UNIT-III

In most of the situation, free space path loss model does not exist. When the height of Tx and Rx is small, two-ray model is more applicable. This is discussed next.

2-Ray Model:

Some realistic example

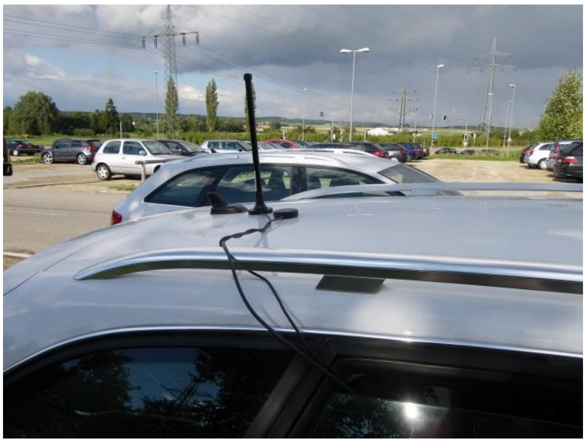


Fig. Omnidirectional Tx and GPS receiver mounted on a Car [1]

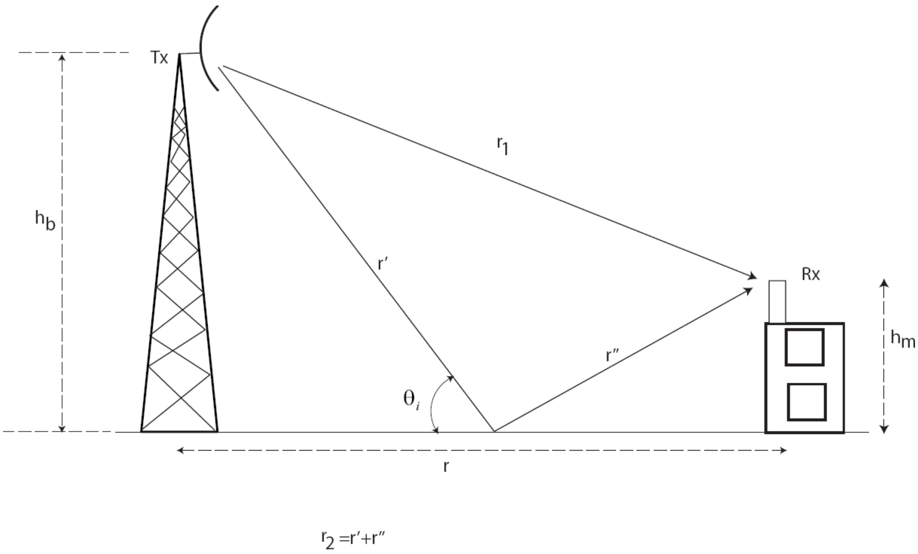


Figure 2: Two-ray model

A practical scenario of two-ray model is shown in Figure 2. Our objective is to compute total field and power at the receiver. Two different path length from Tx to Rx are computed as

$$r_1 = \sqrt{(h_b - h_m)^2 + r^2} \tag{1}$$

$$r_2 = \sqrt{(h_b + h_m)^2 + r^2} \tag{2}$$

Equation (2) is obtain by extending Transmitter below the ground and finding the image source.

$$(r_1 - r_2) = [\sqrt{\{r^2 + (h_b + h_m)^2\}} - \sqrt{\{r^2 + (h_b - h_m)^2\}}\,]$$

$$= r[\sqrt{\left\{1 + \left(\frac{h_b+h_m}{r}\right)^2\right\}} - \sqrt{\{1 + \left(\frac{h_b-h_m}{r}\right)^2\}}]$$

Assuming h_b,h_m<< r, then term $\left(\frac{h_b+h_m}{r}\right)^2 \ll 1$

We know that

$$(1 + x)^{\frac{1}{2}} \cong 1 + \frac{x}{2} \qquad \text{if } x<<1$$

so, $(r_1 - r_2) = \frac{r}{2r^2}[(h_b + h_m)^2 - (h_b - h_m)^2]$

$$(r_1 - r_2) \cong \frac{2h_b h_m}{r}$$

Thus, we note that with $r \to \infty$, the path difference $(r_1 - r_2)$ tends towards zero. That is why, at very large separation, both the paths, that is, direct and reflected path have almost same distance and both the signals at Rx are going through phase reversal only due to reflection from the ground.

$$E_{total} = E_{direct} + E_{reflected} \tag{3}$$

Assuming the initial phase of thetransmitted signal at the Tx is zero, let the phase of LOS component signal at the Rx be ϕ_1 and that of reflected component at the Rx be $\phi_1 + \Delta\phi$ where $\Delta\phi$ is the phase difference between LOS and reflected components. Then we can easily write

$$E_{direct} = E_0 \exp(-jk\phi_1)$$
$$E_{reflected} = E_0 \times R \times \exp(-jk(\phi_1 + \Delta\phi))$$

Thus, $E_{reflected}$ can be written in term of E_{direct} as

$$E_{reflected} = E_{direct} \times R \times \exp\left(jk\frac{2h_b h_m}{r}\right) \tag{4}$$

Here, it is assumed that the receiver is far away from transmitter and the amplitude of direct ray and reflected ray (except the reflection loss R) is almost same.

Putting (4) in (3),

$$E_{total} = E_{direct} + E_{direct} \times R \times \exp\left(jk\frac{2h_b h_m}{r}\right)$$

$=E_{direct}[1+ R \times \exp\,(jk\frac{2h_b h_m}{r})]$

$\frac{|E_{total}|}{|E_{direct}|} = |1+ R \times \exp\left(jk\frac{2h_b h_m}{r}\right)|$

$$\frac{P_r}{P_{direct}} = \frac{|E_{total}|^2}{|E_{direct}|^2} = |1 + \; R \times \exp\left(jk\frac{2h_b h_m}{r}\right)|^2$$

Assuming $G_t = 1, G_r = 1$, (omnidirectional antenna)

$P_{direct} = P_T(\lambda/4\pi r)^2$

$\frac{P_r}{P_T} = (\lambda/4\pi r)^2 \times |1 + \; R \times \exp\left(jk\frac{2h_b h_m}{r}\right)|^2$

Let us consider the mod term

$I = |1 + \; R \times \exp(j\theta)\,|^2$ where $\theta = k\frac{2h_b h_m}{r}$

Since h_b,h_m<< r , θ is very small.

Since the angle of incident with the ground θ_i is very small (almost grazing), the magnitude of the reflection coefficient will be close to one whatever its conductivity (or roughness) may be (this can easily be shown. Please see the next example).

Example 3: Show that if medium-1 is free space and medium-2 is a dielectric, both | R_{par} | and | R_{per} | approach '1' as θi approaches 0° regardless of Єr.

Solution: $R_{par}= \frac{-\text{Єr Sin }\theta+\sqrt{(\text{Єr}-\cos^2\theta)}}{\text{Єr Sin }\theta+\sqrt{(\text{Єr}-\cos^2\theta)}}$

$\theta = 0 \rightarrow| R_{par} | = \frac{\sqrt{(\text{Єr}-1)}}{\sqrt{(\text{Єr}-1)}} =1$

We Know

$R_{per}=\frac{\text{Sin }\theta-\sqrt{(\text{Єr}-\cos^2\theta)}}{\text{Sin }\theta+\sqrt{(\text{Єr}-\cos^2\theta)}}$

$\theta = 0 \rightarrow R_{per}$ =-1 &| R_{per} |=1

This result indicates that the ground may be modeled as a perfect conductor with magnitude of reflection coefficient as 1 when the grazing incidence case is considered.

I=|1- $\exp(j\theta)$|^2=|(1-cos θ)- j sin θ |^2

$=\left(\sqrt{[(1-cos\theta)^2+\sin^2\theta]}\right)^2$

$I=(1-\cos\theta)^2$ as $\sin\theta \cong 0; \theta \to 0$

$= (1+\cos^2\theta - 2cos\,\theta)$

$= (1+1-\sin^2\theta - 2cos\theta)$ again $\sin^2\theta \cong 0$

$= (2-2cos\theta)$

$= 2[1-cos\theta]$

$=4\left[\sin^2(\theta/2)\right]=4(\theta/2)^2=\theta^2$ Here we assume $\sin^2(\theta/2)\approx(\theta/2)^2$ since θ is very low.

Hence

$$\frac{Pr}{PT} \approx \left(\frac{\lambda}{4\pi r}k\frac{2h_b h_m}{r}\right)^2 \approx \frac{{h_b}^2.{h_m}^2}{r^4} \tag{5}$$

Hence, path loss in dB is expressed as

$$(L)_{PEL} = 40\log r - 20\log\, h_m - 20\log h_b \tag{6}$$

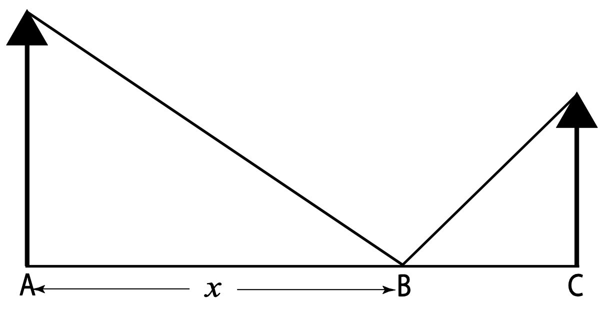
Example 4: Consider a two-ray model with the height of the transmitter as 10 m and that of receiver as 3 m. The distance between Tx and Rx is 2km. Compute the distance of a reflecting point on the ground from the transmitter. Also compute the phase difference between two signals reaching the receiver and the pathloss (Assume transmitted signal having perpendicular polarization).

Solution:

$h_b = 10m, h_m = 3m, r = 2km$

Since

$(h_b + h_m) = r \Rightarrow$ hence approximation holds



$$\frac{10}{x} = \frac{3}{(2000-x)} \Rightarrow \quad x = \frac{20,000}{13} m$$

$$\Rightarrow (r_1 - r_2) ; \; \frac{2h_b h_m}{\gamma} = 30\times10^{-3} m, \lambda = 0.3m (\text{for } f = 1GHz)$$

$$\Delta\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = \frac{2\pi}{0.3}\times\left(30\times10^{-3}\right) = 0.2\pi = 0.628 \text{ radian} = 36^{\circ}$$

Assuming perpendicular polarization, $R = -1 \; (\text{as } \; \theta \to 0)$

$$L_{\text{pathloss}}(dB) = 40\log(\gamma) - 20\log(h_b) - 20\log(h_m)$$
$$= 102.49dB$$

Without approximating

$$R = \frac{\sin\theta - \sqrt{\epsilon_r - \cos^2\theta}}{\sin\theta + \sqrt{\epsilon_r - \cos^2\theta}}$$

$$\theta = \tan^{-1}\left(\frac{10\times13}{20,000}\right) = 0.0065, \; \epsilon_r = 10 \Rightarrow R = -0.9957$$

So, we note that $\theta ; \; 0 \Rightarrow R = -1$ is justified.

Critical Distance:

The plot result can be divided into three sections

(i) d<h_b

(ii) h_b<d<dc where dc is critical distance

(iii) dc<d

For d<h_b only constructive interference takes place. For h_t<d<dc constructive and destructive both interference take place, thus power decrease follows the rule Pr $\alpha \frac{1}{d^2}$. For d>dc only destructive interference takes place and power fall follows the rule Pr $\alpha \frac{1}{d^4}$.

dc is called critical distance given as

$$d_c = \frac{4h_b h_m}{\lambda}$$

Case 1 : Assuming perfect reflecting surface (log(dcritical) = 3.08)

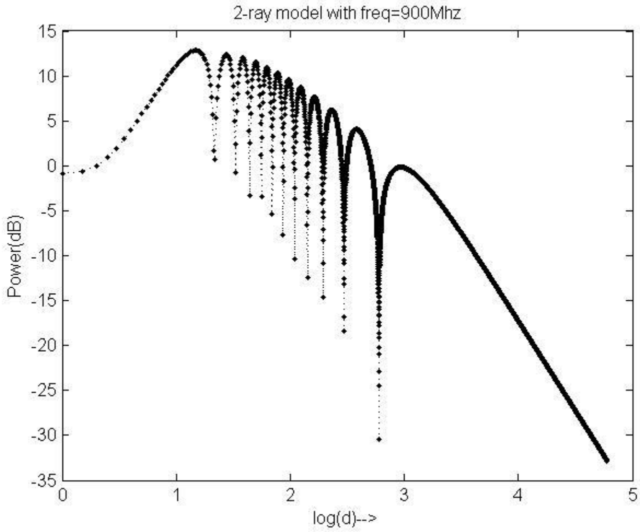


Figure 3

Case II: Taking Reflection coefficient as a function of angle of incidence(log(dcritical) = 3.08)

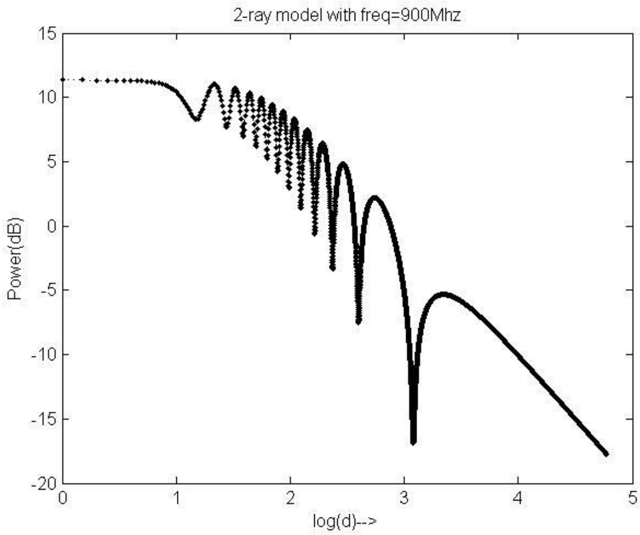


Figure 4

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MATLAB Code:

lambda=1/3;                                % wavelength in metres => f =900MHz
beta=2*pi/lambda;                          % wavenumber
dcritical=4*Tx*Rx/lambda;                  % critical distance for far field

E0=500;
k=1;
D=1:0.5:dcritical*50;

for d=D
% To determine thetha ====
temp=Tx*d/(Tx+Rx);
thetai=pi/2-atan(Tx/temp);
thetat=asin(sin(thetai)/2);

%Case 1: Reflection coefficient: perfect reflector
R(k)=-1;
```

```
%Case 2
R(k)=(2*cos(thetat)-cos(thetai))/(2*cos(thetat)+cos(thetai));

% Let eta2 = 2.eta1 and n2=1.5* `T,CVBNM,  1n1
d2=sqrt(Tx^2+temp^2)+sqrt(Rx^2+(d-temp)^2);
d0=sqrt(abs(Tx-Rx)^2+d^2);


Elos(k) = E0/d0*(exp(complex(0,-beta*d0)));
Eg(k)= E0*R(k)/d2*(exp(complex(0,-beta*d2)));
Etot(k)=Elos(k)+Eg(k);

E(k)=20*log10(abs(Etot(k)));
d3(k)=log10(d);
k=k+1;
end;

%d=dcritical:10:dcritical*5;
plot(d3,E,':k.');
title('2-ray model with freq=900Mhz');
xlabel('log(d)-->');
ylabel('Power(dB)');
axis 'auto x';
```

Example 5: As BS transmit power of 10W into a feeder cable with the loss of 10dB. The transmit antenna has gain of 12dBd in the direction of mobile receiver. With antenna gain 0dBd and feeder loss 2dB the mobile has sensitivity of -104dBm.

(a) Determine the effective isotropic radiated power.

(b) Determine the maximum acceptable path loss.

Solution:

$$P_T = 10W$$

$$(P_T)_{dB} = 10dBW$$

$$(l_f)_{dB} = 10dB$$

$$(G_T)_{dB} = 12dBd$$

Since, 0 dBd=2.15 dBi

12dBd=12+2.15

=14.15dBi

(a) EIRP(Effective isotropic radiated power) in dB =$(P_T)_{dB} + (G_T)_{dB} - (L_f)_{dB}$

= 10+14.15-10