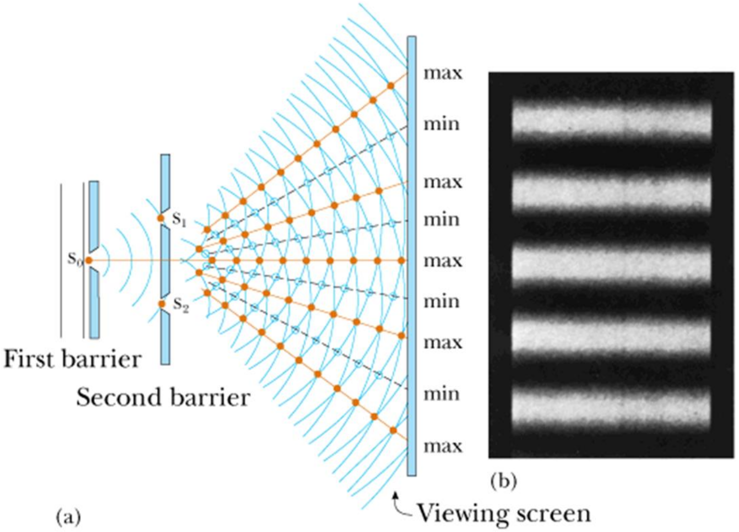
Interference

UNIT III
Optics
Lecture-1



Content of Lecture

- **Brief History**
- **Interference of light**
- **Young’s double slit experiment**

Brief History

- Upto the middle of the seventeenth century it was generally believed that light consisted of a stream of corpuscles.
- These corpuscles could penetrate transparent materials and were reflected from the surfaces of opaque materials.
- When they entered the eye, they caused the sensation of light.
- Huygen in 1670 explained the laws of reflection and refraction could be explained on the basis of wave theory.
- But it was objected that if light is a wave it should show the bending around the corners of the obstacles in their path.
- We know now that the wavelengths of light waves are so small that the bending, which actually does take place can not be observed at ordinary conditions.

- This bending was first observed by Grimaldi and interpreted by Hooke.

- In about 1827, Young's experiments enabled him to measure the wavelength of the waves, and Fresnel showed that the rectilinear propagation of light along with the diffraction effects observed by Grimaldi

- In 1873, Maxwell suggested that light consisted of electromagnetic wave of extremely short wavelengths.

- After some years, Hertz succeeded in producing short wavelength waves of electromagnetic origin and showed that they possessed all the characteristics of light wave, such as refraction, reflection, polarisation.

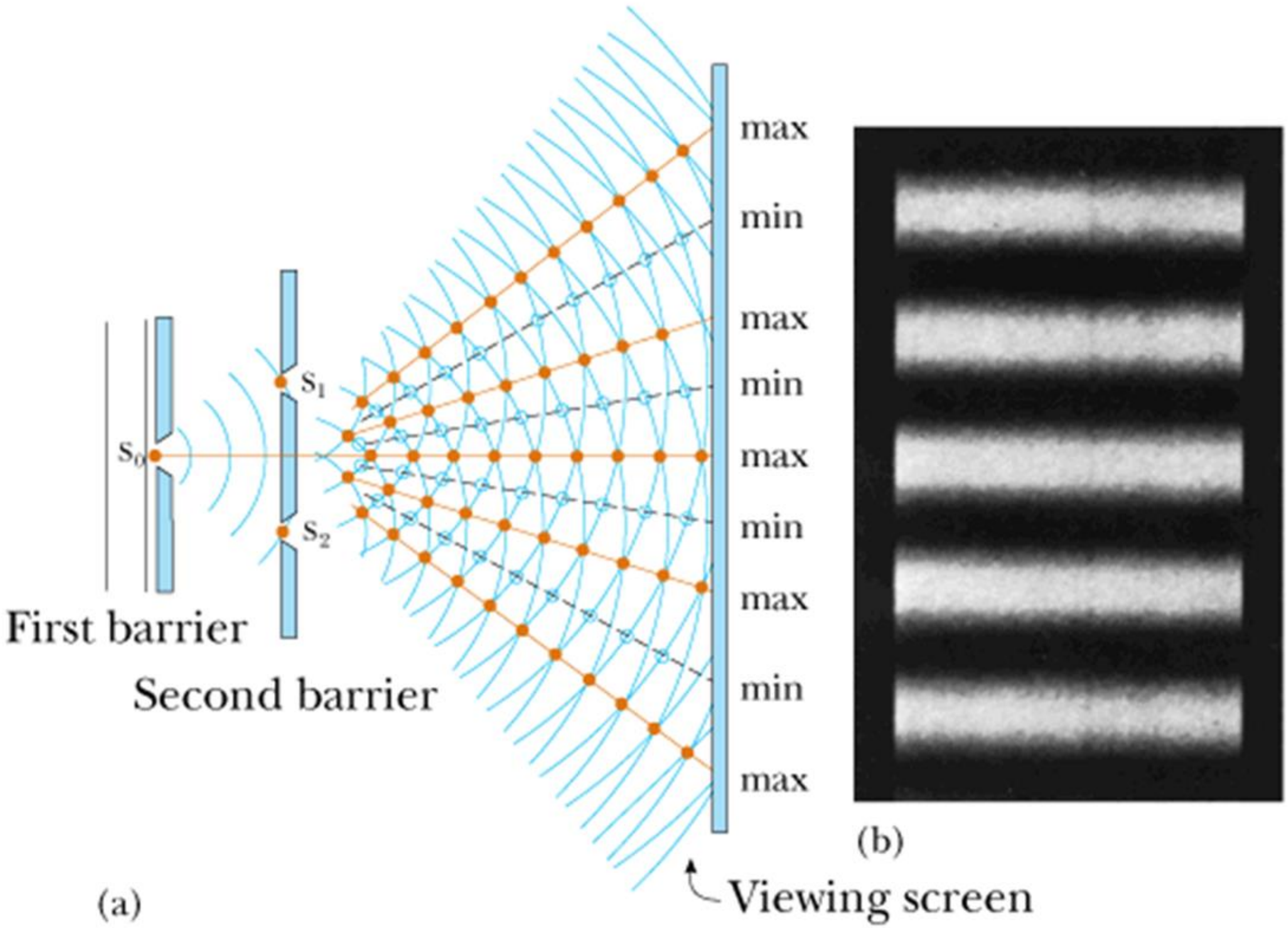
- Einstein (1905) consider the postulated that energy in a light beam was concentrated in small packet or photons.
- The photoelectric effect thus consisted in the transfer of energy from a photon to an electron. Compton, in 1921, determined the motion of a photon and a single electron, using same postulate.
- Thus, the present scenario is to accept the fact that light is dualistic in nature.
- Wave nature
- Particle nature

- **“Here we will consider only wave nature of light to explain the phenomenon of Interference in the later lectures we will focus on Diffraction and Polarization”**

Interference of light

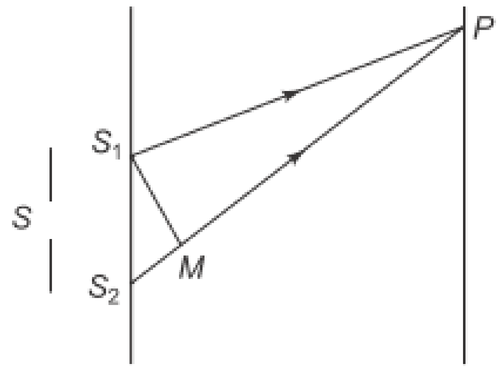
- When two waves of the same frequency having a constant initial phase difference traverse simultaneously in a medium superimpose each other, the resultant intensity of light is not distributed uniformly in space.
- This modification in intensity is called interference.
- At some points of the medium the superposition takes place in such a way that the resultant intensity is greater than the sum of intensities of individual waves.
- This type of interference is called constructive interference, while at some points of the medium the resultant intensity is found to be less than the sum of the intensities of individual waves.
- This is classified as destructive interference.

Young's Double-Slit Experiment



Resultant intensity due to superposition of two interfering waves

- Let S be a narrow slit illuminated by a monochromatic light source, and S_1 and S_2 are two similar parallel slits very close together and equidistant from S.



- Let the waves from S reach at S_1 and S_2 in the same phase such that the waves proceed as if they started from S_1 and S_2. We have to find out the resultant intensity at P on a screen placed parallel to S_1 and S_2.

Analysis

Let a_1 and a_2 be the amplitudes at P due to the waves from S_1 and S_2 , respectively. The waves reaching at P will have different paths S_1P and S_2P. Hence, they will superimpose with a phase difference δ given by

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (S_2P - S_1P) = \frac{2\pi}{\lambda} \times S_2M$$

where λ is the wavelength of light used.

The individual displacements at P will then be represented by

$$y_1 = a_1 \sin \omega t$$

$$y_2 = a_2 \sin (\omega t + \delta)$$

Analysis

$$y = y_1 + y_2$$
$$= a_1 \sin \omega t + a_2 \sin (\omega t + \delta)$$
$$= a_1 \sin \omega t + a_2 (\sin \omega t \cos \delta + \cos \omega t \sin \delta)$$

$$= a_1 \sin \omega t + a_2 \cos \delta \sin \omega t + a_2 \sin \delta \cos \omega t.$$
$$= (a_1 + a_2 \cos \delta) \sin \omega t + (a_2 \sin \delta) \cos \omega t.$$

Let us make a change in constants as

$$a_1 + a_2 \cos \delta = R \cos \theta$$

and $$a_2 \sin \delta = R \sin \theta$$

$$I = R^2$$
$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta^*$$

Conditions for Maxima and Minima

- When

$$\cos\delta = +1$$

or $$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots$$

So, path difference $(S_2P - S_1P) = n\lambda$ [from Eq. (12.1)]
Thus from Eq. (12.6), we have

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$
$$= (a_1 + a_2)^2 > a_1^2 + a_2^2, \text{ i.e., } I_{max} > I_1 + I_2$$

- When

$$\cos\delta = -1$$

or $$\delta = (2n+1)\pi, \quad n = 0, 1, 2, \ldots$$

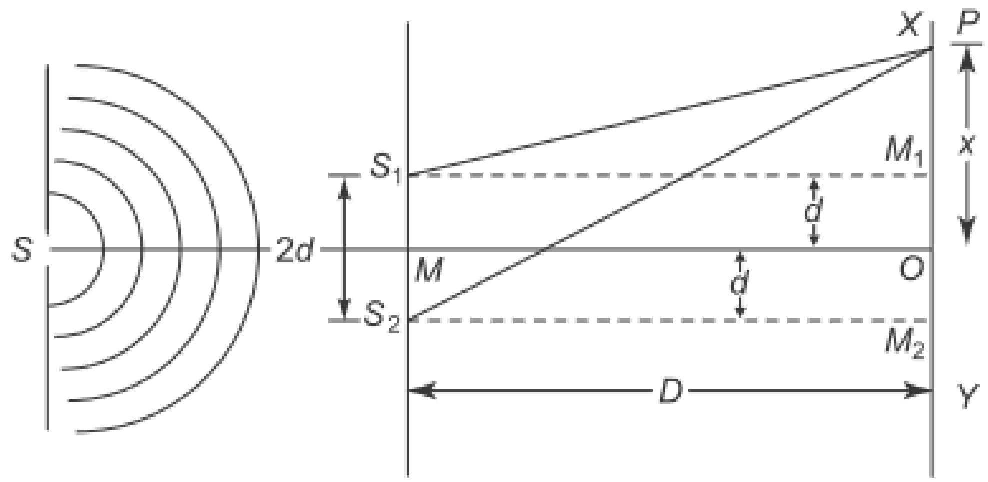
So, path difference $(S_2P - S_1P) = (2n+1)\lambda/2$.

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$
$$= (a_1 - a_2)^2 < a_1^2 - a_2^2, \text{ i.e., } I_{min} < I_1 - I_2$$

Thus, as we move on the screen, the path difference between the two waves gradually changes and there is a variation in the intensity of light being alternately maximum and minimum. This is called interference pattern.

DETERMINATION OF FRINGE WIDTH IN YOUNG'S EXPERIMENT

- Let S be a narrow slit illuminated by monochromatic light and S_1 and S_2 be two parallel slits very close together and equidistant from S. The light waves from S_1 and S_2 produce an interference pattern on a screen XY placed parallel to S_1 and S_2. as shown in Fig.



DETERMINATION OF FRINGE WIDTH IN YOUNG'S EXPERIMENT

$$(S_2P)^2 = (S_2M_2)^2 + (PM_2)^2$$
$$= D^2 + (x+d)^2$$
$$= D^2\left[1+\frac{(x+d)^2}{D^2}\right]$$

$$\therefore \qquad S_2P = D\left[1+\frac{(x+d)^2}{D^2}\right]^{1/2}$$

Since $D >> (x+d)$, the binomial expansion up to two terms will give

$$S_2P = D\left[1+\frac{(x+d)^2}{D^2}\right]^{1/2}$$
$$= D + \frac{(x+d)^2}{2D}$$

Similarly, $\qquad S_1P = D + \dfrac{(x-d)^2}{2D}$

$$\therefore \qquad S_2P - S_1P = \frac{2xd}{D}$$

Now for maxima or bright fringes, the path difference in given by

$$S_2P - S_1P = n\lambda, \text{ where } n = 0, 1, 2, \ldots$$

or $\qquad \dfrac{2xd}{D} = n\lambda$

or $\qquad x = n\dfrac{D\lambda}{2d}$

DETERMINATION OF FRINGE WIDTH IN YOUNG'S EXPERIMENT

$$S_2P - S_1P = (2n+1)\frac{\lambda}{2}, \text{ where } n = 0, 1, 2, \ldots$$

$$\frac{2xd}{D} = (2n+1)\frac{\lambda}{2}$$

or $$x = \frac{D}{2d}(2n+1)\frac{\lambda}{2}$$

Now let x_n and x_{n+1} denote the distances of nth and $(n+1)$th bright fringes, then

$$x_n = n\frac{D\lambda}{2d}$$

and $$x_{n+1} = (n+1)\frac{D\lambda}{2d}$$

∴ Spacing between nth and $(n+1)$th bright fringe or successive bright fringes is

$$x_{n+1} - x_n = (n+1)\frac{D\lambda}{2d} - n\frac{D\lambda}{2d} = \frac{D\lambda}{2d}$$

It is independent of n. Hence, spacing between any two consecutive bright fringes is same. Similarly, it can be shown that spacing between two consecutive dark fringes will also be $D\lambda/2d$.

The spacing between any two consecutive bright and dark fringes is called the fringe width (β). Thus,

$$\beta = \frac{D\lambda}{2d} \qquad (12.7)$$

Example-1:Two coherent sources whose intensity ratio is 100:1 produce interference fringes. Find the ratio of maximum intensity to minimum intensity in the interference pattern.

Solution

The ratio of maximum intensity to the minimum intensity is given by

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

Here, $$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{100}{1}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{10}{1}$$

$\Rightarrow$ $$a_1 = 10a_2$$

Substituting the value of a_1 from Eq. (2) in Eq. (1), we get

$$\frac{I_{\max}}{I_{\min}} = \frac{(10a_2 + a_2)^2}{(10a_2 - a_2)^2} = \frac{(11)^2}{(9)^2} = \frac{121}{81}$$

Two coherent sources of intensity ratio a interfere. Prove that in the interference pattern

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

Solution

Let I_1 and I_2 be the intensities and a_1 and a_2 be the corresponding amplitudes of the two coherent sources.

$$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \alpha$$

or

$$\frac{a_1}{a_2} = \sqrt{\alpha}$$

$$\therefore \quad a_1 = \sqrt{\alpha}\, a_2 \qquad (1)$$

Now,

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2} = \frac{2a_1 a_2}{a_1^2 + a_2^2}$$

$$= \frac{2(a_1/a_2)}{\left(\frac{a_1}{a_2}\right)^2 + 1}$$

$$= \frac{2\sqrt{\alpha}}{1+\alpha}$$

Assignment Based on this Lecture

- Discuss the nature of light.
- Explain the phenomena of Interference.
- Obtain the condition of constructive and destructive interference due to superposition of two waves.
- Obtained the expression for the fringe width of interference pattern.