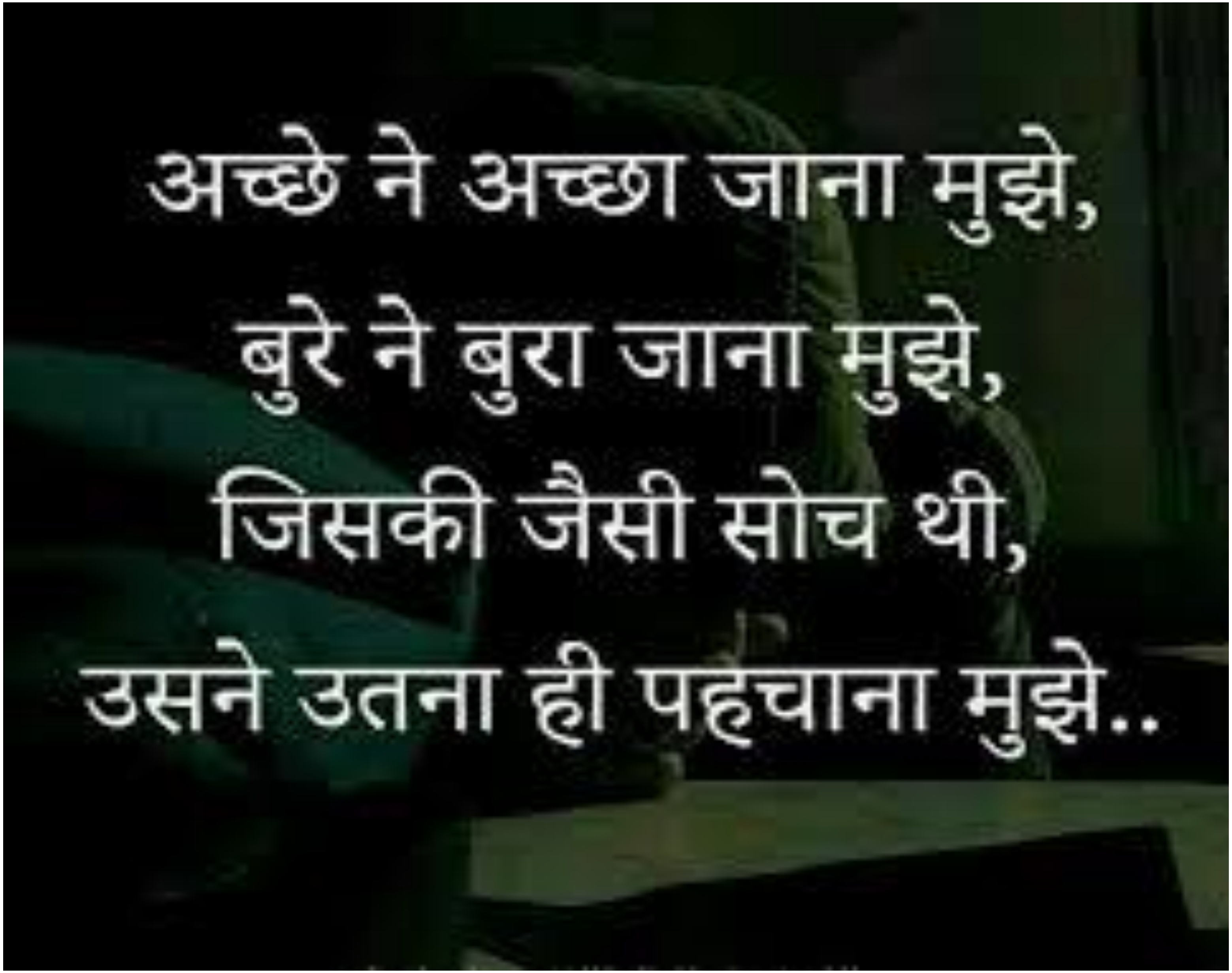
Theory of Relativity

UNIT I
Relativistic Mechanics
Lecture-4


अच्छे ने अच्छा जाना मुझे,
बुरे ने बुरा जाना मुझे,
जिसकी जैसी सोच थी,
उसने उतना ही पहचाना मुझे..

Simultaneity in the observation

- The time of occurrence of the events observed by observer of moving frame of reference

$$t_1' = \frac{t_1 - (vx_1/c^2)}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad t_2' = \frac{t_2 - (vx_2/c^2)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\text{Therefore,} \qquad t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1 - (v^2/c^2)}} - \frac{(v/c^2)(x_2 - x_1)}{\sqrt{1 - (v^2/c^2)}}$$

- If both the events are occurring simultaneously for the observer in stationary frame of reference
- Then $\Delta t = t_1 - t_2 = 0$

$$\Delta t' = \frac{(v/c^2)\,(x_1 - x_2)}{\sqrt{1 - (v^2/c^2)}}$$

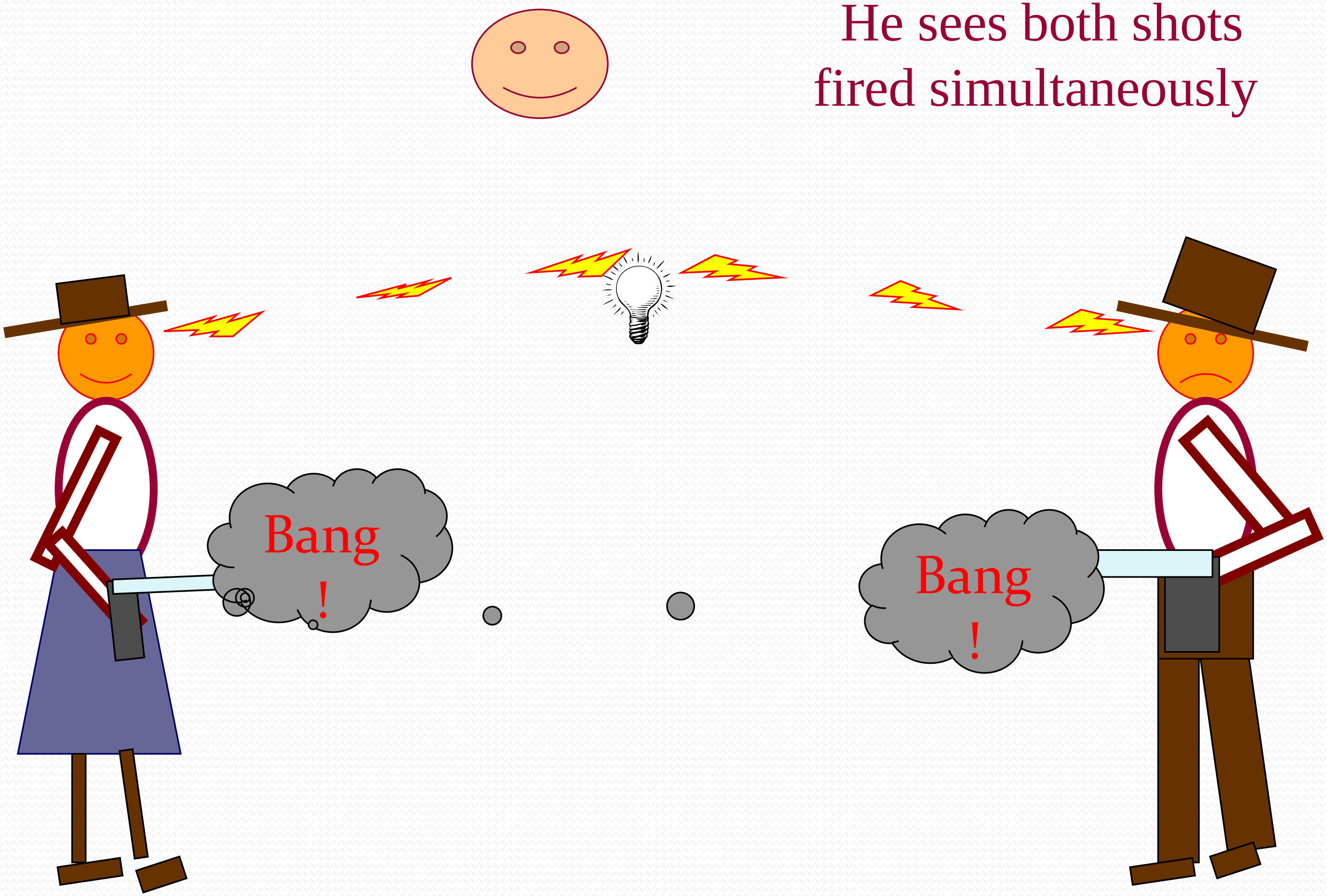
$$\text{i.e.,} \qquad \Delta t' \neq 0$$

Time depends on the state of motion of the observer!!

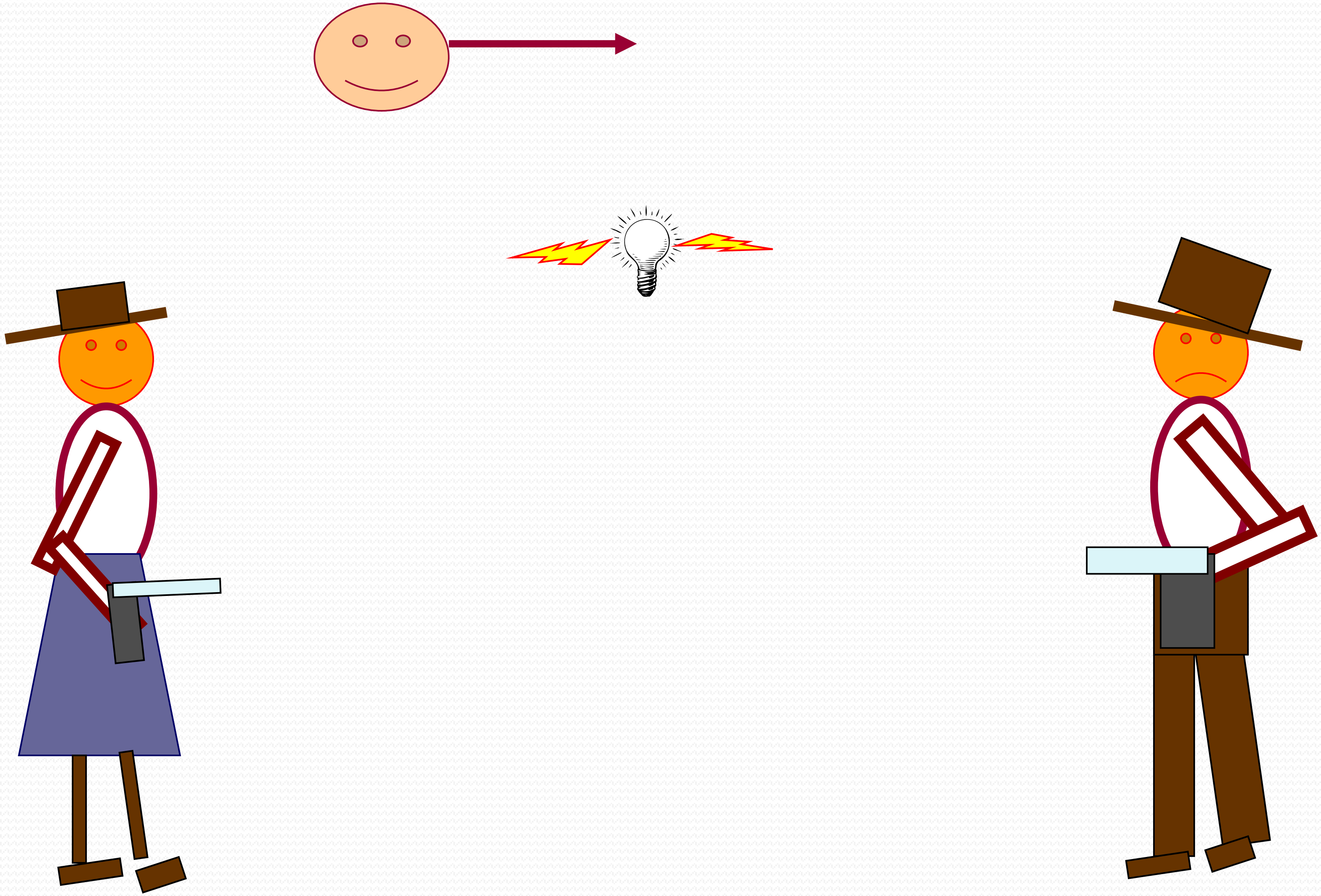
Events that occur **simultaneously** according to one observer can occur at **different times** for other observers

Gunfight viewed by observer at rest

He sees both shots fired simultaneously

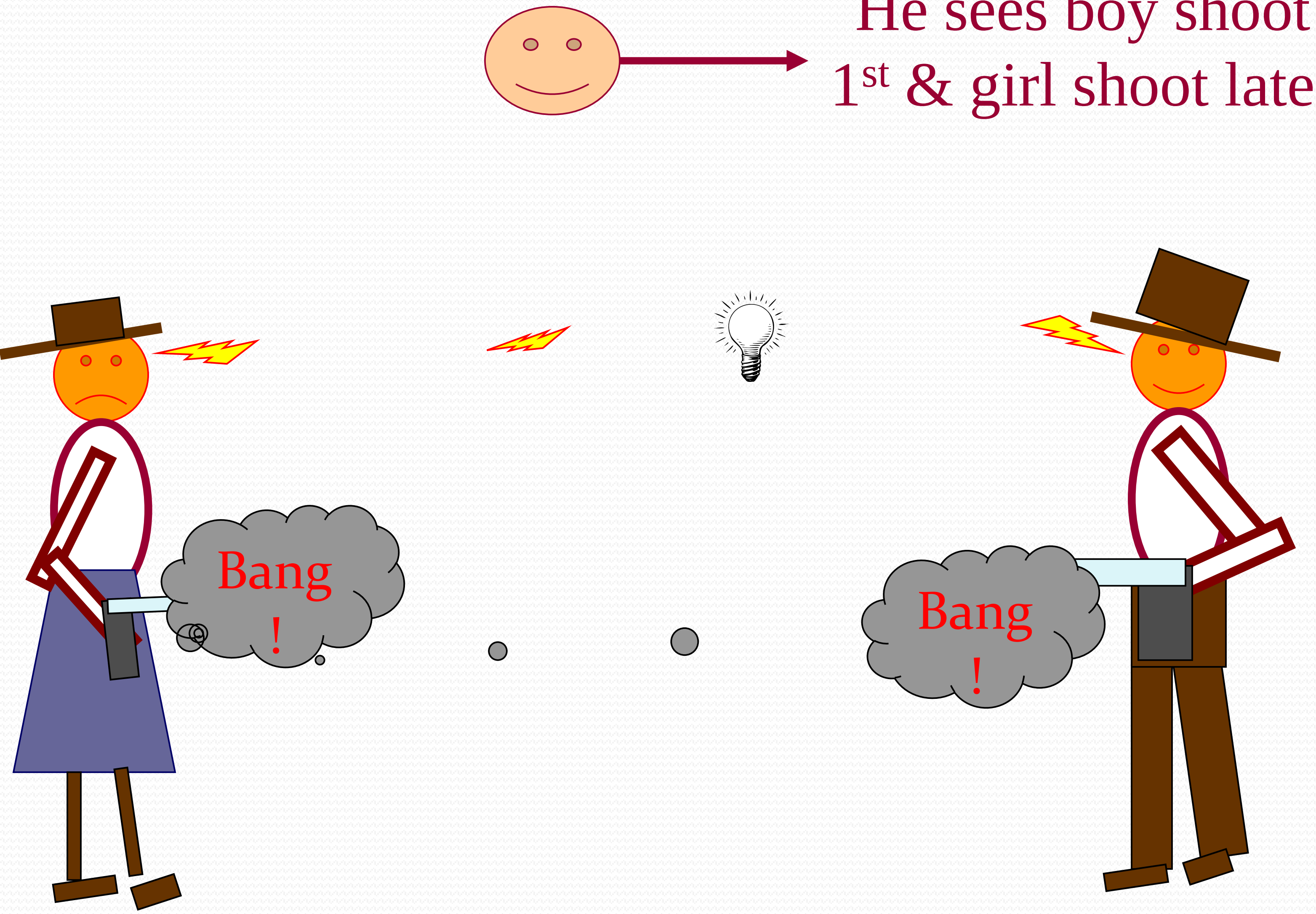


Viewed by a moving observer

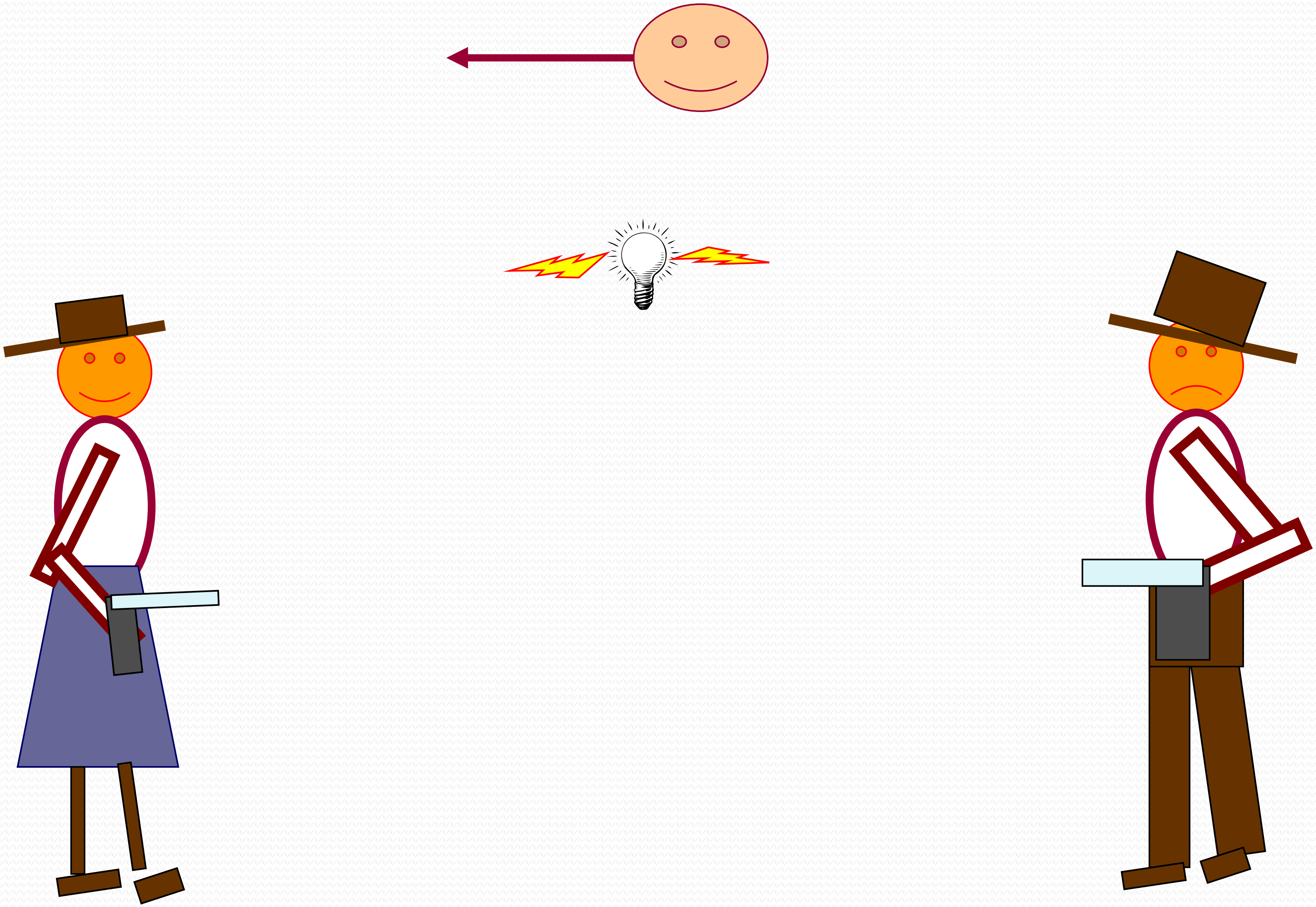


Viewed by a moving observer

He sees boy shoot 1st & girl shoot later

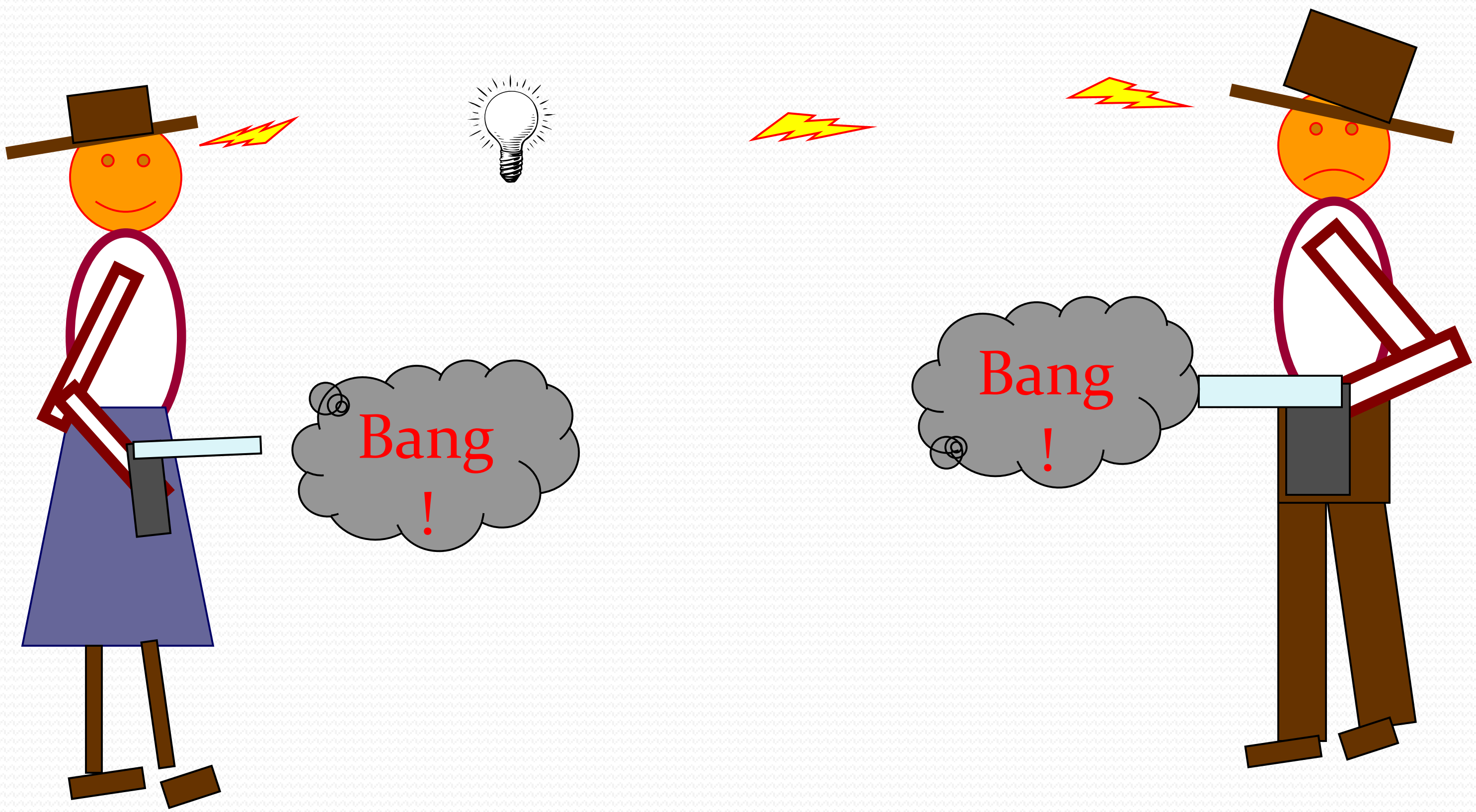


Viewed by an observer in the opposite direction

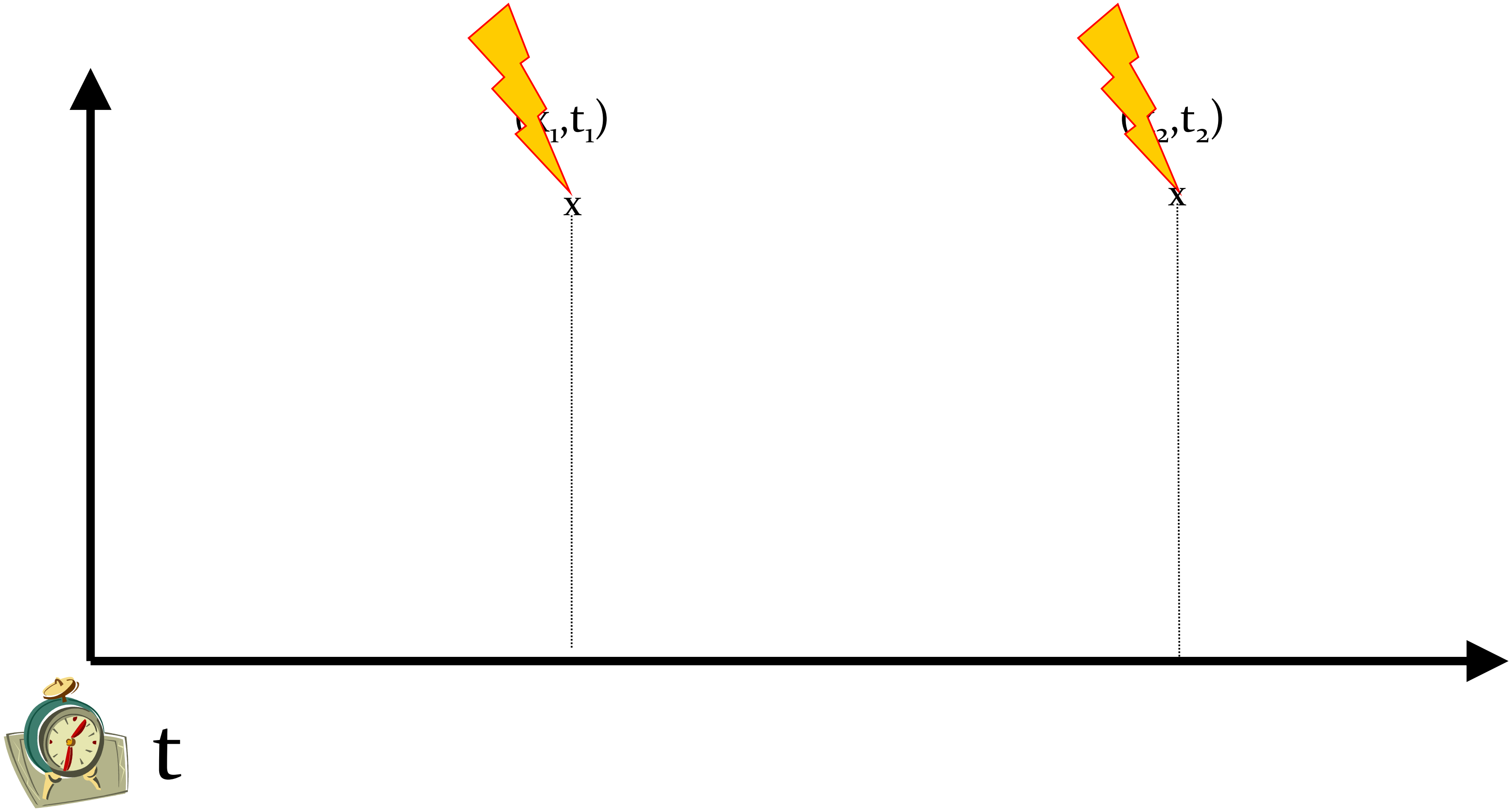


Viewed by a moving observer

He sees girl shoot 1st & boy shoot later

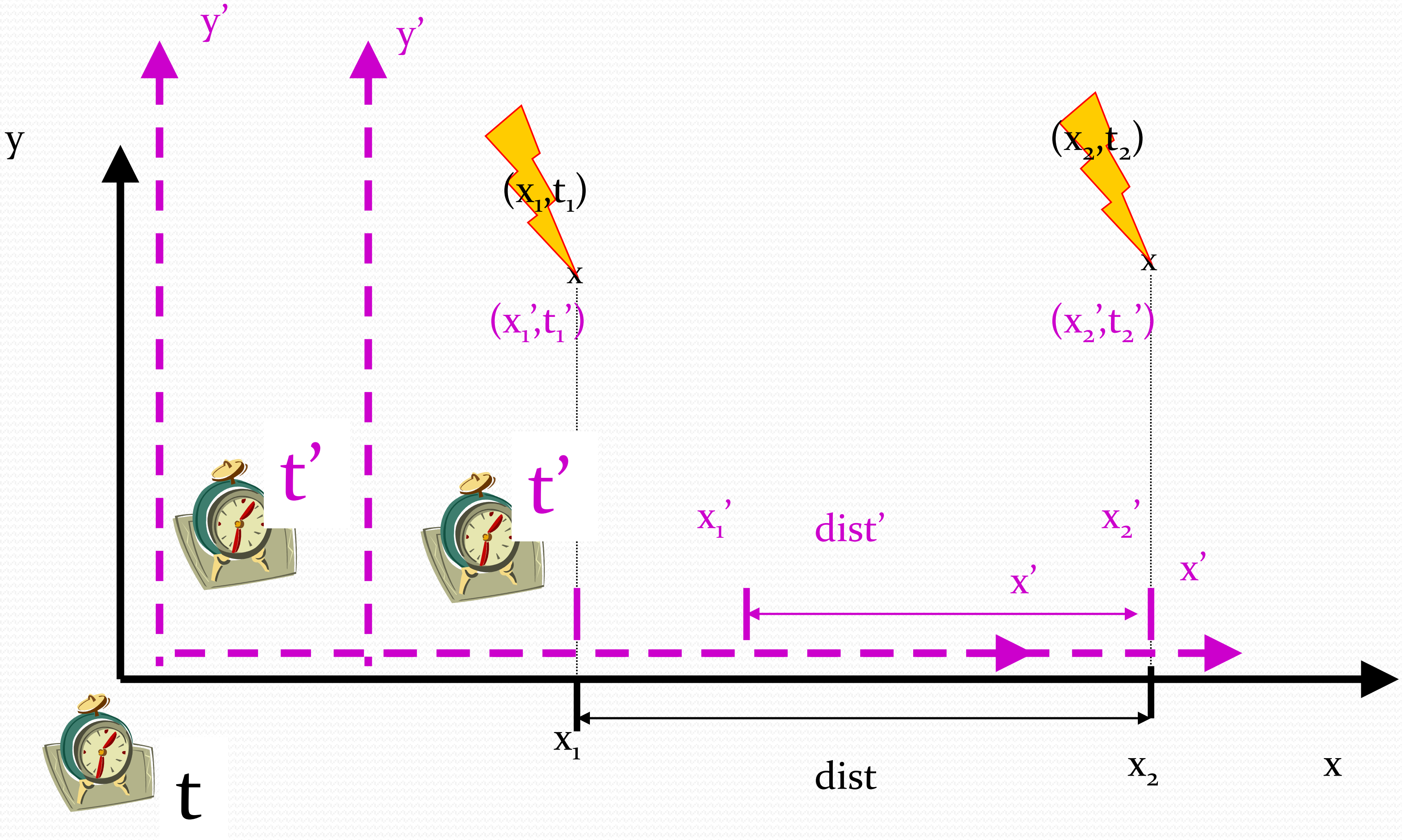


Events

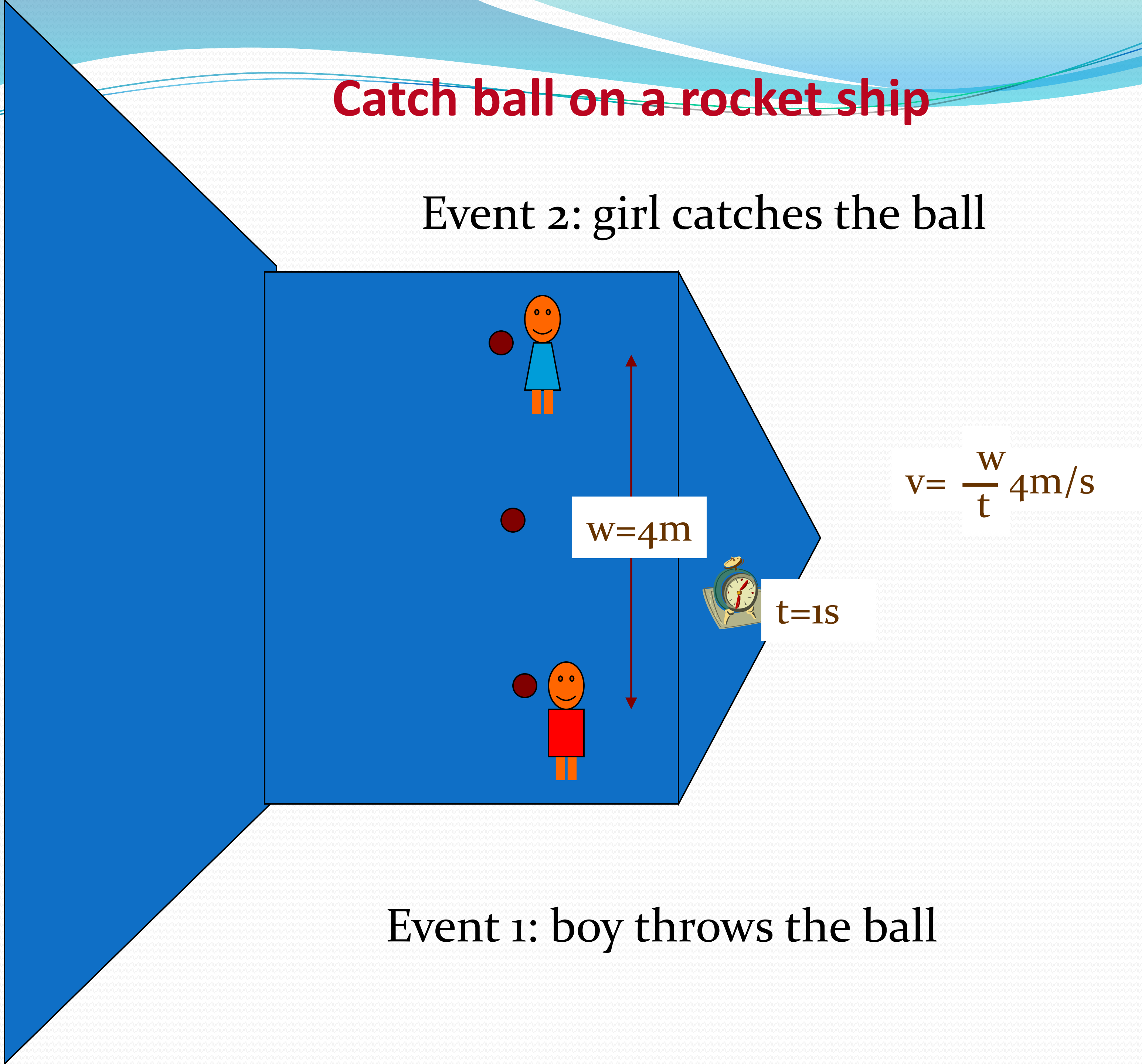


Prior to Einstein, everyone agreed the distance between events depends upon the observer, but not the time.

Same events, different observers

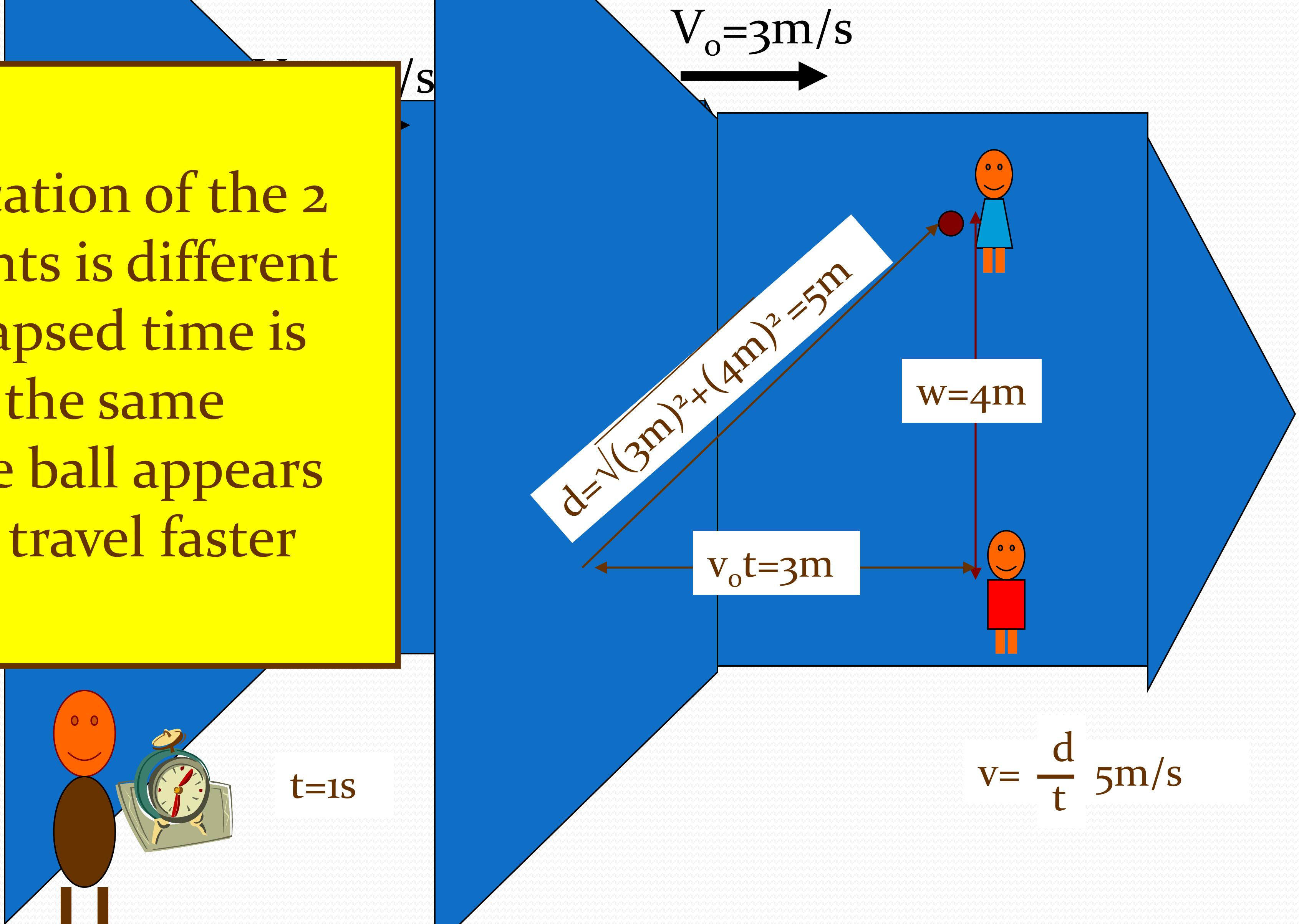


Catch ball on a rocket ship



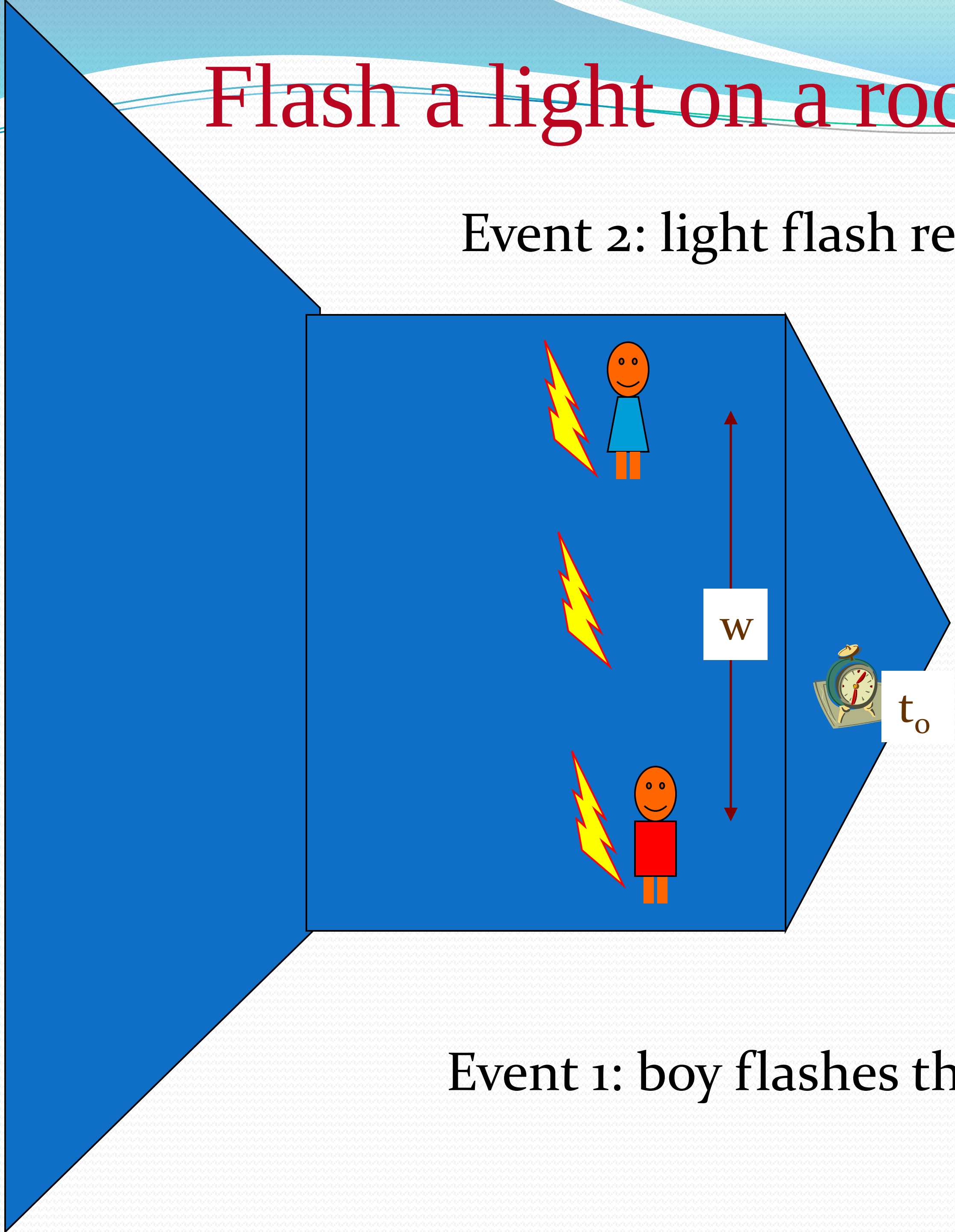
S from earth

Location of the 2 events is different
Elapsed time is the same
The ball appears to travel faster



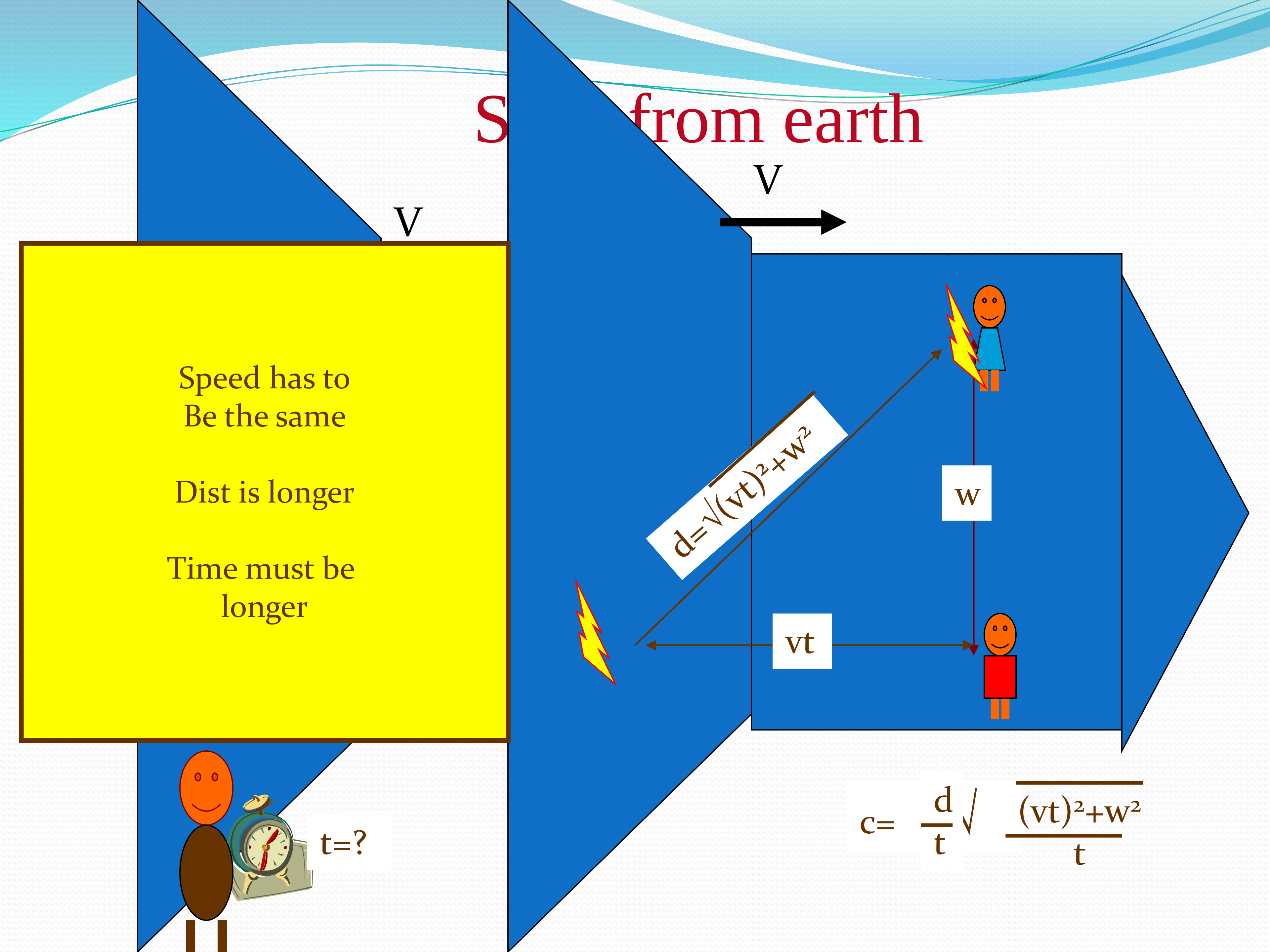
Flash a light on a rocket ship

Event 2: light flash reaches the girl



$$c = \frac{w}{t_o}$$

Event 1: boy flashes the light


S from earth
V
V
Speed has to
Be the same
Dist is longer
Time must be
longer
d=√(vt)²+w²
w
vt
t=?
c= d/t √ (vt)²+w² / t

How is t related to t_0?

t= time on Earth clock

t_o = time on moving clock

$c = \frac{\sqrt{(vt)^2+w^2}}{t}$

$c = \frac{w}{t_o}$

$ct = \sqrt{(vt)^2+w^2}$

$ct_o = w$

$(ct)^2 = (vt)^2+w^2$

$(ct)^2 = (vt)^2+(ct_o)^2$ ➔ $(ct)^2-(vt)^2= (ct_o)^2$ ➔ $(c^2-v^2)t^2= c^2t_o^2$

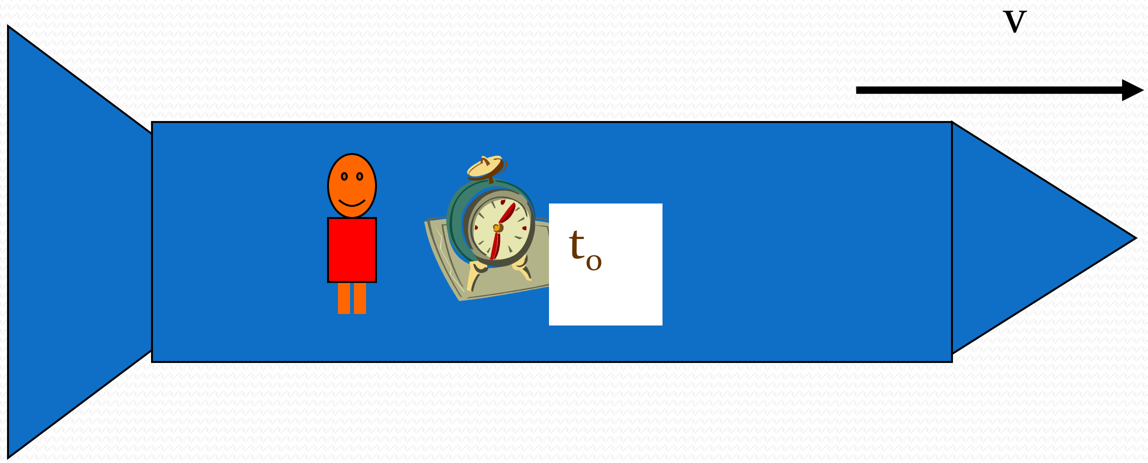
➔ $t^2 = \frac{c^2 t_0^2}{c^2 - v^2}$

$t^2 = \frac{c^2 t_0^2}{1-\frac{v^2}{c^2}}$

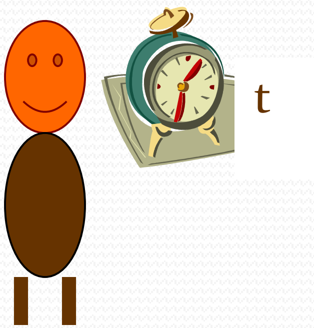
➔ $t = \frac{c \quad t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

➔ $t = k\, t_0$

Moving clocks run slower



$$t = \frac{1}{\sqrt{1 - v^2/c^2}}\, t_o$$



$$t = \kappa\, t_o$$

$\kappa > 1 \Rightarrow t > t_0$

Properties of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.01c (i.e. 1% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.01c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.01)^2 \cancel{c^2}/\cancel{c^2}}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.01)^2}} = \frac{1}{\sqrt{1 - 0.0001}} = \frac{1}{\sqrt{0.9999}}$$

$\kappa = 1.00005$

Properties of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.01c (i.e. 1% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.01c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.01)^2 c^2/c^2}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.01)^2}} = \frac{1}{\sqrt{1 - 0.0001}} = \frac{1}{\sqrt{0.9999}}$$

$\kappa = 1.00005$

Properties of k $= \dfrac{1}{\sqrt{1 - v^2/c^2}}$ (cont'd)

Suppose v = 0.1c   (i.e. 10% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.1c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.1)^2\cancel{c^2}/\cancel{c^2}}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.1)^2}} = \frac{1}{\sqrt{1 - 0.01}} = \frac{1}{\sqrt{0.99}}$$

k = 1.005

Let’s make a chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005

Other values of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.5c (i.e. 50% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.5c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.5)^2\cancel{c^2}/\cancel{c^2}}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.5)^2}} = \frac{1}{\sqrt{1 - (0.25)}} = \frac{1}{\sqrt{0.75}}$$

$\kappa = 1.15$

Enter into chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15

Other values of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.6c (i.e. 60% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.6c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.6)^2 c^2/c^2}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.6)^2}} = \frac{1}{\sqrt{1 - (0.36)}} = \frac{1}{\sqrt{0.64}}$$

$\kappa = 1.25$

Back to the chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25

Other values of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.8c (i.e. 80% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.8c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.8)^2 c^2/c^2}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.8)^2}} = \frac{1}{\sqrt{1 - (0.64)}} = \frac{1}{\sqrt{0.36}}$$

$\kappa = 1.67$

Enter into the chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67

Other values of $k = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.9c (i.e.90% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.9c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.9)^2 \cancel{c^2}/\cancel{c^2}}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.9)^2}} = \frac{1}{\sqrt{1 - 0.81}} = \frac{1}{\sqrt{0.19}}$$

$\kappa = 2.29$

update chart

v	κ $=1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67
0.9c	2.29

Other values of $k = \frac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 0.99c (i.e.99% of c)

$$\kappa = \frac{1}{\sqrt{1 - (0.99c)^2/c^2}} = \frac{1}{\sqrt{1 - (0.99)^2 \cancel{c^2}/\cancel{c^2}}}$$

$$\kappa = \frac{1}{\sqrt{1 - (0.99)^2}} = \frac{1}{\sqrt{1 - 0.98}} = \frac{1}{\sqrt{0.02}}$$

$$\kappa = 7.07$$

Enter into chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67
0.9c	2.29
0.99c	7.07

Other values of $k = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = c

$$\kappa = \frac{1}{\sqrt{1 - (c)^2/c^2}} = \frac{1}{\sqrt{1 - c^2/c^2}}$$

$$\kappa = \frac{1}{\sqrt{1 - 1^2}} = \frac{1}{\sqrt{0}} = \frac{1}{0}$$

$\kappa = \infty$ Infinity!!!

update chart

v	κ $=1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67
0.9c	2.29
0.99c	7.07
1.00c	∞

Other values of $k = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

Suppose v = 1.1c

$$\kappa = \frac{1}{\sqrt{1 - (1.1c)^2/c^2}} = \frac{1}{\sqrt{1 - (1.1)^2 c^2/c^2}}$$

$$\kappa = \frac{1}{\sqrt{1 - (1.1)^2}} = \frac{1}{\sqrt{1 - 1.21}} = \frac{1}{\sqrt{-0.21}}$$

$\gamma =$???

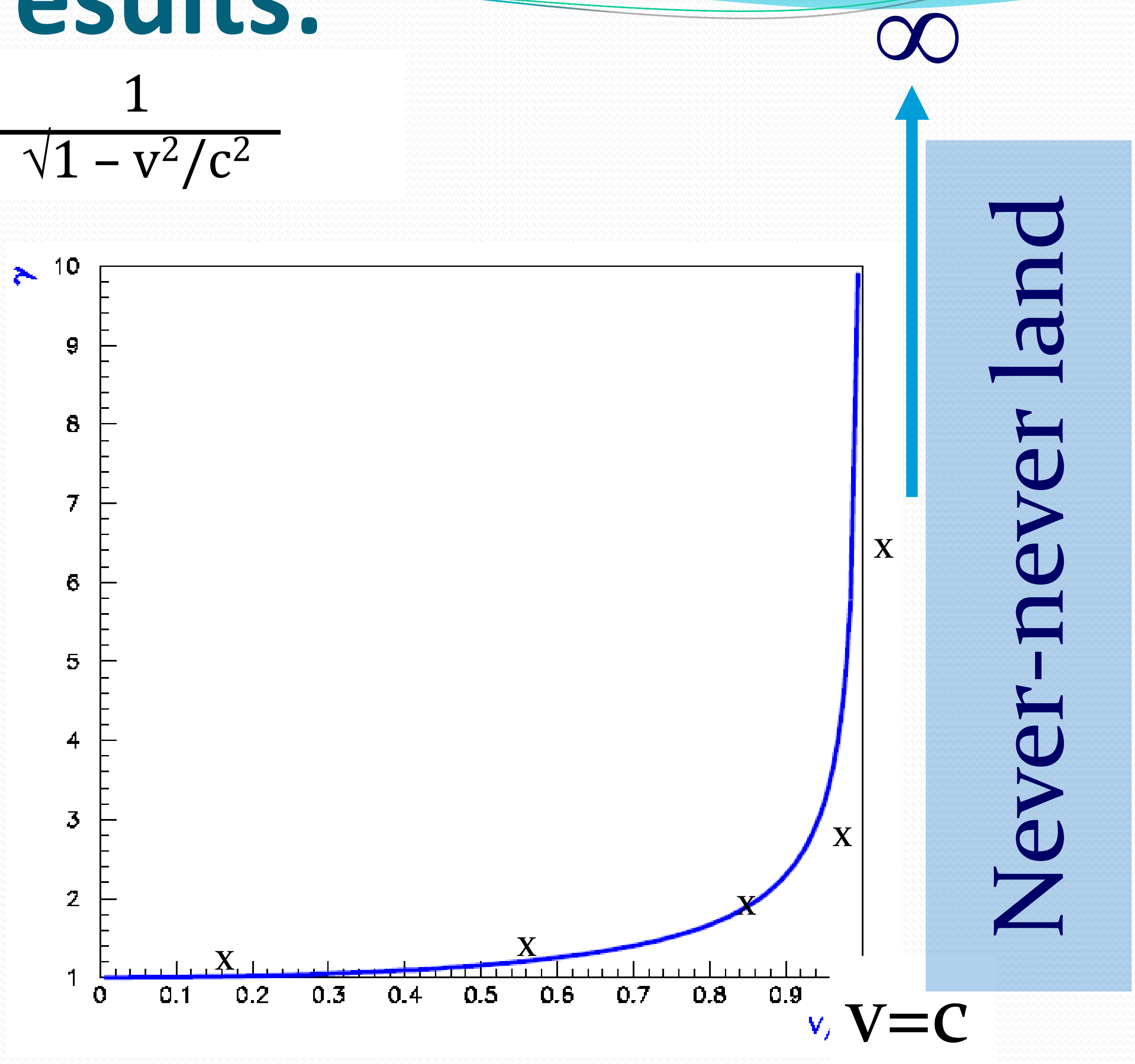
Imaginary number!!!

Complete the chart

v	$\kappa = 1/\sqrt{(1-v^2/c^2)}$
0.01 c	1.00005
0.1 c	1.005
0.5c	1.15
0.6c	1.25
0.8c	1.67
0.9c	2.29
0.99c	7.07
1.00c	∞
Larger than c	Imaginary number

Plot results:

$$\kappa = \frac{1}{\sqrt{1 - v^2/c^2}}$$



Assignment based on what we learnt in this lecture ?

- What will happen when two simultaneous events are observed by the stationary and moving frame of reference?
- Describe the physical significance regarding the observations of simultaneous events observed by moving and stationary observers.
- Discuss the infinite time for the moving observer.