DYNAMICS OF MACHINES

UNIT-II

GOVERNORS

for

B.TECH 5th Semester

Dr Vijay K Singh
Assistant Professor
Mechanical Engineering Department
National Project Implementation Unit (NPIU)
Ministry of Human Resource and Development (MHRD); Govt. of India

MADAN MOHAN MALAVIYA UNIVERSITY OF TECHNOLOGY,

GORAKHPUR-273010 (U.P.)

CONTENTS (UNIT-II)

1. Chapter-I:GOVERNOR

- TEMINOLOGY
- Centrifugal governors-**WATT GOVERNOR**
- Dead weight governor- **PORTER & PROELL GOVERNOR**
- Spring controlled governor- **HARTNELL GOVERNOR**
- Sensitivity
- Stability
- Hunting
- Isochronism
- Effort & power of governor

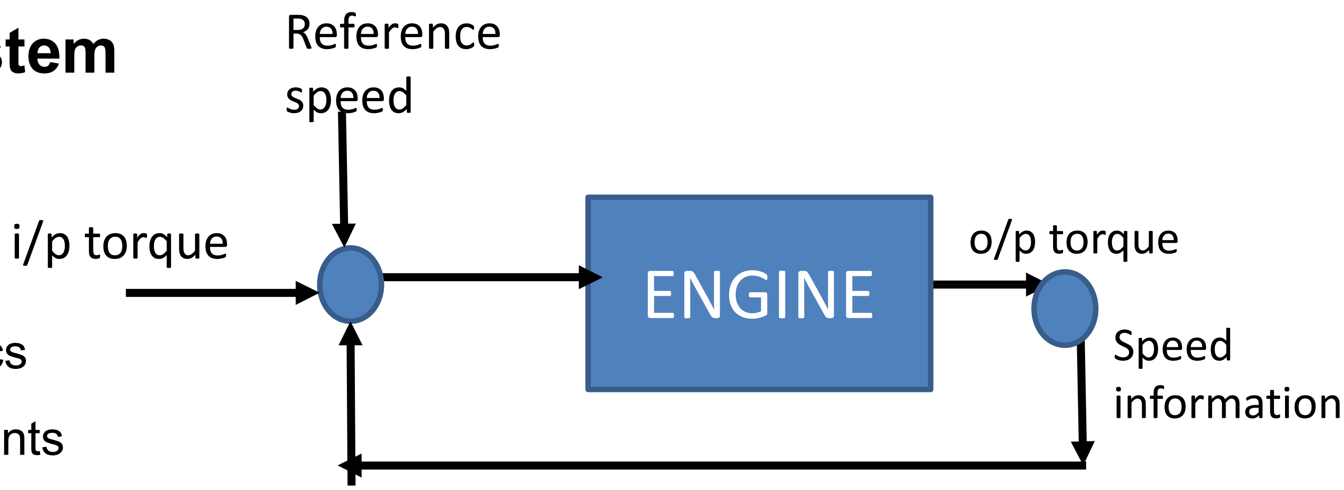
2. Chapter-II:GYROSCOPIC MOTION

- Principles
- Gyroscopic torque
- Effect – aeroplanes & automobiles

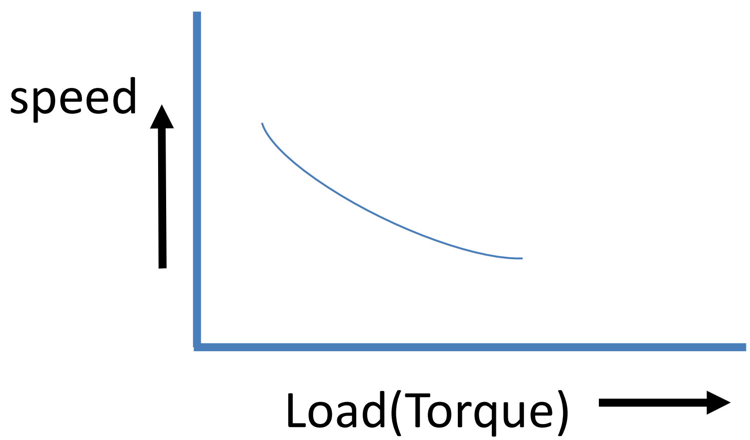
GOVERNOR

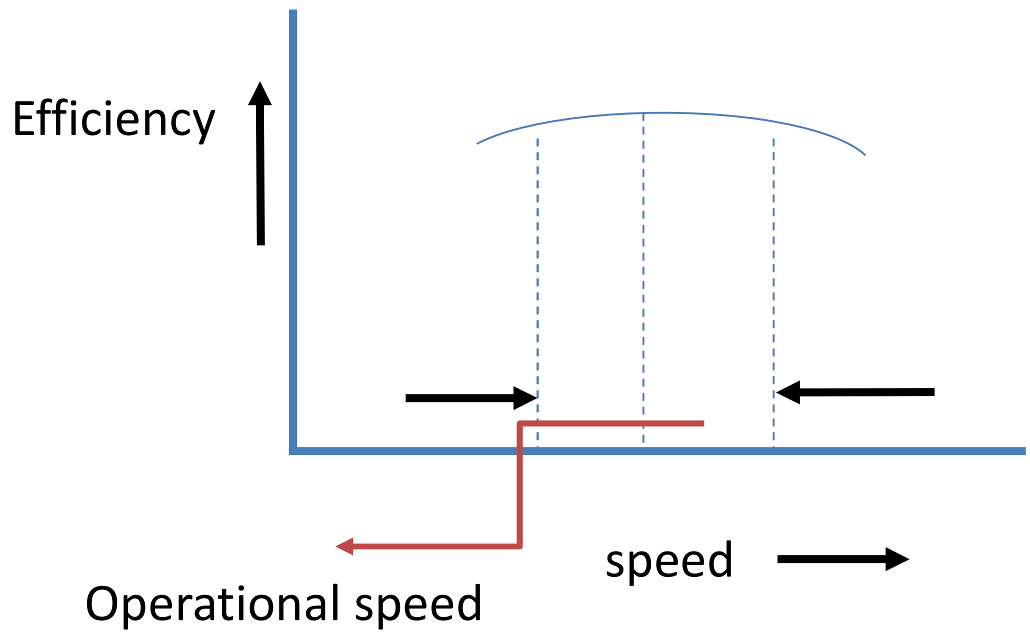
Mechanical System Design

1. Kinematics
2. Dynamics & Statics
3. Forces and moments
4. Component design
5. Inertia force balancing
6. **Power smoothening**



Feedback control



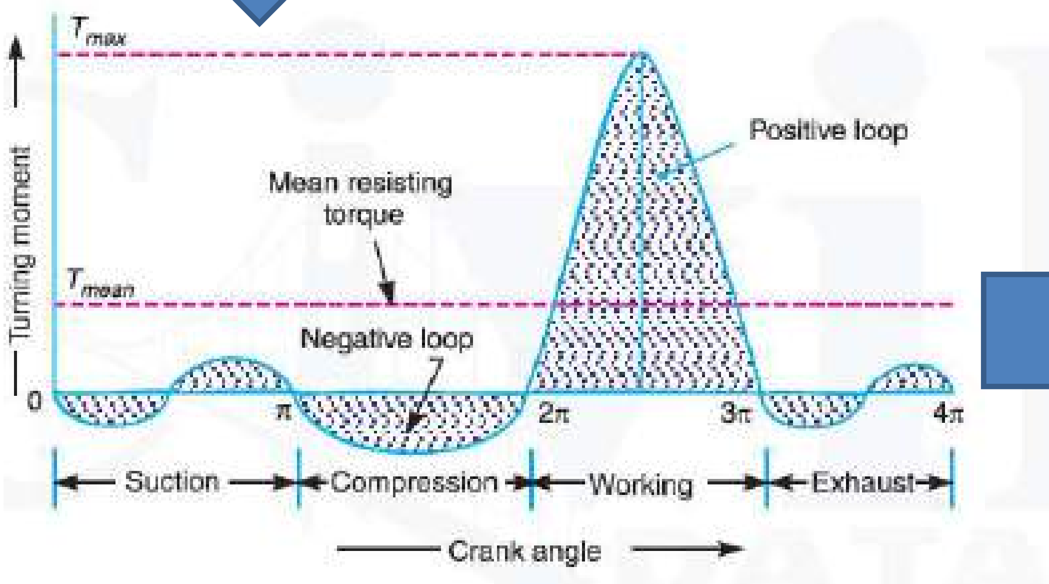


Control of Engine speed

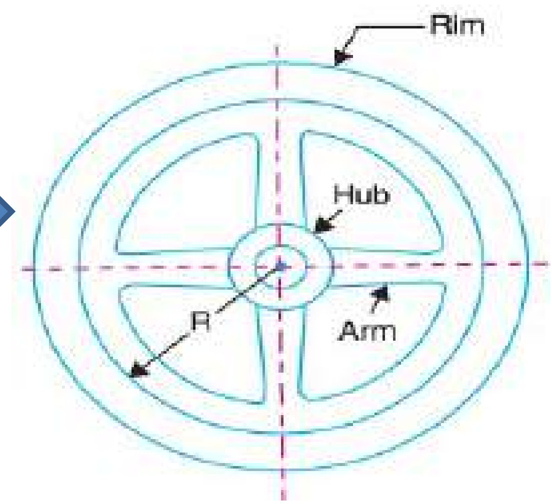
Cyclic variation due to o/p torque

variation over a no. of revolutions due to varying load

GOVERNOR



.2. Turning moment diagram for a four stroke cycle internal combustion engine.



FLYWHEEL

GOVERNOR

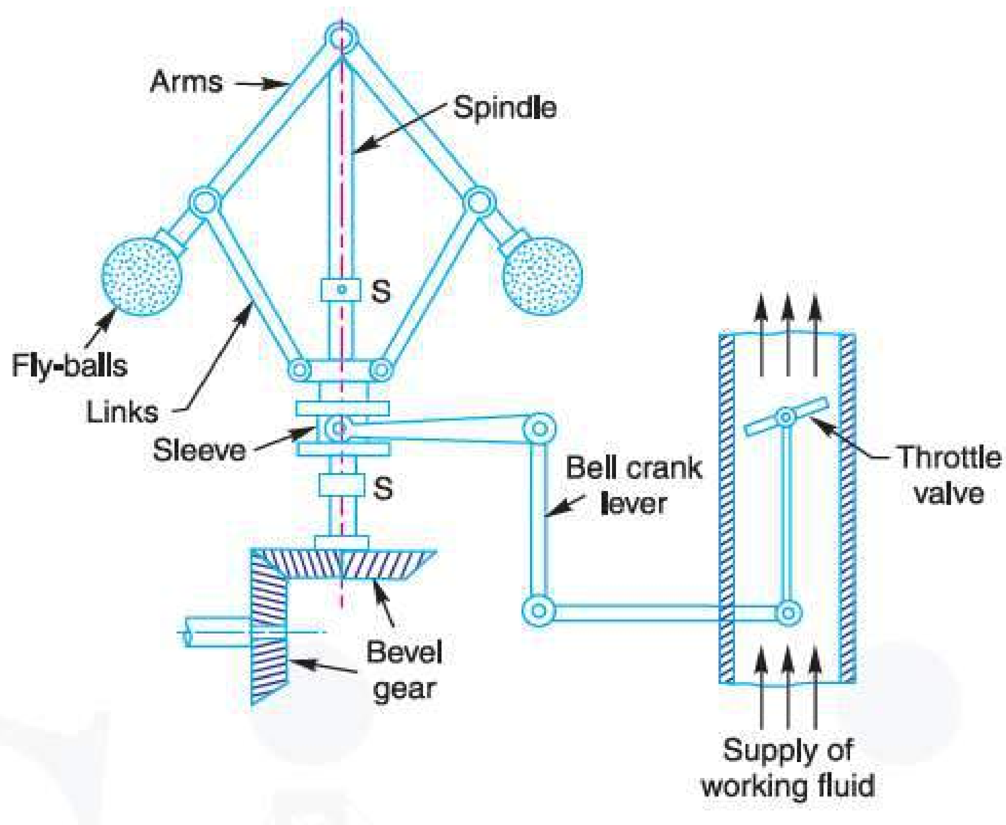
Introduction

- Controls, maintains, and regulates mean speed of an engine w.r.t varying loads

- Increases supply of working fluid if speed of the engine decreases and vice versa

- Keeps the mean speed within certain limits

- Used mainly in engines of generators not in ordinary vehicles



GOVERNOR

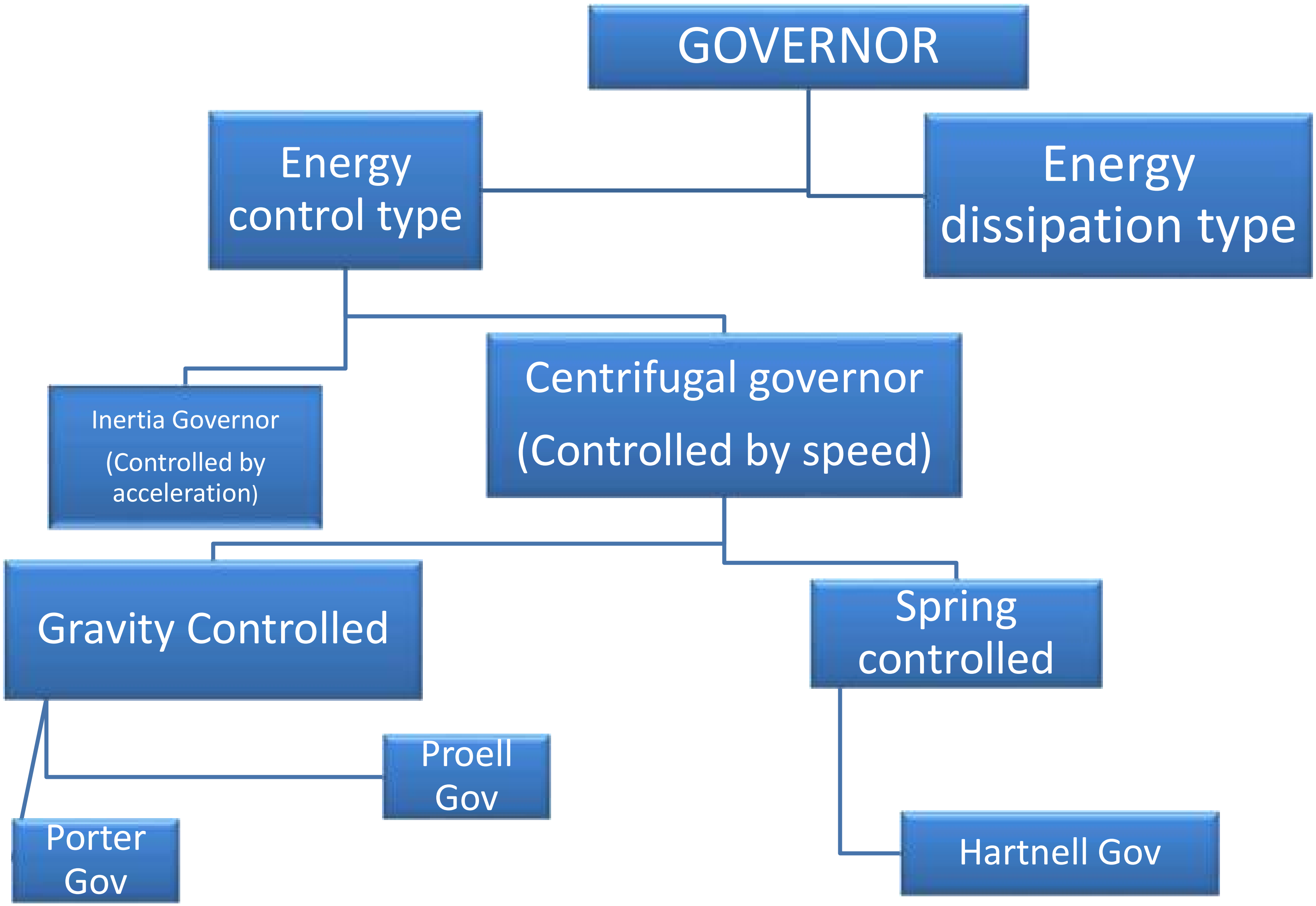
- Desired in case of fluctuating input torque e.g. 4 stroke engine, punching machine rolling mill etc.

- Reduces the fluctuation of speed but does not maintain a constant speed

FLYWHEEL

- Desired where constant speed is required e.g. generator, engines, turbines
- Control mean speed by regulating the fuel supply according to varying load

GOVERNOR TYPES



Terms Used in Governors

- ***Equilibrium Speed***

It is the speed at which the governor balls, arms etc. are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

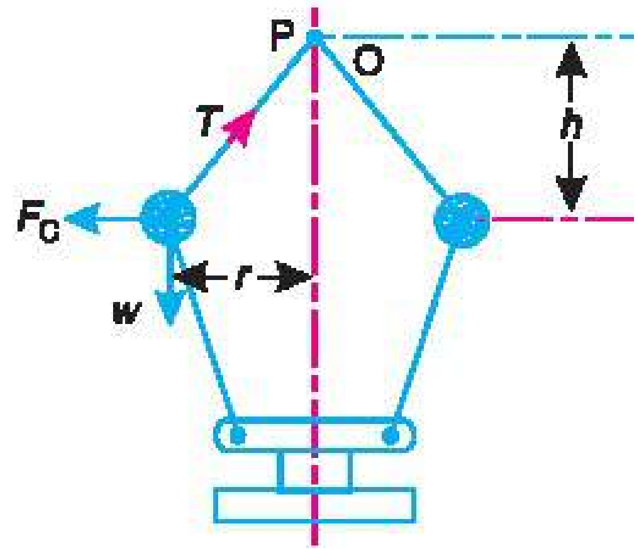
- ***Mean Equilibrium Speed***

It is the speed at the mean position of the balls or the sleeve.

Terms Used in Governors

- **Height of a Governor**

It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by *h*.



- ***Sleeve Lift***

It is the vertical distance which Centrifugal governor the sleeve travels due to change in equilibrium speed.

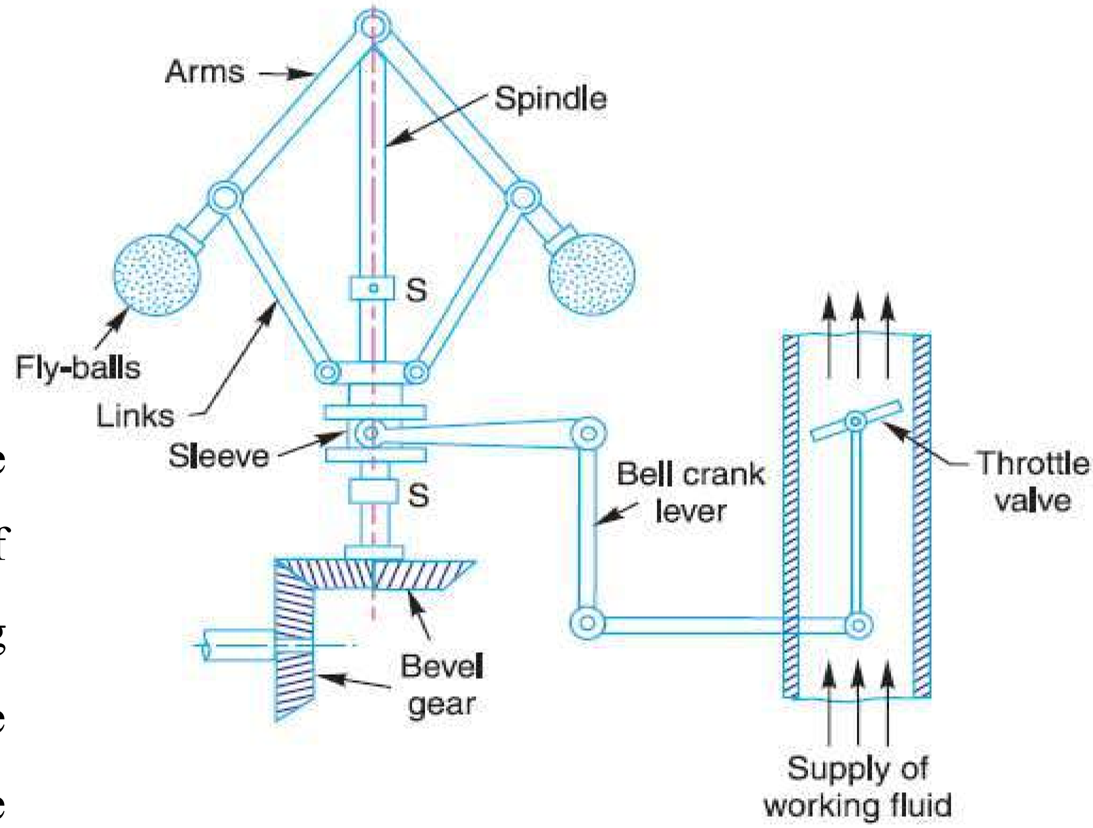
- ***Maximum And Minimum Equilibrium Speeds***

  The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

- **Note :**

  There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

Centrifugal Governors



Principle

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force.***

Centrifugal Governors- Working

- *Governor* ***balls*** **or** ***fly balls*** revolve with a spindle, which is driven by the engine through bevel gears.
- The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the Spring steel vertical axis.
- The downward movement of the sleeve operates a throttle to increase the supply of working fluid and thus the engine speed is increased.

Centrifugal Governors- Working

- The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases.

- This results in Rotating the decrease of centrifugal force on the balls. Hence weight the balls move inwards and the sleeve moves down- wards.

- The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the **controlling force.**
- **It consists** of two balls of equal mass, which are attached to the arms.
- These balls are known as **governor balls or fly balls.**
- The balls revolve with a spindle, which is driven by the engine through bevel gears.

- The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis.
- The sleeve revolves with the spindle; but can slide up and down.
- The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases.
- In order to limit the travel of the sleeve in upward and downward directions, two stops *S, S* are provided on the spindle.
- The sleeve is connected by a bell crank lever to a throttle valve.

- The supply of the working fluid decreases when the sleeve rises and increases when it falls.
- When the load on the engine increases, the engine and the governor speed decreases.
- This results in the decrease of centrifugal force on the balls.
- Hence the balls move inwards and the sleeve moves downwards.

- The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased.
- Hence, the extra power output is provided to balance the increased load.
- When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls.
- Thus the balls move outwards and the sleeve rises upwards.
- This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. Hence, the power output is reduced.

Isochronisms:

This is an extreme case of sensitiveness. When the equilibrium speed is constant for all radii of rotation of the balls within the working range, the governor is said to be in isochronisms.

This means that the difference between the maximum and minimum equilibrium speeds is zero and the sensitiveness shall be infinite.

Stability: Stability is the ability to maintain a desired engine speed without Fluctuating. Instability results in hunting or oscillating due to over correction.

Excessive stability results in a dead-beat governor or one that does not correct sufficiently for load changes

Hunting

The phenomenon of continuous fluctuation of the engine speed above and below the mean speed is termed as hunting. This occurs in over- sensitive or isochronous governors. Suppose an isochronous governor is fitted to an engine running at a steady load.

With a slight increase of load, the speed will fall and the sleeve will immediately fall to its lowest position. This shall open the control valve wide and excess supply of energy will be given, with the result that the speed will rapidly increase and the sleeve will rise to its higher position.

- As a result of this movement of the sleeve, the control valve will be cut off; the supply to the engine and the speed will again fall, the cycle being repeated indefinitely. Such a governor would admit either more or less amount of fuel and so effect would be that the engine would hunt.

Sensitiveness

A governor is said to be sensitive, if its change of speed s from no load to full load may be as small a fraction of the mean equilibrium speed as possible and the corresponding sleeve lift may be as large as possible.

Suppose

ω_1 = max. Equilibrium speed

ω_2 = min. equilibrium speed

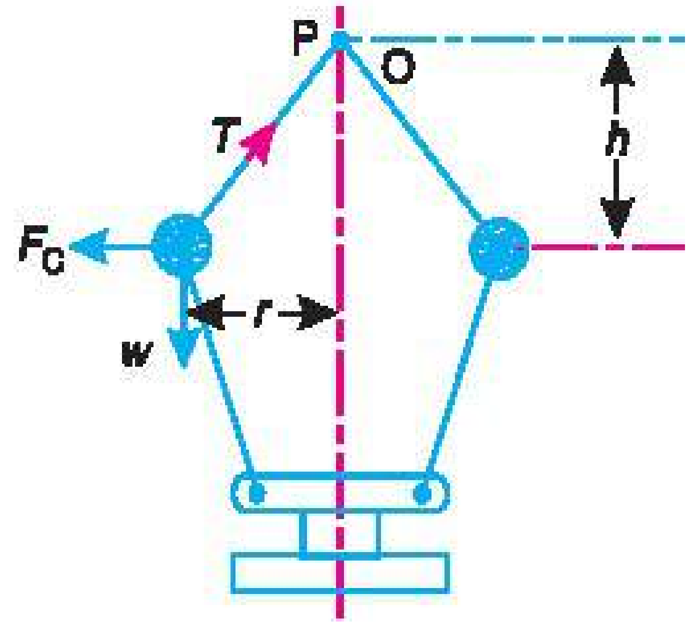
ω = mean equilibrium speed

Therefore sensitiveness = = $(\omega_1 + \omega_2)/2$

Watt governor

Let;

- w = weight of ball in N = m.g,
- T = tension in arms in N
- ω = angular velocity of arm about the spindle axis in rad/s
- r = Radius of the governor
- Fc= Centrifugal force acting on the ball in N= m.ω².r
- h = Height of the governor in metres



Watt governor

$$\left.\begin{aligned} T\cos\theta &= mg \\ T\sin\theta &= m\omega^2 r \end{aligned}\right\} \quad (1/a,b)$$

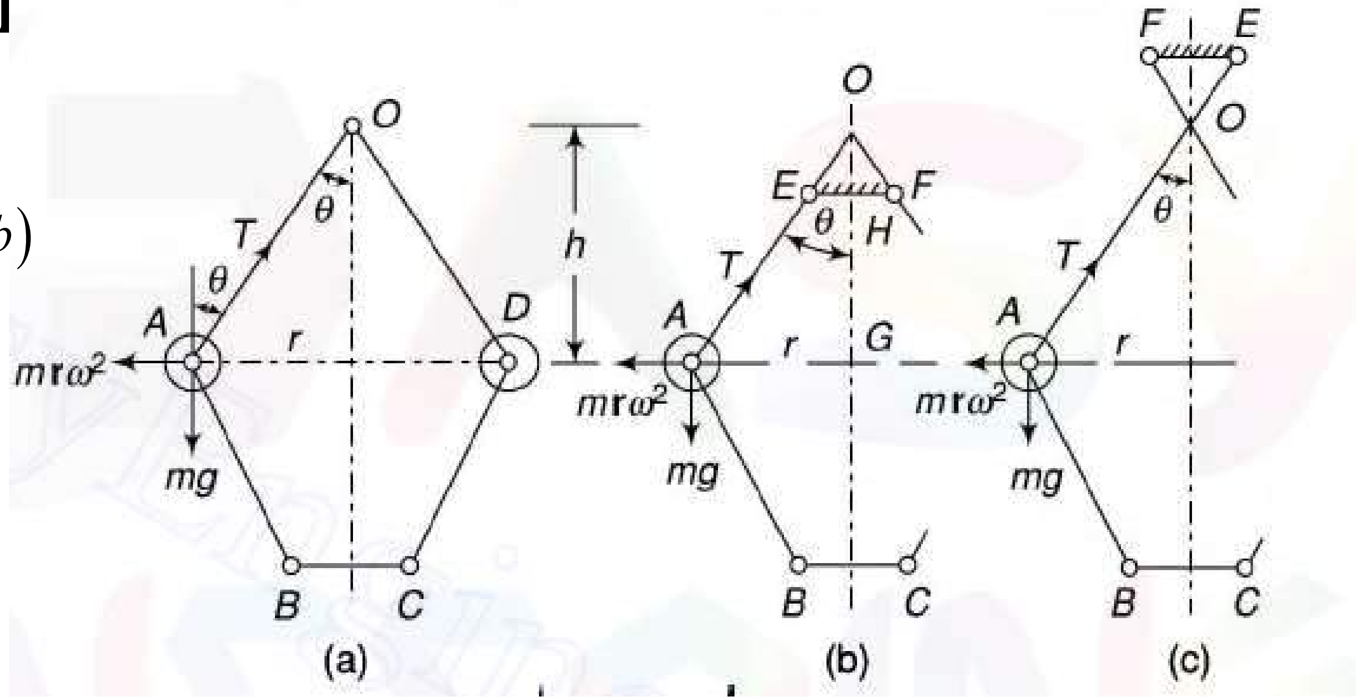
$$\tan\theta = \frac{m\omega^2 r}{mg}$$

$$\frac{r}{h} = \frac{\omega^2 r}{g}$$

$$h = \frac{g}{\omega^2}$$

$$= \left(\frac{60}{2\pi N}\right)^2 g = \frac{895}{N^2} m$$

$$h \propto \frac{1}{N^2}$$

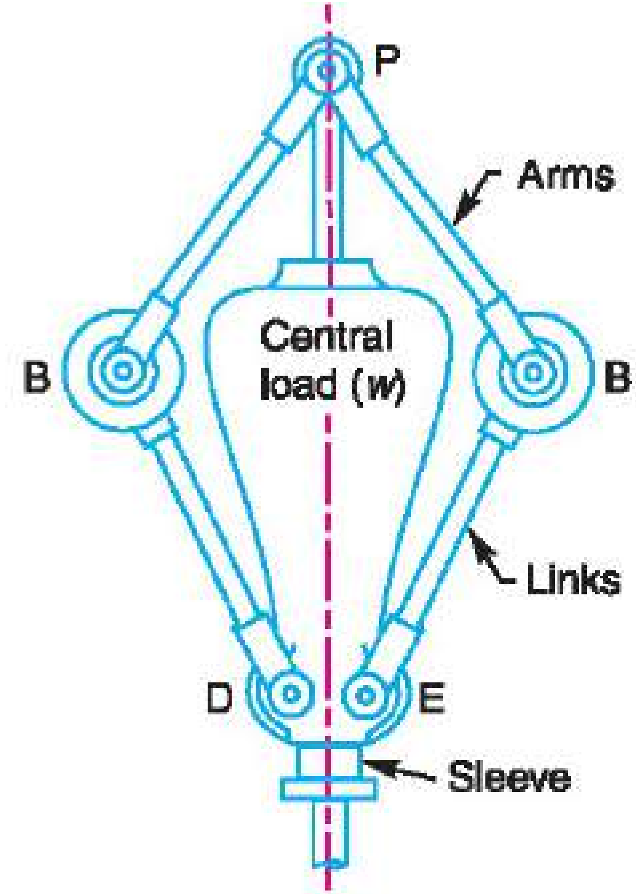


NOTE:

The sleeve movement is very less at high speeds and thus is unsuitable for these speeds however this drawback has been overcome by loading the governor by dead weight or by means of a spring and therefore lead to the development of other centrifugal governor like PORTER, PROELL etc.

PORTOR GOVERNOR

- It is the modification of Watt's governor with a dead weight (load) attached to the sleeve as shown:

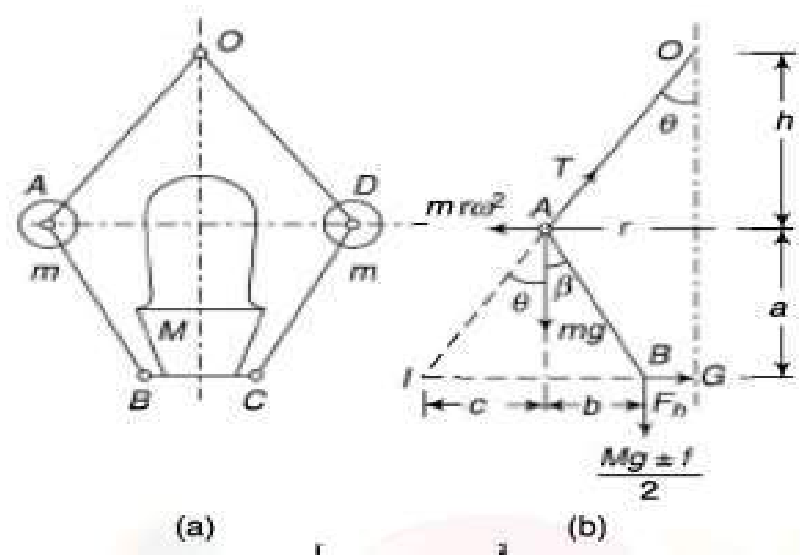


- The additional downward force increases the rpms required to enable the balls to rise to any pre-determined level.

PORTOR GOVERNOR

Let;

- M = mass of the sleeve ,
- m= mass of each ball
- f = force of friction at the sleeve
- r= distance of center of each ball from axis of rotation



$$\sum M_I = 0$$

$$\Rightarrow m\omega^2 ra - mgc - \frac{Mg \pm f}{2}(b+c) = 0$$

$$\Rightarrow m\omega^2 r = mg\frac{c}{a} + \frac{Mg \pm f}{2}\left(\frac{b}{a} + \frac{c}{a}\right)$$

$$= mg\tan\theta + \frac{Mg \pm f}{2}\left(\tan\beta + \tan\theta\right)$$

$$= \tan\theta\left[mg + \frac{Mg \pm f}{2}\left(1 + K\right)\right]$$

$$N = \left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]$$

$taking$

$$\tan\theta = \frac{c}{a};\ \tan\beta = \frac{b}{a};\ k = \frac{\tan\theta}{\tan\beta}$$

$$if\ k = 1$$

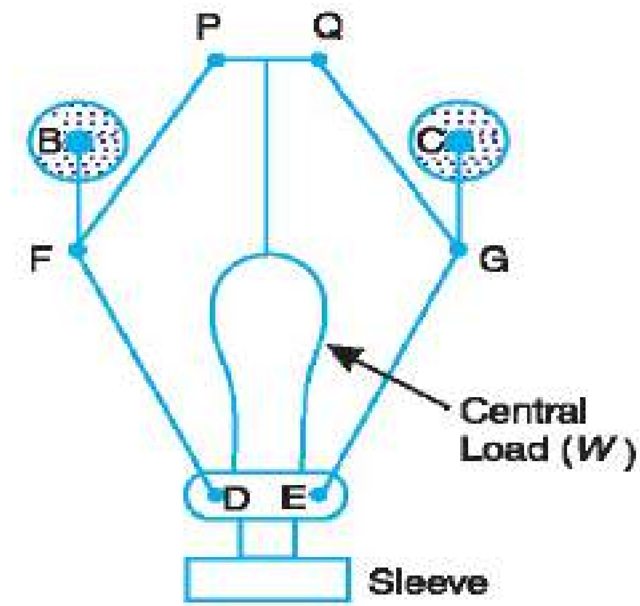
$$N^2 = \frac{895}{h}\left[\frac{mg + (Mg \pm f)}{mg}\right]$$

$$if\ k = 1, f = 0$$

$$N^2 = \frac{895}{h}\left[\frac{m + M}{m}\right]$$

PROELL GOVERNOR

A portor governor is known as Proell governor when two balls(masses) are fixed on the upward extension of the lower links which are in the form of bent links



Considering the equilibrium of the link:

- The weight of the ball ,mg
- the centrifugal force,
- the tension in the link
- the horizontal reaction of the sleeve
- the weight of sleeve and friction

Taking moment about instantaneous centre I

$$m\omega^2 r' e = mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)\Big)$$

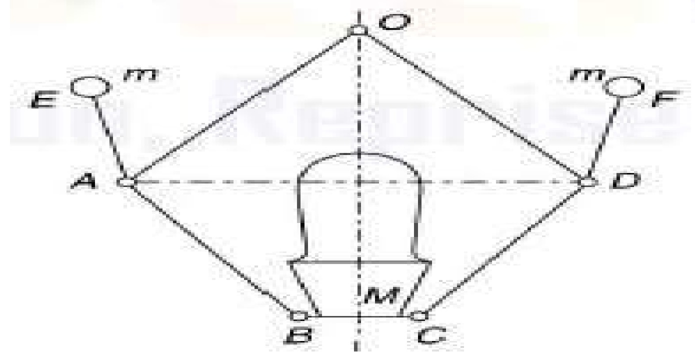
$$m\omega^2 r' = \frac{1}{e}\left[mg(c + r - r') + \frac{Mg \pm f}{2}(c + b)\Big)\right]$$

= a/e. g/h$\left[mg\tan\theta + \frac{Mg \pm f}{2}(\tan\theta + \tan\beta)\Big)\right]$

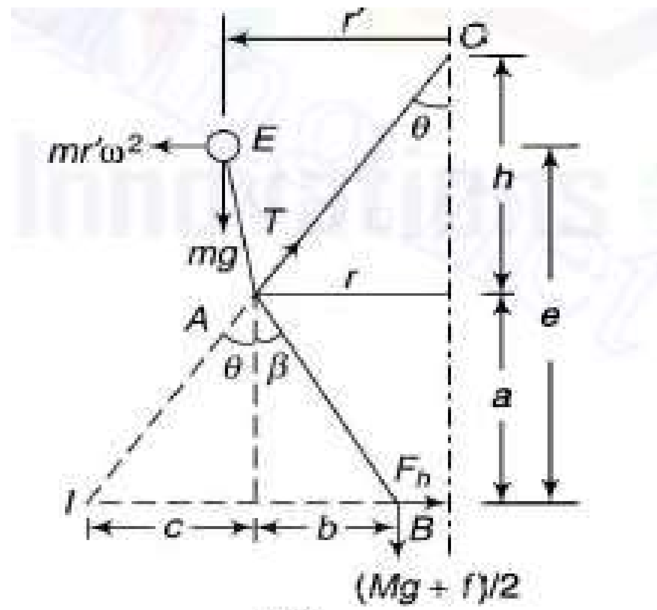
$$N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{2mg + (Mg \pm f)(1+K)}{2mg}\right]$$

$$if\ \ k = 0 \quad N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{mg + (Mg \pm f)}{mg}\right]$$

$$if\ \ k = 1, f = 0, \quad N^2 = \frac{895}{h}\frac{a}{e}\left[\frac{m + M}{m}\right]$$

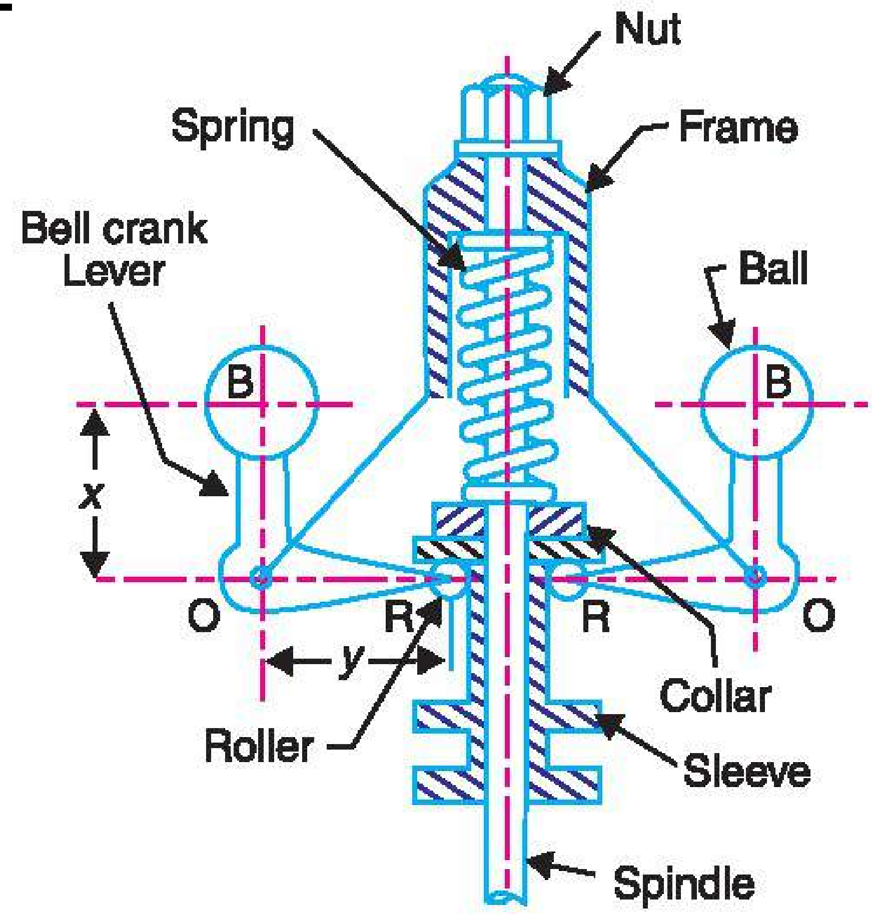


(a)



(b)

Hartnell Governor



- A hartnell governor is a spring loaded governor.
- It consist of two bell crank lever pivoted at point to the frame
- The frame is attached to the governor spindle and therefore rotates with it
- w=weight of flyball

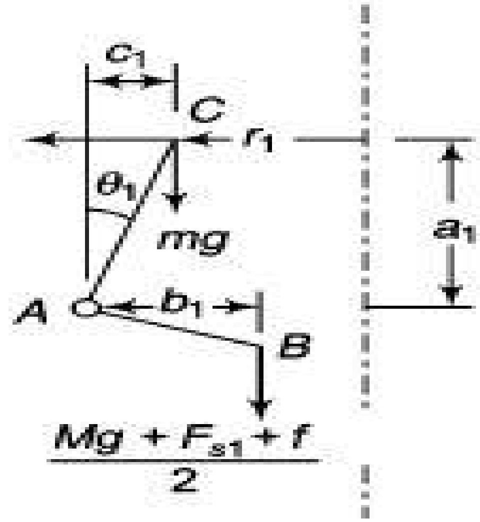
  W=weight on sleeve (Dead weight)

  S=force exerted on the sleeve by the spring that surrounds the spindle axis.

  x,y=vertical & horizontal arm of bell crank.

  R=radius of rotation.
- K=stiffness of spring;

$$\sum M_A = 0$$



$$F_1 a_1 = \frac{1}{2}(Mg + F_{s1} + f)b_1 + mgc_1 \,..\, 1(a)$$

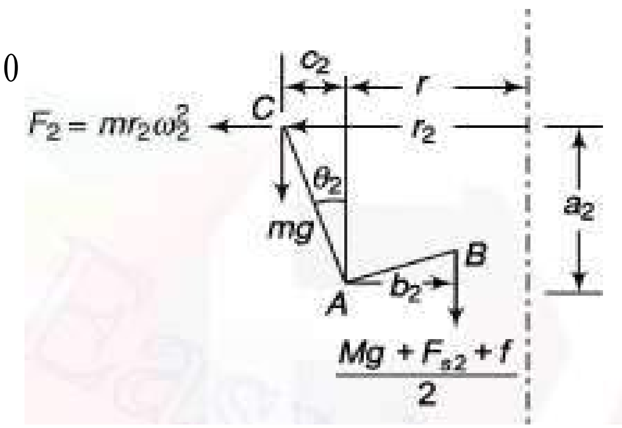
$$F_2 a_2 = \frac{1}{2}(Mg + F_{s2} + f)b_2 - mgc_2 \,..\, 1(b)$$

$$F_1 a = \frac{1}{2}(Mg + F_{s1} + f)b \,..\, 2(a)$$

$$F_2 a = \frac{1}{2}(Mg + F_{s2} + f)b \,..\, 2(b)$$

By neglecting the obliquity effect of arm of the bell crank lever

$a_1 = a_2 = a,\ b_1 = b_2 = b,\ c_1 = c_2 = 0$



Subtracting eqn 2(a) from eqn 2(b)

$$(F_2 - F_1)a = \frac{1}{2}(F_{s2} - F_{s1})b \,\ldots\, 3(a)$$

$$(F_{s2} - F_{s1}) = \frac{2a}{b}(F_2 - F_1) \,\ldots\, 3(b)$$

h1= sleeve movement

$$F_{s2} - F_{s1} = h_1 s = \frac{2a}{b}(F_2 - F_1) \text{ ...4(a)}$$

$$s = \frac{2}{h_1}\frac{a}{b}(F_2 - F_1)$$

$$h_1 = b.\theta = \frac{r_2 - r_1}{a}.b$$

$$s = \frac{2}{r_2 - r_1}\left(\frac{a}{b}\right)^2.(F_2 - F_1) \ = 2\left(\frac{a}{b}\right)^2.\left(\frac{F_2 - F_1}{r_2 - r}\right)$$

Effort of governor

- ‘‘Effort of the governor is the mean force exerted at the seleeve for a given percentage of speed (lift of the sleeve).’’

- It may be noted that when the governor is running steadily, there is no force at the seleeve, it is assumed that this resistance which is equal to effort varies uniformly from maximum value to zero while the governors moves into its new position of equilibrium. It is denoted by ‘Q’

Power of governor

- "The power of governor is the work done at the seleeve for a given percentage change of speed. It is the product of mean value of effort and the distance through which seleeve moves."

Governor Power ≅

$$\frac{4c^2}{1+2c}\left\{\frac{W(1+k)+2w}{2}\right\}h$$

Mathematically,
Power = Mean Effort ×
Lift of the Seleeve

$$P = Q \times x$$

References

Books & References

1. Theory of Machines - Thomas Bevan (CBS Publication)
2. Theory of Machines and Mechanisms- Shigley (Oxford University Press-New Delhi)
3. Theory of Machines and Mechanisms-Ghosh & Mallik (East West Press)
4. Theory of Machines and Mechanisms- Rao & Dukkipati (Wiley)
5. Theory of Machines - S.S. Rattan (Tata McGraw Hill)
6. Theory of Machines – R.K. Bansal (Laxmi)
7. Mechanics of Machines – V. Ramamurti (Alpha Science Intl Ltd.)
8. Theory of Machines – Khurmi & Gupta (S Chand)
9. Theory of Machines – P.L. Ballaney (Khanna)
10. Theory of Machines – V. P. Singh (Dhanpat Rai publisher)

NPTEL video & web lecture URL

- https://nptel.ac.in/courses/112/104/112104114/
- https://nptel.ac.in/courses/112/101/112101096/

Gyroscopic Motion

for

B.TECH 5th Semester

Dr Vijay K Singh
Assistant Professor
Mechanical Engineering Department
National Project Implementation Unit (NPIU)
Ministry of Human Resource and Development (MHRD); Govt. of India

MADAN MOHAN MALAVIYA UNIVERSITY OF TECHNOLOGY,

GORAKHPUR-273010 (U.P.)

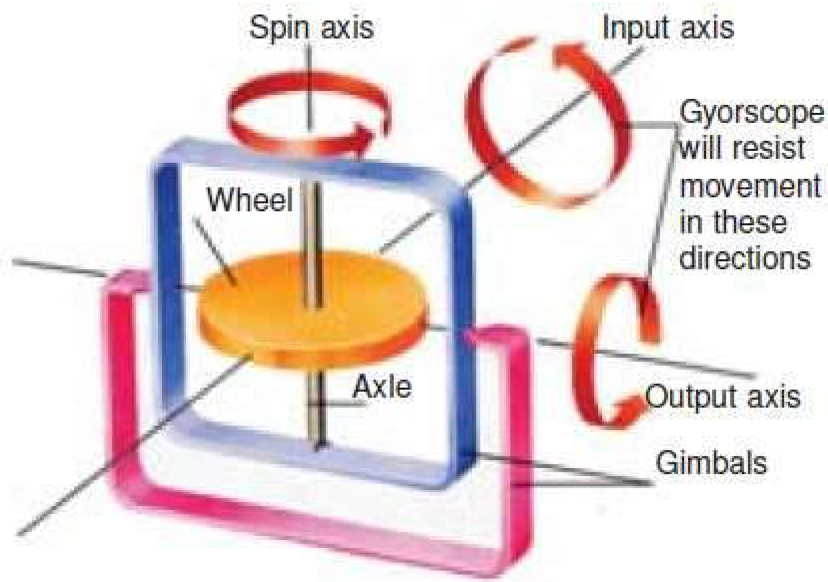
CONTENTS (UNIT-II)

Chapter-II:
GYROSCOPIC MOTION

- ❖ Principles
- ❖ Gyroscopic torque
- ❖ Effect – stability on aeroplanes & automobiles

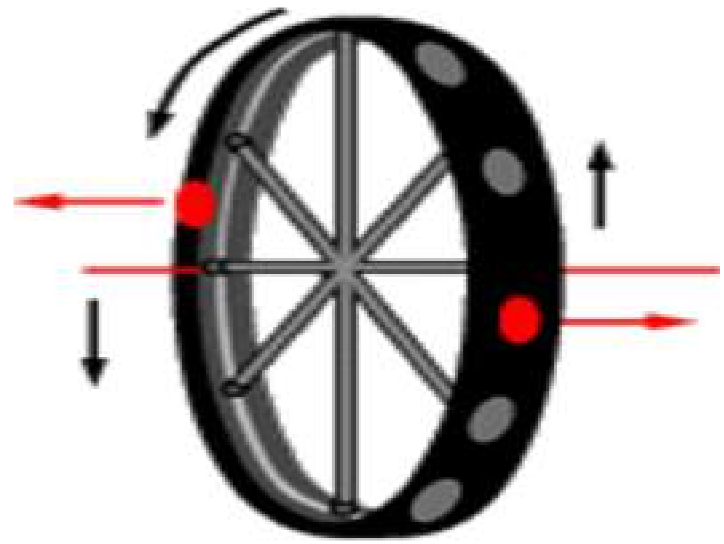
WHAT IS A GYROSCOPE?

Gyroscope is a mechanical system or arrangement having a rotor(usually heavy) spinning at high speed about its axis and being free to turn in any direction.



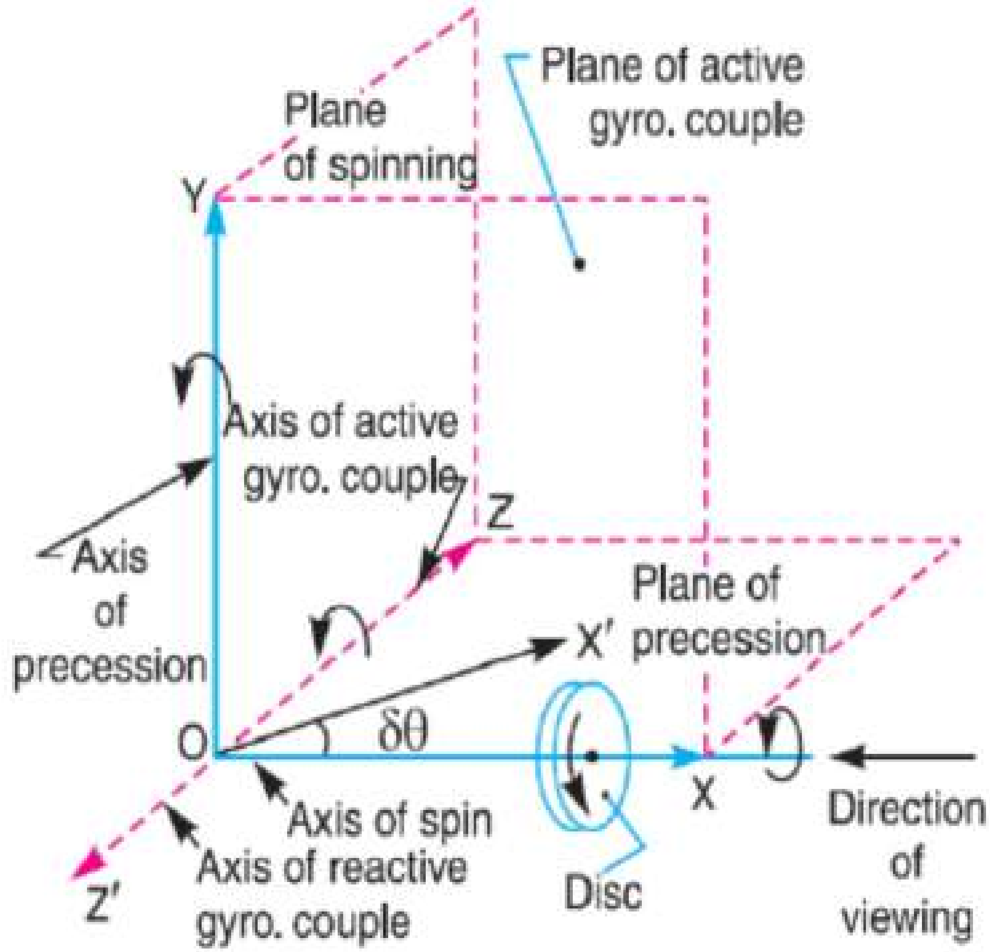
Gyroscopic inertia prevents a spinning top from falling sideways.

- A mechanical gyroscope is essentially a spinning wheel or disk whose axle is free to take any orientation. This orientation changes much less in response to a given external torque than it would without the large angular momentum associated with the gyroscope's high rate of spin.



PLANE OF PRECESSION

Consider a disc spinning with an angular velocity ω rad/s about the axis of spin OX, in anticlockwise direction when seen from the front, as shown in Fig.

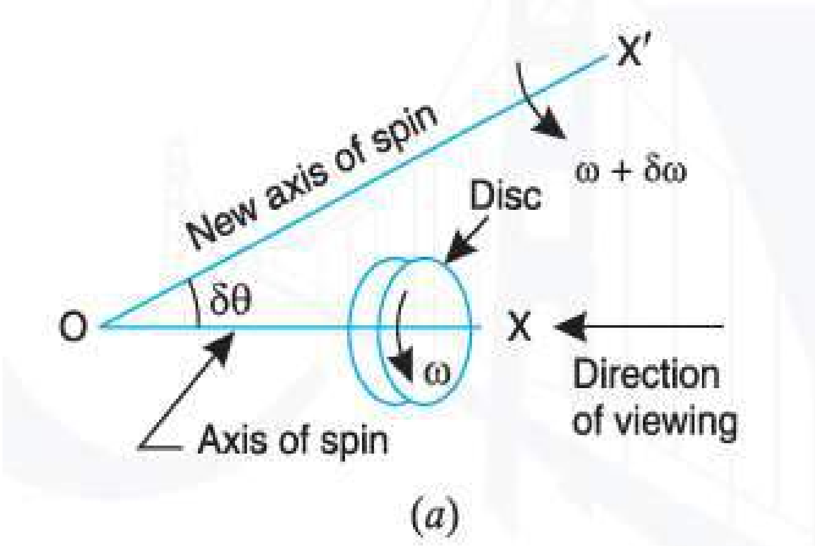


This horizontal plane XOZ is called plane of precession and OY is the axis of precession.

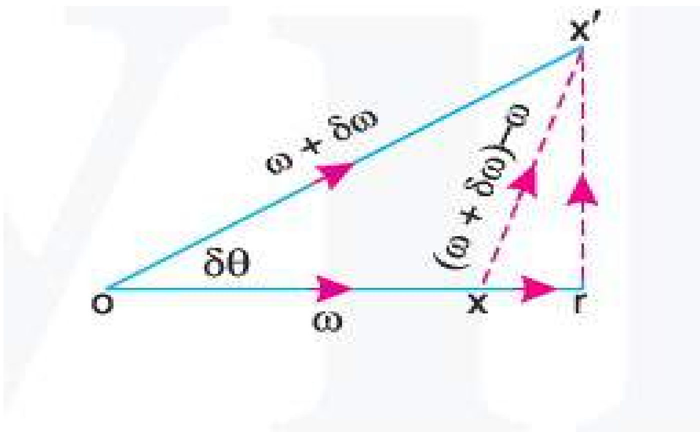
PRECESSIONAL ANGULAR MOTION (VECTORIAL REPRESENTATION OF ANGULAR MOTION)

- Angular acceleration is the rate of change of angular velocity with respect to time.

After a short interval of time t, let the disc be spinning about the new axis of spin OX' (at an angle δθ) with an angular velocity (ω +δω).



(a)



(b)

COMPONENT OF ANGULAR ACCELERATION

$$\alpha_t = \frac{(\omega + \delta\omega)\cos\delta\theta - \omega}{\delta t}$$

$$= \frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}$$

$$= \frac{\omega + \delta\omega - \omega}{\delta t}$$

$$\because \delta\theta \simeq 0, \cos\delta\theta = 1$$

$$= \frac{\delta\omega}{\delta t}$$

$$\delta t \to 0,$$

$$\alpha_t = \lim_{\delta t \to 0}\left(\frac{\delta\omega}{\delta t}\right) = \frac{d\omega}{dt}$$

Gyroscopic acceleration (α_c) = $angular\ velocity\ of\ rotor \times angular\ velocity\ of\ precession$

$$\alpha_c = \frac{(\omega + \delta\omega)\sin\delta\theta - 0}{\delta t} = \frac{\omega\delta\theta + \delta\omega\delta\theta - 0}{\delta t}$$

$$\because \delta\theta \simeq 0, \sin\delta\theta = \delta\theta$$

$$= \omega\frac{\delta\theta}{\delta t}$$

$$\delta t \to 0,$$

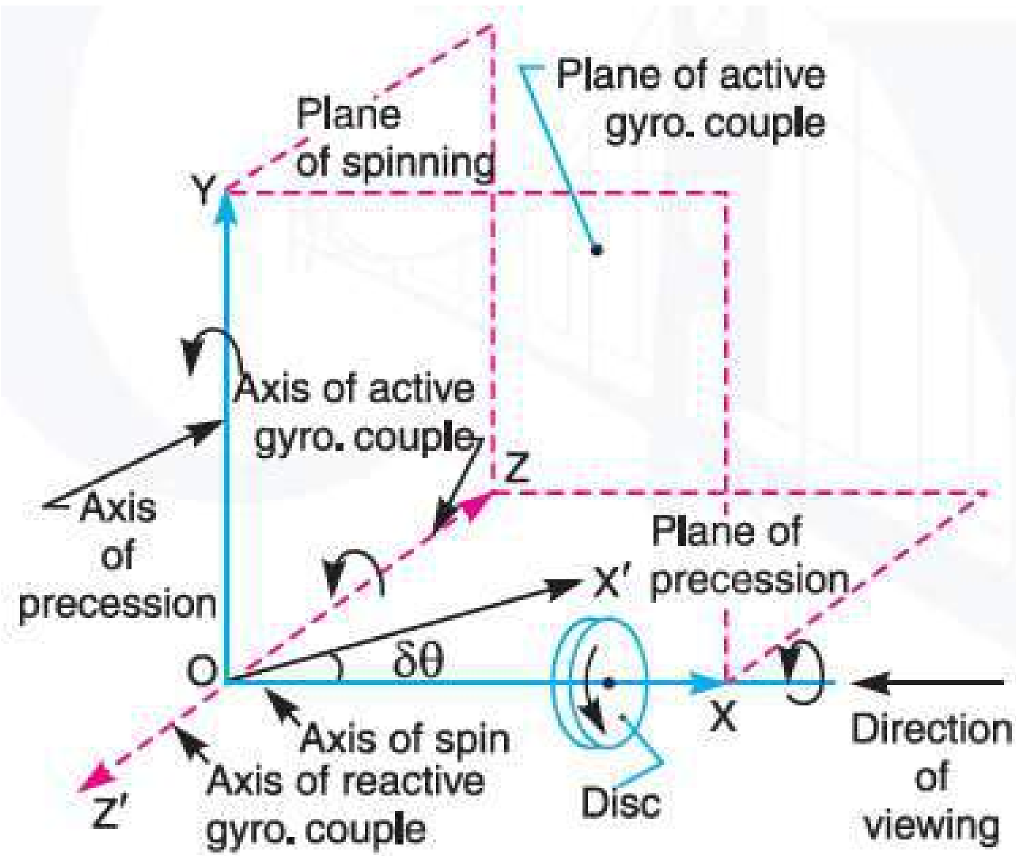
$$\alpha_c = \lim_{\delta t \to 0}\left(\omega\frac{\delta\theta}{\delta t}\right) = \omega\frac{d\theta}{dt} = \omega.\omega_P$$

Total angular acceleration of the disc

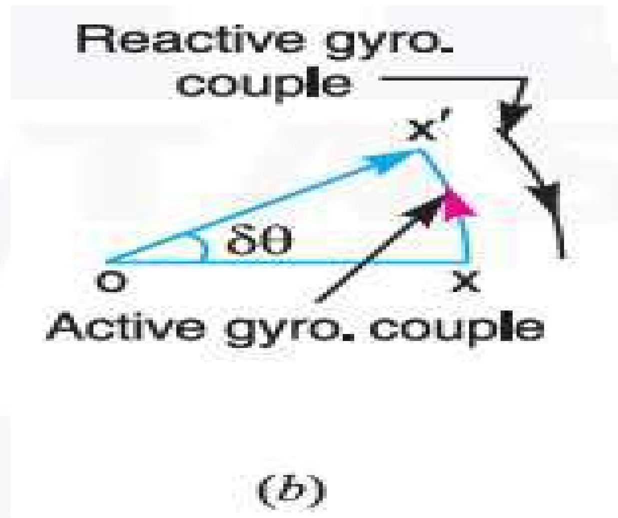
$$\alpha_t + \alpha_c = \frac{d\omega}{dt} + \omega\frac{d\theta}{dt}$$

$$= \frac{d\omega}{dt} + \omega.\omega_P$$

ACTIVE & REACTIVE COUPLE



(a)



(b)

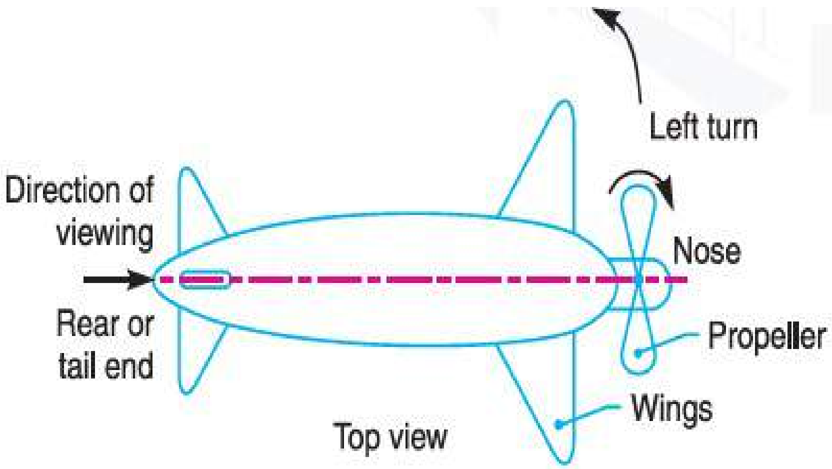
Change in angular momentum

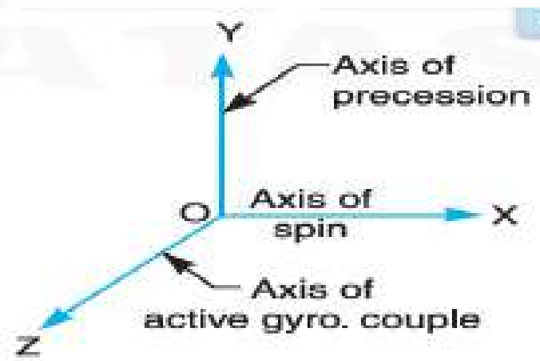
$$= ox.\delta\theta = I\omega\delta\theta$$

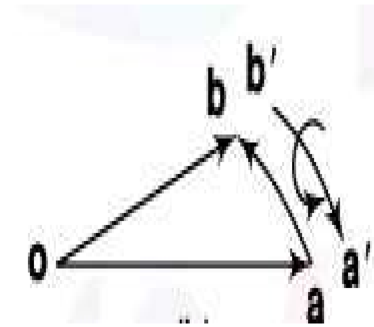
$$C = I\omega\frac{\delta\theta}{\delta t} = I\omega\omega_P$$

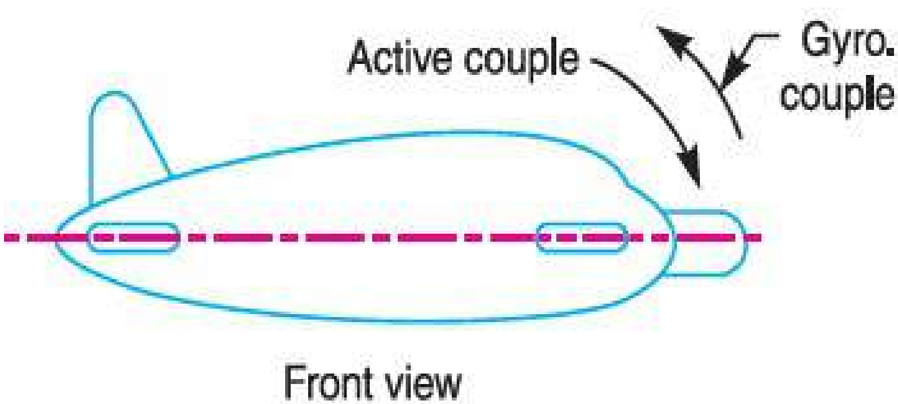
Effect of the Gyroscopic Couple on an Aero-plane

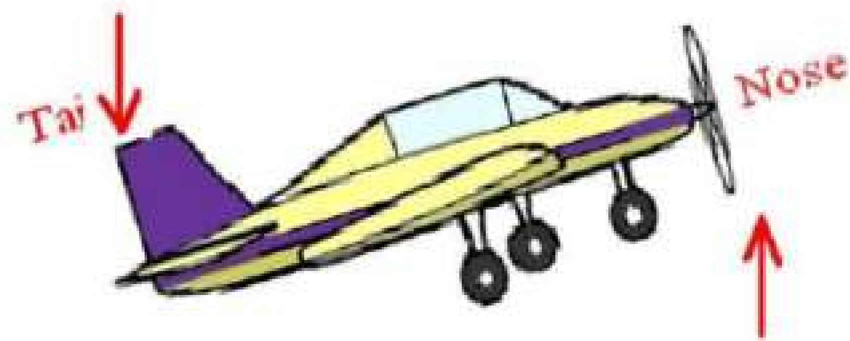
CASE-I: Let engine or propeller rotates in the **CLOCKWISE DIRECTION** when seen from the rear or tail end and the aero-plane takes a turn to the **LEFT**.



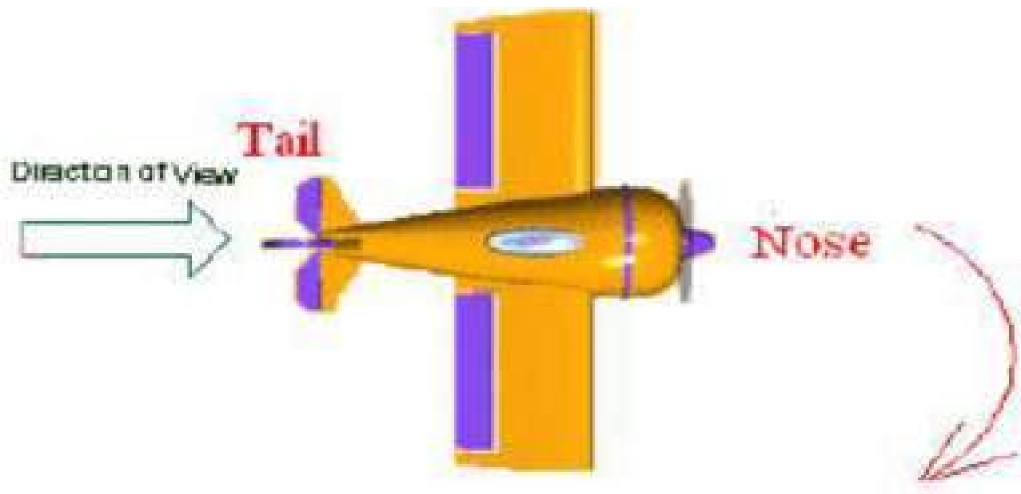


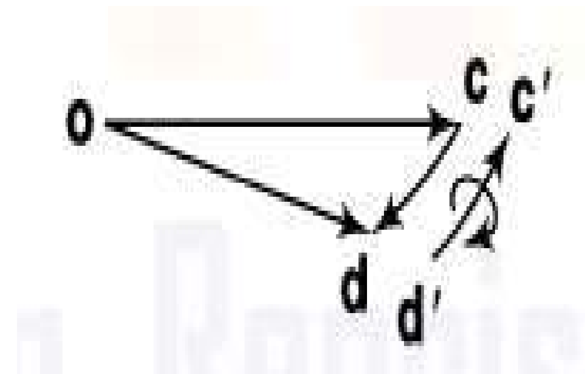






CASE-II: Let engine or propeller rotates in the CLOCKWISE-DIRECTION when seen from the rear or tail end and the aero-plane takes a turn to the RIGHT.

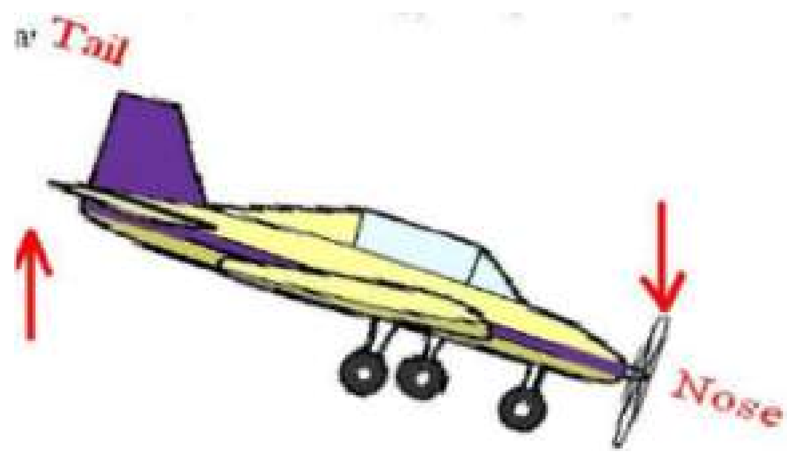




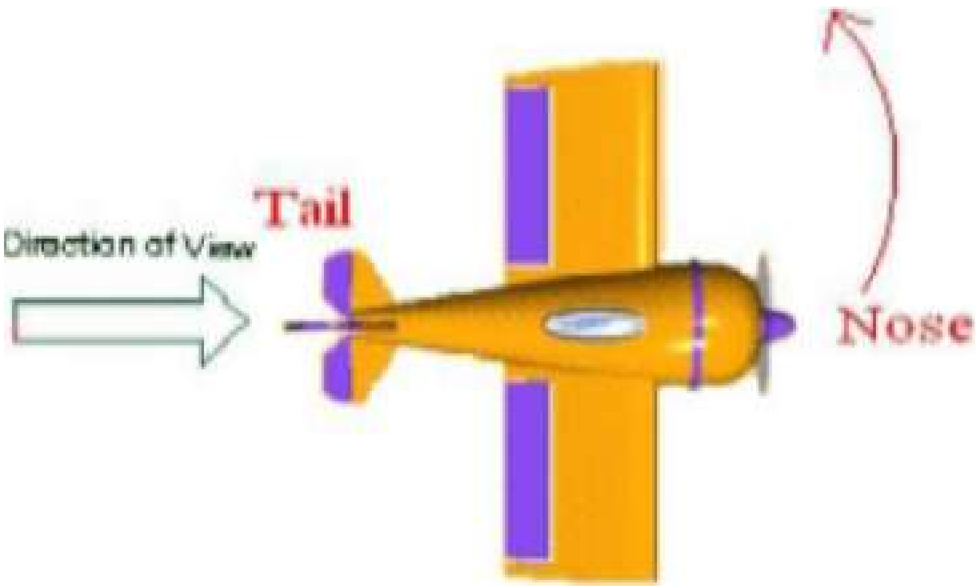
Active couple : Cause

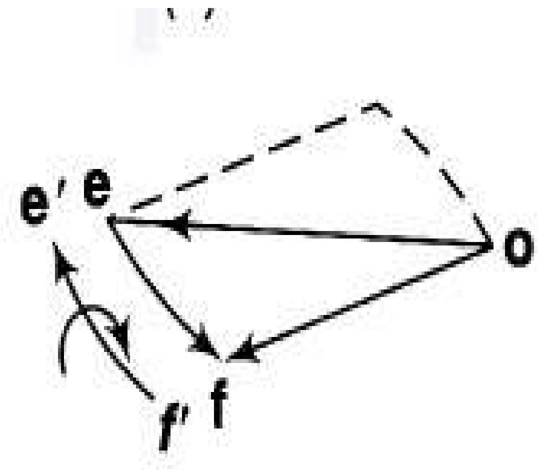
Reactive couple: Effect

Due to Reactive couple(Effect) the NOSE will go down and TAIL will go up



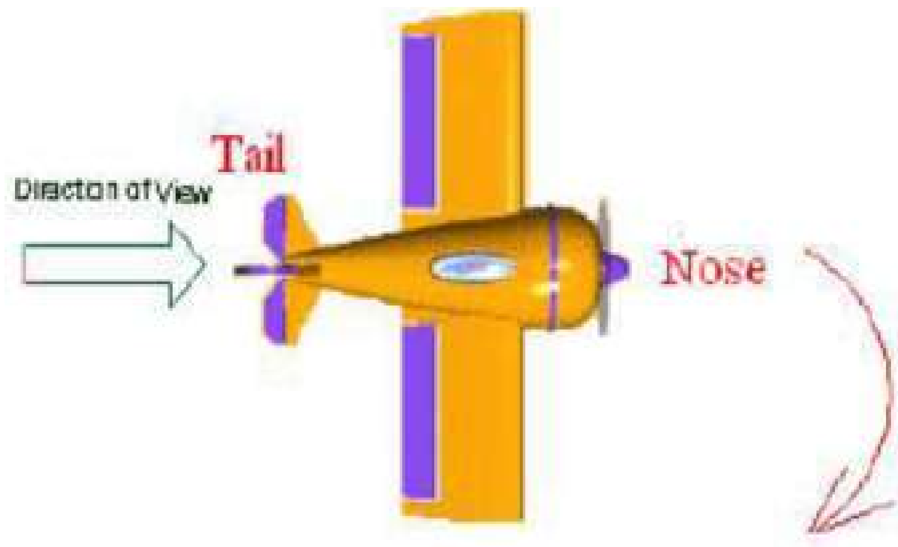
CASE-III: Let engine or propeller rotates in the COUNTER-clockwise direction when seen from the rear or tail end and the aero-plane takes a turn to the LEFT.

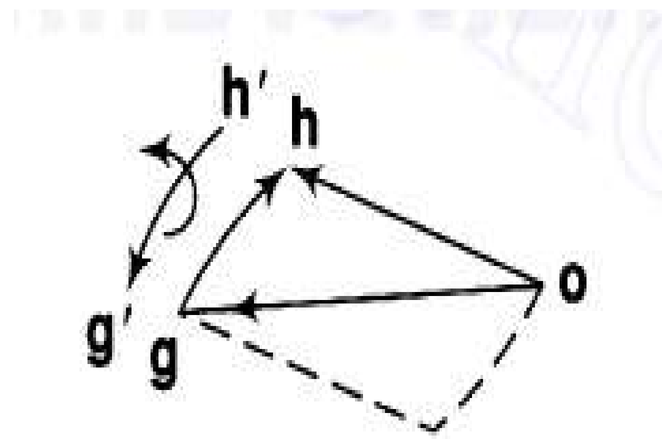




Due to Reactive couple(Effect) the NOSE will go down and TAIL will go up

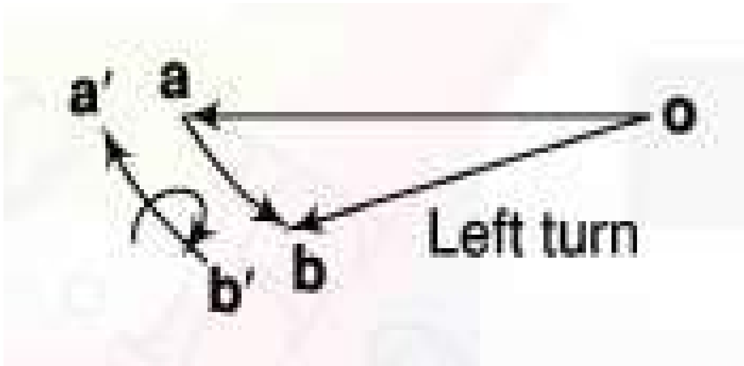
CASE-IV: Let engine or propeller rotates in the COUNTER-clockwise direction when seen from the rear or tail end and the aero-plane takes a turn to the RIGHT.





NOTE: It can be concluded from the above cases that if the direction of either the spin of the rotor or of the precession is changed, the gyroscopic effect is reversed, but if both are changed the effect remains the same

Stability of a Four Wheel Drive Moving in a Curved Path



Let

m= mass of the vehicle in kg

W=weight of the vehicle in newtons
 = mg

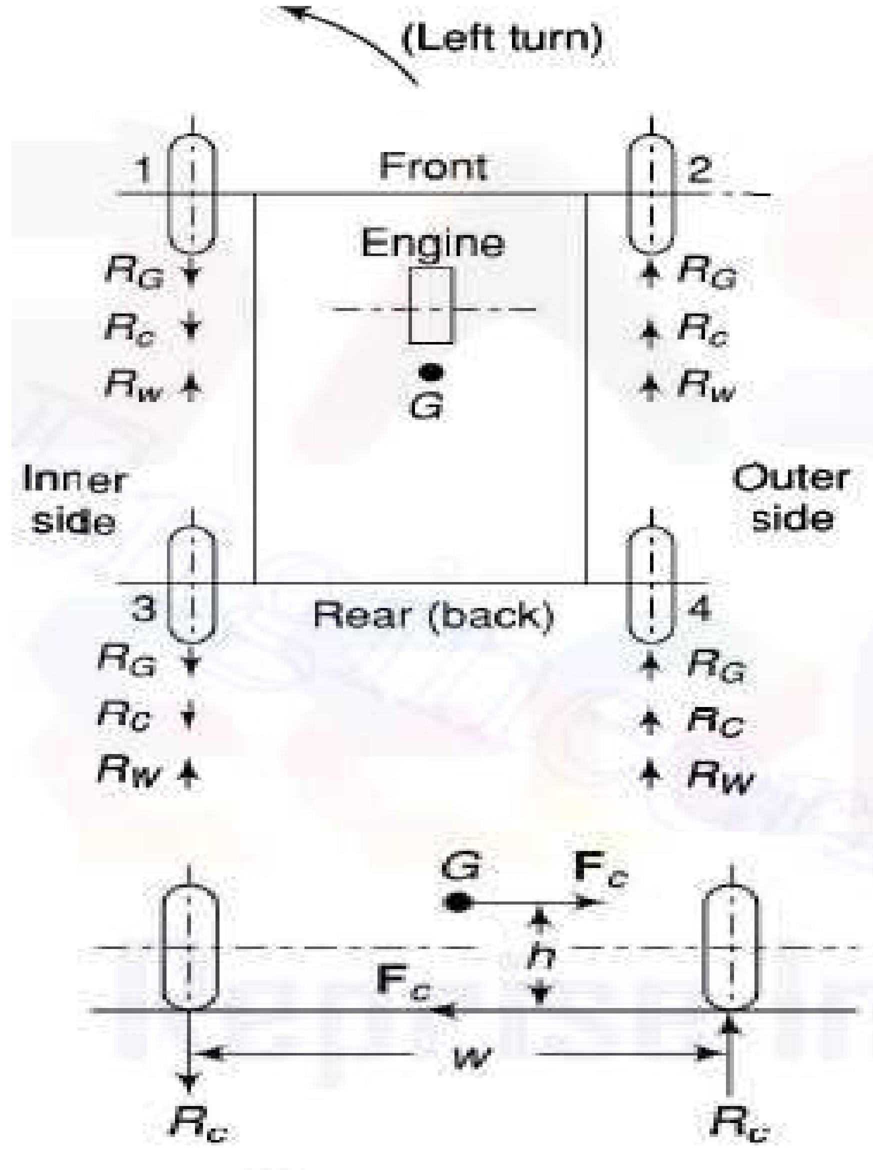
r_w = radius of the wheels in meters

R= radius of curvature in meters ($R>r_w$)

h = distance of centre of gravity vertically above the road surface in meters

x= width of track in meters

I_w = mass moment of inertia of one of the wheels in kg-m^2



$$Angular\ velocity\ of\ wheel\left(\omega_w\right)=\frac{v}{r}$$

$$Angular\ velocity\ of\ axis\ of\ precession\left(\omega_p\right)=\frac{v}{R}$$

$$Gyroscopic\ couple\ due\ to\ 4\ wheels$$

$$C_w = 4I_w\omega_w\omega_p = 4I_w\frac{v^2}{rR}$$

$$Gyroscopic\ couple\ due\ to\ engine\ rotating\ parts$$

$$C_e = I_e\omega_e\omega_p = I_eG\omega_w\omega_p$$

$$G = \frac{\omega_e}{\omega_w}$$

$$C_G = C_w \pm C_e$$

$$centrifugal\ couple\left(C_C\right)=mR\omega_p^2\times h = m\frac{v^2}{R}h$$

$$R_w = W/4$$

$$R_G\ _{(outer)} = C_G/2x\ (upward)$$

$$R_G\ _{(inner)} = C_G/2x\ (downward)$$

$$R_C\ _{(outer)} = C_c/2x\ (upward)$$

$$R_C\ _{(inner)} = C_c/2x\ (downward)$$

$$R_{(outer)} = R_w + R_C + R_G$$

$$= W/4 + C_C/2x + C_G/2x\ \ (upward)$$

$$R_{(inner)} = R_w - R_C - R_G$$

$$= W/4 - C_C/2x - C_G/2x\ \ (upward)$$

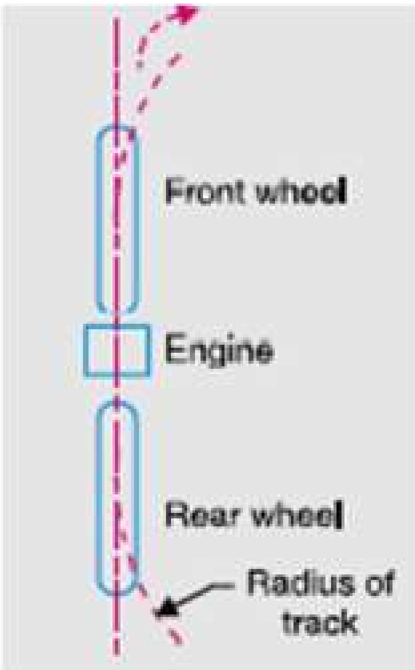
$$W/4 \geq \frac{C_C + C_G}{2x}$$

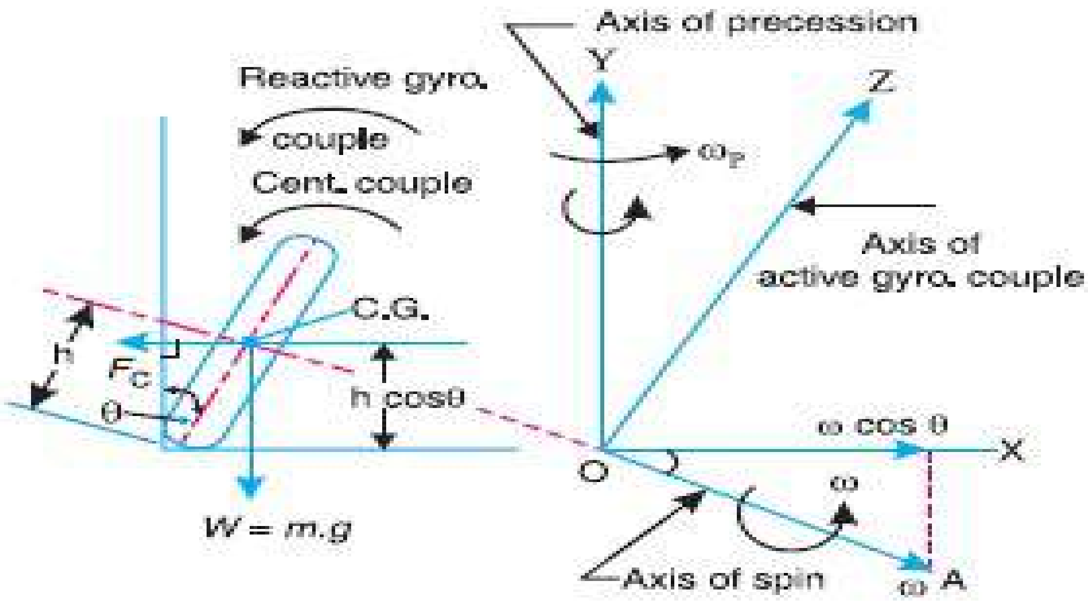
Stability of a Two Wheel Vehicle Taking a Turn

Let

- *m = Mass of the vehicle and its*
  rider in kg,
- *W = Weight of the vehicle and* its rider in newtons = *$m.g$,*
- *h = Height of the centre of* gravity of the vehicle and rider,
- *r_W = Radius of the wheels,*
- *R = Radius of track or* curvature,
- *I_W = Mass moment of inertia of each wheel,*
- *I_E = Mass moment of inertia of the rotating parts of the engine,*
- ω_W = Angular velocity of the wheels,
- ω_E = Angular velocity of the engine,
- *G = Gear ratio = ω_E / ω_W,*
- *v = Linear velocity of the vehicle = $\omega_W \times r_W$,*
- θ = Angle of heel. It is inclination of the vehicle to the vertical for equilibrium.

1.Effect of gyroscopic couple





We know that

$$v = \omega_W \times r_W \qquad \text{or} \qquad \omega_W = v / r_W$$

And

$$\omega_E = G.\omega_W = G \times \frac{v}{r_W}$$

∴ Total

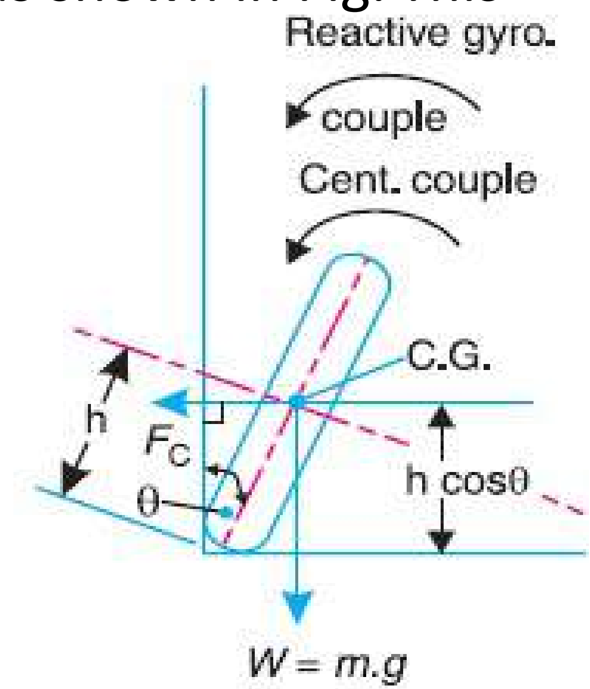
$$(I \times \omega) = 2 I_W \times \omega_W \pm I_E \times \omega_E$$

$$= 2 I_W \times \frac{v}{r_W} \pm I_E \times G \times \frac{v}{r_W} = \frac{v}{r_W}(2 I_W \pm G I_E)$$

and velocity of precession, $\omega_P = v / R$

When the wheels move over the curved path, the vehicle is always inclined at an angle θ with the vertical plane as shown in Fig. *This angle is known* as

angle of heel.



∴ Gyroscopic couple

$$C_1 = I.\omega \cos\theta \times \omega_P = \frac{v}{r_W}(2\,I_W \pm G.I_E)\cos\theta \times \frac{v}{R}$$

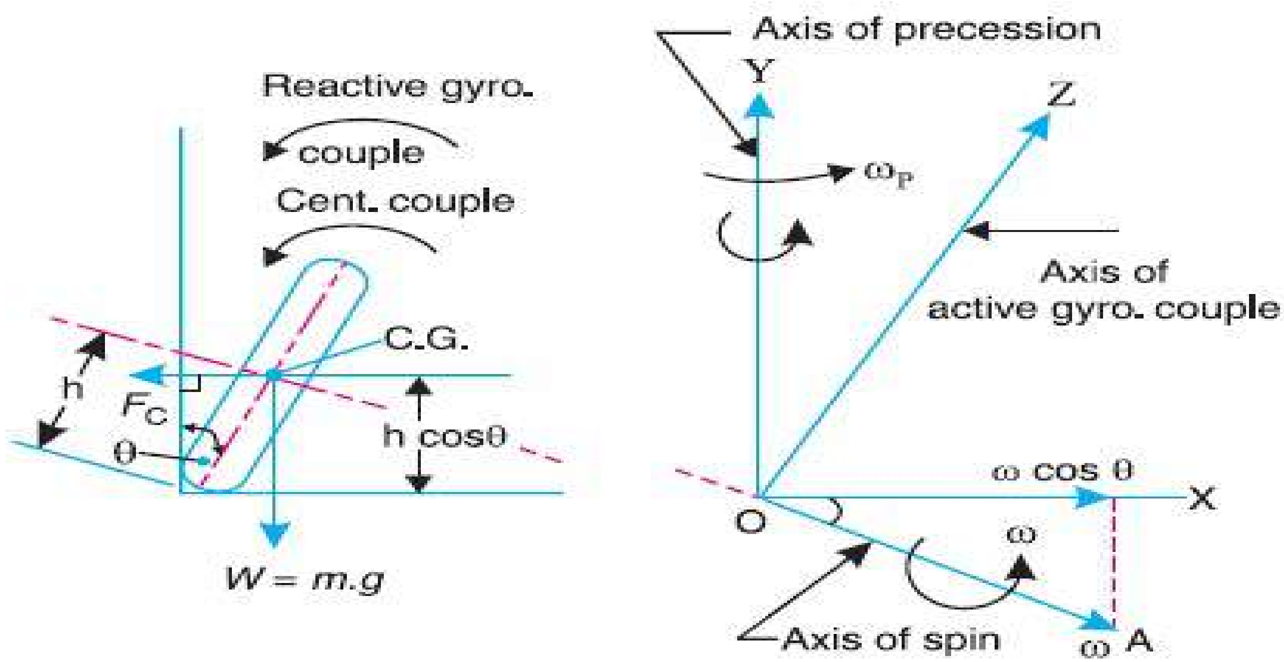
$$= \frac{v^2}{R.r_W}(2\,I_W \pm G.I_E)\cos\theta$$

Notes :

(*a*) ***When the engine is rotating in the same direction as that of wheels, then the positive sign is used in*** the above expression and if the engine rotates in opposite direction, then ***negative sign is used.***

(*b*) ***The gyroscopic couple will act over the vehicle outwards i.e. in the anticlockwise direction*** when seen from the front of the vehicle. The tendency of this couple is to overturn the vehicle in outward
direction.

2. Effect of centrifugal couple



Centrifugal force ,

Centrifugal Couple ,

$$C_2 = F_C \times h \cos \theta = \left(\frac{m.v^2}{R} \right) h \cos \theta$$

Total overturning couple,

C_o = *Gyroscopic couple + Centrifugal couple*

$$= \frac{v^2}{R\,r_W}(2\,I_W + G\,I_E)\cos\theta + \frac{m.v^2}{R} \times h\cos\theta$$

$$= \frac{v^2}{R}\left[\frac{2\,I_W + G\,I_E}{r_W} + m.h\right]\cos\theta$$

We know that balancing couple = *m.g.h sin ϑ*

As the stability, the overturning couple must be equal to the balancing couple, i.e

$$\frac{v^2}{R}\left(\frac{2\,I_W + G\,I_E}{r_W} + m.h\right)\cos\theta = m.g.h\sin\theta$$

From this expression, the value of the angle of heel (θ) may be determined.

References

Books & References

1. Theory of Machines - Thomas Bevan (CBS Publication)
2. Theory of Machines and Mechanisms- Shigley (Oxford University Press-New Delhi)
3. Theory of Machines and Mechanisms-Ghosh & Mallik (East West Press)
4. Theory of Machines and Mechanisms- Rao & Dukkipati (Wiley)
5. Theory of Machines - S.S. Rattan (Tata McGraw Hill)
6. Theory of Machines – R.K. Bansal (Laxmi)
7. Mechanics of Machines – V. Ramamurti (Alpha Science Intl Ltd.)
8. Theory of Machines – Khurmi & Gupta (S Chand)
9. Theory of Machines – P.L. Ballaney (Khanna)
10. Theory of Machines – V. P. Singh (Dhanpat Rai publisher)

NPTEL video & web lecture URL

- https://nptel.ac.in/courses/112/104/112104114/
- https://nptel.ac.in/courses/112/101/112101096/