Balancing.

→ All the reciprocating & revolving parts should be balanced

→ If not balanced dynamic forces are set-up which increases the load on bearings & stresses in the various members. It also produces dangerous vibration.

→ Vibration at high speed produce excessive noise, cause wear & tear of m/c parts and result in faulty performance

→ Balancing is defined as the process of designing / modifying a machine in which unbalance forces are minimum.

→ A machine is said to be perfectly balanced if all the resultant forces and couples between the frame & its foundation are zero.

→ Forces due to the pressure of the working fluid is called statical forces and forces due to acceleration of component parts is called Inertia forces.

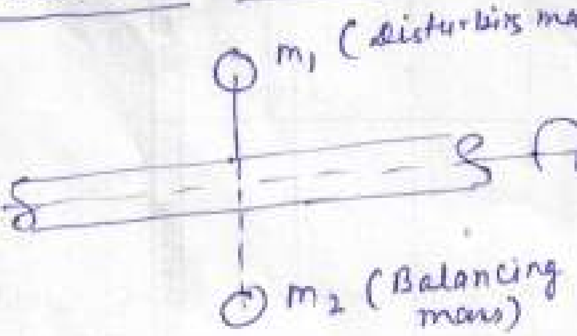
(2) Balancing of Rotating Masses

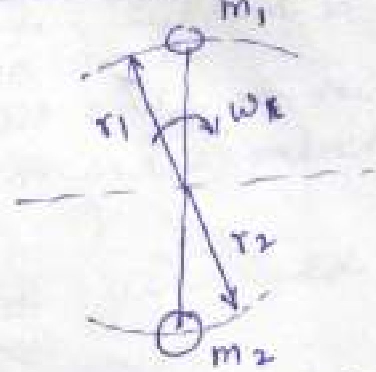
The process of providing a second mass in order to counteract the effect of centrifugal force of first mass is known as Balancing of rotating mass

dynamic Balancing :
(a) centre of masses of system lie on axis of rotation
(b) Net couple due to dynamic forces acting on shaft is zero
sum of moments about any

Balance of Single Rotating mass < balance mass rotating in same plane / balance mass is not rotating in same plane

Balance mass is Rotating in the same plane





[r = distance of c.a. of mass from axis of rot^n]

centrifugal force acting radially outwards on disturbing mass

$$\frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = m\omega^2 r = \boxed{m_1 \omega^2 r_1} \quad \text{— (a)}$$

This centrifugal force produce bending moment on shaft.

m_2 is attached to the shaft in same plane of rotation as that of (m_1) such that centrifugal force of two masses are equal & opposite.

centrifugal force due to m_2 $= m_2 \omega^2 r_2$ — (b)

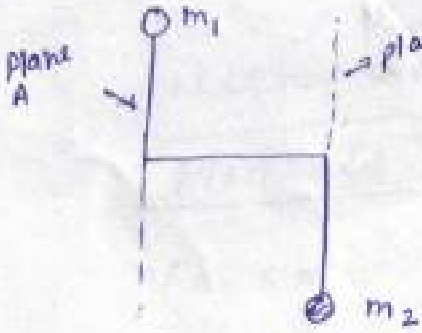
$$m_1 \omega^2 r_1 = m_2 \omega^2 r_2$$

$$\boxed{m_1 r_1 = m_2 r_2}$$

m_2 can be reduced by increasing r_2

[m_1 & m_2 are in the same plane i.e. plane of paper.]

Balance mass not rotating in same plane



due to different plane couple will create. Since couple is balanced by couple, hence a second balancing mass will be required to produce the couple of opposite sence so that the system is in pefect balance.

For perfect balance two balancing masses must be used.

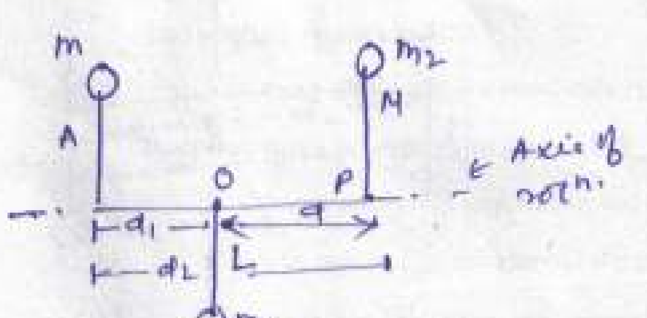
Now the three masses must be arranged such that:

(a) resultant dynamic force on shaft is zero.

(b) " couple " shaft is zero.

It is only possible if line of action of three centrifugal forces are parallel & the algebraic sum of their moments about any point in same plane is zero.

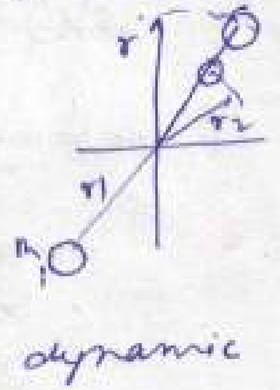
(a) When the plane of disturbing mass lies on one end of the planes of balancing masses



$F_c = m\omega^2 r$

$F_{c_1} = m_1\omega^2 r_1$

$F_{c_2} = m_2\omega^2 r_2$



For perfect balancing, the resultant dynamic force on shaft is zero.

$$F_c + F_{c_2} = F_{c_1}$$

$$\Rightarrow \boxed{m \times r + m_2 \times r_2 = m_1 r_1}$$

Algebraic sum of the moments of centrifugal force about any point on the shaft should be zero.

[O & P are pt of intsection of plane L & plane M with axis of rotn.]

$\boxed{\text{Moments @ O}}$

$$F_{c_2} \times d = F_c \times d_1$$

$$m_2 \omega^2 r_2 \times d = m \omega^2 r \, d_1$$

$$\Rightarrow m_2 r_2 d = m \cdot r d_1$$

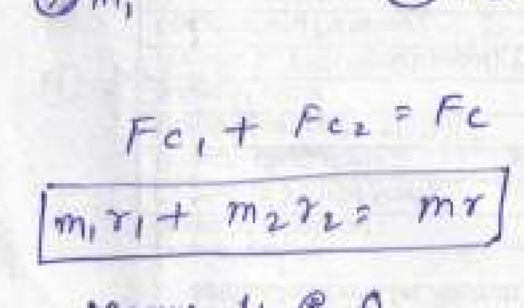
$$m_2 r_2 = \frac{m r d_1}{d}$$

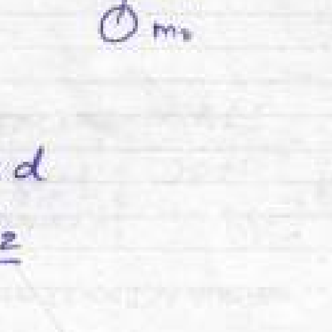
$\boxed{\text{Moments @ P}}$

$$F_{c_1} \times d = F_c \times d_2$$

$$\boxed{m_1 r_1 = \frac{m r d_2}{d}}$$

when plane of disturbing mass lies in between the plane of two balancing masses





$$F_{c_1} + F_{c_2} = F_c$$

$$\boxed{m_1 r_1 + m_2 r_2 = m r}$$

Moments @ O

$$F_c \times d_1 = F_{c_2} \times d$$

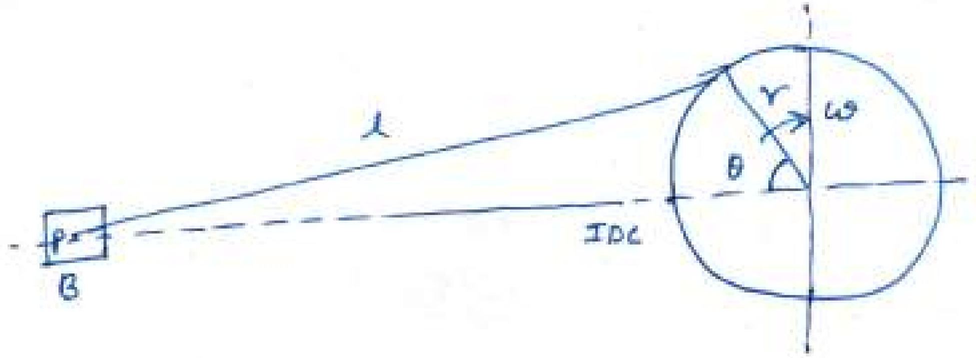
$$m_2 r_2 = \frac{m r d_1}{d}$$

@ P

$$F_c \times d_2 = F_{c_1} \times d$$

$$m_1 r_1 = \frac{m r d_2}{d}$$

Balancing of Reciprocating Mass



Acc^n of piston / reciprocating mass

$$f_p = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Force reqd to accelerate the reciprocating mass

$$F = \text{mass of reciprocating parts} \times \text{acc}^n$$
$$= m_R \times f_p$$
$$= m_R \times \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

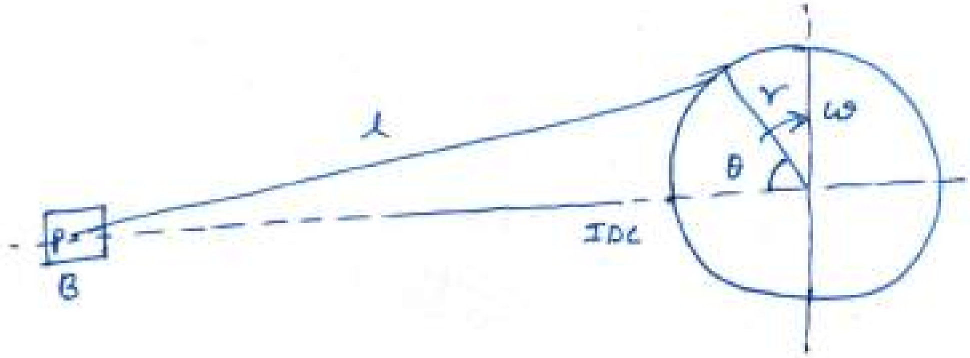
∴ Inertia force:

$$F_i = - m_R \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$F_i = - (F_p + F_s)$$ [+ve = inertia force directed away from main bearing; −ve = directed toward bearing]

The inertia force (F_i) acts along the line of stroke of reciprocting engine hence primary and secondary disturbing forces will be acting along the line of stroke.

Balancing of Reciprocating Mass



Acc^n of piston / reciprocating mass

$$f_p = \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Force reqd to accelerate the reciprocating mass

$$F = \text{mass of reciprocating parts} \times \text{acc}^n$$
$$= m_R \times f_p$$
$$= m_R \times \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

∴ Inertia force:

$$F_i = - m_R \omega^2 r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$F_i = -(F_p + F_s)$$ [+ve = inertia force directed away from main bearing; −ve = directed toward bearing]

The inertia force (F_i) acts along the line of stroke of reciprocating engine hence primary and secondary disturbing forces will be acting along the line of stroke.

When $\theta = 0$, $F_P = m_R \times \omega^2 \times r \cos 0^\circ = m_R \omega^2 r$ (Max)

$$F_S = \frac{m_R \omega^2 r}{n} \text{ (Max)}$$

$\theta = 90^\circ$; $F_P = m_R \omega^2 r \cos 90^\circ = 0$

$$F_S = m_R \omega^2 r \frac{\cos 180^\circ}{n} = \frac{-m_R \omega^2 r}{n} \left[\text{Mg. Max}^n\right]$$

$\theta = 180^\circ$

$$F_P = m_R \omega^2 r \cos 180^\circ = -m_R \omega^2 r \text{ — } \left[\text{Mag. Max}^n\right]$$

$$F_S = \frac{m_R \omega^2 r}{n} \text{ — Max. value}$$

$\theta = 270^\circ$

$$F_P = 0$$

$$F_S = \cancel{\frac{m_R \omega^2 r}{n}} \left(\frac{-m_R \omega^2 r}{n} \right. \text{ (Mag. max}^n)$$

Secondary force has twice the frequency of primary force, therefore in high speed engine it is necessary to consider this.

From the above

Max^m value of secondary force

$$F_S = \frac{m_R \omega^2 r}{n}$$

Max^m value of primary force

$$F_P = m_R \omega^2 r$$

$$\therefore F_S = \frac{1}{n} F_P.$$

$\because n >> 1$

$\therefore$ secondary force is small compared to F_p.

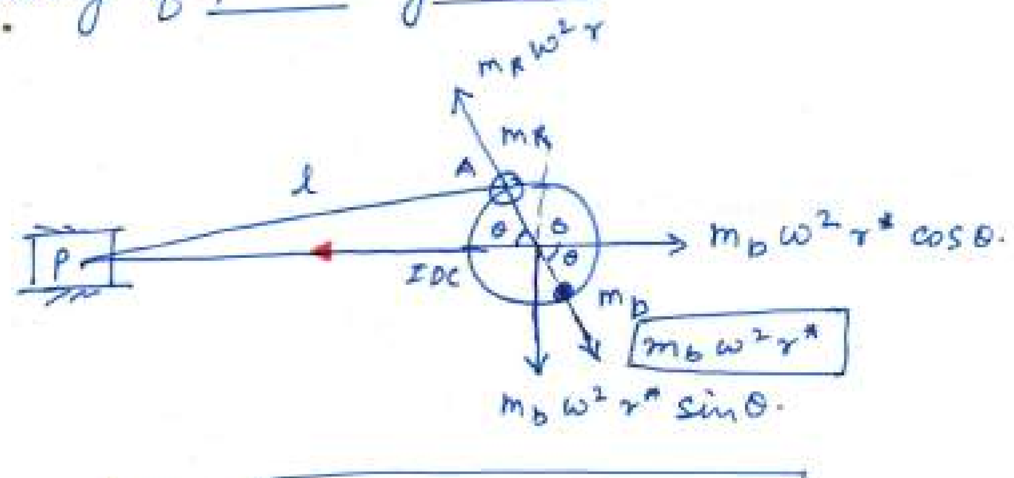
(a) For one revoⁿ of crank

Max^m value of primary force occurs two time whereas the max^m value of secondary force occurs four times.

$\because$ Inertia force acts from $O \rightarrow P$ along line of stroke

$\therefore$ Primary & secondary force also act along line of stroke from O – P.

Partial balancing of primary forces



Primary disturbing force $\boxed{F_p = m_R \omega^2 r \cos\theta}$

The primary disturbing force acts from O to P.

Centrifugal force due to rotating mass (m_R) placed at A at the crank radius 'r' is $\boxed{m_R \omega^2 \times r}$

The horizontal component of this centrifugal force is $m_R \omega^2 r \cos\theta$. The magnitude of this component is equal to primary disturbing force.

If there is no mass at A but the balance mass (m_b) is fixed at radius r^* directly opposite to crank.

centrifugal force of this mass is
$m_b \omega^2 r^*$ ~~cosθ~~

component parallel to line of stroke $= m_b \omega^2 r^* \cos\theta$ (acts along $O \rightarrow Q$ .ie opposite to primary disturbing force.

Resultant disturbing force from O to P

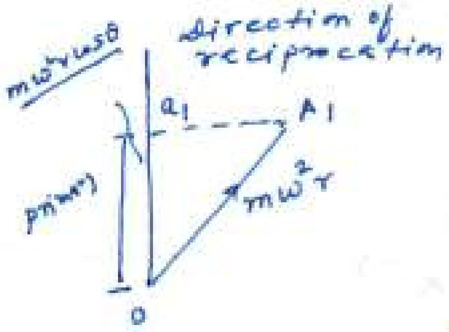
$$= m_R \omega^2 r \cos\theta - m_b \omega^2 r^* \cos\theta$$

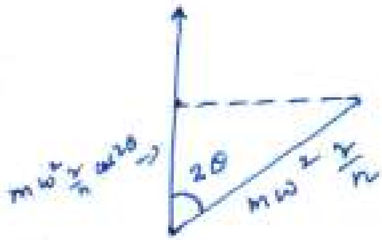
$$= \omega^2 \cos\theta (m_R \times r - m_b \times r^*)$$

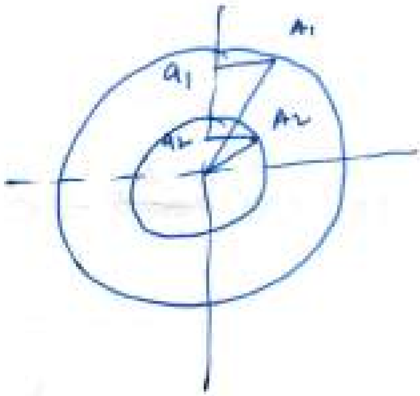
$\therefore$ Resultant disturbing force is zero if $m_R \times r = m_b \times r^*$

[primary force is completely balanced.]

Note: unbalanced force due to a reciprocating mass varies in magnitude but constant in direction. and unbalanced force due to revolving mass varies in direction but constant in magnitude. Therefore single revolving mass cannot be used to balance a reciprocating mass or vice versa.







Even though at $m_R \times r = m_b \times r^*$, primary force is completely balanced. But the centrifugal force produced by rotating mass m_b has a vertical component perp. to line of stroke. having magnitude $m_b \times \omega^2 \times r^* \sin\theta$. This component remain unbalance. The max^m value of this force is at 90° or 270°.

The max^m value of primary disturbing force is $m_R \omega^2 r$ or $m_b \omega^2 r^*$ when $\theta = 0°$ or $180°$

The introduction of ~~bal~~ rotating balance mass (m_b) has only served to change the direction of disturbing force.

The disturbing force was previously along the line of stroke, now exists perpendicular to the line of stroke. Only a fraction of reciprocating mass is balanced

$$\cancel{m_b \times r^* = c\, m_R}$$

$$m_b \times r^* = c \times m_R \times r \qquad [c<1] \rightarrow 0.5 \text{ to } 0.75$$

Resultant unbalanced force along the line of stroke is

$$= m_R \times \omega^2 \times r \cos\theta - c\, m_R\, r\, \omega^2 \cos\theta$$

$$= m_R\, \omega^2 \cos\theta\, r(\overset{1}{\cancel{\cos\theta}} - c \cancel{\cos\theta})$$

$$= \boxed{m_R\, \omega^2\, r \cos\theta\, (1-c)} \quad \text{—— (I)}$$

Resultant unbalanced force at right angles to the line of stroke

$$m_b \times r^* \times \omega^2\ \cancel{\cos\theta}\ \sin\theta$$

$$= c\, m_R\, r\, \omega^2 \sin\theta \quad \text{—— (II)}$$

Resultant unbalance force on engine frame at any instant

$$= \sqrt{[(1-c)\, m_R\, \omega^2\, r \cos\theta]^2 + [c\, m_R\, r\, \omega^2 \sin\theta]^2}$$

$$= m_R\, \omega^2\, r \sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

If $c = 0.5$, then resultant force on engine frame is

$$= m_R \omega^2 r \sqrt{0.5^2 (\sin^2\theta + \cos^2\theta)}$$

$$= 0.5 \times m_R \times \omega^2 \times r$$

(a) If case of locomotive, higher value of c may be used because in case of locomotives disturbing forces parallel to the line of stroke is more harmful than one perpendicular to it.

(b) If the balancing mass is required to balance the revolving masses as well as give a partial balance of reciprocating masses then

$$m_b \times r^* = M \times r + c \times m_R \times r$$
$$= r \boxed{(M + c m_R)}$$

Total equivalent revolving mass at crank radius r which is to be balanced

M = revolving masses (magnitude)

m_R = Mass of reciprocating parts

c = fraction of reciprocating parts which is to be balanced.

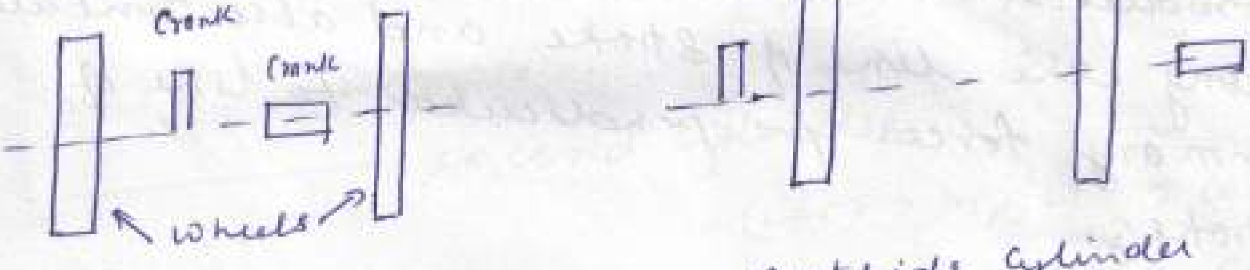
r = radius of crank.

* due to primary unbalanced force, the mechanism slides to & fro on its mounting
* due to unbalanced force perpendicular to line of stroke, the mechanism jumps up & down.

Partial balancing of Locomotives

Locomotives usually have two cylinders ~~with~~ of same dimensions placed at right angles to each other to have uniformity in Turning moment diagram. This also ensure that at least one crank is away from the dead centre and it is always possible to start the engine.

It may be of ① Inside cylinder Locomotives ⑪ Outside Cylinder Locomotives



Inside cylinder Locomotive.

Outside cylinder Locomotive.

$\boxed{\text{It may also be coupled or uncoupled.}}$

uncoupled : The effort is transmitted to one pair of wheel only.

coupled Locomotives : the driving wheels are connected to leading & trailing wheel by an outside coupling rod which connect the crank pins of the wheels. The coupling rod ~~rotates~~ ~~with~~ revolve with crank pin & hence the proportionate mass of coupling rod can be considered as a revolving mass.

Important: The ratio of length of connecting rod to the crank length is generally large. so that the secondary force is small.

The crank at right angles, the secondary forces for one set of reciprocating parts are equal & opposite to those for the other set.

Effect of Partial balancing of Locomotive

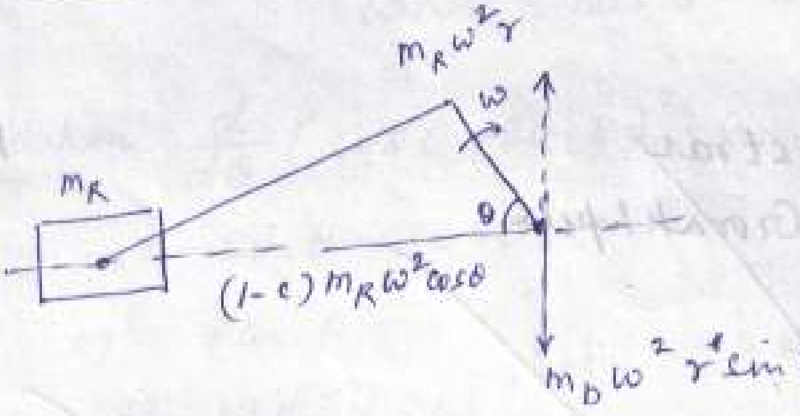
Partial balancing of Reciprocating parts produces unbalanced primary forces along the line of stroke and also unbalanced primary forces perpendicular to line of stroke.

unbalanced primary force along line of stroke produce — (a) variation in tractive force along line of stroke (b) swaying couple.

unbalanced primary force perpendicular to line of stroke produce

Hammer Blow

Variation of Tractive Force (or Effort)



Variation of Tractive force is the resultant unbalance force along the line of stroke.

Unbalanced primary force along line of stroke for first cylinder

$$= (1-c) \times m_R \times \omega^2 \times r\cos\theta.$$

________ second cylinder

$$= (1-c) \times m_R\,\omega^2\, r\cos(90+\theta)$$

$$= -(1-c)\, m_R\,\omega^2 r \sin\theta.$$

Resultant unbalanced force along line of stroke

$$(1-c)\, m_R\,\omega^2 r\cos\theta - (1-c)\, m_R\,\omega^2\, r\sin\theta$$

$$= \cancel{(1-c)\,\omega^2\, r\,(\cos}$$

$$= (1-c)\, m_R\,\omega^2\, r\,(\cos\theta - \sin\theta)$$

$\therefore$ variation of tractive force

$$= (1-c)\, m_R\,\omega^2 r\,(\cos\theta - \sin\theta)$$

$\therefore$ Max^m var^n of tractive force is when $\cos\theta - \sin\theta$ is max^m

when $\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$

$\Rightarrow -\sin\theta - \cos\theta = 0$

$\Rightarrow \tan\theta = -1$

$\theta = 135^\circ / 315^\circ$

at $\underline{\theta = 135^\circ}$

Max^m variation of tractive force

$= (1-c)\, m_R \omega^2 \times r(\cos\theta - \sin\theta)$

$= (1-c)\, m_R \omega^2 \times r(\cos 135^\circ - \sin 135^\circ)$

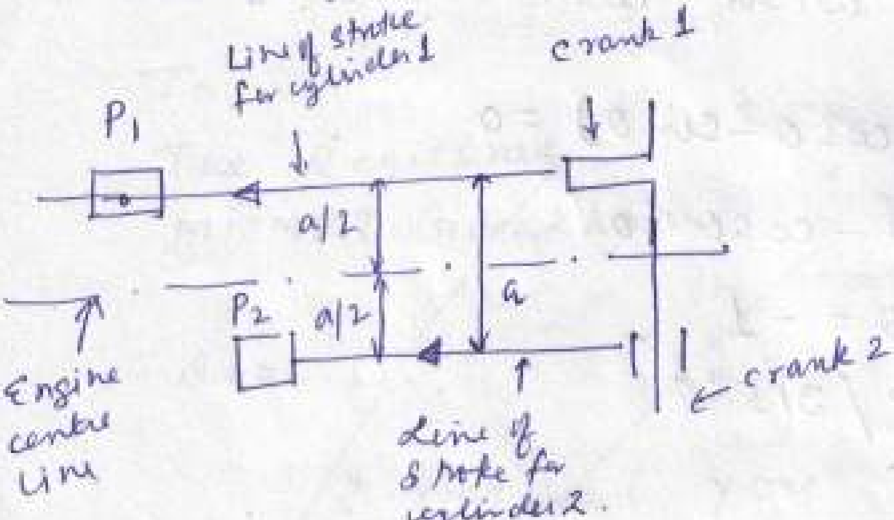
$= -\sqrt{2}\,(1-c)\, m_R \omega^2 r.$

$\theta = 315^\circ$

Max^m V.T.F $= \sqrt{2}\,(1-c)\, m_R \omega^2 r.$

$\therefore$ Max^m V.T.F $= \boxed{\pm\sqrt{2}\,(1-c)\, m_R \omega^2 r}$

Swaying Couple



unbalance parts of the primary disturbing forces along the line of stroke for the two cylinder consitute a ~~couple~~ horizontal couple about the engine centre line which is known as swaying couple. This couple tends to make the leading wheel away from side to side.

Unbalanced force along line of stroke for cylinder-1

$$= (1-c)\, m_R \omega^2 r \cos\theta$$

________ cylinder 2.

$$= (1-c)\, m_R \omega^2 r \cos(90+\theta)$$

Take moment about centre line

$$= \left[(1-c) m_R \omega^2 r \cos\theta\right] \times \frac{a}{2} - \left[(1-c) m_R \omega^2 r \cos(90+\theta)\right] \times \frac{a}{2}$$

$$\therefore \text{Swaying couple} = (1-c)\, m_R \omega^2 r \times \frac{a}{2} (\cos\theta + \sin\theta).$$

Swaying couple is max^m when
$\cos\theta + \sin\theta = \max^m$.

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$$

$$\Rightarrow -\sin\theta + \cos\theta = 0$$

$\boxed{\tan\theta = 1}$ $\therefore \theta = \cancel{90^\circ}\ 45^\circ / 225^\circ$

when $\theta = 45^\circ$

max^m swaying couple

$$= (1-c)\, m_R\, \omega^2 r \times \frac{a}{2}(\cos 45^\circ + \sin 45^\circ)$$

$$= \frac{a}{\sqrt{2}}(1-c)\, m_R\, \omega^2 r$$

when $\theta = 225^\circ$

$$= -\frac{a}{\sqrt{2}}(1-c)\, m_R\, \omega^2 r$$

$\therefore$ Maximum swaying couple $= \pm \frac{a}{\sqrt{2}}(1-c)\, m_R \omega^2 r$

Hammer Blow: The unbalanced force perpendicular to the line of stroke is $= m_b \times r^* \times \omega^2 \sin\theta$.

It is max^m when $\theta = 90^\circ / 270^\circ$

$\therefore$ Max^m unbalanced force $\perp^r$ to line of stroke (hammer blow)

$$= \pm m_b r^* \omega^2$$

Hammer Blow causes variation in pressure betn the wheel & ~~xxx~~ rail.

Net pressure betn wheel & rail = $P \pm m_b r^* \omega^2$

The wheel will lift from the rail if $P - m_b r^* \omega^2$ is negative.

The limiting condition when wheel does not lift from rail is

$$P - m_b \times r^* \omega^2 = 0$$

$$P = m_b r^* \omega^2$$

$$\omega = \sqrt{\frac{P}{m_b r^*}}$$

Gives the permissible value of angular

#1) The cranks of two cylinder uncoupled inside cylinder locomotive are at right angle and are 300 mm long. The distance between centre line of the cylinder is 650 mm. The wheel centre line are 1.6 m apart. The reciprocating mass per cylinder is 300 kg. The driving wheel diameter is 1.8 m. If the hammer blow is not to exceed 45 kN at 100 km/hr. determine

a) the fraction of reciprocating ~~balance~~ masses to be balanced.

b) the variation in tractive effort

c) the max^m swaying couple.

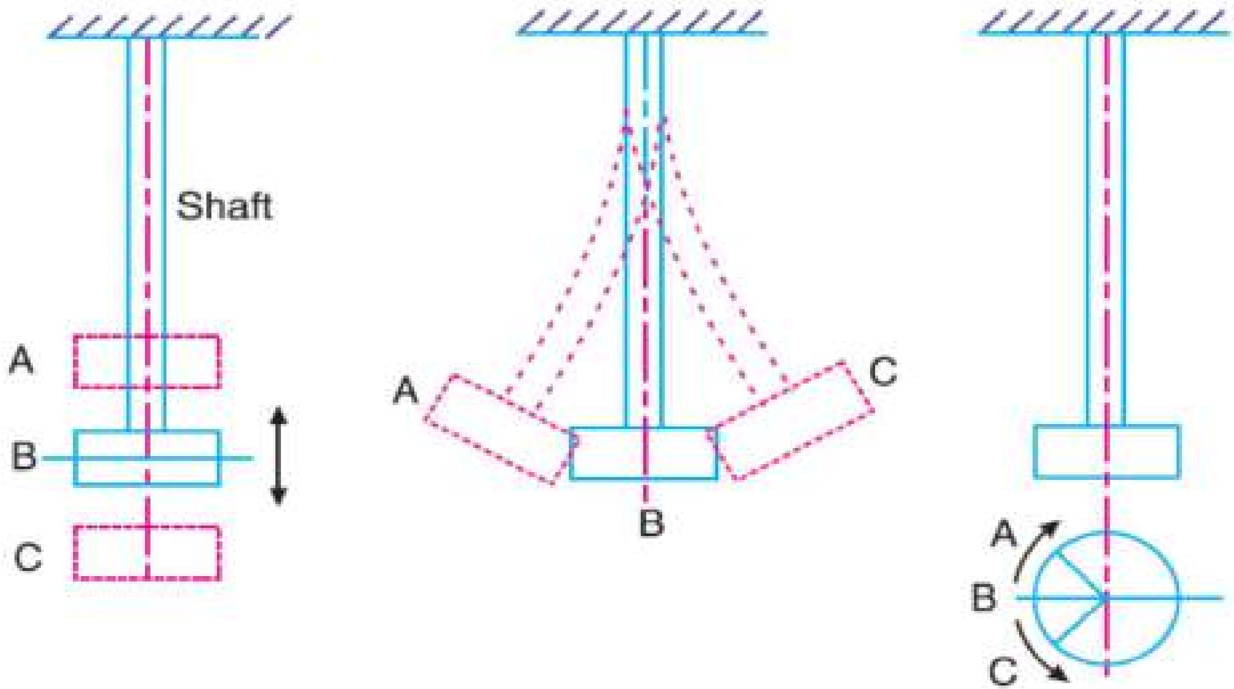
Vibrations

Definition: Vibrations are oscillations of a system about an equilibrium position

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion.

- When a body is displaced, the internal forces in the form of **elastic or strain energy** are present in the body.
- At release, these forces bring the body to its original position.
- When the body reaches the **equilibrium position**, the whole of the elastic or strain energy is converted into **kinetic energy** due to which the body continues to move in the opposite direction.
- The whole of the kinetic energy is again converted into **strain energy** due to which the body again returns to the equilibrium position.

Types of Vibrations



B = Mean position ; A and C = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

Types of Vibrations

Longitudinal vibrations. When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations are known as ***longitudinal vibrations.*** In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

Transverse vibrations. When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft. then the vibrations are known as ***transverse vibrations.*** In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.

Torsional vibrations. When the particles of the shaft or disc move in a circle about the axis of the shaft then the vibrations are known as torsional vibrations. In this case, the shaft is twisted and untwisted alternately, and the torsional shear stresses are induced in the shaft.

Damped vibrations. A vibration in which there is reduction in amplitude over every cycle of vibration is called damped vibration. Energy of vibrating system gradually dissipated by friction and other resistances.

Types of Vibrations

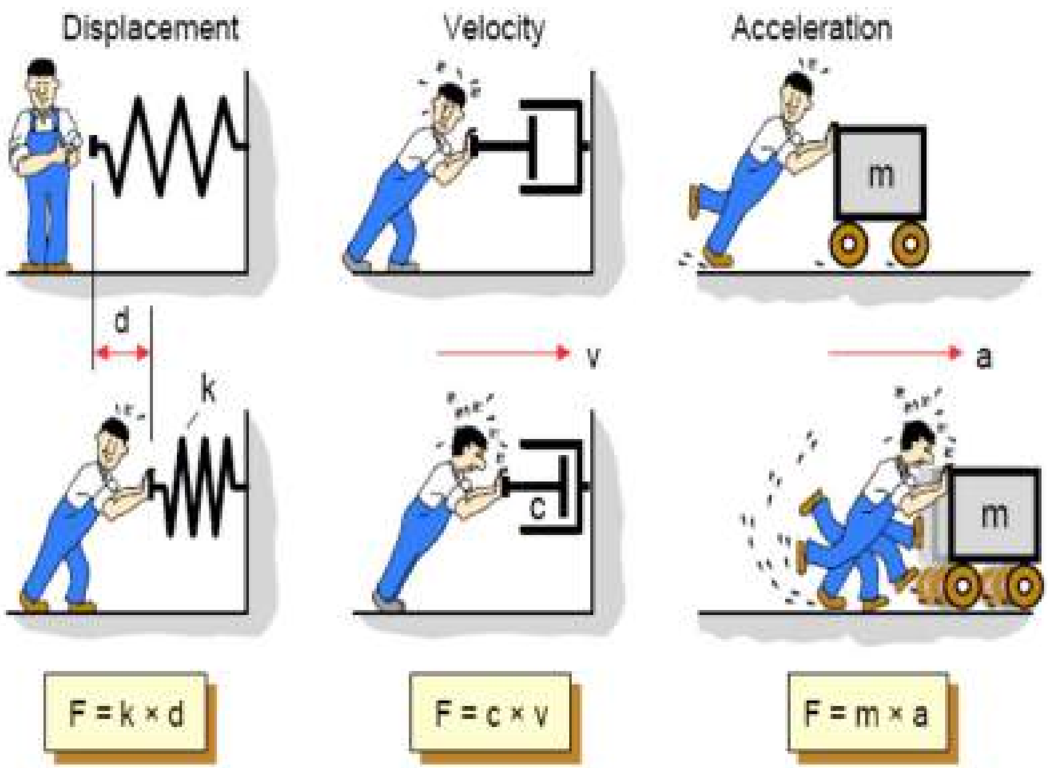
***Period of vibration or time period*.** It is the time interval after which the motion is repeated itself.

***Cycle*.** It is the motion completed during one time period.

***Frequency*.** It is the number of cycles described in one second. In S.I. units, the frequency is expressed in hertz (briefly written as Hz) which is equal to one cycle per second

If the limit of proportionality (i.e. stress proportional to strain) is not exceeded in the three types of vibrations, then the **restoring force in longitudinal and transverse vibrations** or the **restoring couple in torsional vibrations** which is exerted on the disc by the shaft (due to the stiffness of the shaft) is directly **proportional to the displacement of the disc from its equilibrium or mean position**. Hence it follows that the acceleration towards the equilibrium position is directly proportional to the displacement from that position and the vibration is, therefore, simple harmonic.

Vibration parameters



All mechanical systems can be modeled by containing three basic components:

spring, damper, mass

When these components are subjected to *constant* force, they react with a *constant* displacement, velocity and acceleration

Vibration parameters

Basic Elements of Vibrating System: Require for mathematical analysis of vibratory system

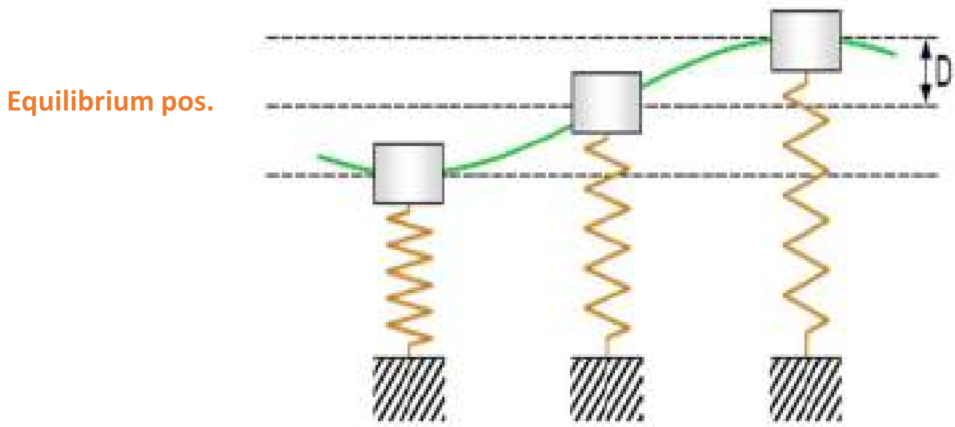
Inertial elements: Lumped mass for rectilinear motion or by lumped moment of inertial for angular motion.

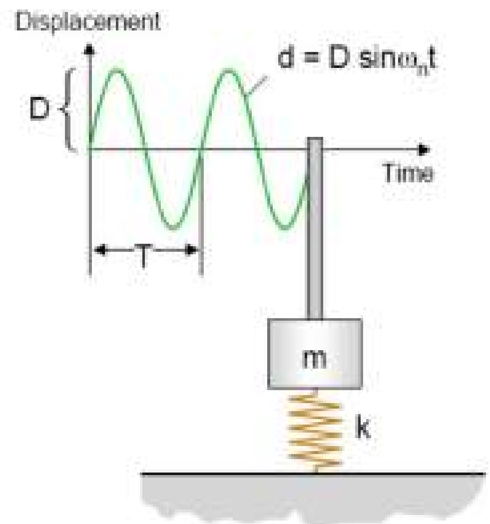
Restoring elements: Massless linear or torsional springs

Damping elements: Massless damper of rigid elements

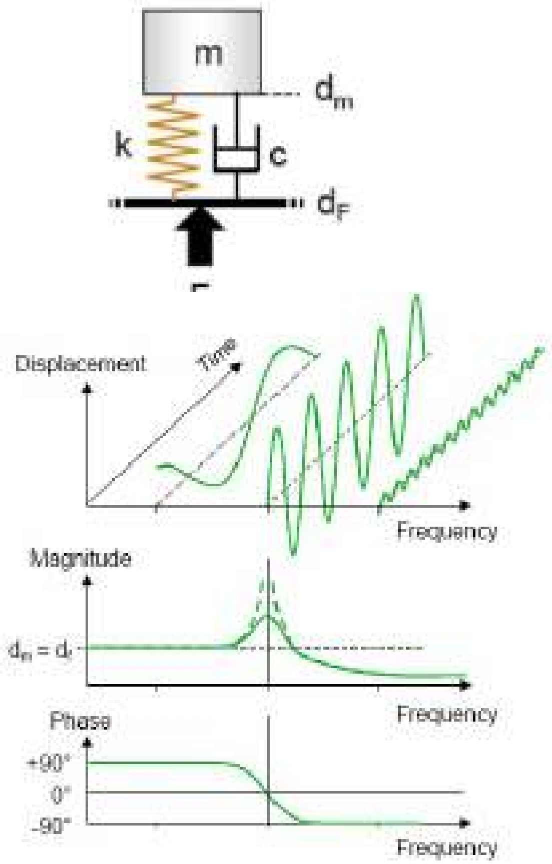
Free vibration

- When a system is initially disturbed by a displacement, velocity or acceleration, the system begins to vibrate with a constant amplitude and frequency depend on its stiffness and mass.
- This frequency is called as **natural frequency,** and the form of the vibration is called as **mode shapes**





Forced Vibration



If an external force applied to a system, the system will follow the force with the same frequency.

However, when the force frequency is increased to the system's natural frequency, amplitudes will dangerously increase in this region. This phenomenon called as **"Resonance"**

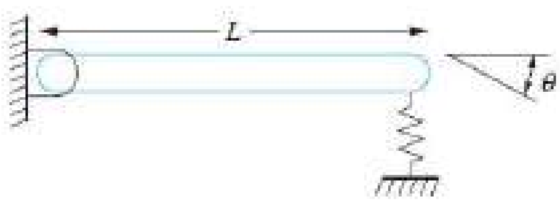
Degree of Freedom (DOF)

• Mathematical modeling of a physical system requires the selection of a set of variables that describes the behavior of the system.

• **The number of *degrees of freedom* for a system is the number of kinematically independent variables necessary to completely describe the motion of every particle in the system**

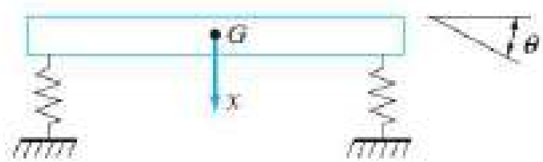
DOF=1

Single degree of freedom (SDOF)



DOF=2

Multi degree of freedom (MDOF)

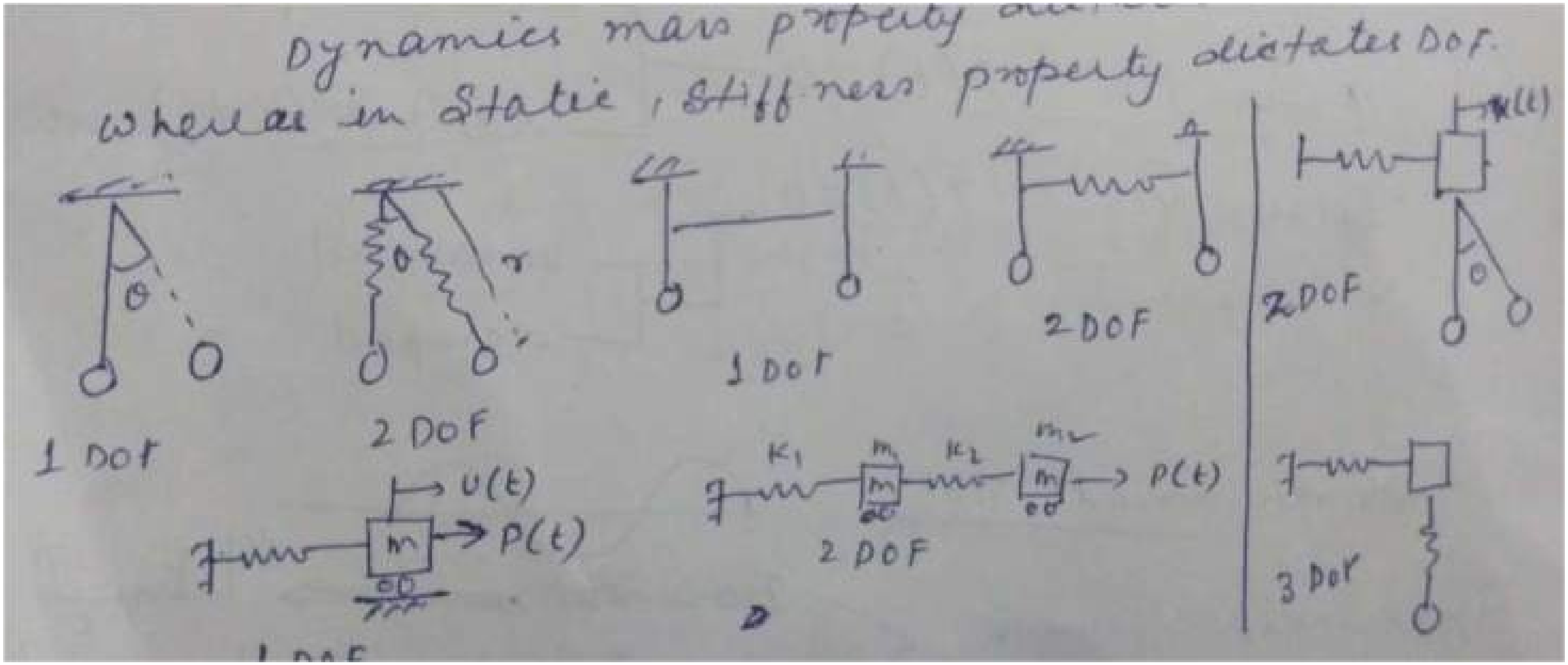


Degree of Freedom (DOF)

Number of independent coordinates (displacements) required to define the displaced position of all the masses relative to their all position

Dynamics: mass property dictates DOF
Statics: Stiffness property dictates DOF

Degree of Freedom (DOF)



Basic Definitions

Simple Harmonic Motion: Motion of particle with time that moves round a circle with uniform angular velocity. Trigonometric functions can be used to represent such motion.

Free Vibration: Vibration of a system because of its own elastic property. No external force is required for this vibration and only initiation of vibration may be necessary

Forced Vibration: A system that vibrates under an external force at the same frequency as that of external force

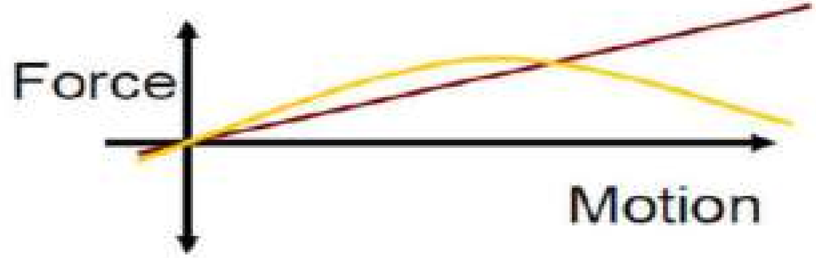
Natural frequency: It is the frequency of free vibration of a system. It is constant for a system. In fact, it is an inherent property of a system. It depends on the elastic properties, mass and stiffness of the system.

Basic Definitions

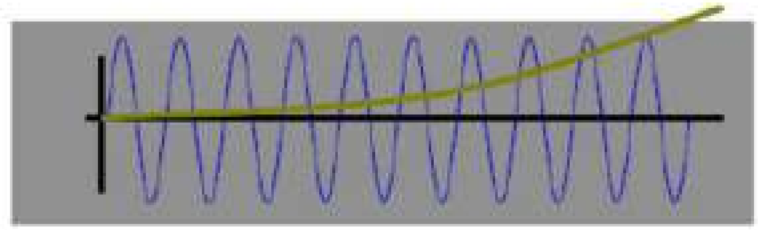
Resonance: Vibration of a system when the frequency of external force is equal to the natural frequency of the system. The amplitude of vibration at resonance becomes excessive. During resonance, with minimum input, there will be a maximum output. Hence both displacement and the stresses in the vibrating body become very high.

Dynamics

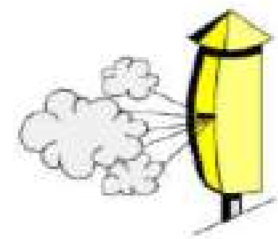
- Linear
- Non-Linear

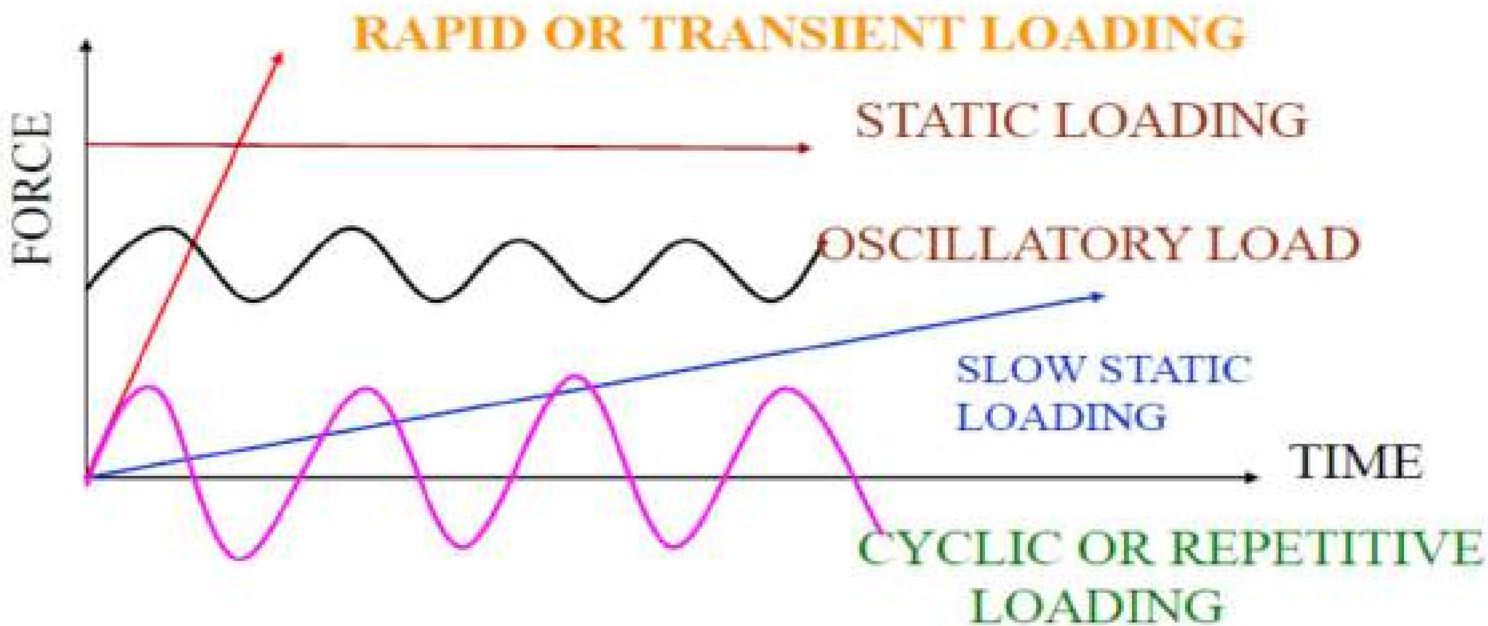


- Arbitrary motion
- Harmonic Motion



- Mechanical Vibrations
- Sound (Acoustics)




RAPID OR TRANSIENT LOADING
STATIC LOADING
FORCE
OSCILLATORY LOAD
SLOW STATIC
LOADING
TIME
CYCLIC OR REPETITIVE
LOADING