MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclear Stability

Lecture-16

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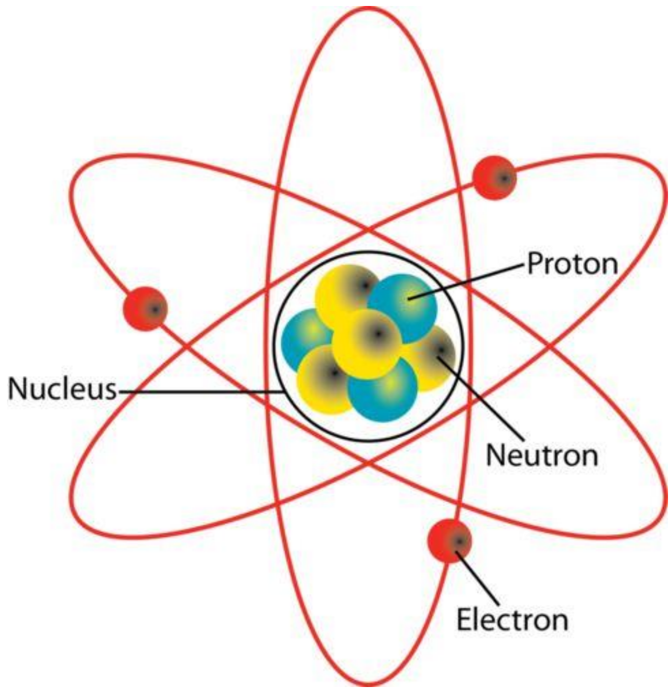

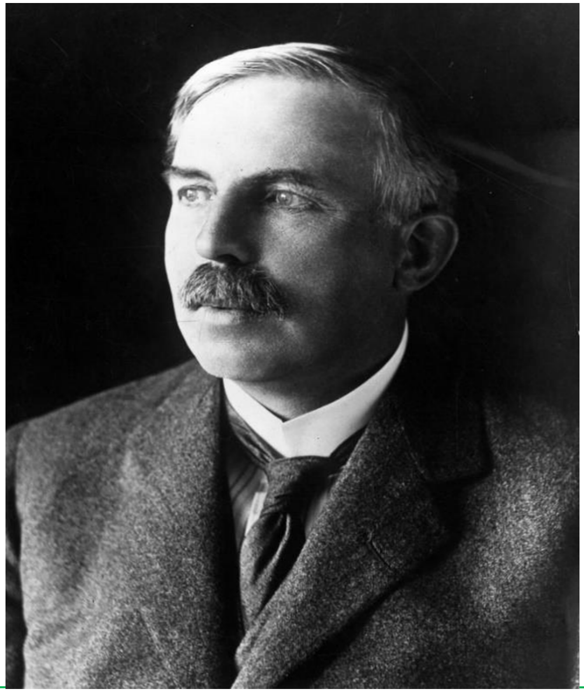

Conservation Laws in Nuclear Reactions

Parity

- In any nuclear reaction the total parity is conserved.

- If $\pi_{X,}$ $\pi_{a,}$ $\pi_{Y,}$ and $\pi_{b,}$ are intrinsic parities of nuclei taking part in the interaction, then for initial and final states of reaction the parities are

- $\pi_i = \pi_X \; \pi_a (-1)^{l_a}$ and $\pi_f = \pi_Y \; \pi_b (-1)^{l_b}$
- $\pi_X \; \pi_a (-1)^{l_a} = \pi_Y \; \pi_b (-1)^{l_b}$

- For example we consider the reaction $^{13}B\,(\alpha, p)C^{13}$ In the above example parity of the ground level of ^{13}B ***and*** ^{4}He (α-particle) *and proton*$\left({}^{1}_{1}H\right)$*is is even, while that of* $^{13}_{6}C$ is odd.

Conservation Laws in Nuclear Reactions

Parity

- In the reaction the input parity for S-wave capture of α- particle is therefore even.

- Then the particular excited level in the compound nucleus ^{14}C must have even parity

- In the exit channel C^{13} has odd parity, hence the requirement of total even parity puts an additional restrictions on l_f and allows only l_f = 3 if l_i =0.

Conservation Laws in Nuclear Reactions

Mass- Energy

- In the nuclear reactions neither kinetic energy nor mass is conserved separately. But the total mass-energy is always conserved.

- The kinetic energy Q- liberated in any reaction is always equal to the reduction of the total rest mass of all the constituents of the reaction, the mass energy equivalence relation is

- E = m c^2 *accordingly,* 1u (or amu) = 931.5 MeV Conversely 1 MeV = 0.001074 u

Conservation Laws in Nuclear Reactions

Mass- Energy

- In the exit channel C^{13} has odd parity, hence the requirement of total even parity puts an additional restrictions on l_f and allows only l_f = 3 if l_i =0.
- For example in the nuclear reaction $^{10}_{5}B\ (\alpha, p)\ ^{13}_{6}C$. *The Q value of the reaction is*
- $M_{B^{10}} + M_\alpha = M_p + M_{C^{13}} + Q$
- Where all the masses for the corresponding neutral atoms.
- If relativistic masses were used in above equation, then Q-value for reaction would be identically zero.

Conservation Laws in Nuclear Reactions

Statistics

- In the reaction ${}^{10}_{5}B\ (\alpha, p)\ {}^{13}_{6}C.$ i.e.

$$ {}^{10}_{5}B + {}^{4}_{2}He \rightarrow ({}^{14}_{7}N) \rightarrow {}^{13}_{6}C + {}^{1}_{1}H + Q $$

- Both Side contain same number of fermians: hence the statistics that holds in either Fermi Dirac throughout (for odd $\sum A$) or Bose-Einstein throughout (for even $\sum A$).

Conservation Laws in Nuclear Reactions

Isotopic Spin

- The isotopic spin in nuclear reaction involving strong interaction must remain conserved. If T_i *and* T_f are the isotopic spins in initial and final states, then the conservation law of isotopic spin gives $T_i = T_f$
- For the reaction, a+X = Y+b

$$T_i = T_X + T_a \text{ and } T_f = T_Y + T_b$$

- So the conservation law gives ($T_i = T_f$)
- $$T_X + T_a = T_Y + T_b$$
- Isotopic spin is characteristic of nuclear level. Hence this conservation law can be used to identify the levels of nuclei produced in the reaction.

Reaction Energy: Q-Value

- The Laws of conservation of energy and momentum also holds good in the nuclear reactions.

- The Q-value of a reaction is the change in total kinetic energy of system or the change in total rest mass energy system

- As rest mass is an invariant in relativistic mechanics, therefore Q value is same in laboratory and center of mass reference systems.

- If we consider the general nuclear reaction X (a,b)Y, the total initial energy in the reaction is

$$E_i = K_a + M_a c^2 + K_X + M_X c^2 \quad ....(i)$$

Where K_a and K_X and the kinetic energies of particles a and X respectively

Reaction Energy: Q-Value

- The total potential energy is zero as particles ‘a’ and X are assumed for apart, M_a and M_X are rest masses of ‘a’ and X respectively.

- The final energy of reaction is

$$E_f = K_b + M_b c^2 + K_Y + M_Y c^2 \text{ ....(ii)}$$

- Here K_b = kinetic energy of particle ‘b’, K_Y = kinetic energy of particle Y, $M_b = Rest\ mass\ of\ particle\ b, M_Y$ = Rest mass of particle Y.

- According to law of conservation of energy
- $$E_i = E_f$$
- The difference between initial and final rest mass energies is called the reaction energy or Q- value of the reaction i.e.

Reaction Energy: Q-Value

- $$Q = [(M_a + M_x) - (M_b + M_Y)]\, c^2 \quad \ldots\ldots\ldots\ldots(3)$$
- Now from Eq. (i) and (ii)

$$Q = [(K_b + K_Y) - (K_a + K_x)] \quad \ldots\ldots\ldots\ldots(4)$$

- Equation (iii) may also be expressed as
- $$(M_a + M_x)\, c^2 = (M_b + M_Y)\, c^2 + Q \quad \ldots\ldots\ldots(5)$$
- If Q is positive (i.e. Q>0) the reaction is exoergic and the energy liberated appears in the form of kinetic energies of reaction products; but if Q<0, the reaction is endoergic i.e., it can occur only if the required energy is supplied by thr kinetic energy of the projectile.

Threshold Energy

- The minimum kinetic energy required by an incident particle to bring about necessarily an endoergic reaction is called the threshold energy (E_{th}) for the nuclear reaction

- The incident particle has threshold energy, the reaction products have zero kinetic energy in the center of mass system(CM).

- However in lab (L) system the kinetic energy of the emerging particle b and recoil nucleus Y is not zero.

- In L-system the target X is stationary; therefore, the total kinetic energy is simply that of incident particle a.

Threshold Energy

- For simplicity of the calculations let us assume the information of compound nucleus having mass M_c and velocity v_c in lab system we have

- $$M_a\, v_a \;=\; M_c\, v_c \;\ldots\ldots\ldots\ldots(1)$$

- Since the target nucleus X is at rest in L system

- The sharing of incident particle energy takes place as
- $$Q+\frac{1}{2}\,M_a v_a^2 = \frac{1}{2}\,M_c v_c^2$$
- $$-Q = \frac{1}{2}\,M_a v_a^2 - \frac{1}{2}\,M_c v_c^2$$
- Using (1), we get
- $$-Q = \frac{1}{2}\,M_a v_a^2 - \frac{1}{2}\,M_c\left(\frac{M_a v_a}{M_c}\right)^2$$

Threshold Energy

- But $M_c = M_a + M_X$, therefore we have
- $-Q = \frac{1}{2} M_a v_a^2 \left(\left(1 - \frac{M_a}{M_c}\right)\right)$
- $= \frac{1}{2} M_a v_a^2 \left(\left(1 - \frac{M_a}{M_a + M_X}\right)\right)$
- $= \frac{1}{2} M_a v_a^2 \left(\frac{M_X}{M_a + M_X}\right)$
- Thus Threshold energy
- $E_{th} = \frac{1}{2} M_a v_a^2 = -Q \left(\frac{M_a + M_X}{M_X}\right)$

Threshold Energy

- This equation determines the threshold energy for a reaction
- For a reaction induced by gamma rays, M_a =0
- $E_{th} = -Q$
- In equation (3) all masses are nuclear masses but in actual calculations masses of neutral atoms may be used