MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclear Stability

Lecture-10

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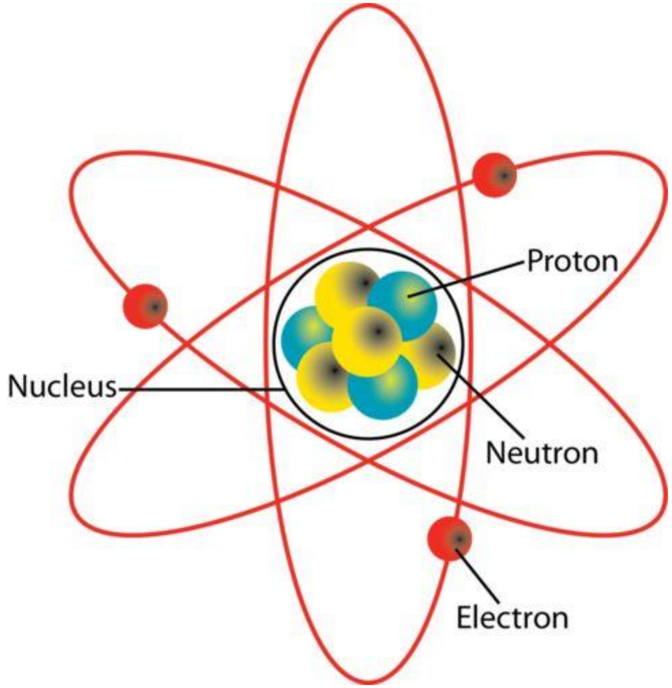
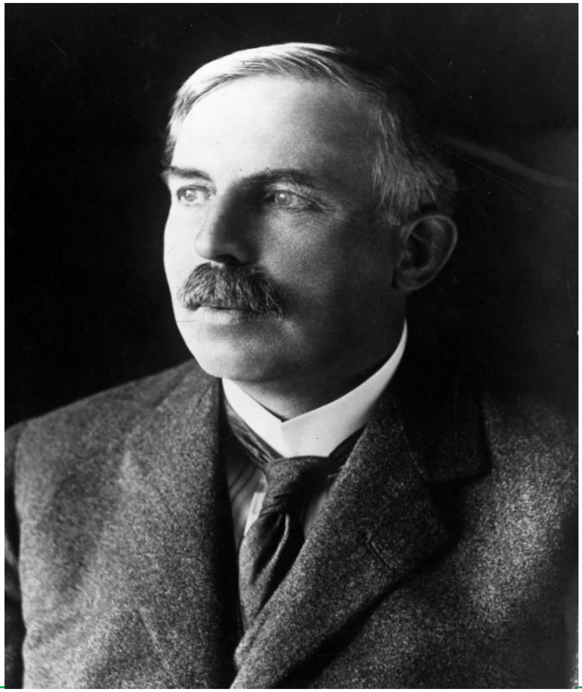




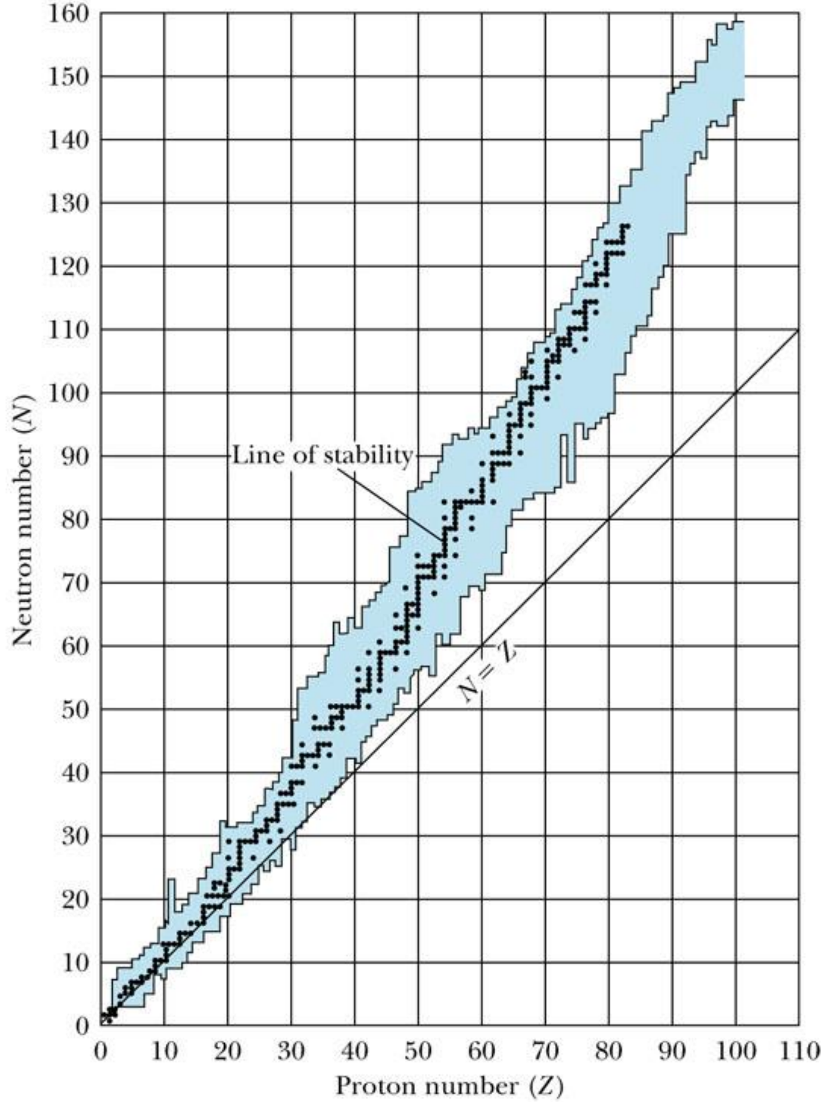
Nuclear Stability

- The binding energy of a nucleus against dissociation into any other possible combination of nucleons. Example nuclei *R* and *S*.

$$B = \left[M(R) + M(S) - M\left({}^{A}_{Z}X \right) \right] c^2$$

- Proton (or neutron) *separation energy*:
  - The energy required to remove one proton (or neutron) from a nuclide.

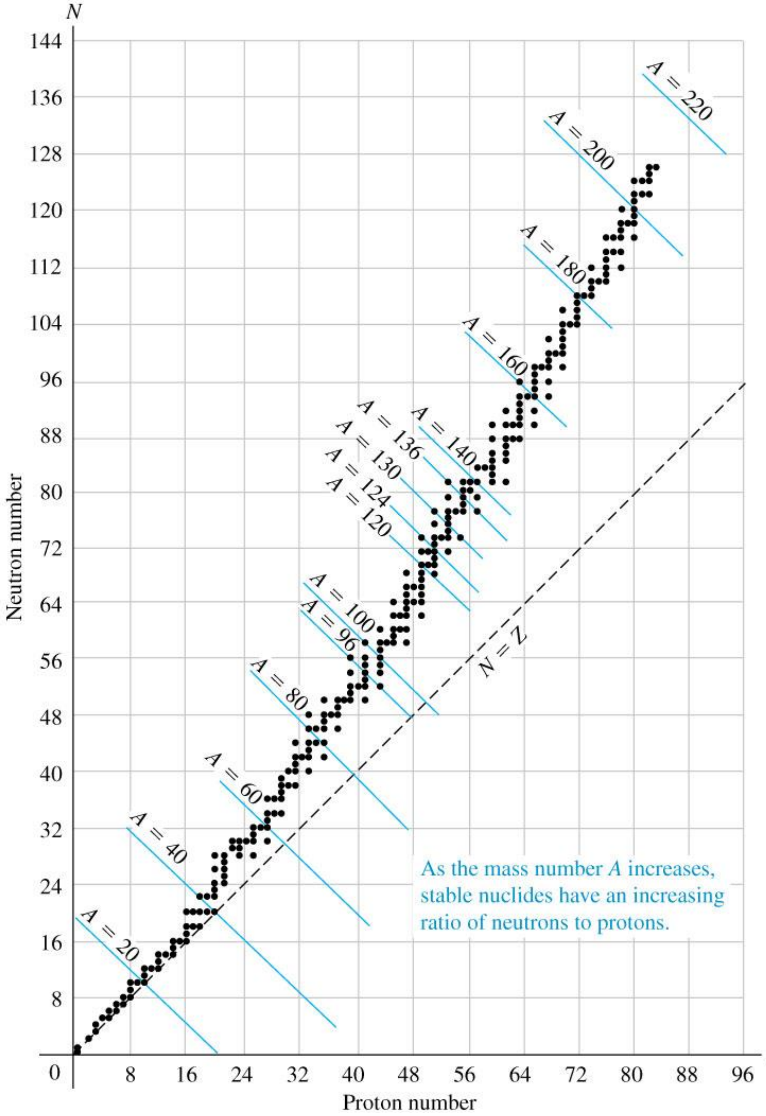
- All stable and unstable nuclei that are sufficiently long-lived to be observed.



Of the more than 3000 nuclides known, only 266 are stable.

Nuclear Stability

- *Radioactivity* is the decay of unstable nuclides by the emission of particles and electromagnetic radiation.
- Figure (right) is a Segrè chart showing *N* versus *Z* for stable nuclides. Notice that as A increases, the stable nuclides leave the *N* = *Z* line and prefer more neutrons that protons. This is probably due to the increasing influence of repulsion among the protons.
- There are no stable nuclides with *A* = 5 or 8. The nuclide decays immediately into two He nuclei.
- The highest *A* is 209. There are so-called islands of stability above this.



Nuclear Stability

- The line representing the stable nuclides is the **line of stability**.

- It appears that for $A \leq 40$, nature prefers the number of protons and neutrons in the nucleus to be about the same $Z \approx N$.

  However, for $A \geq 40$, there is a decided preference for $N > Z$ because the nuclear force is independent of whether the particles are *nn*, *np*, or *pp*.

- As the number of protons increases, the Coulomb force between all the protons becomes stronger until it eventually affects the binding significantly.

Nuclear Stability

Interestingly, most stable nuclei have even values of A. In fact, certain values of Z and N correspond to unusually high stability in nuclei. These values of N and Z, called **magic numbers,** are

$$Z \text{ or } N = 2, 8, 20, 28, 50, 82, 126 \quad (13.2)$$

- Most stable nuclides have both even *Z* and even *N* (even-even nuclides), e.g. ${}_{2}^{4}He$

Only four stable nuclides have odd *Z* and odd *N* (odd-odd nuclides).

$${}_{1}^{2}\mathrm{H},\ {}_{3}^{6}\mathrm{Li},\ {}_{5}^{10}\mathrm{B}, \text{ and } {}_{7}^{14}\mathrm{N}.$$

Radioactive Decay

- In the stability diagram above a particular value number of there are no nuclei.

- This limit is approximately Z=85 and N= 125. Above which there is no stable nuclei which is clear from the figure.

- This suggest that the heavy nuclei above to the mentioned values of Z and N is not stable.

- This can be be understood with help of SEMF based on BE.

- B = a_vA $-a_s(A^{2/3})$ - $a_c \frac{Z(Z-1)}{A^{1/3}}$ - $a_a \frac{(A-2Z)^2}{A} \pm \delta$

α- Decay

- Emission of particle from the nucleus means on part of it is very small and other is very large.
- The Smaller part may be **α**- particle, **β**- particle or **γ**-particles.
- If splitting is in comparable size then we will call it as fission.
- Emission of **α**- particles is the most dominant.
- The nucleus ^{4}He has a binding energy of 28.3 MeV.
- If two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle (alpha decay) is possible.

α- Decay

- ${}_{Z}^{A}X \;=\; {}_{Z-2}^{A-4}X$ + ${}_{2}^{4}He$ +Q (Energy) where Q is positive
- Q = [M(${}_{Z}^{A}X$) $- M({}_{Z-2}^{A-4}X$) – $\mathrm{M}({}_{2}^{4}He)$]c^2
- Q = [BE(${}_{Z-2}^{A-4}X$) +$\mathrm{BE}({}_{2}^{4}He)$ – BE(${}_{Z}^{A}X$)]
- Binding Energy of Helium is fixed and very high it can not be estimated by SEMF.
- Now Using SEMF we can find the values of Q for different combinations
- From the beata- stability mass parabola
- Z = $\frac{A/2}{1+0.0078A^{2/3}}$

α- Decay

- If we calculate the value of Q for different combinations of A and Z then it is observed that after 145 or 150, Q > 0 and below these values Q < 0.

- But from N-Z diagram it is clear that the nuclei having their values more than 150-225 are also stable.

Energy of the α- particles

- After the disintegration the of parent nuclei we get

- Parent Nuclei = Daughter Nuclei + Alfa Particle + Q

- Actually, Q is distributed among the N_d and $N_{\boldsymbol{\alpha}}$

- The linear momentum of the particles will be conserved.
- Both the fragments will have the momentum equal and opposite, thus

- $p_d = p_{\boldsymbol{\alpha}}$ = p Now the kinetic energy can be given as
- $\frac{p_d^2}{2m_d} + \frac{p_{\boldsymbol{\alpha}}^2}{2m_{\boldsymbol{\alpha}}}$ = Q

Energy of the α- particles

- $\frac{p^2}{2}\left(\frac{m_\alpha+m_d}{m_\alpha m_d}\right)$ = Q ; or $\frac{p^2}{2}$ = Q$\frac{m_\alpha m_d}{m_\alpha+m_d}$

$$K_\alpha = \frac{p^2}{2m_\alpha} = Q\frac{m_d}{m_\alpha+m_d} = \frac{Q(A-4)}{A} = Q\left(1-\frac{4}{A}\right)$$

$$K_d = \frac{p^2}{2m_d} = Q\frac{m_\alpha}{m_\alpha+m_d} = \frac{Q(4)}{A} = \frac{4Q}{A}$$

- To estimate the energy of alfa particle let A=200

- Now K_α = Q(1-$\frac{1}{50}$) = 98%

Alpha Decay

- From the conservation of energy and conservation of linear momentum, determine a unique energy for the alpha particle.

$$Q = K_\alpha + K_D$$

$$p_\alpha = p_D$$

$$K_\alpha = Q - K_D = Q - \frac{{p_D}^2}{2M_D} = Q - \frac{{p_\alpha}^2}{2M_D}$$

$$K_\alpha = Q - \frac{2M_\alpha K_\alpha}{2M_D} = Q - \frac{M_\alpha}{M_D} K_\alpha$$

$$K_\alpha \left(1 + \frac{M_\alpha}{M_D}\right) = Q$$

$$K_\alpha = \frac{M_D}{M_D + M_\alpha} Q \approx \left(\frac{A-4}{A}\right) Q$$

Radioactive Decay

- An empirical law that is fulfilled only statistically

- Marie Curie and her husband Pierre discovered polonium and radium in 1898.
  - The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.
  - Other modes of decay include emission of α particles, *β* (– and +) particles, protons, neutrons, and fission.

- The decays per unit time (**activity**).

$$\text{Activity} = -\frac{dN}{dt} = R$$

where *dN* / *dt* is negative because total number *N* decreases with time.

Radioactive Decay

- If N(t) is the number of radioactive nuclei in a sample at time *t*, and λ (**decay constant**) is the probability per unit time that any given nucleus will decay:

$$R = \lambda N(t)$$

$$dN(t) = -R\,dt = -\lambda N(t)\,dt$$

$$\int \frac{dN}{N} = -\int \lambda\,dt$$

$$\ln N = -\lambda t + \text{constant}$$

$$N(t) = e^{-\lambda t + \text{constant}}$$

$$-\frac{dN}{dt} = R$$

$$N(t) = N_0 e^{-\lambda t}$$

----- radioactive decay law

Radioactive Decay

The activity R is also

$$-\frac{dN}{dt} = R$$

$$R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

where R_0 is the initial activity at $t = 0$.

half-life $t_{1/2}$ or the "mean lifetime" τ are defined on basis of decay constant.

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\ln\left(\tfrac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

Radioactive Decay

The mean lifetime is

$$\tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln(2)}$$

$$-\frac{dN}{dt} = R$$

The time it takes to arrive at N_o/e

The Activity of Carbon $^{11}_{6}C$

The number of radioactive nuclei as a function of time

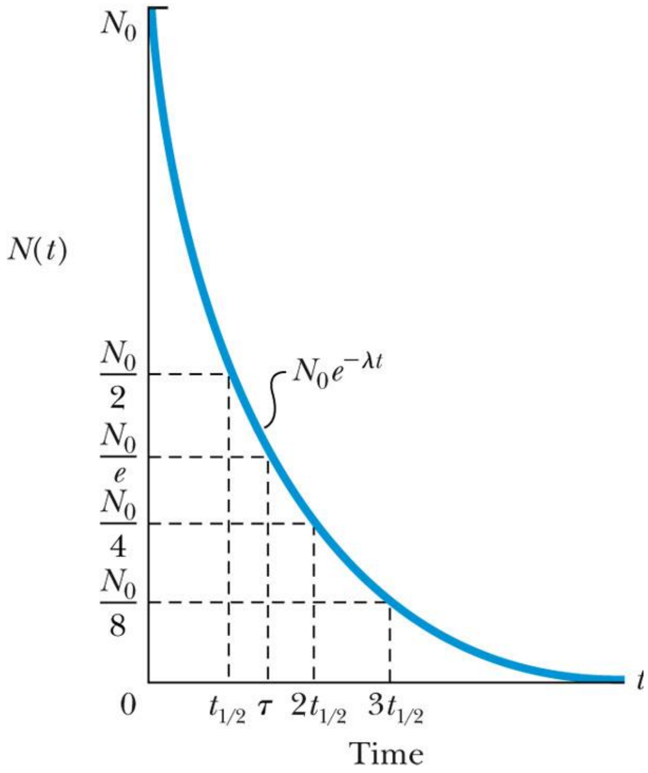


Table 13.3 Activity Versus Time for the Sample Described in Example 13.6

t (h)	R (decays/s)
0	1.08×10^{14}
1	1.41×10^{13}
2	1.84×10^{12}
3	2.39×10^{11}
4	3.12×10^{10}
5	4.06×10^{9}
6	5.28×10^{8}
7	6.88×10^{7}
8	8.96×10^{6}

EXAMPLE 13.7 A Radioactive Isotope of Iodine

A sample of the isotope ^{131}I, which has a half-life of 8.04 days, has an activity of 5 mCi at the time of shipment. Upon receipt of the ^{131}I in a medical laboratory, its activity is 4.2 mCi. How much time has elapsed between the two measurements?

Solution We can make use of Equation 13.10 in the form

$$\frac{R}{R_0} = e^{-\lambda t}$$

Taking the natural logarithm of each side, we get

$$\ln\left(\frac{R}{R_0}\right) = -\lambda t$$

$$(1) \qquad t = -\frac{1}{\lambda} \ln\left(\frac{R}{R_0}\right)$$

To find λ, we use Equation 13.11:

$$(2) \qquad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{8.04 \text{ days}}$$

Substituting (2) into (1) gives

$$t = -\left(\frac{8.04 \text{ days}}{0.693}\right) \ln\left(\frac{4.2 \text{ mCi}}{5.0 \text{ mCi}}\right) = 2.02 \text{ days}$$

EXAMPLE 13.5 The Activity of Radium

The half-life of the radioactive nucleus ${}^{226}_{88}\mathrm{Ra}$ is about 1.6×10^3 yr. (a) What is the decay constant of ${}^{226}_{88}\mathrm{Ra}$?

Solution (a) We can calculate the decay constant λ by using Equation 13.11 and the fact that

$$\begin{aligned} T_{1/2} &= 1.6 \times 10^3 \text{ yr} \\ &= (1.6 \times 10^3 \text{ yr})(3.16 \times 10^7 \text{ s/yr}) \\ &= 5.0 \times 10^{10} \text{ s} \end{aligned}$$

Therefore,

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.0 \times 10^{10}\text{ s}} = 1.4 \times 10^{-11}\text{ s}^{-1}$$

Note that this result gives the probability that any single ${}^{226}_{88}\mathrm{Ra}$ nucleus will decay in 1 s.

(b) If a sample contains 3.0×10^{16} such nuclei at $t = 0$, determine its activity at this time.

Solution (b) We can calculate the activity of the sample at $t = 0$ using $R_0 = \lambda N_0$, where R_0 is the decay rate at $t = 0$ and N_0 is the number of radioactive nuclei present at $t = 0$. Since $N_0 = 3.0 \times 10^{16}$, we have

$$\begin{aligned} R_0 = \lambda N_0 &= (1.4 \times 10^{-11}\text{ s}^{-1})(3.0 \times 10^{16}) \\ &= 4.2 \times 10^5 \text{ decays/s} \end{aligned}$$

Since $1 \text{ Ci} = 3.7 \times 10^{10}$ decays/s, the activity, or decay rate, at $t = 0$ is

$$R_0 = 11.1\ \mu\text{Ci}$$

(c) What is the decay rate after the sample is 2.0×10^3 yr old?

Solution (c) We can use Equation 13.10 and the fact that $2.0 \times 10^3 \text{ yr} = (2.0 \times 10^3 \text{ yr})(3.15 \times 10^7 \text{ s/yr}) = 6.3 \times 10^{10}$ s:

$$\begin{aligned} R &= R_0 e^{-\lambda t} \\ &= (4.2 \times 10^5 \text{ decays/s})\, e^{-(1.4\times 10^{-11}\text{ s}^{-1})(6.3\times 10^{10}\text{ s})} \\ &= 1.7 \times 10^5 \text{ decays/s} \end{aligned}$$

EXAMPLE 13.11 Radioactive Dating

An archaeologist finds a 25.0-g piece of charcoal in the ruins of an ancient city. The sample shows a ^{14}C activity of 250 decays/min. How long has the tree from which this charcoal came been dead?

Solution First calculate the decay constant for ^{14}C, which has a half-life of 5730 yr.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5730\text{ yr})(3.16 \times 10^7\text{ s/yr})}$$
$$= 3.83 \times 10^{-12}\text{ s}^{-1}$$

The number of ^{14}C nuclei can be calculated in two steps. (1) The number of ^{12}C nuclei in 25.0 g of carbon is

$$N(^{12}\text{C}) = \frac{6.02 \times 10^{23}\text{ nuclei/mol}}{12.0\text{ g/mol}}(25.0\text{ g})$$
$$= 1.26 \times 10^{24}\text{ nuclei}$$

Knowing that the ratio of ^{14}C to ^{12}C in the live sample was 1.3×10^{-12}, we see that the number of ^{14}C nuclei in 25.0 g *before* decay is

$$N_0(^{14}\text{C}) = (1.3 \times 10^{-12})(1.26 \times 10^{24})$$
$$= 1.6 \times 10^{12}\text{ nuclei}$$

Hence the initial activity of the sample is

$$R_0 = N_0\lambda = (1.6 \times 10^{12}\text{ nuclei})(3.83 \times 10^{-12}\text{ s}^{-1})$$
$$= 6.13\text{ decays/s} = 370\text{ decays/min}$$

(2) We can now calculate the age of the charcoal, using Equation 13.10, which relates the activity R at any time t to the initial activity R_0:

$$R = R_0 e^{-\lambda t} \qquad \text{or} \qquad e^{-\lambda t} = \frac{R}{R_0}$$

Because it is given that $R = 250$ decays/min and because we found that $R_0 = 370$ decays/min, we can calculate t by taking the natural logarithm of both sides of the last equation:

$$-\lambda t = \ln\left(\frac{R}{R_0}\right) = \ln\left(\frac{250}{370}\right) = -0.39$$

$$t = \frac{0.39}{\lambda} = \frac{0.39}{3.84 \times 10^{-12}\text{ s}^{-1}}$$
$$= 1.0 \times 10^{11}\text{ s} = 3.2 \times 10^3\text{ yr}$$

Table 11-8 Selected naturally occurring isolated radioactive nuclides

Nuclide	$t_{1/2}$ (y)	Abundance (%)	Daughter
^{14}C	5730	1.35×10^{-10}	r^{14}N
^{40}K	1.25×10^{9}	0.0117	^{40}A
^{87}Rb	4.88×10^{10}	27.83	^{87}Sr
^{147}Sm	1.06×10^{11}	15.0	^{143}Nd
^{176}Lu	3.59×10^{10}	2.59	^{176}Hf
^{187}Re	4.30×10^{10}	62.60	^{187}Os

αdecay

Note that the total number of nucleons does not change