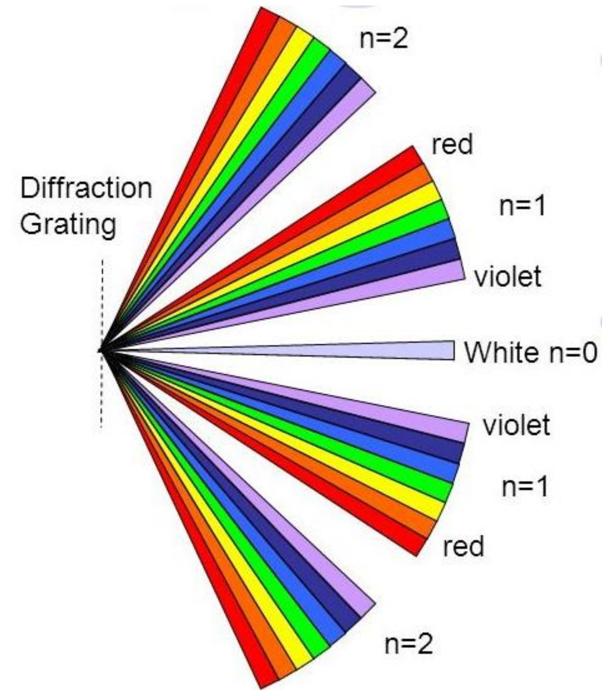
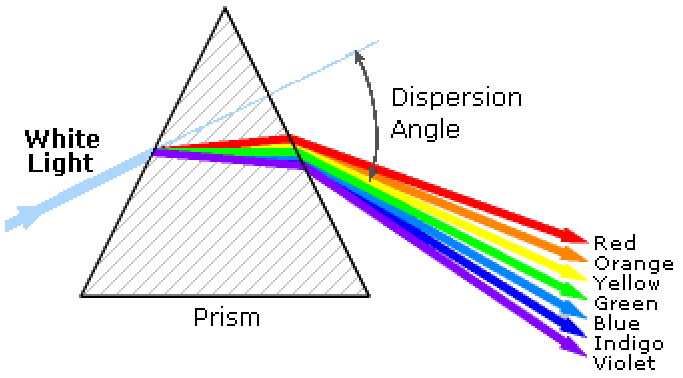
Diffraction



UNIT III
Optics
Lecture-6



Content of Lecture

- DISPERSIVE POWER OF A GRATING
- DIFFERENCE BETWEEN SPECTRA OF GRATING AND PRISM
- RESOLVING POWER OF AN OPTICAL INSTRUMENT: RAYLEIGH'S CRITERION OF RESOLUTION
- NUMERICAL PROBLEMS

DISPERSIVE POWER OF A GRATING

- The dispersive power of a diffraction grating is defined as the rate of change of the angle of diffraction with the wavelength of light.

- It is expressed as $d\theta/d\lambda$.

- The angle of diffraction θ for the principal maximum is related to the corresponding wavelength by

- $$(e+d)\,\mathrm{Sin}\theta = n\,\lambda$$

Differentiating it with respect to λ,

$$(e+d)\cos\theta\,\frac{d\theta}{d\lambda} = n$$

or
$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

From the expression of dispersive power it is clear that

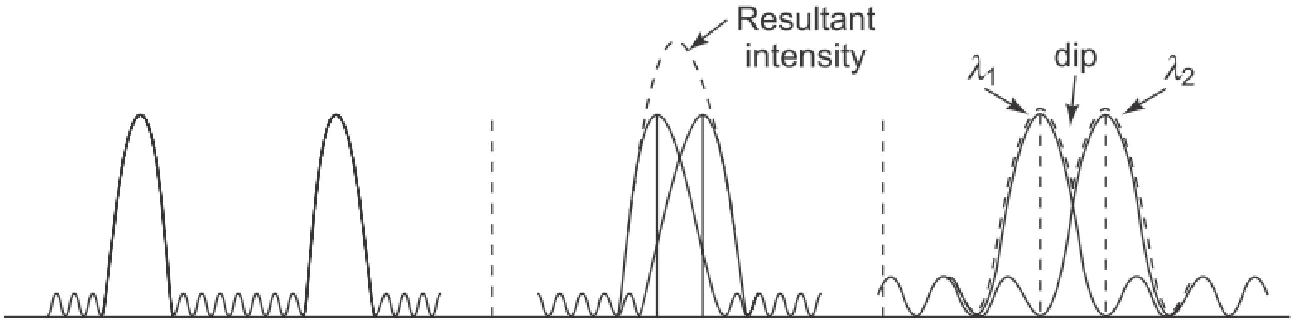
- Dispersive power is directly proportional to the order n
- Dispersive power is inversely proportional to the grating element or directly proportional to the number of rulings
- Dispersive power is inversely proportional to cos θ, i.e. larger the value of θ higher is the dispersive power.

DIFFERENCE BETWEEN PRISM AND GRATING SPECTRA

Prism spectra	*Grating spectra*
(i) Formed by dispersion, which is due to different velocities of light of different wavelenghts through the prism.	(i) Formed by diffraction, the angle of diffraction for maximum intensity being differed for different wavelengths.
(ii) Form only one spectrum.	(ii) A number of spectra of different orders on each side of the central maximum are seen.
(iii) Depends on the material of prism.	(iii) Independent of material of the grating.
(iv) Dispersion is given by $\frac{d\delta}{d\lambda} \times \frac{1}{\lambda^3}$. Thus, there is greater dispresion in violet region than in the red region.	(iv) $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$. As $\theta_v \leq \theta_r'$, therefore $\cos\theta_v < \cos\theta_r$. Hence, there is greater dispersion in the red region than in the violet region.
(v) Prism spectra are bright because all the incident light is distributed in a single spectrum.	(v) Higher-order spectra are much fainter because most of the intensity goes to the zero-order maxima.

RESOLVING POWER OF AN OPTICAL INSTRUMENT: RAYLEIGH'S CRITERION OF RESOLUTION

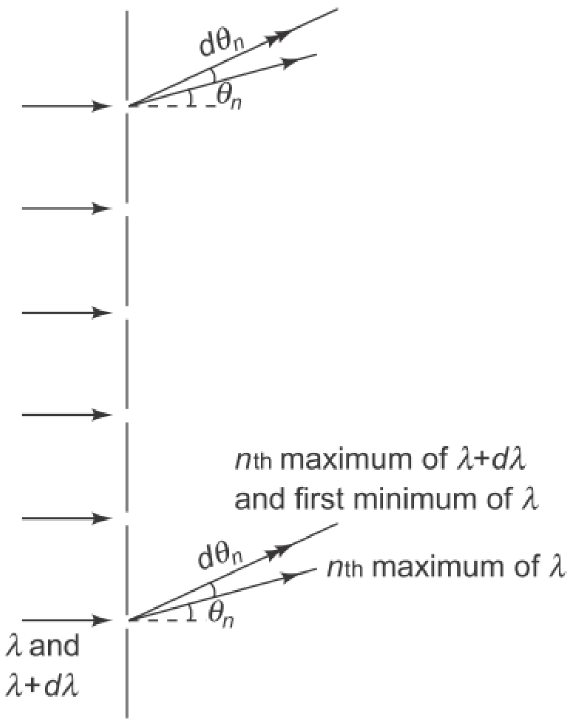
- The resolving power of an optical instrument represents its ability to produce distinctly separate spectral lines of light having two or more close wavelengths.
- "Two spectral lines of equal intensities are just resolved by an optical instrument when the principal maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other."

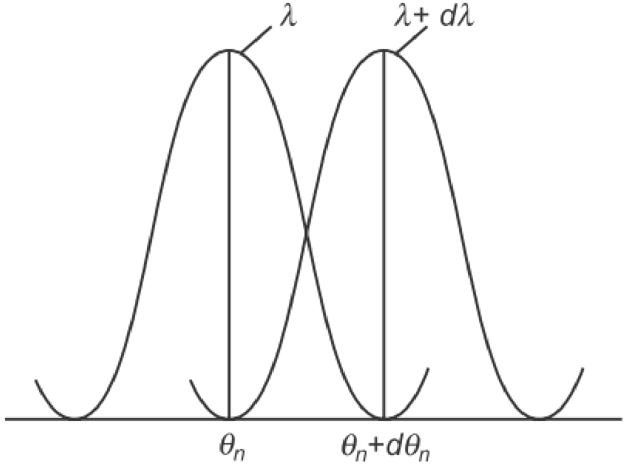


(a) Resolved (b) Not resolved (c) Just resolved

RESOLVING POWER OF AN OPTICAL……..

- If an optical instrument just resolves two spectral lines of wavelengths λ and λ + d λ, then λ /dλ is taken as a measure of resolving power of the instrument.





RESOLVING POWER OF AN OPTICAL…….

- Let a parallel beam of light of two wavelengths λ and λ+dλ be incident normally on the grating.

- If the nth principal maximum of λ is formed in the direction θ_n, we will have

$$(e + d) \sin \theta_n = n\lambda$$

where $(e + d)$ is the grating element.

Now the grating equation for the minima is

$$N(e + d) \sin \theta = m\lambda$$

where N is the total number of rulings on the grating and m can take all integral values except 0, N, 2N,…, nN, because these values of m give respective principal maxima.

RESOLVING POWER OF AN OPTICAL……..

- It is clear from the above Fig. that the first minimum of λ adjacent to nth principal maximum of (λ+dλ) in the direction of increasing θ will be obtained for m = nN + 1.

- Therefore, for this minimum we have

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda$$

$$\Rightarrow \quad (e+d)\sin(\theta_n + d\theta_n) = \left(\frac{nN+1}{N}\right)\lambda$$

According to Rayleigh criterion, the wavelengths λ and $\lambda + d\lambda$ are just resolved by the grating when the nth maximum of $\lambda + d\lambda$ is also obtained in direction $\theta_n + d\theta_n$, i.e.,

$$(e+d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda)$$

RESOLVING POWER OF AN OPTICAL…….

Comparing Equations we get

$$\left(\frac{nN+1}{N}\right)\lambda = n(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

or $$(\lambda/d\lambda) = nN$$

But $(\lambda/d\lambda)$ is the resolving power R of the grating. Therefore,

$$R = nN$$

or $$R = \frac{N(e+d)\sin\theta_n}{\lambda}$$

As expected, the resolving power is zero for the central principal maximum (n = 0), all wavelengths being is undiffracted in this order.

Example-1 A plane transmission grating has N= 40000 lines and (e+d) = 12.5 x 10^{-5} cm. Calculate the maximum resolving power, if the wavelength of light used is 5000 Å.

Solution

The maximum order is given by

$$n_{max} = \frac{(e+d)}{\lambda}$$

Given $e + d = 12.5 \times 10^{-5}$ cm, $\lambda = 5000$ Å $= 5 \times 10^{-5}$ cm, and $N = 40000$.

$$\therefore \quad n_{max} = \frac{12.5 \times 10^{-5}}{5 \times 10^{-5}} = 2.5$$

Hence maximum order possible = 2

$\therefore$ The maximum resolving power $= n_{max} N$

$= 2 \times 40000$

$= 80000$

Example-2 Calculate the minimum number of lines in a grating which will just resolve the lines of wavelengths 5890 Å and 5896 Å in the second order.

Solution

Given, $\lambda_1 = 5890$ Å, $\lambda_2 = 5896$ Å, and $d\lambda = 6$ Å.

The average wavelength $\lambda = (\lambda_1 + \lambda_2)/2 = 5893$ Å and $n = 2$.

$$\text{RP} = \frac{\lambda}{d\lambda} = \frac{5893\times10^{-8}}{6\times10^{-8}} = 982$$

Further $\quad \text{RP} = nN$

$\Rightarrow \quad nN = 982$

$\Rightarrow \quad 2N = 982 \times N = \dfrac{982}{2} = 491$

Example: 3 A diffraction grating used at normal incidence gives a line λ_1 = 6000 Å in a certain order superimposed on another line λ_2= 4500 Å of the next higher order. If the angle of diffraction is 30^0, calculate the number of lines in 1 cm of the grating.

Solution

Given $\lambda_1 = 6000$ Å $= 6 \times 10^{-5}$ cm, $\lambda_2 = 4500$ Å $= 4.5 \times 10^{-5}$ cm, and $\theta_1 = \theta_2 = \theta = 30°$.

Now, from $(e + d) \sin \theta = n\lambda$,

$$(e + d) \sin \theta_1 = n \times \lambda_1$$

$$(e + d) \sin \theta_2 = (n + 1) \times \lambda_2$$

On dividing Eq. (2) by Eq. (1), we get

$$\frac{(e + d) \sin \theta_2}{(e + d) \sin \theta_1} = \frac{(n + 1)\lambda_2}{n\, \lambda_1}$$

$$\Rightarrow \quad n\lambda_1 = n\lambda_2 + \lambda_2$$

$$\Rightarrow \quad \lambda_2 = n(\lambda_1 - \lambda_2)$$

$$4.5 \times 10^{-5} = (6 \times 10^{-5} - 4.5 \times 10^{-5})\, n$$

$$\Rightarrow \quad = (1.5 \times 10^{-5}) n$$

$$\Rightarrow \quad n = \frac{4.5 \times 10^{-5}}{1.5 \times 10^{-5}}$$

$$\Rightarrow \quad n = 3$$

On putting the value of n in Eq. (1), we get

$$(e + d) \sin \theta_1 = 3 \times \lambda_1$$

$$\Rightarrow \quad (e + d) = \frac{3 \times 6 \times 10^{-5}}{\sin 30} = \frac{3 \times 6 \times 10^{-5}}{1/2}$$

$$= 36 \times 10^{-5}$$

Hence, the number of lines per centimetre $= \dfrac{1}{e + d}$

$$= \frac{1}{36 \times 10^{-5}}$$

$$= 2778$$

Two spectral lines have wavelengths λ and $\lambda + d\lambda$. Show that if $d\lambda << \lambda$, their angular separation $d\theta$ in a grating spectrum is given by

$$d\theta = \frac{d\lambda}{\sqrt{[(e+d)/n]^2 - \lambda^2}}$$

Solution

The condition for spectral maximum

$$(e + d)\sin\theta = n\lambda$$

But from dispersive power of grating, we have

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

$$\Rightarrow \qquad d\theta = \frac{n d\lambda}{(e+d)\cos\theta}$$

$$= \frac{n\,d\,\lambda}{(e+d)\sqrt{1-\sin^2\theta}}$$

[on putting the value of sin θ from Eq. (1)]

$$= \frac{n\,d\,\lambda}{(e+d)\sqrt{1-[n\lambda/(e+d)]^2}}$$

$$= \frac{n\,d\,\lambda}{(e+d)\sqrt{[(e+d)^2-(n\lambda)^2]/(e+d)^2}}$$

$$= \frac{n\,d\lambda}{n\sqrt{[(e+d)/n]^2-\lambda^2}}$$

$$= \frac{d\lambda}{\sqrt{[(e+d)/n]^2-\lambda^2}}$$

Example-5: Light of wavelength 5000 Å falls normally on a plane transmission grating having 15000 lines in 3 cm. Find the angle of diffraction for maximum intensity in the first-order.

Solution

$$\lambda = 5000\ \text{Å} = 5 \times 10^{-5}\,\text{cm}$$

$$(e+d) = \frac{3}{15000}\,\text{cm} = \frac{1}{5000}\,\text{cm}$$

$$n = 1$$

$$\theta = ?$$

Condition for principal maximum is

$$(e+d)\sin\theta = n\lambda$$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{e+d}$$

$$= \frac{1 \times 5 \times 10^{-5}}{1/5000}$$

$$= 5000 \times 5 \times 10^{-5}$$

$$= 25 \times 10^{-2}$$

$$= 0.25$$

$$\Rightarrow \quad \theta = \sin^{-1}(0.25) = 14.48^\circ$$

Example:6: A plane transmission grating has 15000 lines/in. Find

(i) the resolving power of the grating and

(ii) the smallest wavelength difference that can be resolved with a light of wavelength 6000 Å in second order.

Solution

Here, $N = 15000$, $\lambda = 6000 \times 10^{-8}$ cm $= 6 \times 10^{-5}$ cm, and $n = 2$.

(i) The resolving power of the grating $= nN$

$$= 2 \times 15000$$
$$= 30000$$
$$= 3 \times 10^4$$

(ii) The resolving power of the grating is also given by

$$\frac{\lambda}{d\lambda} = nN$$

$$\Rightarrow \quad \frac{\lambda}{d\lambda} = 30000$$

or $$d\lambda = \frac{\lambda}{30000} = \frac{6\times10^{-5}}{30000} = 2 \times 10^{-9}\ \text{cm}$$

$$= 20 \times 10^{-8}\ \text{cm}$$
$$= 20\ \text{Å}$$

Assignment Based on this Lecture

- Define dispersive power of prism. Obtain the expression for the resolving power of prism.
- Differentiate between spectra of grating and prism.
- Discuss the resolving power of optical instruments.
- Obtain the expression of resolving power of grating.