Lec-1

"Perturbation theory"

What is perturbation – ?

Perturbation is small change or a small disturbance.

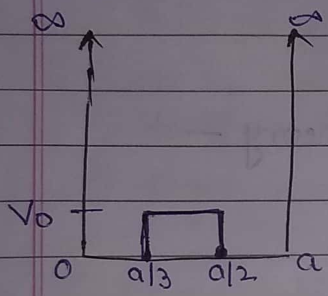
Perturbation in Quantum mechanics –

A slight change in Hamiltonian

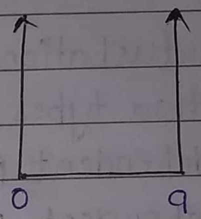
$$\hat{H} = \hat{T} + \hat{V}$$

(a slight change in potential is perturbation)

asymmetric well:



symmetric well :–



SE. solutions $\left\langle \begin{matrix}\psi_n \\ E_n\end{matrix}\right.$

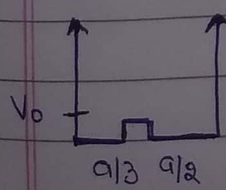
Schrodinger equation solution: $\left\langle \begin{matrix}\psi_n \\ E_n\end{matrix}\right. = ?$

SE → X

Solutions obtain by approximation method.
Perturbation Th. is one of them.

What is perturbation theory ?

Perturbation theory is a tool oR a systemetic procedure for obtaining approximate solutions to perturbed problems by using solutions for unperturbed case.

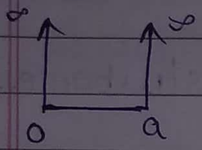


Solutions $\left\langle \begin{matrix}\psi_n \\ E_n\end{matrix}\right. = ?$

Approximate solution find (Near to exact soln)

Perturbed system.

Note :- Solution in a Unperturbed system :-



$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2m(a)^2}$$

These solutions of unperturbed system are used to find the approximate solutions in case of perturbed system.

Types of perturbation theory :→

There are two types of perturbation theory –

(i) Time independent perturbation theory

(ii) Time dependent perturbation theory.

(i) Time independent perturbation theory :

$$H' \neq H'(t)$$

eg' $\quad H' = V_0 \; ; \; \frac{a}{3} < x < \frac{a}{2}$

$$H' = g \cos kx$$

$$H' = \delta(x-a)$$

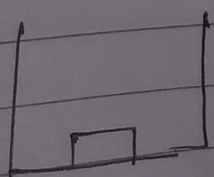
(ii) Time dependent PT :-

$$H' = H(t)$$

eg – $\quad H' = V_0 \cos \omega t$

$$H' = V_0 \sin \omega t$$

Note : –



$$H = H_0 + H'$$

H = Perturbed Hamiltonian

H_0 = Unperturbed Hamiltonian

H' = perturbation term

Further classifications –

* Time Independent PT :–

(i) Non degenerate time Independent PT

(ii) Degenerate time Independent PT.

**(i) Non degenerate –

if Unperturbed Hamiltonian H_0 have non-degenerate eigen value then it will be the case of Non degenerate TI

- What do you mean by Non-degenerate –

$$H_0\psi_1 = E_1\psi_1$$
$$H_0\psi_2 = E_2\psi_2$$
$$H_0\psi_3 = E_3\psi_3$$

all eigen wave functions have different eigen values.

ii) Degenerate TIPT :–

if H_0 have denerate energy eigen values

- What do you mean by denerade –

$$H_0\psi_1 = E_1\psi_1$$
$$H_0\psi_2 = E_1\psi_2$$

Note :→

(i) Perturbation term H' should be small (जितना small होगा solution उतना ही exact solution के पास होगा otherwise vary

(ii) By PT we find approximate solutions not exact solutions (exact solutions are solutions obtained by solving schroding equation for perturbed hamiltonian. ($H = H_0 + H'$)

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Non Degenerate Time independent perturbation theory :-

Topics —
- schematic of PT
- Various formulas
- possible Question & strategy to handle

Schematic of perturbation theory :—

$$H = H_0 + H'$$

By PT, we obtain energy eigen values E_n for perturbed.

$$E_n = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \cdots$$

$$|\psi_n\rangle = |\psi\rangle_n^0 + |\psi_n\rangle^1 + |\psi_n\rangle^2 + |\psi_n\rangle^3 + \cdots$$

Where;

E_n = Energy eigen values for perturbed Hamiltonian For nth state.

$E_n^{(0)}$ = Energy eigen value in/for unperturbed Hamiltonian for nth state

$E_n^{(1)}$ = First order energy correction to energy eigen value for nth state.

$E_n^{(2)}$ = 2nd order energy correction to energy eigen value for nth state

$|\psi_n\rangle$ = Energy eigen function for perturbed Hamiltonian for nth state.

$|\psi_n^{(0)}\rangle$ = Energy eigen function for unperturbed Hamitonian for nth state

$|\psi_n^{(1)}\rangle$ = 1st order correction to energy eigen function for nth state

$|\psi_n^{(2)}\rangle$ = 2nd order correction to energy eigen function for nth system.