, \ldots)$$

where (e + d) is the grating element and n is the order of the maximum.

The minima in a single slit pattern are obtained in the directions given by

$$e \sin \theta = m\lambda \qquad (m = 0, 1, 2, 3, \ldots)$$

- If both above conditions are simultaneously satisfied, a particular maximum of order n will be missing in the grating spectrum.
- The condition of missing order is
- $$\frac{e+d}{e} = \frac{n}{m}$$
- This is the condition for the spectrum of the order n to be absent.
- If d = e, then n = 2m = 2, 4, 6, … (for m = 1, 2, 3, …)

DETERMINATION OF WAVELENGTH OF LIGHT BY GRATING

- For grating, the condition of maximum intensity is
  - $(e + d) \sin \theta = n\lambda \qquad (n = 0, 1, 2, 3, \ldots)$
- Thus, if the grating element (e + d) and the angle of diffraction θ for a particular order n are determined, the wavelength can be obtained.
- Determination of (e + d): The grating element (e + d) is determined from the number of rulings per inch on the grating (written on the grating as LPI).

➢ If this number is N, then

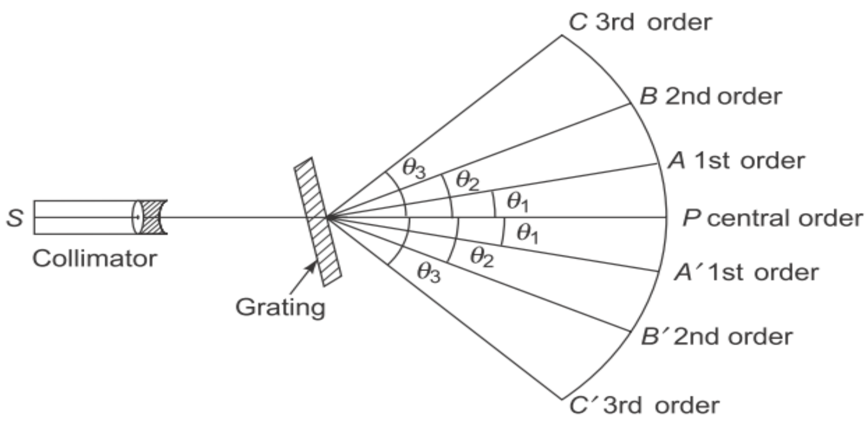
- $$N(e + d) = 1 \text{ in.} = 2.54 \text{ cm}$$

$$(e + d) = \frac{2.54}{N} \text{ cm}$$

Determination of θ

This is done with the help of a spectrometer whose slit is illuminated by the given light and the following adjustment are made:

- (a) The eyepiece of the telescope is focused on the cross-wires.
- (b) The collimator and telescope are adjusted for parallel rays.
- (c) The grating is adjusted on the prism table such that light from collimator falls normally on it with the help of levelling screws A and B.
- (d) The rulings of the grating are adjusted parallel to the axis of the spectrometer.
- (e) The rulings are adjusted parallel to the slit.



Assignment Based on this Lecture

- Discuss the Fraunhofer Diffraction at a Double Slit. Also obtain the expression of Principle Maxima and Minima.
- Discuss principle and working of plane diffraction grating.
- Obtain the expression of Principle Maxima and Minima.
- What do you mean by missing orders or absent spectra, Also obtain the condition for the same.
- Discuss the experimental arrangement for the determination of wavelength of light by grating.