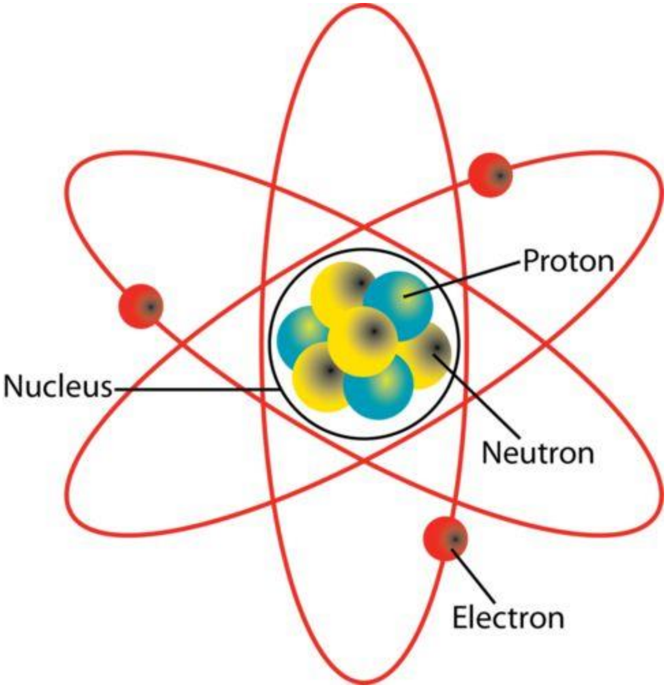
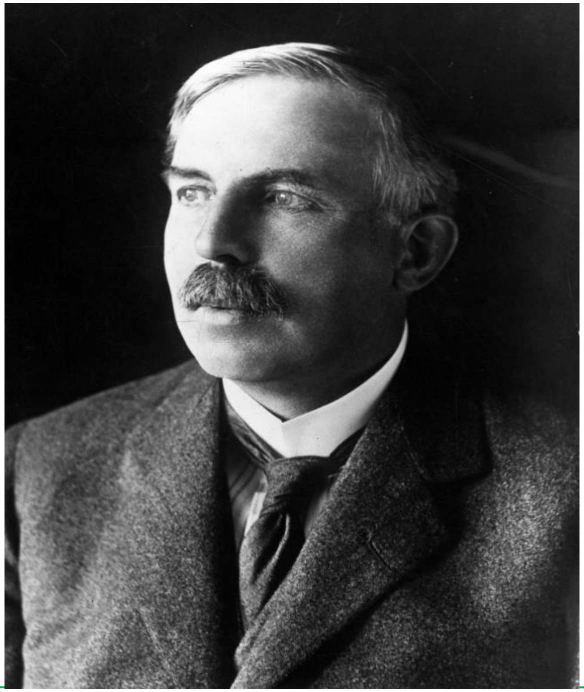
MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-7

By Prof. B. K. Pandey, Dept. of Physics and Material Science





Mass Defect and Packing Fraction

- The **atomic number,** Z (sometimes called the *charge number*), which equals the number of protons in the nucleus
- The **neutron number,** N, which equals the number of neutrons in the nucleus.
- The **mass number,** A, which equals the number of nucleons (neutrons plus protons) in the nucleus.

The isotopes of an element have the same Z value but different N and A values.

The natural abundances of isotopes can differ substantially. For example, ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$, and ${}^{14}_{6}C$ are four isotopes of carbon. The natural abundance of the ${}^{12}_{6}C$ isotope is about 98.9%, whereas that of the ${}^{13}_{6}C$ isotope is only about 1.1%. Some isotopes do not occur naturally but can be produced in the laboratory through nuclear reactions. Even the simplest element, hydrogen, has isotopes: ${}^{1}_{1}H$, the ordinary hydrogen nucleus; ${}^{2}_{1}H$, deuterium; and ${}^{3}_{1}H$, tritium.

Important Parameters

Table 13.1 Masses of the Proton, Neutron, and Electron in Various Units

	Mass		
Particle	**kg**	**u**	**MeV/c^2**
Proton	$1.672\,623 \times 10^{-27}$	1.007 276	938.272 3
Neutron	$1.674\,929 \times 10^{-27}$	1.008 665	939.565 6
Electron	$9.109\,390 \times 10^{-31}$	$5.48\,579\,9 \times 10^{-4}$	0.510 999 1

$$r = r_0 A^{1/3}$$

Table 13.2 Masses, Spins, and Magnetic Moments of the Proton, Neutron, and Electron

Particle	Mass (MeV/c^2)	Spin	Magnetic Moment
Proton	938.28	$\frac{1}{2}$	$2.7928\mu_n$
Neutron	939.57	$\frac{1}{2}$	$-1.9135\mu_n$
Electron	0.510 99	$\frac{1}{2}$	$-1.0012\mu_B$

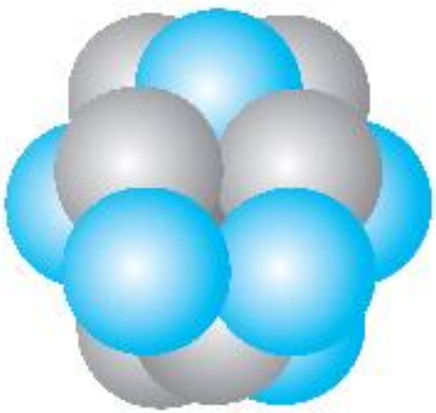


Figure 13.3 A nucleus can be modeled as a cluster of tightly packed spheres, each of which is a nucleon.

The nuclear density is approximately 2.3×10^{14} times as great as the density of water ($\rho_{\text{water}} = 1.0 \times 10^3\ \text{kg/m}^3$)!

Energy-related Issues

- What do you think $E = mc^2$ means?

- I prefer to think of it as:

$$\Delta E = \Delta mc^2$$

- Whenever there is an energy change in the system, there is an associated mass change. ???

  - Whenever a system loses energy (gives off energy [e.g., exothermic process], the energy “comes from” converting a bit of its *mass* to create that energy!

  - Mass is not “absolutely” conserved—just “almost”!

Energy-related Issues

- For any exothermic chemical reaction, the mass of the products is ever so slightly *less than* the mass of the reactants!
  - It's just such a tiny mass that you can almost never detect it!
- Consider that our balances can measure to the nearest 0.0001 g. Let's consider how much energy would have to be released if an amount of mass 100x smaller than what it can detect were to be converted to energy: 0.000001 g (10^{-6} g).

Unit Considerations!

$$\Delta E = \Delta mc^2$$

- Consider SI units:
- Mass is in __ kg___
- c is in ___m/s__
- Δ E is in _ kgx(m/s)^2_= J___________
- Now the energy corresponding to mass 0.000001 g (10^{-6} g).
- $\Delta E = \Delta mc^2$
- $\Delta E = 10^{-9}$ kg x $(3 \, x \, 10^8)^2$ $(m/s)^2$ = $9x10^5$ J

Binding Energy and Mass Defect (for a nuclide)

- Imagine 2 nucleons coming together to form a nucleus (e.g. p + n → ^{2}H nucleus)
  - Energy released or absorbed?
- What about 6 nucleons (or 10, or 100, or *any* number)?
  - Energy released or absorbed?
- So…what should happen to mass during this process (i.e., whenever "free" nucleons form a nucleus)?

Energy "lost" = "binding energy"

- Mass of nucleus is always _Smaller_ than the combined mass of the free nucleons!

Mass "lost" = "mass defect"

Mass Defect for C-16 (example)

Difference is "mass defect"!

- See Board
  - Consider an atom of C-16
    - # p's? _6__ # n's? _10__ # e's? _6__
  - If the *atomic* mass of C-16 is 16.014701 amu, how much mass does (just) the nucleus have?

16.014701- (6 x 0.00054858) =16.01141 amu

    - mass of an electron = 0.00054858 amu
  - How much mass is in 6 p's and 10 n's?
    - mass of a proton = 1.00728 amu **x 6**
    - mass of a neutron = 1.00866 amu **x 10**

16.13028 amu

To calculate Mass Defect From "mass data"…

(Mines method in some answer keys [and some Mastering problems!])

- Let mass defect be abbreviated Δm_{md}

Δm_{md} = mass of free <u>nucleons</u> – mass of nucleus

= m(p's + n's) – m(nucleus)

≈ m(p's + n's) – [m(***atom***) – m(e's)]

[(6 x 1.00782) + 10 x 1.00866] – [16.014701- (6 x 0.00054858)]

16.13028 – 16.01141

= 0.11887 amu

I'll describe how Tro does it in a minute…

Important Clarification

- Note: Although binding energy technically refers to the E required to separate a **nucleus** into free **nucleons**, and thus "mass defect" represents the difference between the "mass of free **nucleons**" and the "mass of the **nucleus**", the way we *calculate* mass defect from mass data usually involves a slightly different quantity because experimentally it is the mass of an ***atom*** that is known, not the mass of "just" the nucleus.

To calculate Mass Defect From "mass data"...(rationalizing Tro's approach)

- Let mass defect be abbreviated Δm_{md}

Δm_{md} = mass of free nucleons – mass of nucleus

= m(p's + n's) – m(nucleus)

≈ m(p's + e's + n's) – m(nucleus + e's)

(bonded) (bonded)

= m(H ***atoms*** + n's) – m(***atom***) **Tro**

This "works" because the energy lowering associated with binding the electrons to the nucleus (electrostatic force at large distance) is almost negligible relative to the energy lowering associated with binding the nucleons to one another (strong force at small distance)

EXAMPLE: Mass Defect and Nuclear Binding Energy

Calculate the mass defect and nuclear binding energy per nucleon (in MeV) for C-16, a radioactive isotope of carbon with a *mass of 16.014701 amu.**

***Means *atomic* mass here, not nuclear mass!**

= m(H *atoms* + n's) – m(*atom*) [prior slide]

SOLUTION

Calculate the mass defect as the difference between the mass of one C-16 atom and the sum of the masses of 6 hydrogen atoms and 10 neutrons.	$\text{Mass defect} = 6(\text{mass } {}^1_1\text{H}) + 10(\text{mass } {}^1_0\text{n}) - \text{mass } {}^{16}_6\text{C}$ $= 6(1.00783)\text{ amu} + 10(1.00866)\text{ amu} - 16.014701\text{ amu}$ $= 0.118879\text{ amu}$
Calculate the nuclear binding energy by converting the mass defect (in amu) into MeV. (Use 1 amu = 931.5 MeV.)*	$0.118879\text{ amu} \times \frac{931.5\text{ MeV}}{\text{amu}} = 110.74\text{ MeV}$

***Tro's solution disappoints me! I want you to be able to use E = mc²! Otherwise there's little "learning value". So:**

$$\underbrace{\underbrace{0.118879\,\text{amu} \times \frac{1.6605\times10^{-27}\text{ kg}}{1\,\text{amu}}}_{\text{m (in kg)}} \times \left(\frac{2.9979\times10^{8}\text{ m}}{\text{s}}\right)^2}_{\text{mc}^2\text{ (in J)}} \times \underbrace{\frac{1\,\text{MeV}}{1.6022\times10^{-13}\text{ J}}}_{\text{converts to MeV}} = 110.729..\text{MeV}$$

EXAMPLE 19.7 Mass Defect and Nuclear Binding Energy

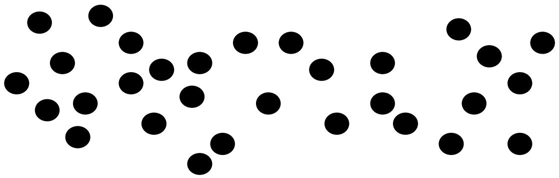
Calculate the mass defect and nuclear binding energy per nucleon (in MeV) for C-16, a radioactive isotope of carbon with a mass of 16.014701 amu.

SOLUTION

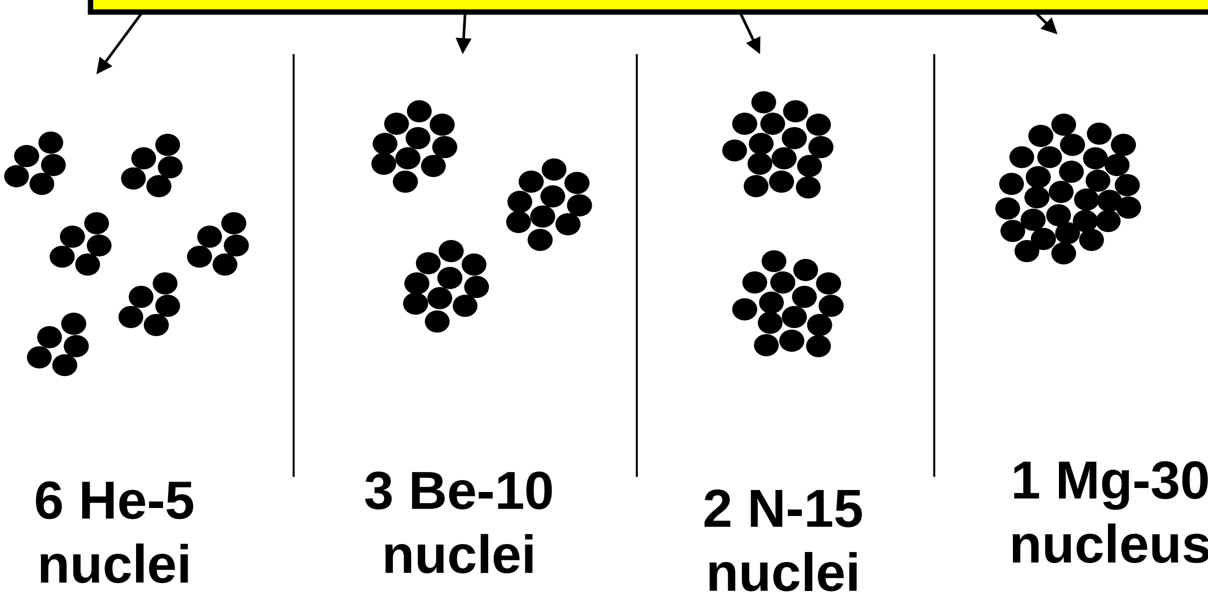
Calculate the mass defect as the difference between the mass of one C-16 atom and the sum of the masses of 6 hydrogen atoms and 10 neutrons.	$\text{Mass defect} = 6(\text{mass } {}^{1}_{1}\text{H}) + 10(\text{mass } {}^{1}_{0}\text{n}) - \text{mass } {}^{16}_{6}\text{C}$ $= 6(1.00783)\text{ amu} + 10(1.00866)\text{ amu} - 16.014701\text{ amu}$ $= 0.118879\text{ amu}$
Calculate the nuclear binding energy by converting the mass defect (in amu) into MeV. **(Use 1 amu = 931.5 MeV.)***	$0.118879\text{ amu} \times \frac{931.5\text{ MeV}}{\text{amu}} = 110.74\text{ MeV}$
Determine the nuclear binding energy per nucleon by dividing by the number of nucleons in the nucleus.	$\text{Nuclear binding energy per nucleon} = \frac{110.74\text{ MeV}}{16\text{ nucleons}}$ $= 6.921\text{ MeV/nucleon}$

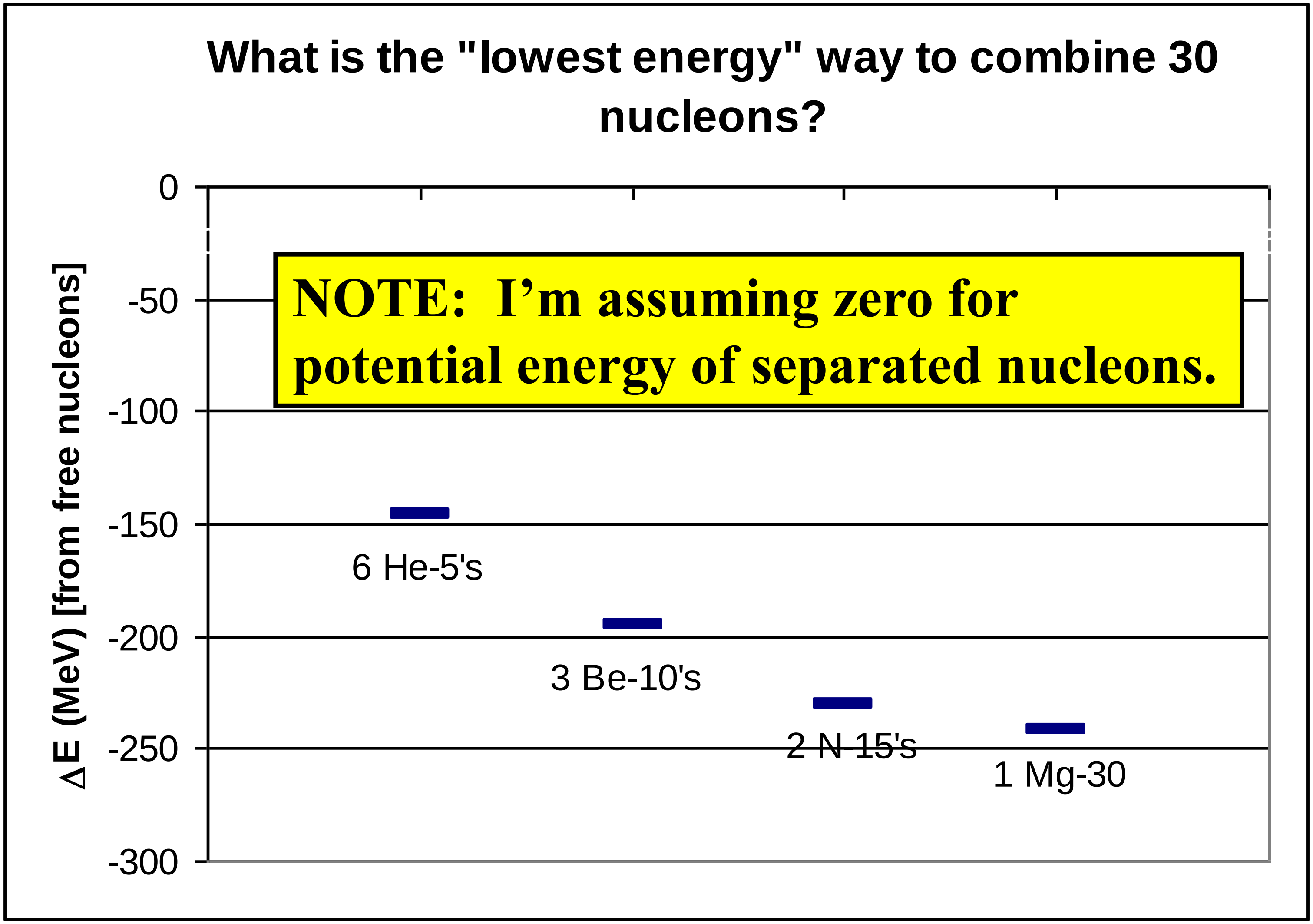
Binding Energy *per nucleon* indicates the thermodynamic stability of a nucleus

- Although we typically think that being "low in (potential) energy" is associated with more stability, that isn't quite so for nuclei.
  - The different number of nucleons in different nuclei make E_b an "unfair" comparison.
- Dividing E_b by the number of nucleons (E_b per nucleon) allows for a fair comparison!
  - It's like comparing the price of two boxes of cereal, one with 11 oz and one with 16 oz. If you find the "price per ounce" you can tell which is the better buy!



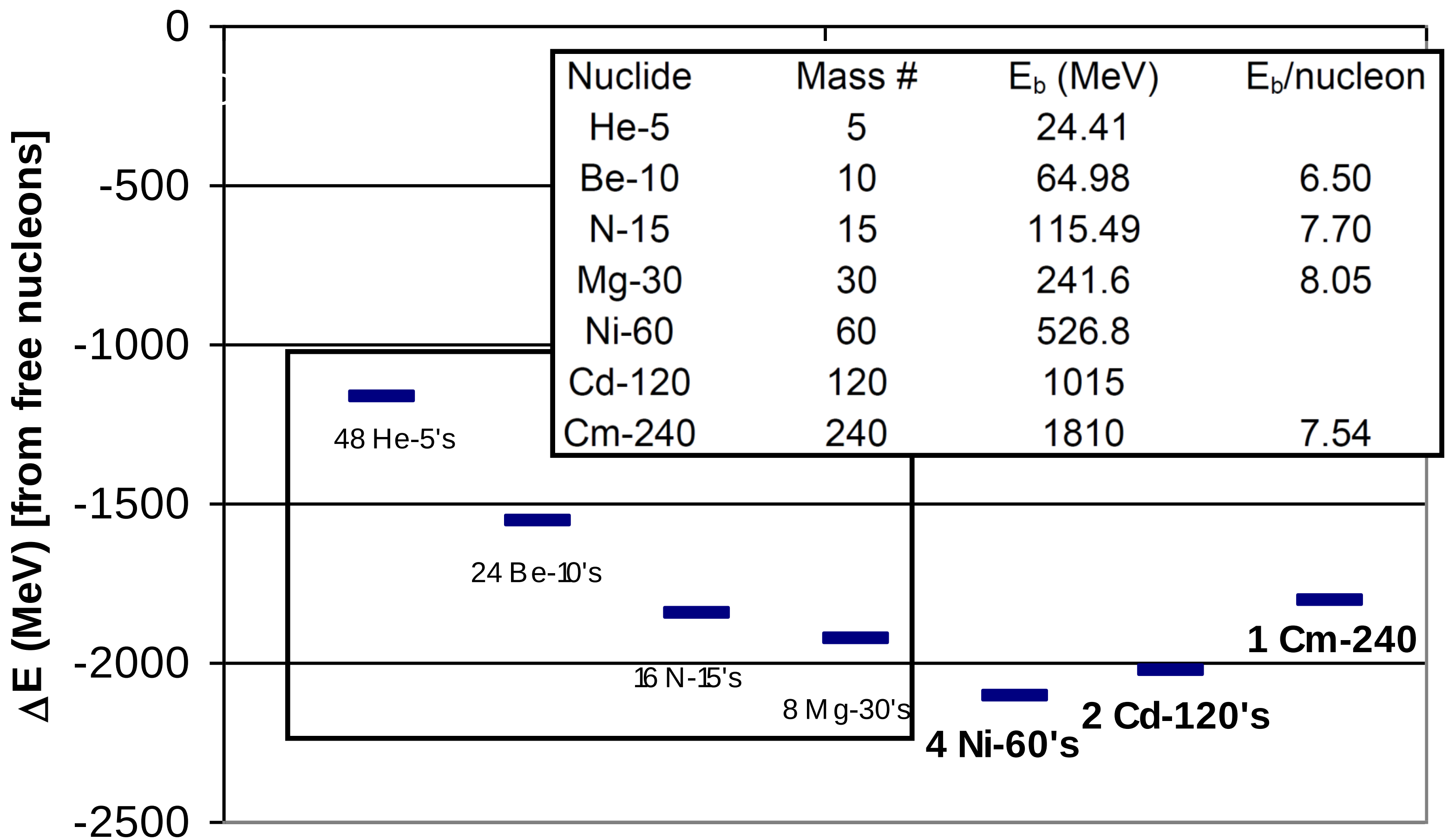
If separated nucleons had zero potential energy, the nuclides (bound nucleons) would have *negative* potential energy (lower than zero).




What is the "lowest energy" way to combine 30 nucleons?
NOTE: I'm assuming zero for potential energy of separated nucleons.
ΔE (MeV) [from free nucleons]
0
-50
-100
-150
-200
-250
-300
6 He-5's
3 Be-10's
2 N 15's
1 Mg-30

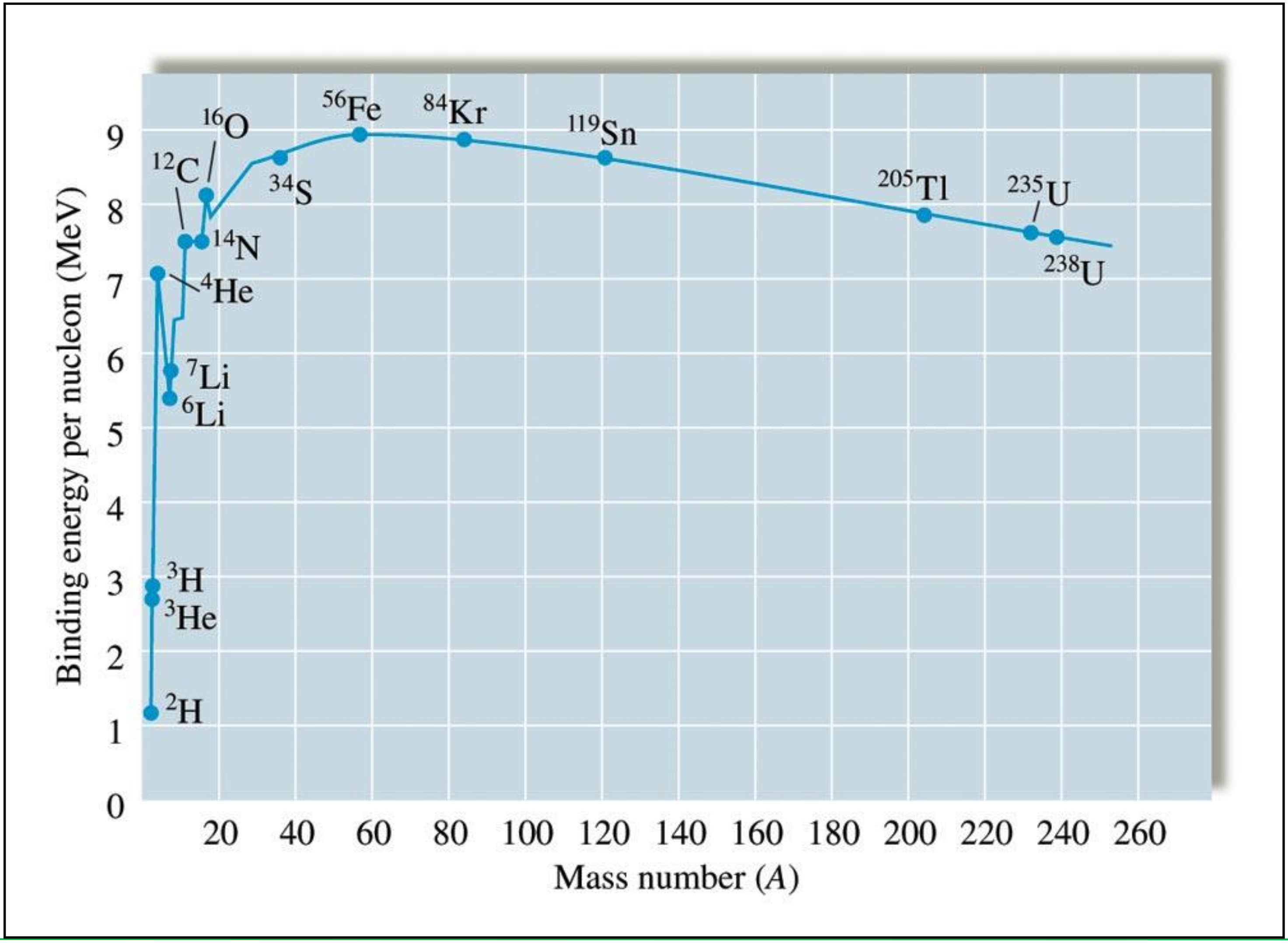
Does it continue this way if we consider combining *larger* amounts of nucleons? Say, six times more (i.e., 240)?

What is the "lowest energy" way to combine 240 nucleons?

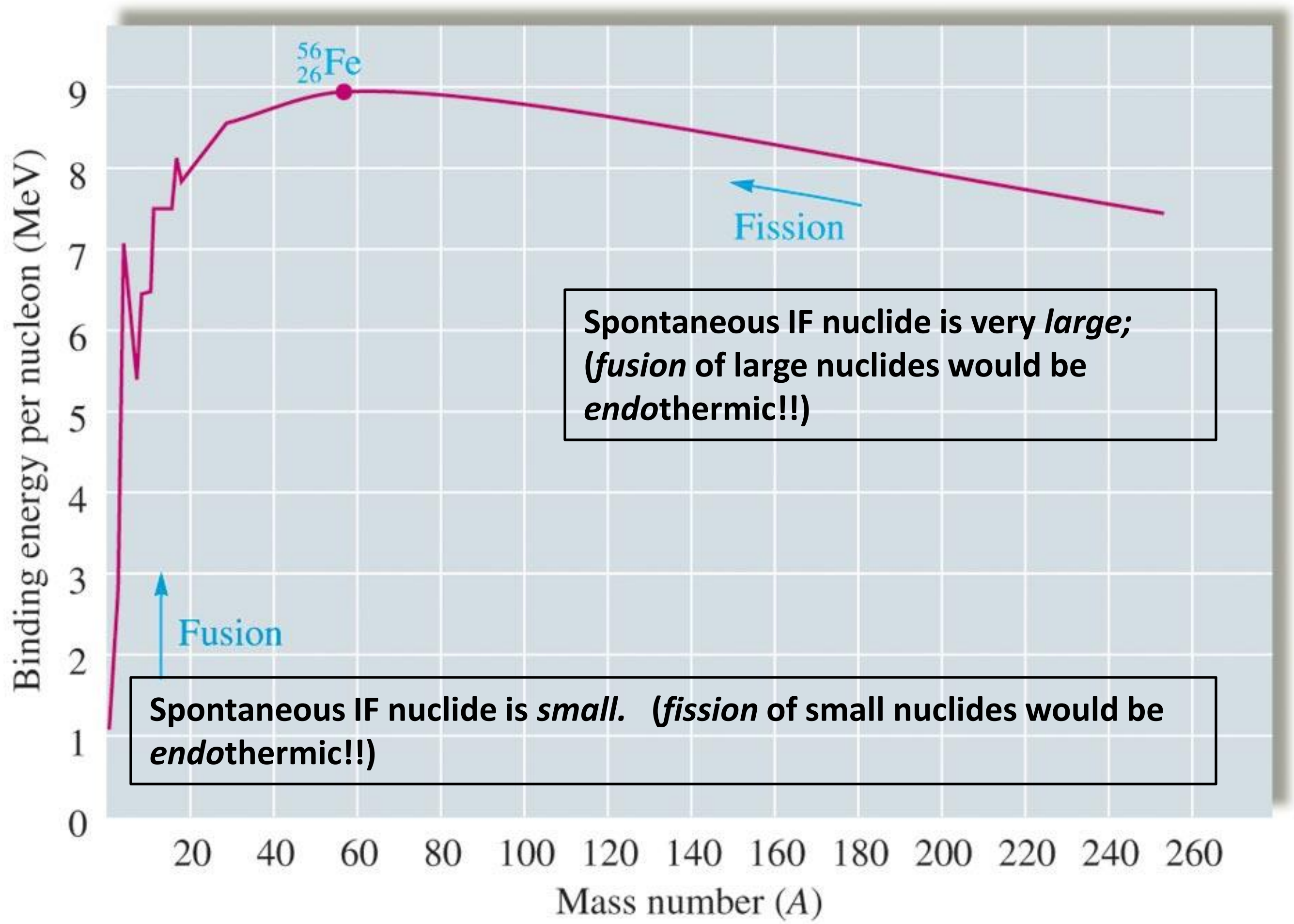


Nuclide	Mass #	E_b (MeV)	E_b/nucleon
He-5	5	24.41	
Be-10	10	64.98	6.50
N-15	15	115.49	7.70
Mg-30	30	241.6	8.05
Ni-60	60	526.8	
Cd-120	120	1015	
Cm-240	240	1810	7.54

The Binding Energy per Nucleon as a Function of Mass Number



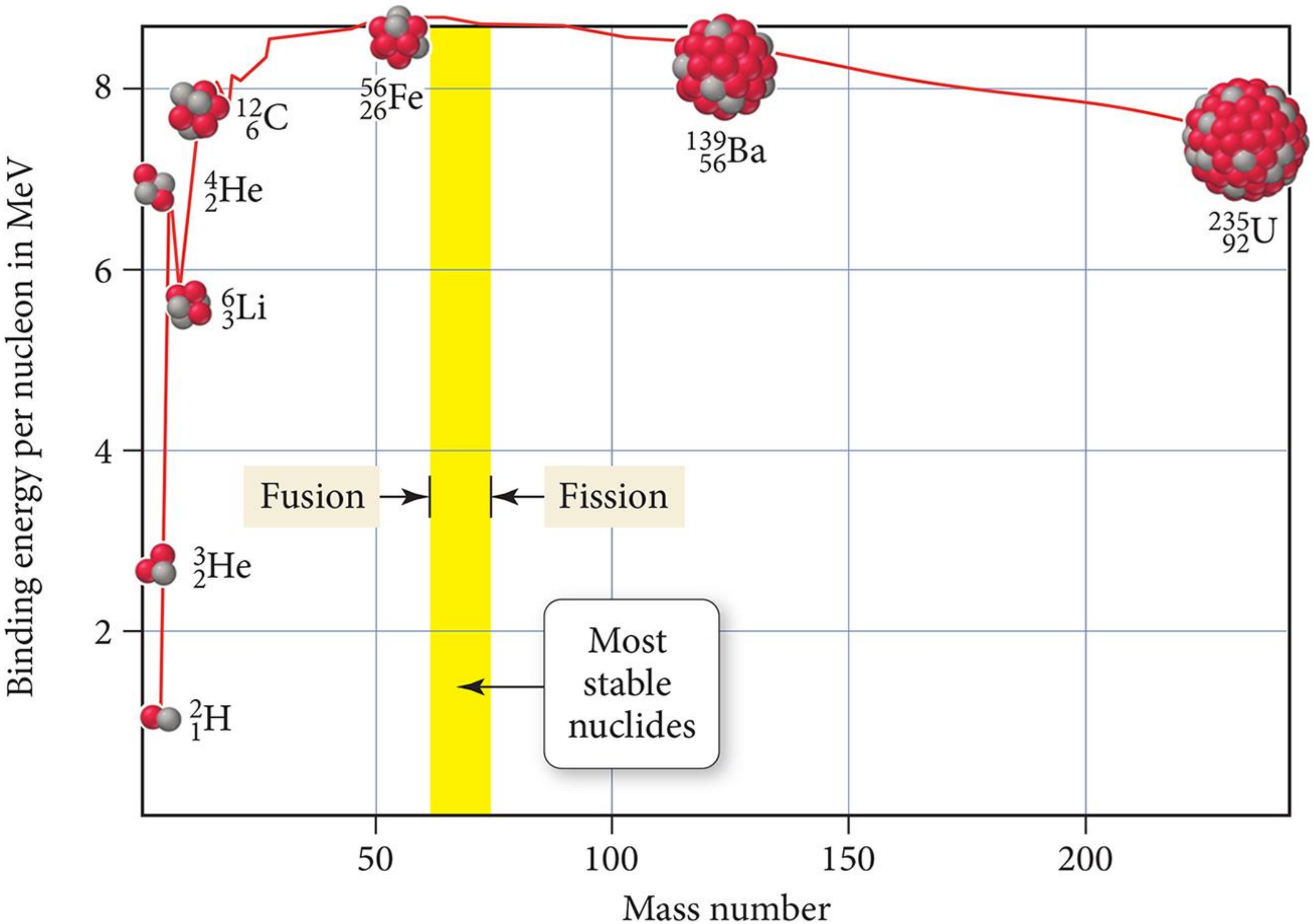
Both Fission and Fusion CAN Produce More Stable Nuclides and are thus Exothermic



Spontaneous IF nuclide is very *large;* (*fusion* of large nuclides would be *endo*thermic!!)

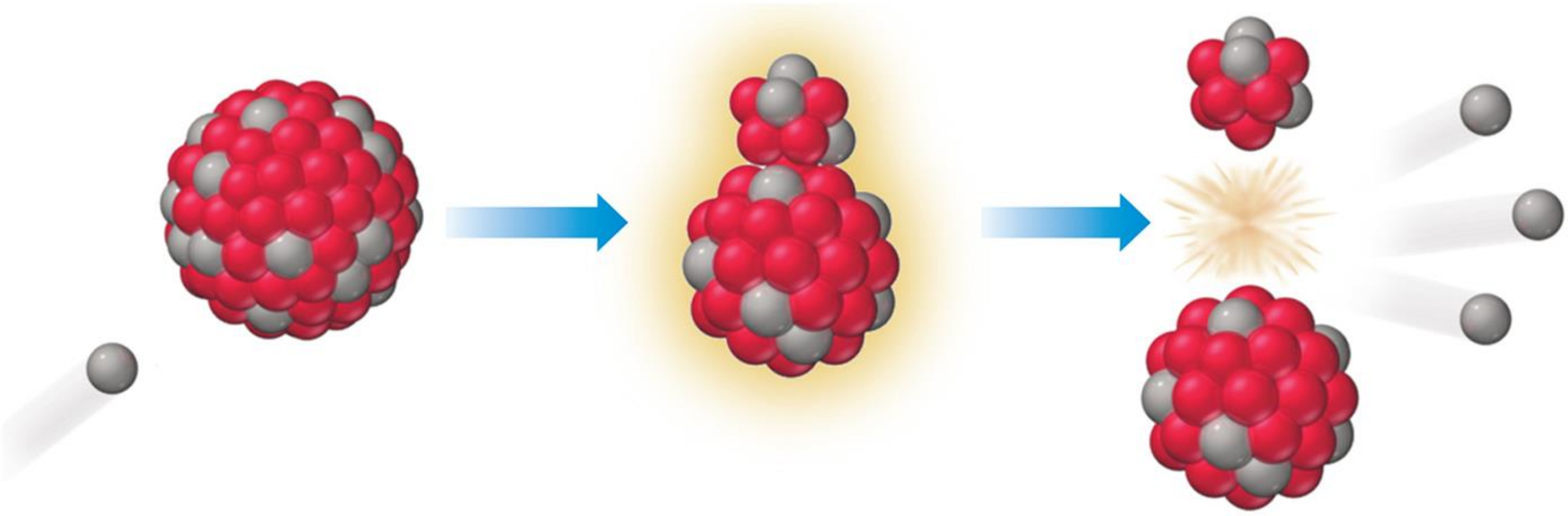
Spontaneous IF nuclide is *small.* (*fission* of small nuclides would be *endo*thermic!!)

The Curve of Binding Energy



Fission

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \longrightarrow ^{140}_{56}\text{Ba} + ^{93}_{36}\text{Kr} + 3\,^{1}_{0}\text{n} + \text{energy}$$



Fission Chain Reaction