~~04/03/19~~ 20/02/19

Coherent Binary PSK :-

In coherent binary PSK $s_1(t)$ and $s_2(t)$ are used to represent binary symbols 1 and 0, respectively.

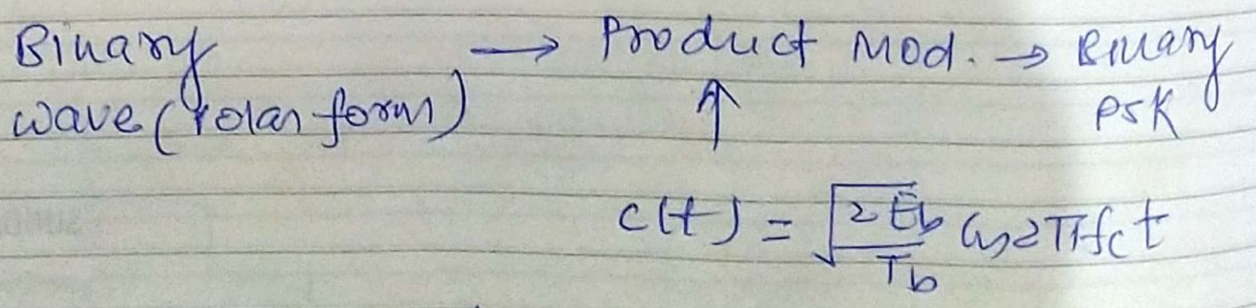
$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

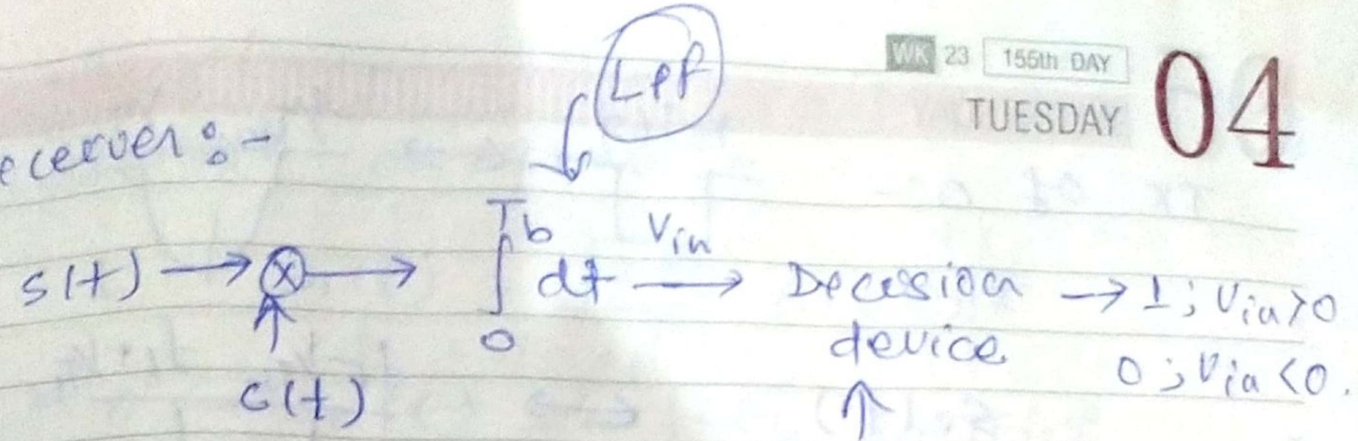
Transmitter :-

To generate binary PSK signal the i/p binary sequence is represented in polar form.



Binary wave and sinusoidal carrier signal $c(t)$ with freq $f_c = n_c/T_b$ are applied to product modulator. The desired PSK wave is obtained at the o/p of modulator.

Receiver :-



$V_{th} = \frac{A_c^2}{2} + \left(-\frac{A_c^2}{2}\right)$

$= 0$

Multiplier O/P corresponding to transmission of 1 :-

$(MUL)_{O/P} = A_c^2 \cos^2 2\pi f_c t$

0 tx :-

$(MUL)_{O/P} = -A_c^2 \cos^2 2\pi f_c t$

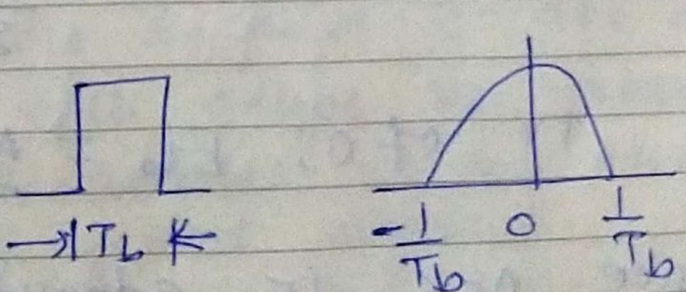
✓

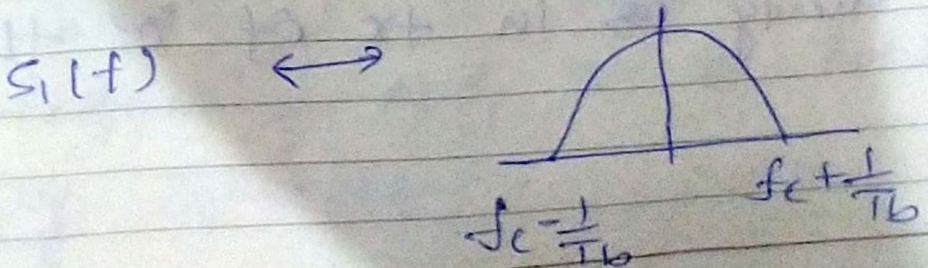
let $c(t) = A_c \cos(2\pi f_c t + \phi)$

Demodulatⁿ of PSK is affected by QNE.

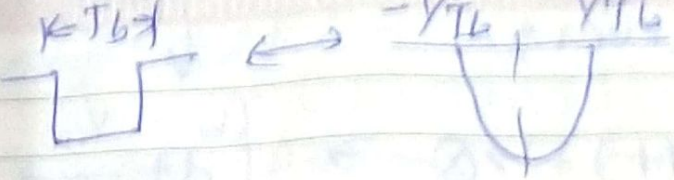
BW of PSK :-

TX of 1 :-



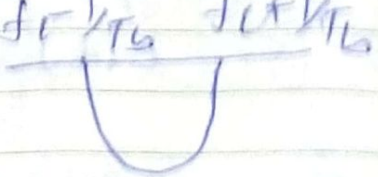


TX of 0:-



9

10 $s_2(t)$ ⟷



11 BW of PSK = $2R_b$

12 since it is bit by bit transmission

1 so either '0' or '1' will be txed at a time.

2 → channel BW requirement for ASK and

3 PSK is same.

4 Energy per bit:-

5 TX of 1; $E_b = \int_0^{T_b} s_1^2(t)\,dt$

6 $= \frac{A_c^2}{2} T_b$

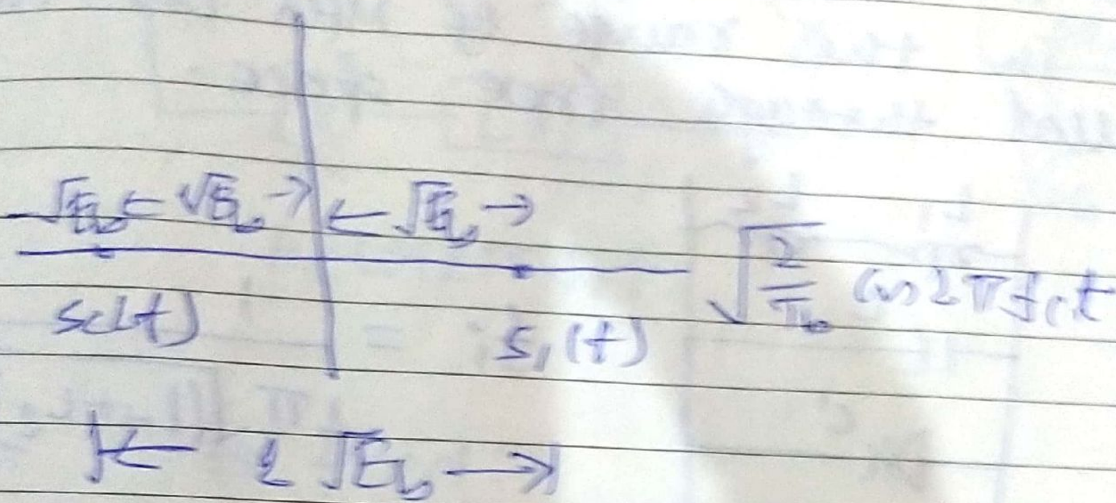
TX of 0; $E_b = \frac{A_c^2}{2} T_b$

Hence ASK is advantageous here as no energy is in tx of '0' there.

constellatⁿ diagram:-

$1 \rightarrow s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$

$0 \rightarrow s_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t$



Distance betⁿ the signalling points

$d_{12} = 2\sqrt{E_b}$.

Coherent Binary FSK:-

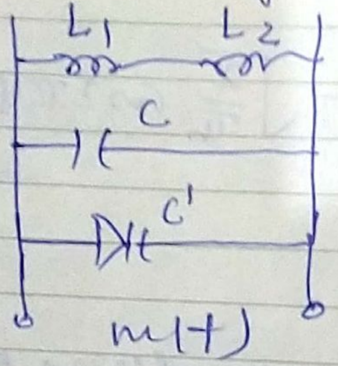
→ In FSK symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in freq. by a fixed amount.

→ Here binary 1 is represented by high frequency and 0 by low frequency component.

$1 \to s_1(t) = A_c \cos 2\pi f_{c1} t$

$0 \to s_2(t) = A_c \cos 2\pi f_{c2} t$

~~wst~~ Here, $f_{c1} > f_{c2}$ and both should be in the range of MHz as they passed through free space.



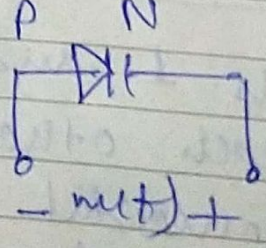
$$f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}$$

→ NRZ

1 ⇒ +ve

0 ⇒ -ve.

TX of 1 :- m(t) is positive.



RB → W↑ → C'↓ → $f_1 \uparrow = f_1 = \frac{n_1}{T_b}$

TX of 0 :-

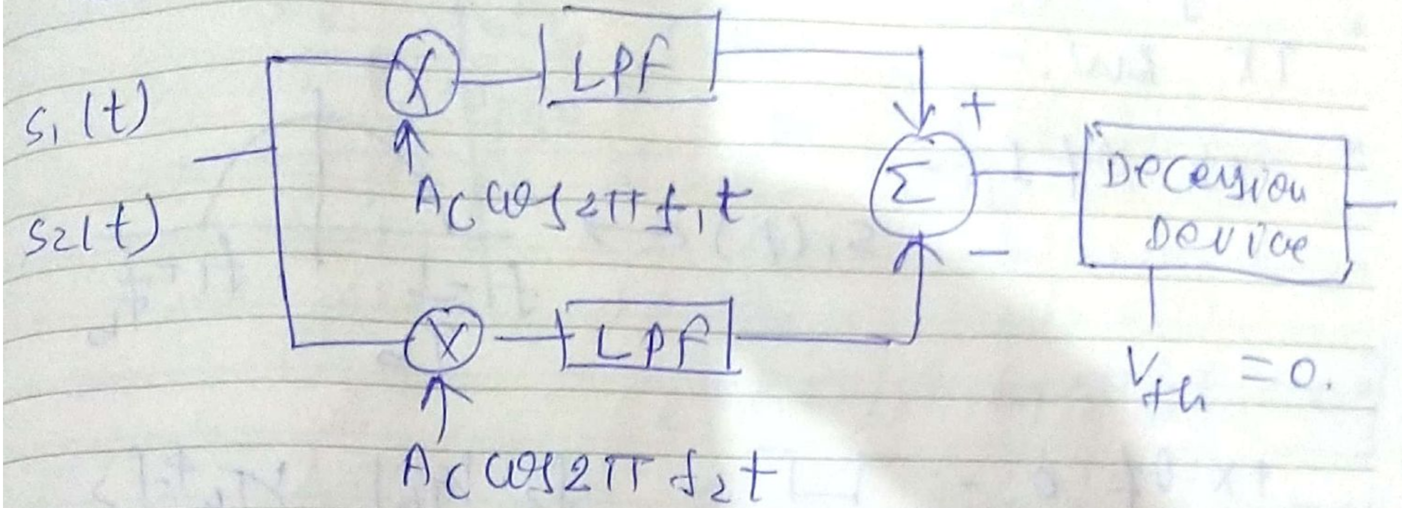
m(t) is -ve.

FB → W↓ ⇒ C'↑ → $f_i \downarrow \Rightarrow f_2 = \frac{n_2}{T_b}$

~~ON-off FX:-~~ ~~Date 21/03/19.~~

08/03/19

FSK Rx:-



when 1 is txed:-

$$\frac{A_c^2}{2} - \left[\cancel{A_c^2}\ 0\right] = \frac{A_c^2}{2}$$

$$(LPF)_{i/p} = A_c^2 \cos 2\pi f_1 t \cos 2\pi f_2 t$$

$$= \frac{A_c^2}{2}\left[\cos 2\pi (f_1+f_2)t + \cos(2\pi(f_1-f_2)\right]$$

$$(LPF)_{o/p} = 0.$$

SUNDAY 09

when 0 is txed then

$$(LPF)_{2\,o/p} = \frac{A_c^2}{2}$$

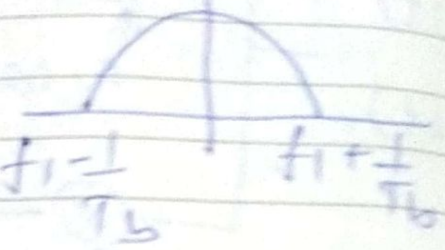
$$V_{th} = -\frac{A_c^2}{2}. \qquad o/p \rightarrow 0.$$

→ The Demodulation of FSK is affected by QNE

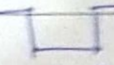
Tx BW:-

Tx of 1:-

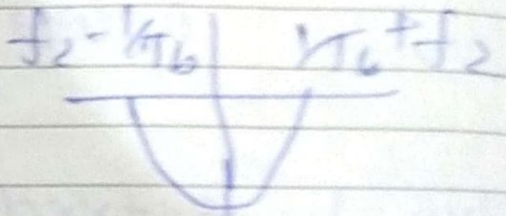
$s_1(t) \longleftrightarrow$



Tx of 0:-



$s_2(t) \longleftrightarrow$



$$\text{FSK BW} = \left(f_1 + \frac{1}{T_b}\right) - \left(f_2 - \frac{1}{T_b}\right)$$

$$= f_1 - f_2 + 2R_b$$

→ FSK needs high tx BW compared to ASK and PSK.

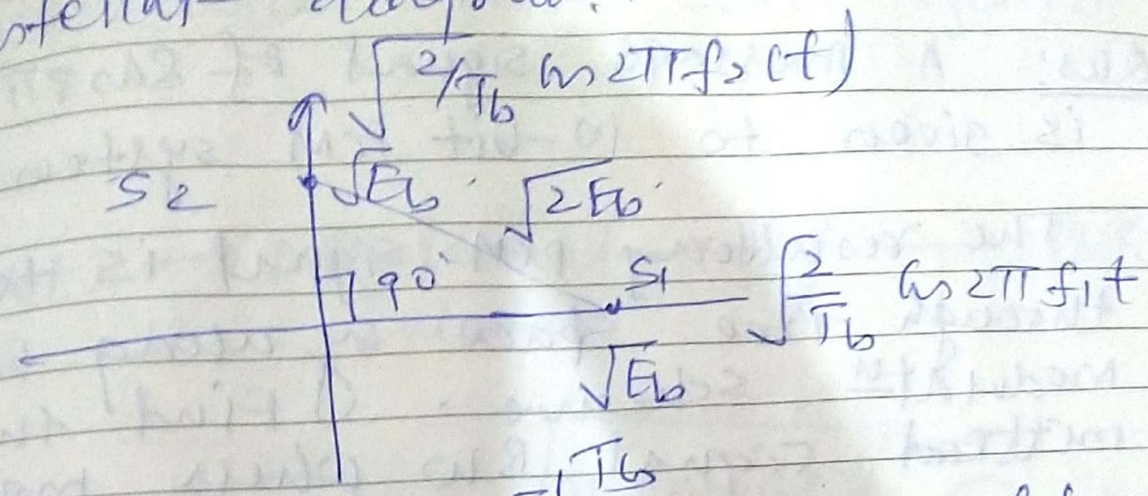
Energy per bit:-

Tx of 1:- $E_b = \frac{A_c^2}{2} T_b$

Tx of 0:- $E_b = \frac{A_c^2}{2} T_b$.

Transmitter energy requirements of PSK and FSK will be same.

constellation diagram :



$$\left[\int_0^{T_b}\cos 2\pi f_1 t\cdot\cos 2\pi f_2 t = 0\right]$$

$\therefore \cos 2\pi f_1 t$ & $\cos 2\pi f_2 t$ are orthogonal.

f_1 & f_2 are integral multiple of R_b.

Distance betn signaling point $= \sqrt{2E_b}$.

$$P_e \propto \frac{1}{\text{distance betn the signalling point in constellation diagram}}$$

P_e : PSK < FSK < ASK.

Qus: A message signal of $8\cos 8\pi \times 10^3 t$ is given to 10-bit PCM system.

The resulting PCM signal is txed through free space by using band pass modulatn scheme. Find the transmitted signal BW when modulatn scheme used is—

(a) ASK (b) PSK (c) FSK with

$f_H = 4$ MHZ and $f_L = 1$ MHZ

Soln:

$f_m = 4K$; $f_s = 8K$

$R_b = n f_s = 10 \times 8 = 80K$.

(a) ASK BW $= 2R_b$

(b) PSK BW $= 2R_b$

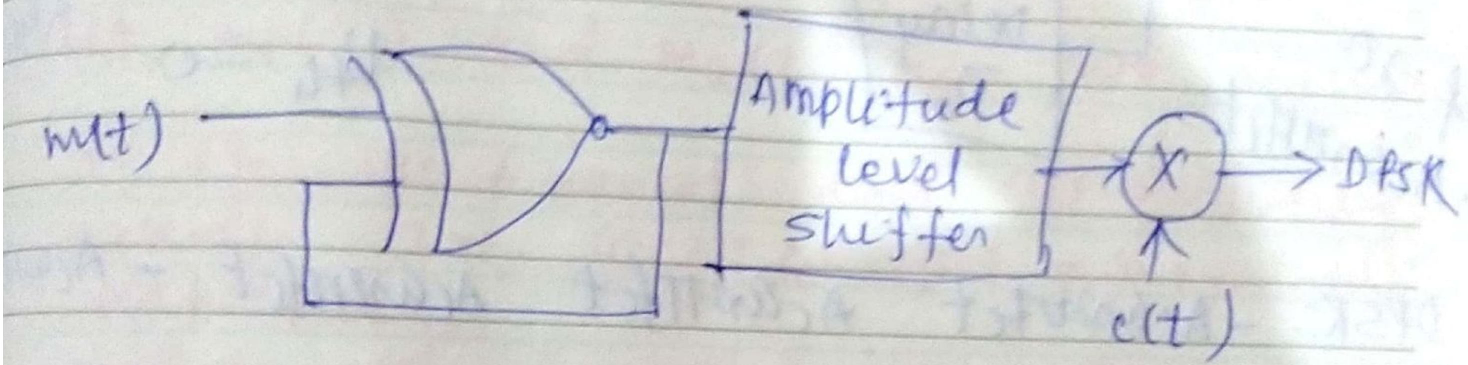
(c) FSK BW $= (4-1) + 2 \times 80K$

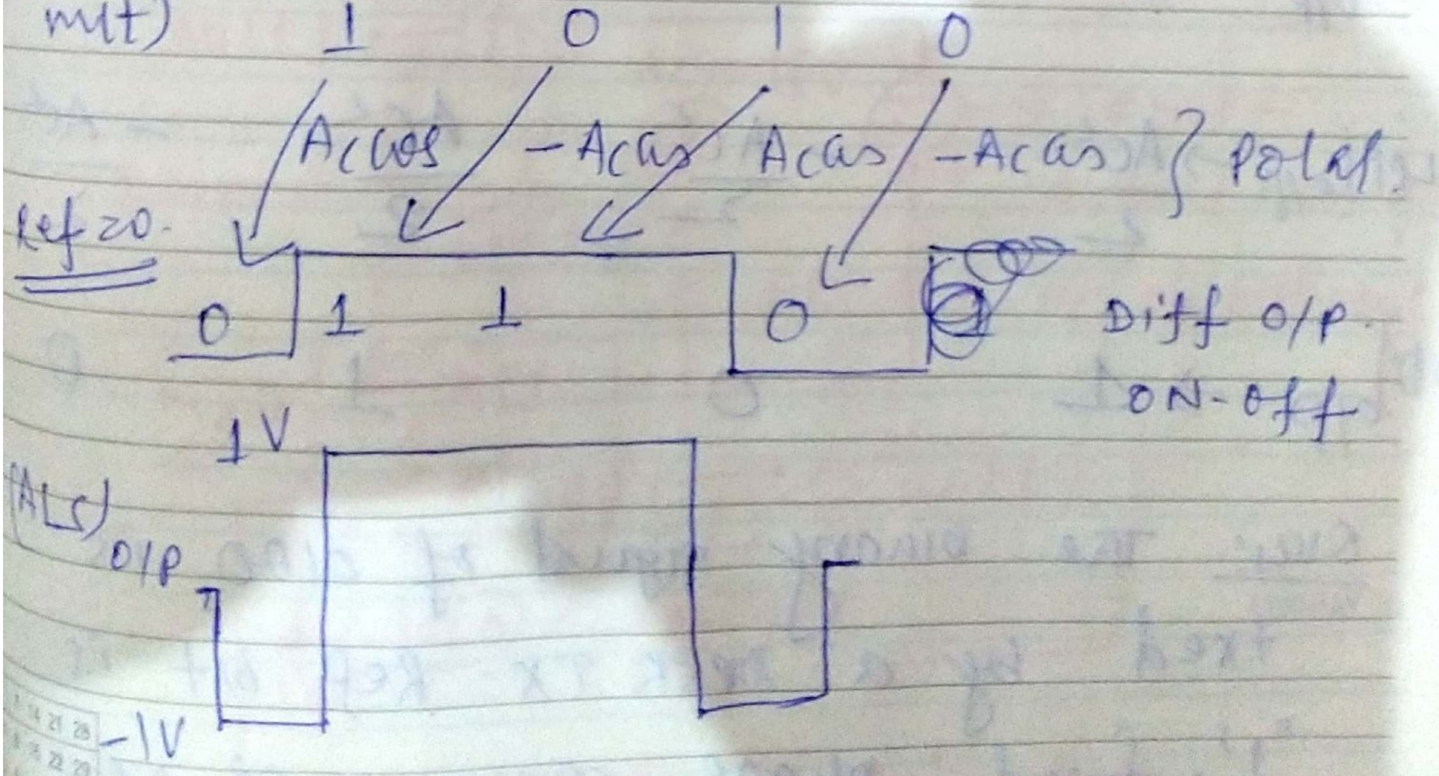
$=$ [illegible]

Differential phase shift keying:-

TX:-

m(t) $\longrightarrow$ Differential Encodes — PSK TX $\longrightarrow$ DPSK

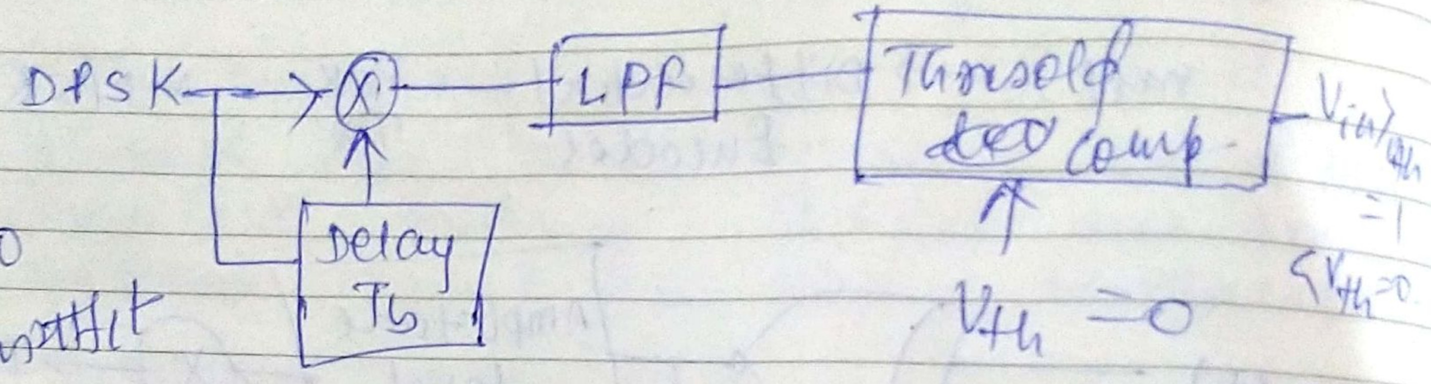




(DPSK) $-A_c\cos2\pi f_c t$ $A_c\cos2\pi f_c t$ $A_c\cos2\pi f_c t$ $-A_c\cos$

ϕ π 0 0 π

DPSK RX:-



ref → 0

$-A_c\cos 2\pi f_c t$

DPSK:- $-A_c\cos 2\pi f_c t$ $A_c\cos 2\pi f_c t$ $A_c\cos 2\pi f_c t$ $-A_c\cos 2\pi f_c t$

$(MUL)_{o/p} \rightarrow A_c^2\cos^2 2\pi f_c t$ $-A_c^2\cos^2$ $A_c^2\cos^2$ $-A_c^2\cos^2$

$(LPF)_{o/p} \rightarrow \frac{A_c^2}{2}$ $-\frac{A_c^2}{2}$ $\frac{A_c^2}{2}$ $-\frac{A_c^2}{2}$

$(DD)_{o/p} \rightarrow 1$ 0 1 0

Ques:- The binary signal of 0100 is fxed by a DPSK TX. Ref bit is '1'. fend phase seeuence of DPSK

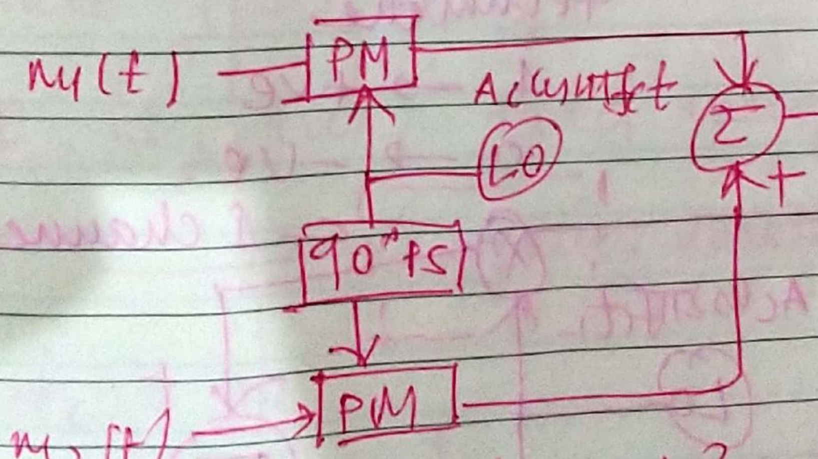
Date 12/03/12

Quadrature carrier multiplexing:-

→ Using this technique two signals will be multiplexed where the corresponding carrier will have same freq and having 90° phase shift betw them.

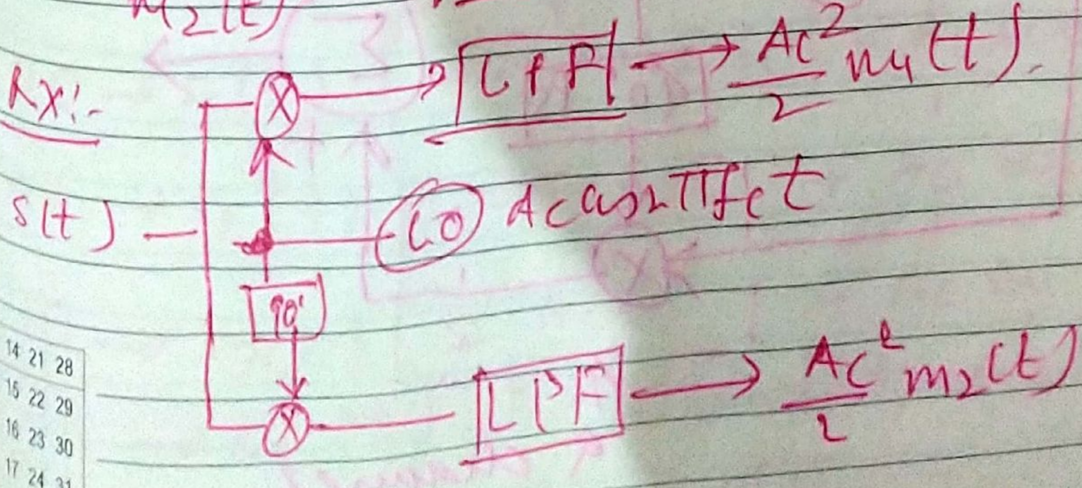
Let $m_1(t)$ and $m_2(t)$ are two signals to be multiplexed and corresponding carrier are $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ respectively.

TX:-



$s(t) = A_c m_1(t) \cos + A_c m_2(t) \sin(\ )$.

RX:-



M - array signalling:-

$M = 2^n$; n = no. of bits in symbol

M = 2 → Binary

M = 4 → Quadrature

PTO

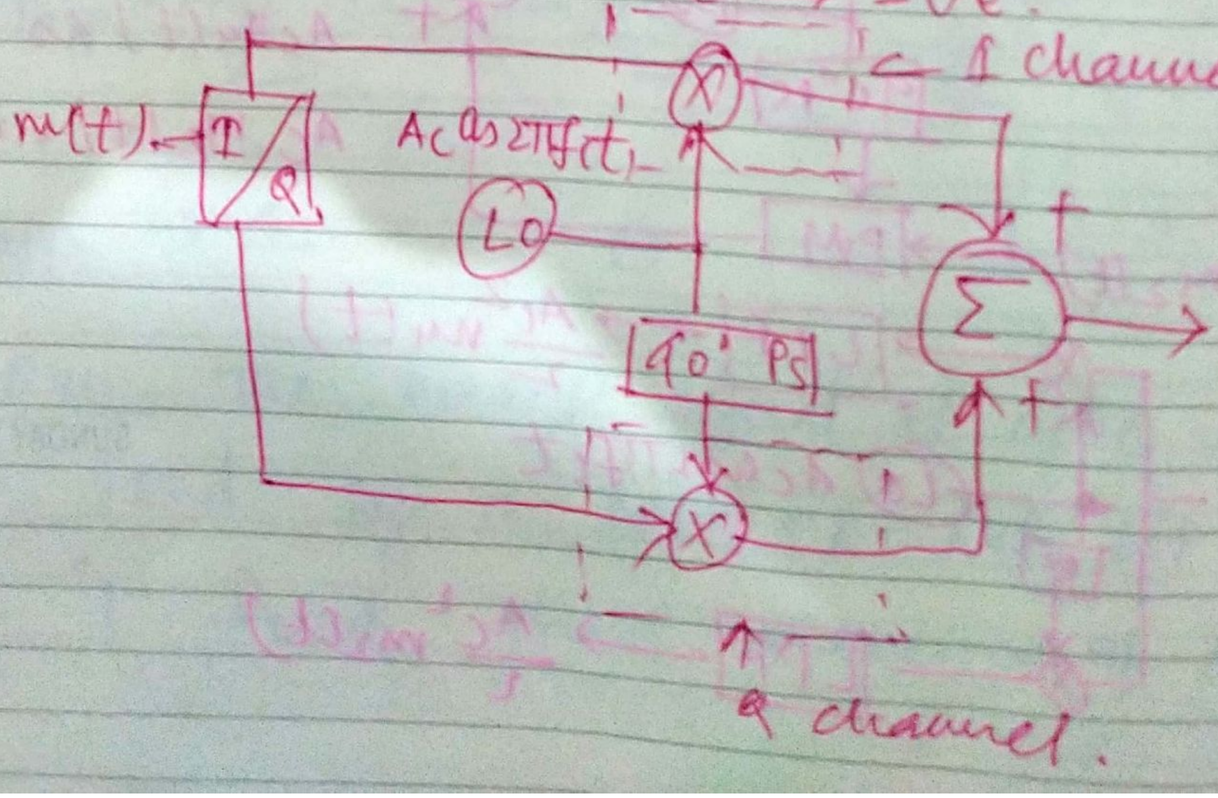
QPSK:-

M = 4; 4 symbols each having 2 bits.
message

$m_1 = 00$
$m_2 = 01$
$m_3 = 10$
$m_4 = 11$

we use NRZ coding technique

1 → +ve
0 → -ve.



Depending upon the dibit sent during the signaling interval $[-T_b, T_b]$, the inphase component may be $g(t)$ or $-g(t)$ and similarly for the quadrature component.

QPSK signal is given by -

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right]$$

$0 \leq t \leq T$

$= 0$, otherwise

where $i = 1, 2, 3, 4$

E is the txed energy per symbol.

T ______ symbol duratn.

$$f_c = \frac{n_c}{T}$$

$00 \rightarrow s_1(t) = -A_c' \cos 2\pi f_c t - A_c' \sin 2\pi f_c t$

$01 \rightarrow s_2(t) = -A_c' \cos 2 + A_c' \sin$

$10 \rightarrow s_3(t) = A_c' \cos - A_c' \sin$

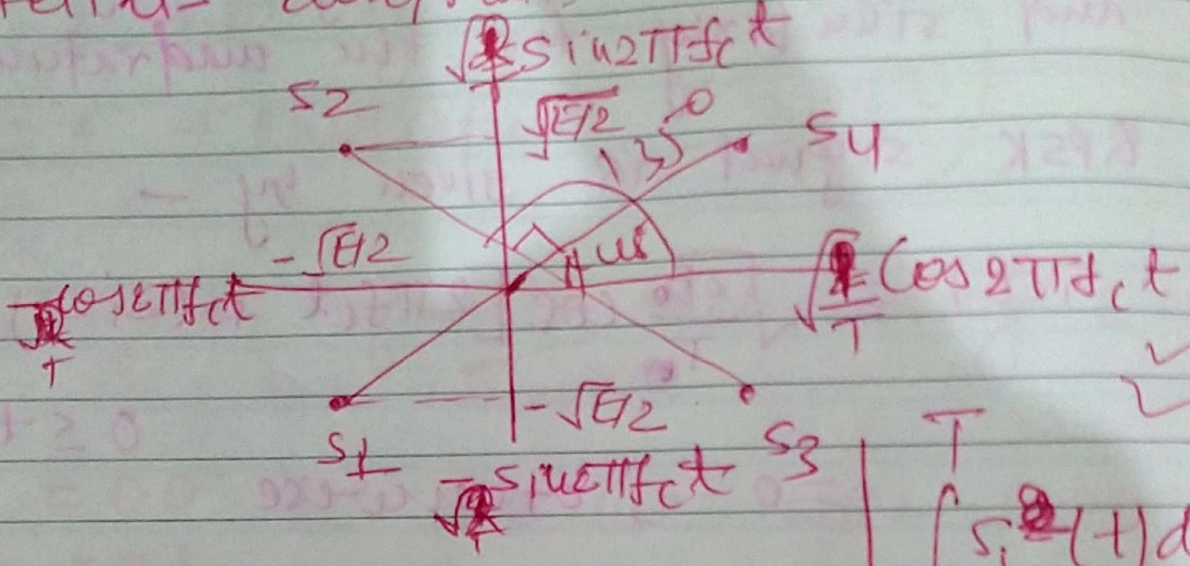
$11 \rightarrow s_4(t) = A_c' \cos 2 + A_c' \sin$

$\oplus\ A_c' = \sqrt{\frac{E}{T}}$

$A \cos 2\pi f_c t + B \sin 2\pi f_0 t = \sqrt{A^2+B^2} \cos(2\pi f_0 t + \phi)$

$\phi = \tan^{-1}(B/A)$,

Constellation diagram.



✓

✓

$s_1 = -\sqrt{\frac{E}{T}} \cos - \sqrt{\frac{E}{T}} \sin$

$= -\frac{\sqrt{E}}{\sqrt{2}} \cdot \sqrt{\frac{2}{T}} \cos - \sqrt{\frac{E}{2}} \cdot \sqrt{\frac{2}{T}} \sin$

$BW = R_b$.

$\int_0^T s_i(t)\,dt = 1$

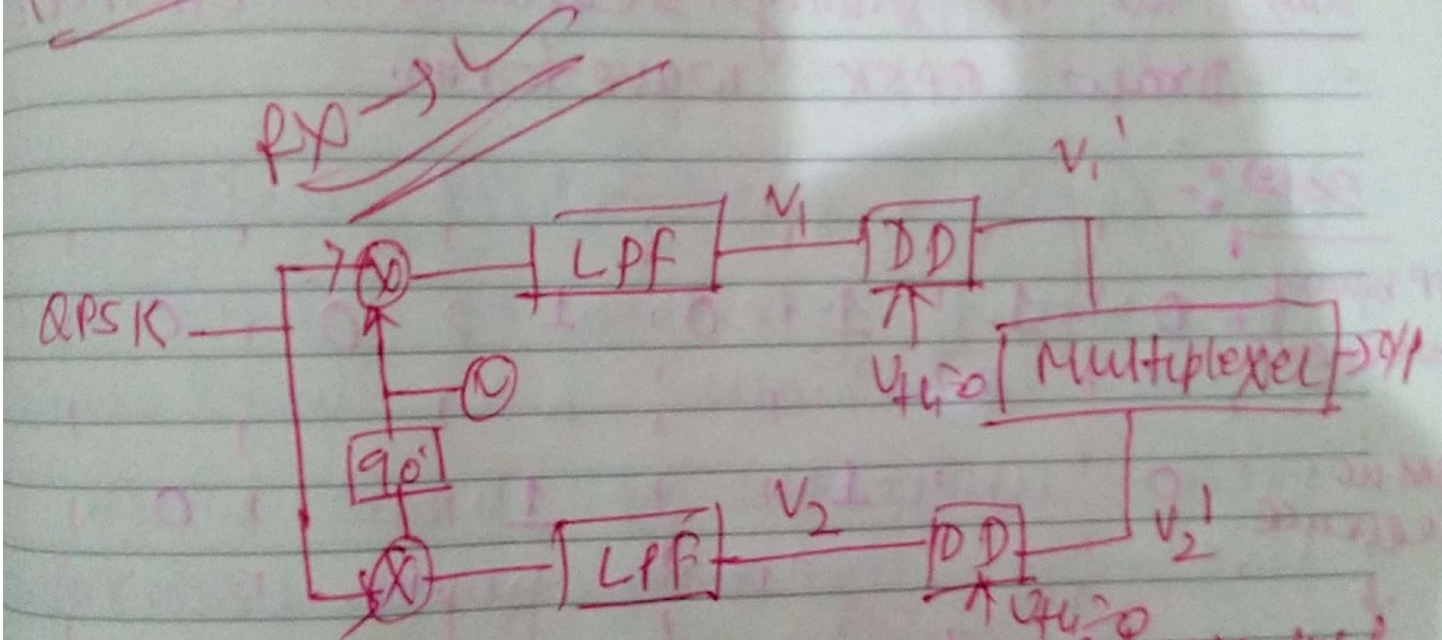
$A^2 \int_0^T \cos^2 + \sin^2 + 2\cos\sin \to 0$

$A^2 \int_0^T 1 =$

$A^2 T = 1$

$A = \frac{1}{\sqrt{T}}$

~~I/P bit~~	~~phase~~		~~coordinates~~
~~00~~	~~(S1)~~	~~$\frac{3\pi}{4}$~~	
~~01~~	~~(S2)~~		



→ QPSK Rxer consist of pair of product modulators with common i/p and quadrature carriers.

When $V_1' > 0$ then inphase channel o/p 1

$V_1' < 0$ —— 0

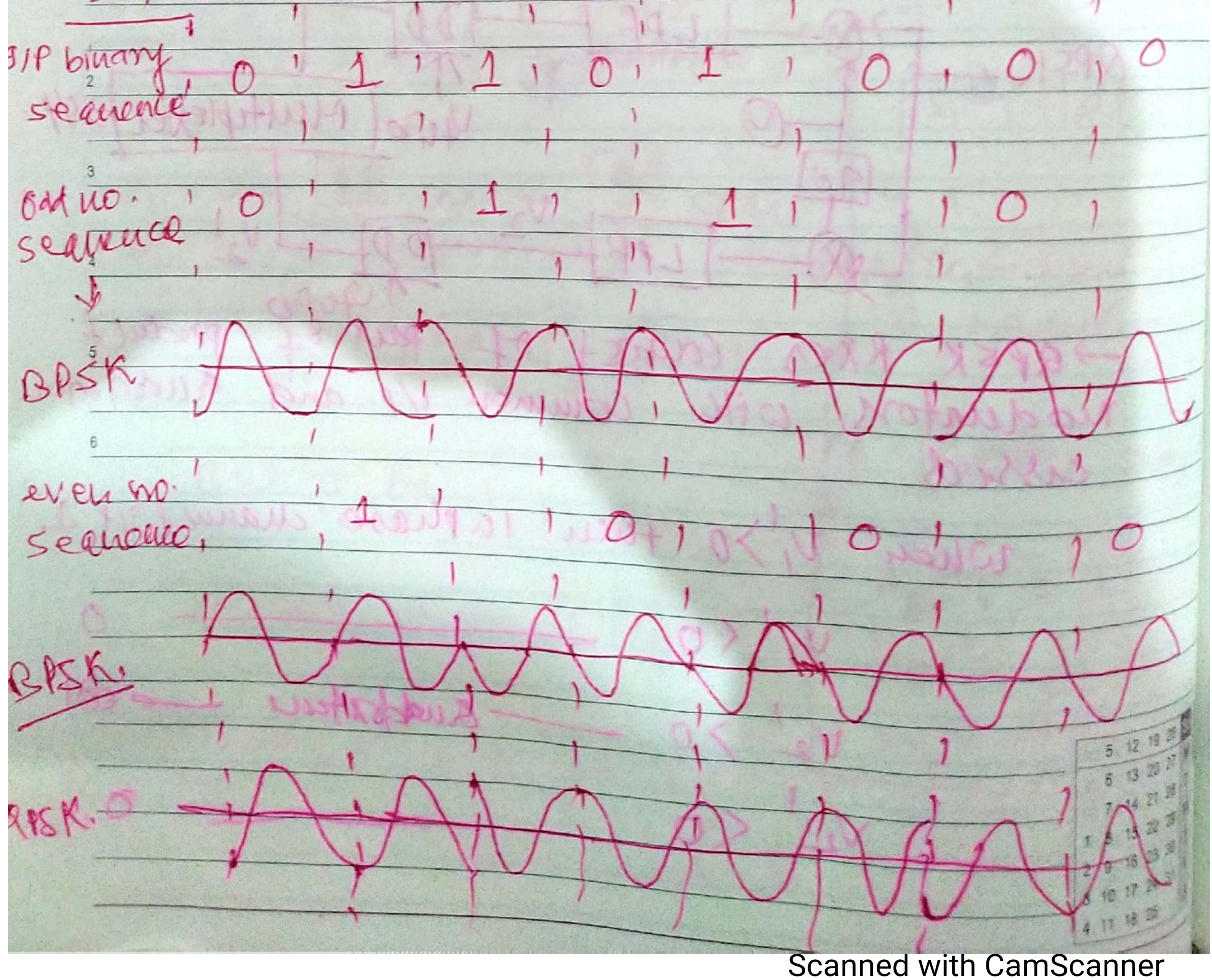
$V_2' > 0$ —— quadrature —— 1

$V_2' < 0$ —— 0

finally these two binary sequence at the inphase and quadrature channel o/ps are Combined in multiplexer to reproduce the original binary sequence

Ans: The i/p binary sequence is 01101000. Draw QPSK waveform.

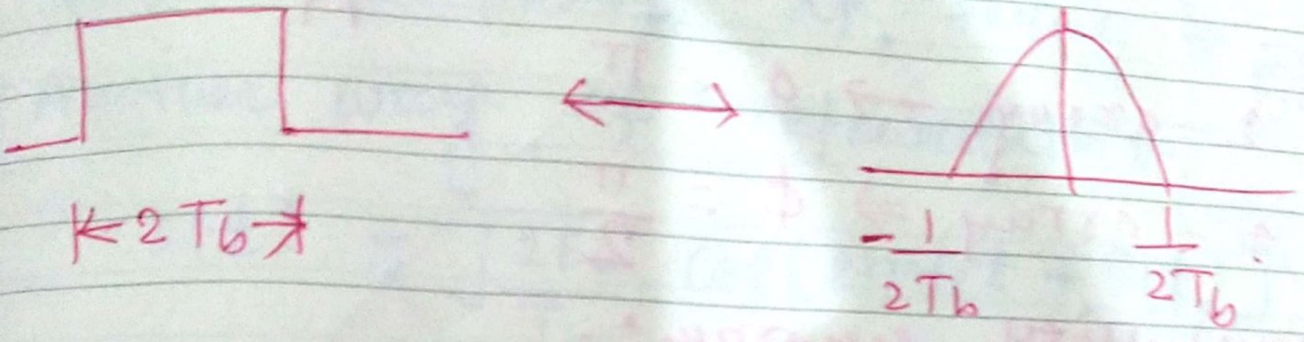
Soln:

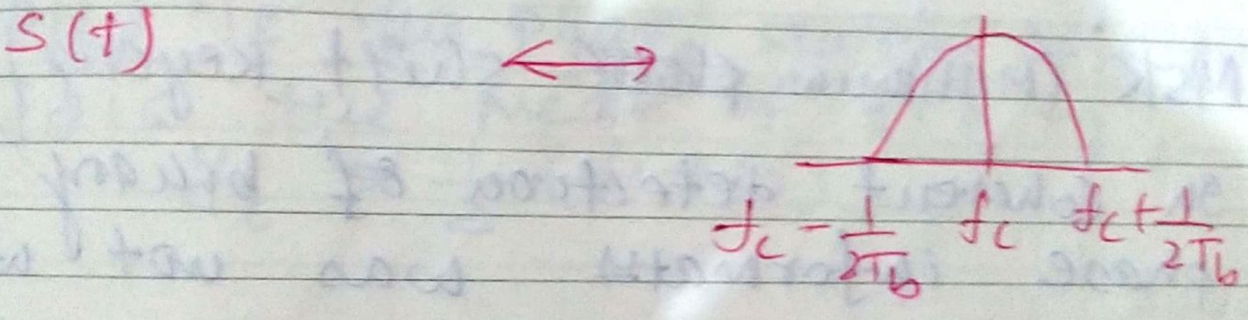


date 22/6/13 (1)

M-array signalling :-

4-array PSK (n = 2)





$BW = R_b$.

for 8-array PSK $BW = \frac{2R_b}{3}$

$\therefore$ M-array PSK, $BW = \frac{2R_b}{\log_2 M}$

→ As the no. of bits to be txed in a specific time increases, transmission BW required will be decreased, but corresponding complexity of the system will be increased.

phase shift in M-array PSK $= \frac{2\pi}{M}$

2-array $\to \phi = \pi$

4-array $\Rightarrow \phi = \frac{\pi}{2}$

Constellatn diagram :-

MSK (Minimum ~~phase~~ shift keying) :-

In coherent detection of binary FSK phase informatn was not utilize proper utilizatn of phase when performing detection, noise performance of receiver can be improved.

Consider a continuous-phase FSK signal

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_1 t + \theta(0)\right)$$

for symbol 1

$$= \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_2 t + \theta(0)\right)$$

for symbol 0

E_b is the txed energy per bit, and T_b is the bit duration.

$\theta(0)$ is phase at time $t = 0$.

Another way of representation:-

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta(t))$$

$\theta(t)$ is the phase shift of $s(t)$.

→ f_c is the mean of f_1 & f_2.

$$f_c = \frac{1}{2}(f_1 + f_2)$$

$$\theta(t) = \theta(0) \pm \frac{\pi t h}{T_b} \quad ; \quad 0 \le t \le T_b.$$

$$h = T_b(f_1 - f_2)$$

+ → when 1 txed

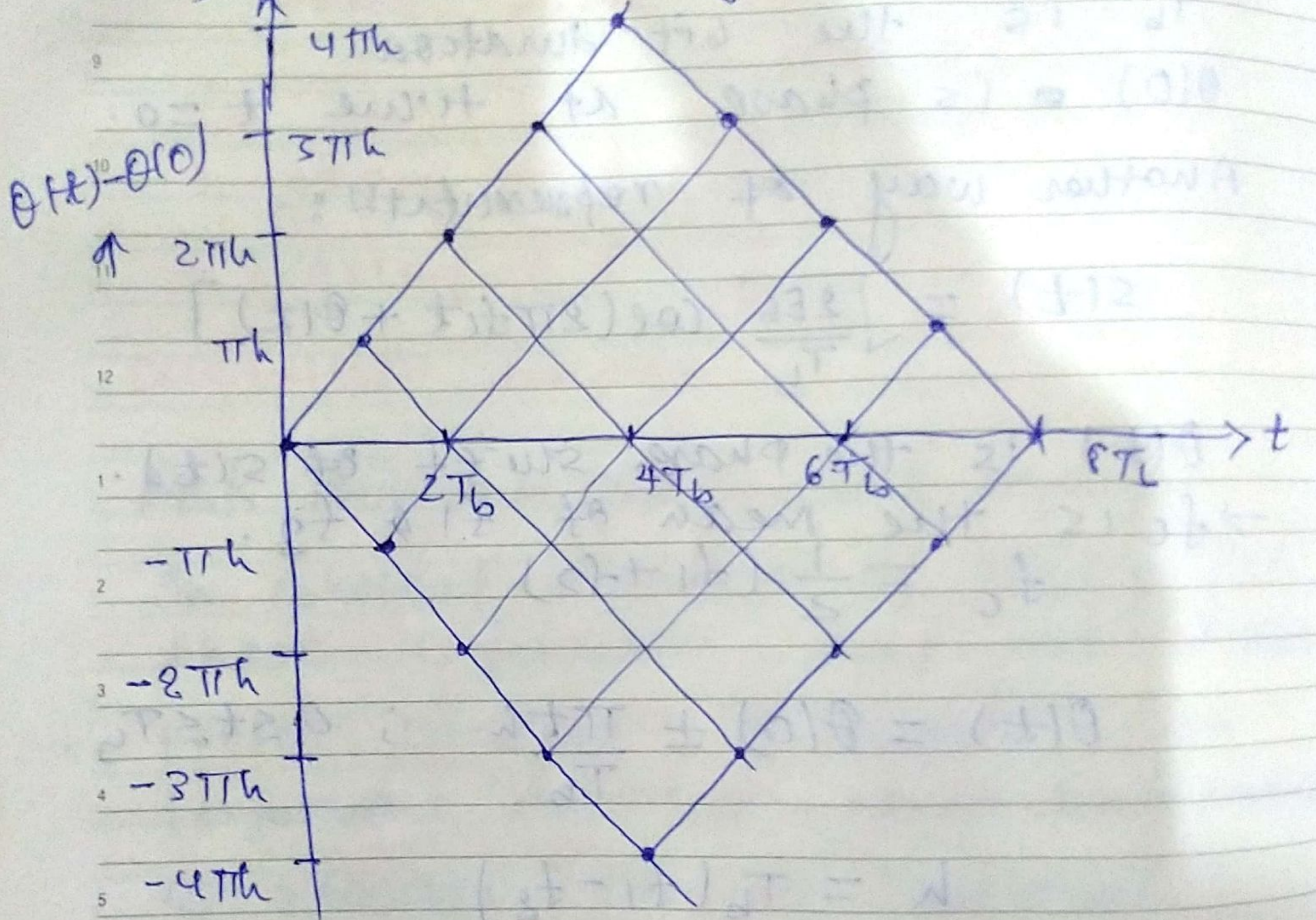
− → when 0 txed.

h is the deviation ratio.

$$\theta(T_b) - \theta(0) = \pi h \quad \text{for '1'}$$

$$= -\pi h \quad \text{for '0'}.$$

i.e. transmission of 1 increase the phase of CPFSK signal s(t) by πh radians, whereas sending of θ reduces by an equal amount.



phase tree.

Above figure shows possible paths starting from time t = 0. It is called phase tree.

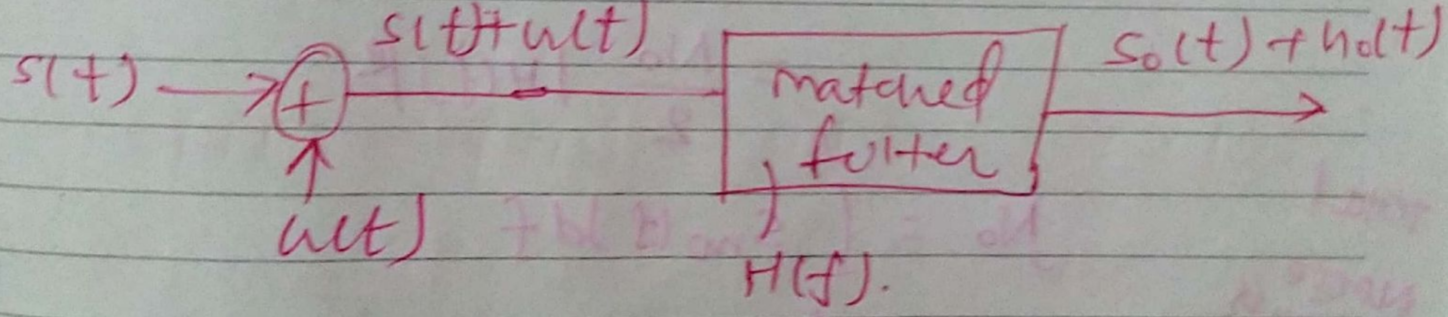
It shows the transition of phase across interval boundaries of the incoming sequence of data bits.

→ CPFSK signal with deviation ratio one-half is reffered as MSK.

Date 13/03/19.

Matched filter

Matched filter is used for decreasing probability of error by increasing signal to noise ratio.



Assume that n(t) is a white noise of having two sided power spectral density $\frac{N_0}{2}$ watts/Hz.

$$\left(\frac{S}{N}\right)_0 = \frac{|s_0(t)|^2}{N_0}$$

$$s_0(t) = s(t) \otimes h(t)$$

$$S_0(f) = S(f) \cdot H(f)$$

$$\therefore s_0(t) = IFT[S_0(f)]$$

$$= \int_{-\infty}^{\infty} S_0(f)\, e^{j2\pi f t}\, df$$

$$S_0(t) = \int_{-\infty}^{\infty} S(f)\cdot H(f)\, e^{j2\pi f t}\, df$$

$$(\text{Noise PSD})_{o/p} = |H(f)|^2\, (\text{Noise PSD})_{i/p}$$

$$S_{no}(f) = S_{ni}(f)\cdot |H(f)|^2$$

$$= \frac{N_0}{2}\cdot |H(f)|^2$$

total Noise power

$$N_0 = \int_{-\infty}^{\infty} S_{no}(f)\, df$$

$$= \int_{-\infty}^{\infty} \frac{N_0}{2}\, |H(f)|^2\, df$$

$$\left(\frac{S}{N}\right)_0 = \frac{|S_0(t)|^2}{N_0}$$

$$\text{at } t = T;\ \left(\frac{S}{N_0}\right) = \frac{|S_0(T)|^2}{N_0}$$

$$= \frac{\left|\int_{-\infty}^{\infty} S(f) H(f)\, e^{j2\pi f T}\, df\right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df}$$

$$\sqrt{A^2+A^2} = \sqrt{\frac{2E}{T}}$$

$$\sqrt{2A^2} = \frac{2E}{T}$$

$$A = \boxed{\frac{E}{T}}$$

According to Schwartz's inequality theorem :-

$$\left|\int_{-\infty}^{\infty} X_1(f)X_2(f)\,df\right|^2 \leq \int_{-\infty}^{\infty} |X_1(f)|^2 \int_{-\infty}^{\infty} |X_2(f)|^2 df$$

$$\therefore \left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi fT} df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)e^{j2\pi fT}|^2 df$$

↓ max^m when

$$H(f) = S^*(f)e^{-j2\pi fT}$$

$$\therefore \left|\int_{-\infty}^{\infty} |S(f)|^2 df\right|^2 \leq \int_{-\infty}^{\infty} |S(f)|^2 df \int_{-\infty}^{\infty} |H(f)|^2 df$$

Provided $H(f) = S^*(f)e^{-j2\pi fT}$, the equality of the above relation holds good.

from ①:-

$$\left(\frac{S}{N}\right)_o \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_o}{2}\int_{\infty}^{\infty} |H(f)|^2 df}$$

provided $H(f) = S^*(f) e^{-j2\pi f T}$

$\frac{S}{N}$ reaches to max^m value

$$\therefore \left(\frac{S}{N}\right)_{max} = \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$= \frac{\int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}}$$

where $S(f)$ is the FT of $s(t)$ given at the o/p of i/p of matched filter

$$\left(\frac{S}{N}\right)_{max} = \frac{E}{\frac{N_0}{2}}$$

E = energy of $s(t)$

when $n(t)$ is having one sided PSD of N_0 then

$$\left(\frac{S}{N}\right)_{max} = \frac{E}{N_0}$$

Impulse response of matched filter:-

$$h(t) = IFT\,[H(f)]$$

$$= \int_{-\infty}^{\infty} H(f)\, e^{j2\pi ft}\, df$$

$$= \int_{-\infty}^{\infty} S^*(f)\, e^{-j2\pi fT} . e^{j2\pi ft}\, df$$

When $s(t)$ is real

$$S^*(f) = S(-f)$$

$$\therefore h(t) = \int_{-\infty}^{\infty} S(-f)\, e^{-j2\pi fT} . e^{j2\pi ft}\, df$$

$$-f \rightarrow f$$

$$= \int_{\infty}^{-\infty} S(f)\, e^{j2\pi fT}\, e^{-j2\pi ft}\, (-df)$$

$$= \int_{-\infty}^{\infty} S(f)\, e^{j2\pi fT}\, e^{-j2\pi ft}\, df$$

$$\therefore \boxed{h(t) = S(T-t)}$$

9

$$\left(\frac{S}{N}\right)_0 = \frac{|s_0(t)|^2}{N_0}$$ when $h(t) = s(T-t)$

10

then,

11

$$= \frac{E}{\frac{N_0}{2}}$$

12

1

2

Ques: The i/p to a matched filter is given by

3

4

$s(t) = 10 \sin(2\pi \times 10^6)$; $0 < t < 10^{-4}s$

5

$= 0$; otherwise.

6

find the peak amplitude of the filter output.

Gate-2 marks. (2003).

Qus:: If Eb, the energy per bit of a binary digital signal is 10^{-5} watt-sec and the one sided power spectral density of the white noise $N_0 = 10^{-6}$ W/Hz. found output SNR of the matched filter.

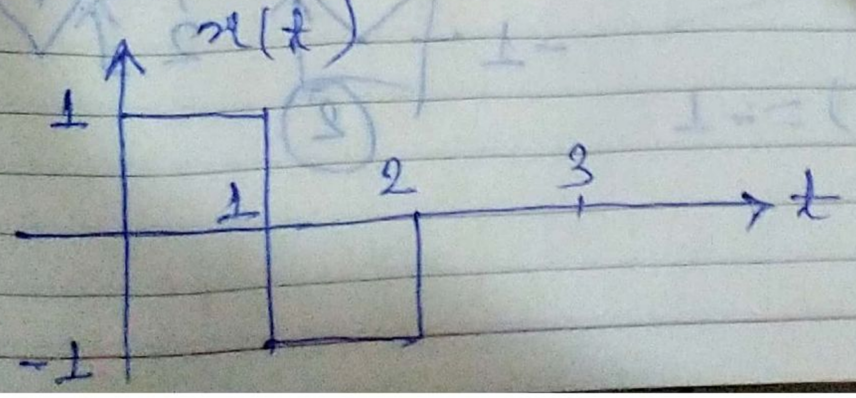
Solⁿ::

$$SNR = \frac{E}{N_0/2} = \frac{2E_b}{N_0}$$

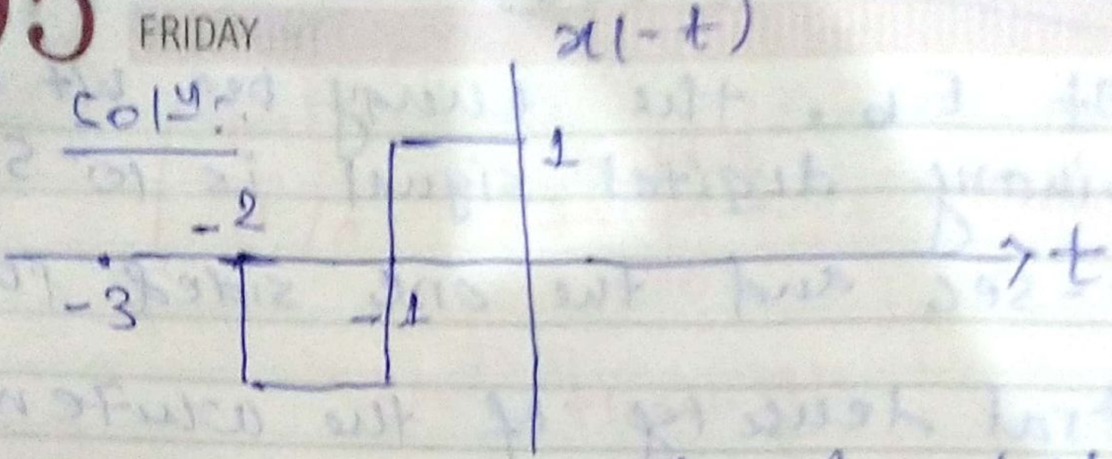
$$= 20$$

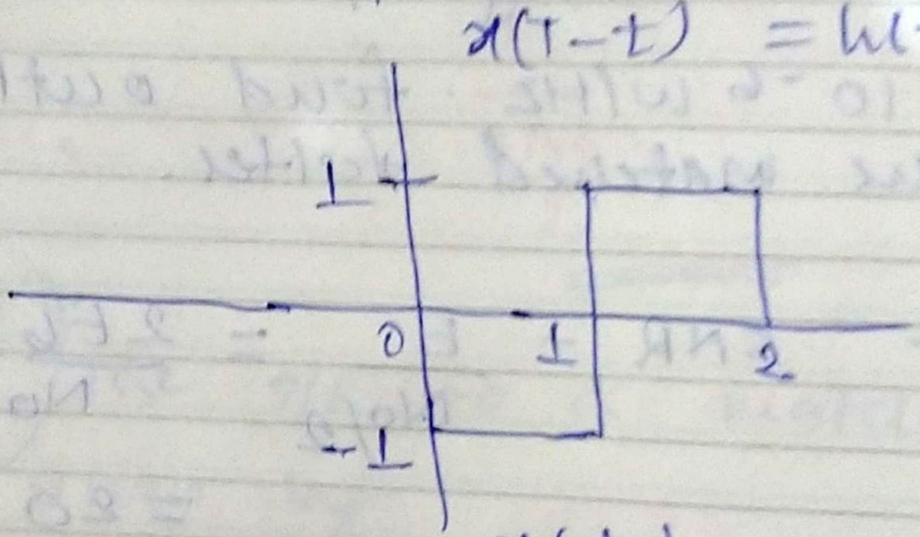
$$(SNR)_{dB} = 10\log 20 = 13 \text{ dB}.$$

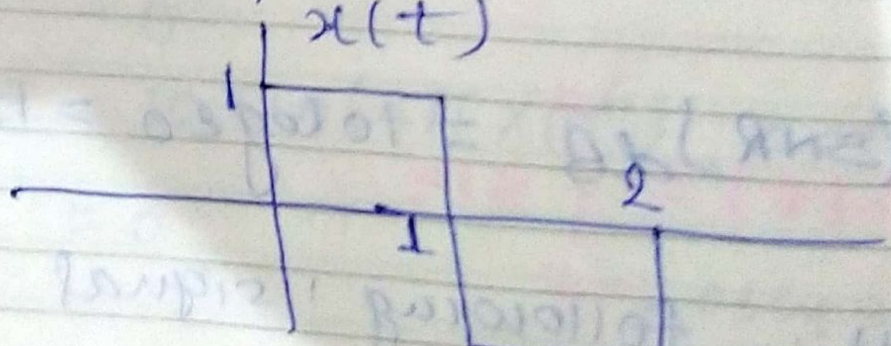
Qus:: following signal is applied to the matched filter. ~~which~~ found the waveform at the output of matched filter.

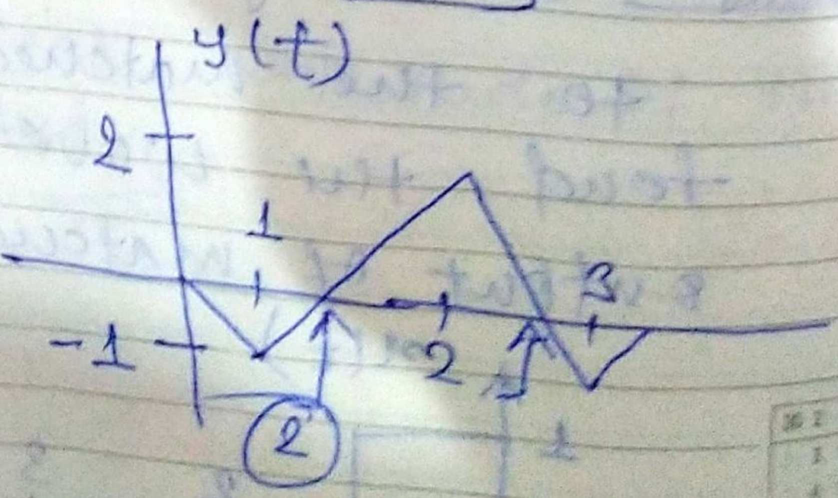


Sol^n:









$\frac{1}{2} \times 2 \times (-1) = -1$

Safe-2marks

Que: An analog pulse $s(t)$ is txed over an AWGN channel. The Rxed signal is $r(t) = s(t) + n(t)$ where $n(t)$ is AWGN with psd $\frac{N_0}{2}$. The Rxed signal is passed through a filter with impulse response $h(t)$. Let E_s and E_h denote the energy of the pulse $s(t)$ and filter $h(t)$, respectively. When SNR is maximized find relatn betn E_s and E_h. Calculate SNR_{max}.

Soln: When SNR is maximum.

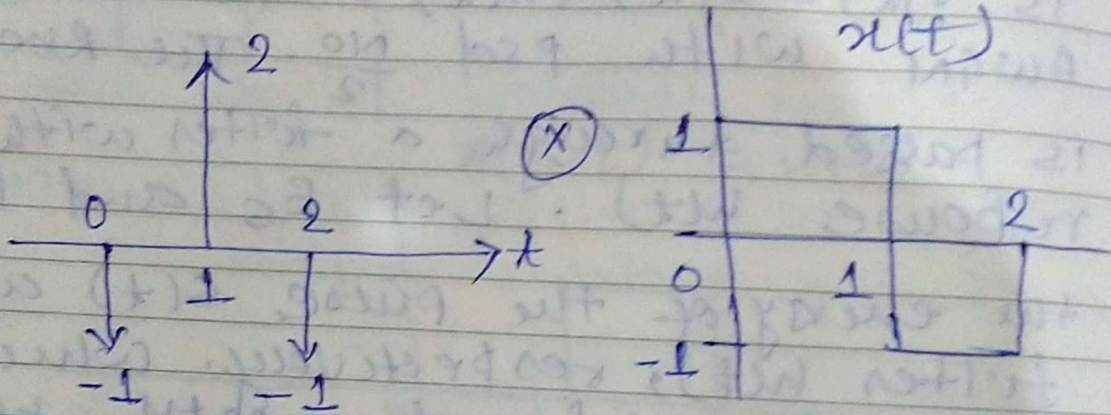
$$h(t) = s(T-t)$$

shifting does not change the energy.

$$E_h = E_s$$

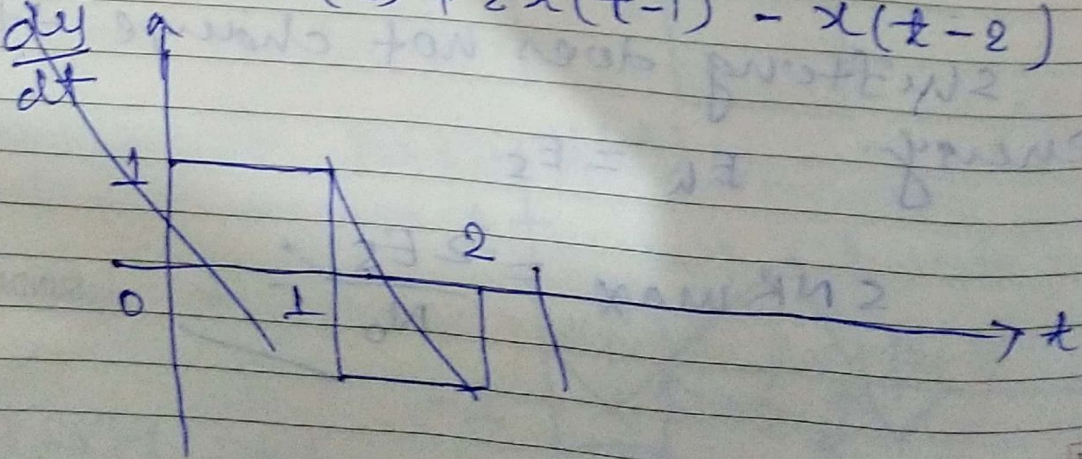
$$SNR_{max} = \frac{2E_s}{N_0}$$

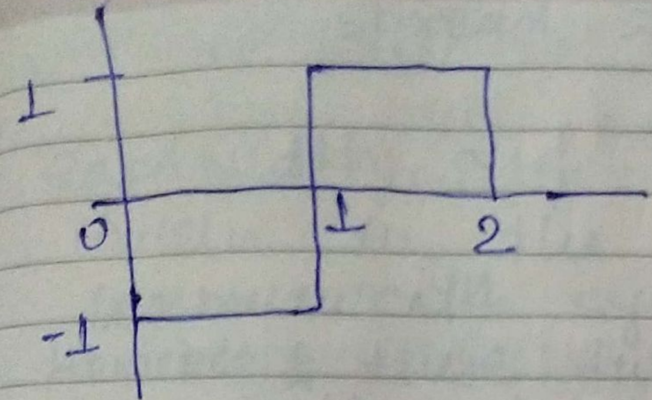
$$\frac{dh(t)}{dt} = -\delta(t) + 2\delta(t-1) - \delta(t-2)$$



$$\frac{dy}{dt} = [-\delta(t) + 2\delta(t-1) - \delta(t-2)] \circledast x(t)$$

$$= -x(t) + 2x(t-1) - x(t-2)$$

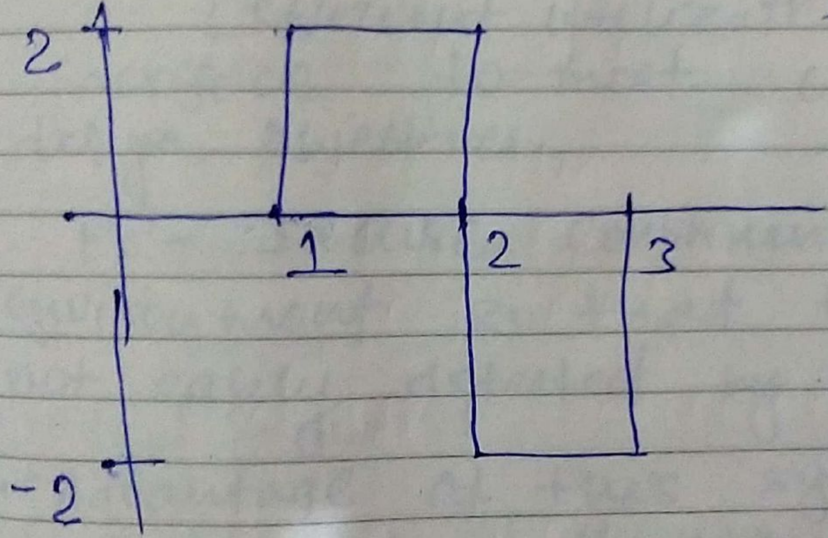




1→2: 1+2+0
=3

2→3: 0-2+1
=-1

3→4:



h→④

①

②