UNIT III
SOLID STATE PHYSICS

CHAPTER-07
X-rays and Compton eEffect

- In 1985 Professor Wilhelm Condrad Röntgen found some highly penetrating radiations named as X-rays.
- X-Rays are electromagnetic rays having shorter wavelength lying in the range from 0.01 Å to 10 Å.

PRODUCTION OF X-RAYS

When electrons with high value of kinetic energy collide with the target metal of high melting point and large atomic weight, X-rays are produced.

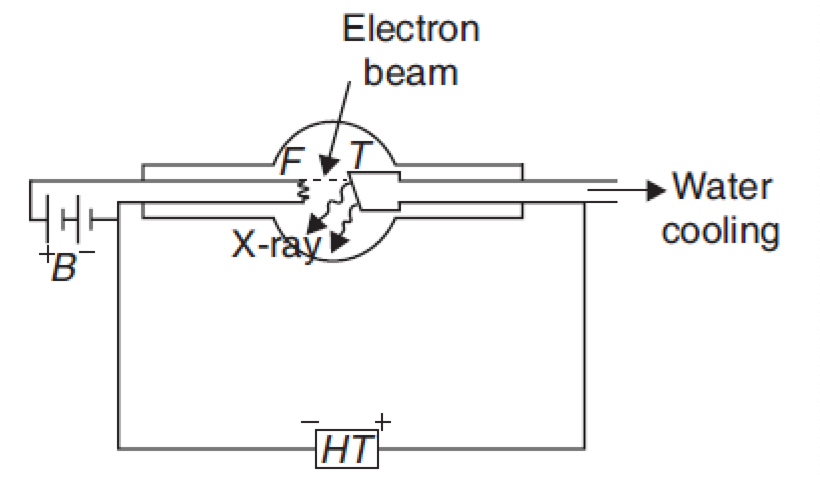
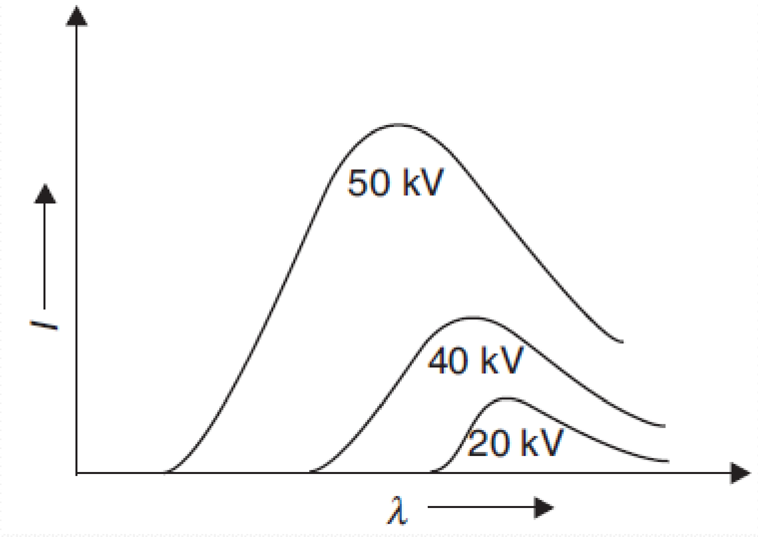


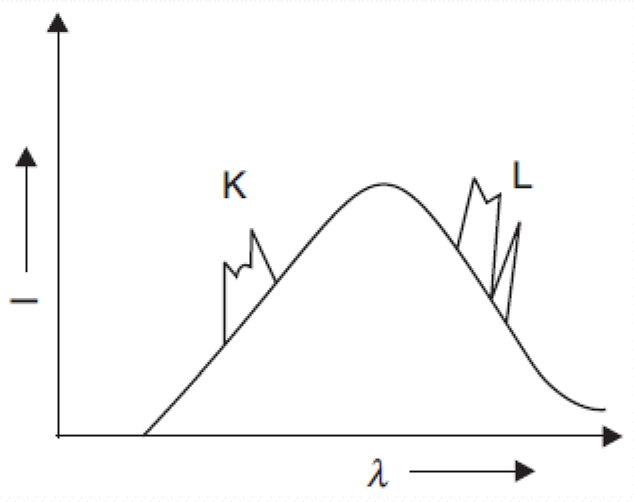
Fig 1: Coolidge tube producing X-rays

X-RAY SPECTRA

Continuous Spectra

Characteristic Spectra





Origin of continuous X-ray spectra:

- When incoming electron reaches near to the target atom, it experiences a strong repulsive coulomb force and is suddenly slowed down and also suffer deflection in path.
- Due to different losses in velocity of the incident electrons, radiations of all possible wavelengths are emitted which give a continuous spectrum of X-rays. *Minimum wavelength* of emitted X-Ray photon is-

$$\lambda_{min} = \frac{c}{\nu_{max}} = \frac{hc}{eV}$$

Origin of characteristic X-ray spectra:

- When high velocity electrons penetrates through the outer shells of the target atom and strikes with an electron in the inner shell.
- Due to collision electron from the inner shell is knocked out vacancy is created and to fill it, an electron from higher energy state jumps into this vacancy.
- The difference in energy is emitted as X-rays photon of energy $\Delta E = h\nu$.

PROPERTIES OF X-RAYS

- X-rays are electromagnetic radiations like light.
- They possess reflection, refraction, diffraction, and interference.
- Have very short range of wavelength, lying between 0.01 Å and 10 Å.
- They are not deviated by either electric or magnetic field.
- Ionize the gas through which they pass.
- Penetrate the solid materials.
- Affect photographic plate.
- Cause fluorescence.

DIFFRACTION OF X-RAYS

- For testing the wave nature of X-rays, scientists tried to obtain the diffraction pattern of X-rays.

- But due to short wavelength of X-rays, man-made transmission gratings is practically impossible.

- The diffraction pattern of a wave is obtained only if the aperture is of the order of the wavelength of the diffracting wave.

- Therefore, transmission gratings having 40 million lines/cm are required for diffracting X-rays. Thus, natural crystals can be used as grating for diffraction of X-rays.

LAUE'S EXPERIMENTAL DEMONSTRATION

Laue obtained diffraction pattern of X-rays passing through a three-dimensional crystal grating. The experimental arrangement of Laue's demonstration is shown in figure.

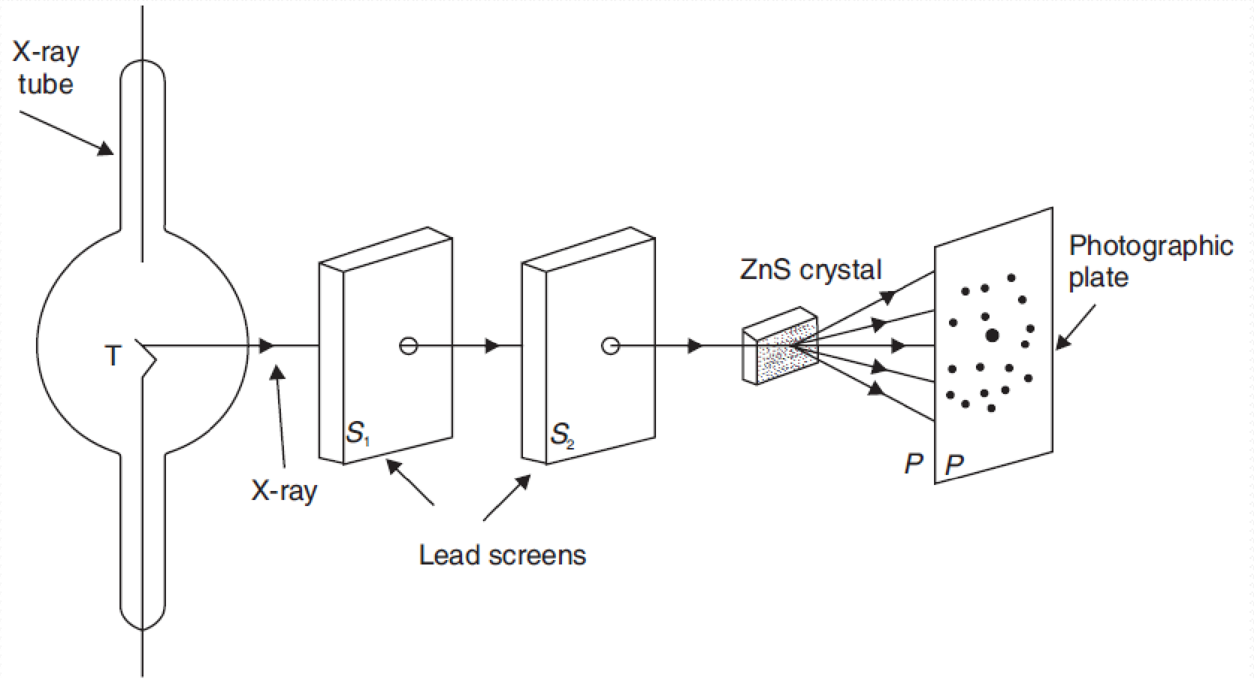


Fig 2: Laue's Experiment

• Diffraction pattern is obtained at photographic plate. This pattern consists of a central spot, surrounded by a series of fainter spots in a definite pattern.

• The symmetrical pattern of spots is called ***Laue's pattern or Laue's spots***.

• Laue's spots prove that X-rays are electromagnetic waves.

Laue's experiment established the following two important facts:
(i) X-rays are electromagnetic waves of short wavelength.
(ii) In crystals, atoms are arranged in a three-dimensional lattice.

BRAGG'S LAW

• Bragg considered that a crystal is made up of a number of parallel planes on which atoms are arranged in a regular fashion.

• When X-rays are allowed to fall on the crystal, reflections from different planes take place. These reflected rays interfere and produce diffraction pattern.

• Let us consider a narrow beam of X-rays of wavelength λ. *It is allowed to incident on a crystal of parallel lattice planes having inter-atomic (lattice planes) separation of d. Let us assume the glancing angle to be* θ.

- It is known that incident rays are reflected from various parallel planes of atoms in the crystal.
- The diffraction pattern of reflected radiation is observed only when reflections from various planes of atoms interfere constructively.

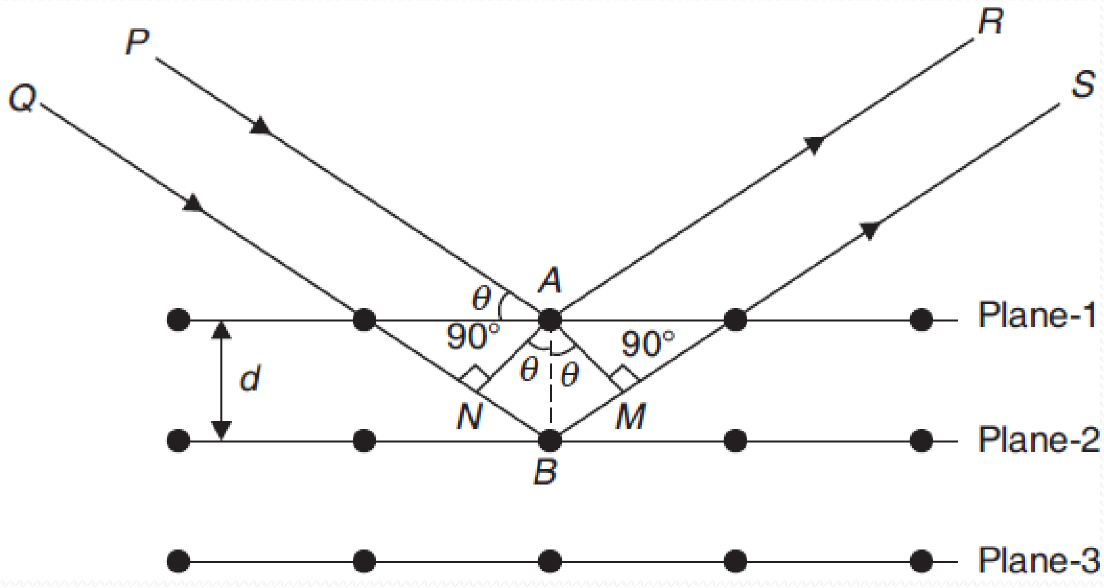


Fig 3: Diffraction of X-rays through a crystal lattice

- Let us suppose that ray *PA is reflected from atom A in the direction AR and ray QB is reflected from atom B in the direction BS.*
- If the path difference between *AR and BS satisfies the condition of constructive interference, then a diffraction pattern will be observed.*
- The path difference between the reflected rays *AR and BS can be given as (NB + BM).*

From the fig.

$$NB = BM = d \sin \theta$$

Hence, the path difference can be given as

$$(NB + BM) = 2d \sin \theta$$

- If this path difference is an even multiple of $\lambda/2$ *(i.e., n* λ*), then the two rays will reinforce each other and produce an intense spot. Thus, the condition of reinforcement can be given as*

$$2d \sin \theta = n \lambda \qquad \text{where } n = 1, 2, 3, \text{etc.}$$

It is known as ***Bragg's law.***

BRAGG'S SPECTROMETER

- For the structural studies of crystals, W.H. Bragg and his son W.L. Bragg devised a spectrometer in which a crystal is used as a reflection grating instead of a transmission grating. With the help of their spectrometer, inter atomic separation in the crystal can be calculated.

- Bragg's spectrometer consists of a source of X-rays S, slits S1 and S2, and crystal C mounted on a prism table. A round scale, consisting of two vernier scales V1 and V2 to note down the angle, is attached with the prism table. An ionization chamber is attached with the prism table, along with a galvanometer.

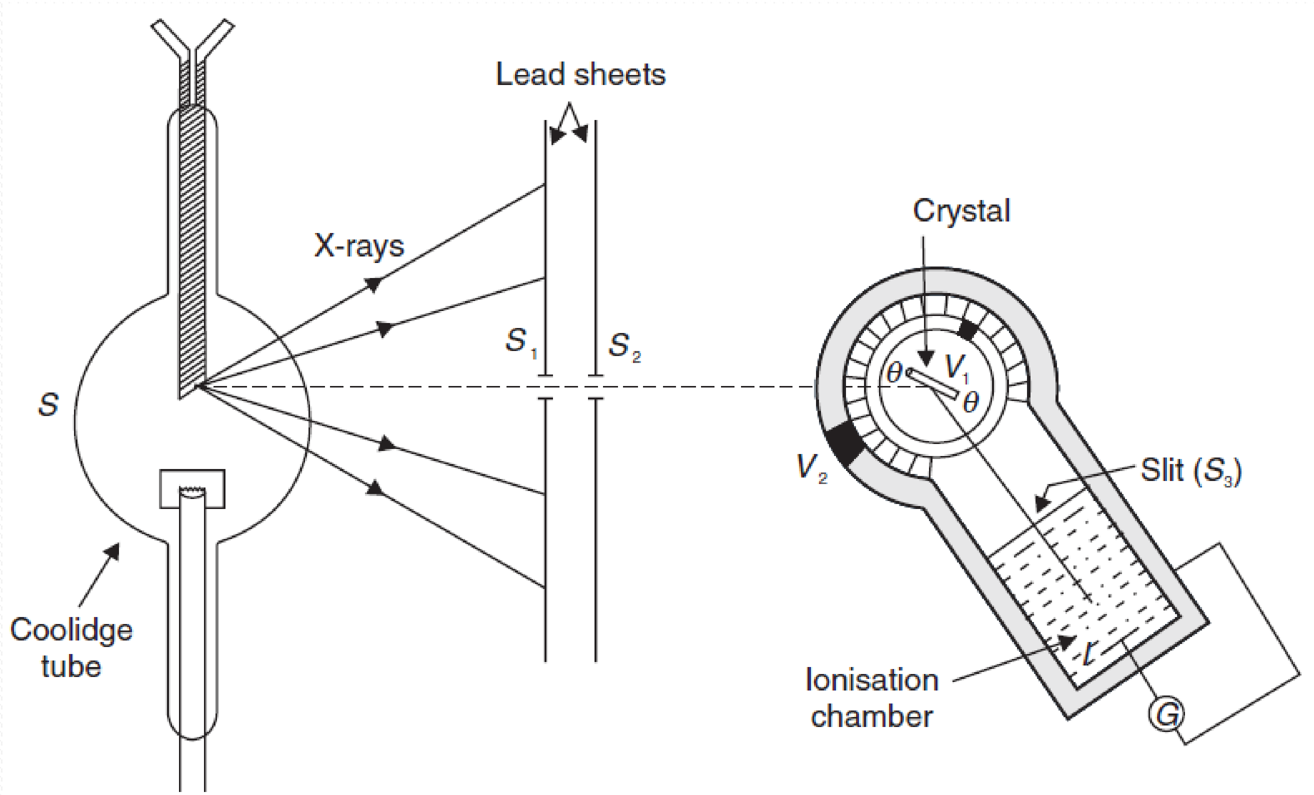
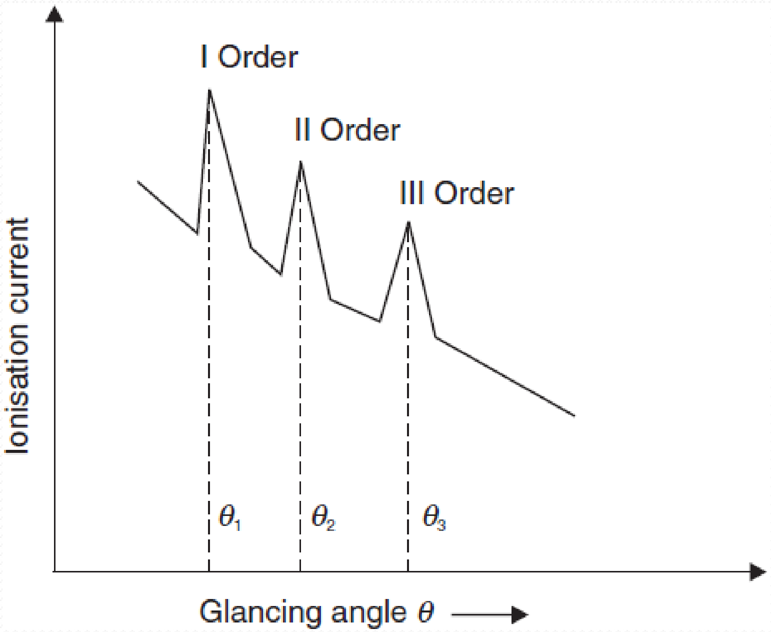


Fig. 4 Bragg's spectrometer

The graph plotted between the ionization current *I and glancing angle θ for the NaCl crystal is shown in Figure.*



The number of peaks in the curve corresponds to those glancing angles which satisfy Bragg's equation, i.e.,

$$2d \sin \theta = n \lambda$$

From the experimental observations, the following facts have been established:

(i) As the order of spectrum increases, the intensity of the reflected X-rays decreases.

(ii) The ionization current does not fall to zero for any value of glancing angle θ, but it does attain a maximum value for certain glancing angles. It indicates that there is continuous spectrum over which the characteristic line spectrum is superimposed.

COMPTON EFFECT

- Compton effect gives direct and conclusive evidence in support of the particle nature of electromagnetic radiations.

- *When a monochromatic beam of X-rays (or the electromagnetic radiations of short wavelength) of wavelength λ is allowed to incident on scattering material, the scattered beam contains radiations of longer wavelength λ' in addition to the radiations of incident wavelength λ.*

- *The difference between λ' and λ, i.e., $\lambda' - \lambda$, is known as Compton shift, and this effect is called Compton effect.*

EXPRESSION FOR COMPTON SHIFT

In order to derive an expression for change in wavelength of the scattered X-rays, Compton made some basic assumptions as given below:

(i) Incident X-rays or electromagnetic radiations consist of photons of same energy, i.e.,

$$E = h\nu$$

Momentum (*P) of electron can be calculated from the relativistic energy expression,*

$$E^2 = P^2c^2 + m_0^2 c^4$$

where m_0 *is the rest mass of the particle. Since the rest mass of photon is zero, hence the above expression becomes*

$$E = Pc$$

$$P = \frac{h\nu}{c}$$

(ii) Scattered electrons are treated as free particles because the energy of X-ray photons is very large (~ 10 keV) as compared to the binding energy of the electrons in the atom.

(iii) The electrons of the scattering material are treated as at rest, because the energy of incident X-ray photons is very high as compared to that of electrons in atoms.

(iv) Collisions between incident X-ray photons and free electrons are perfectly elastic. There is no loss of energy and momentum during their collision, and hence the laws of conservation of energy and momentum hold good.

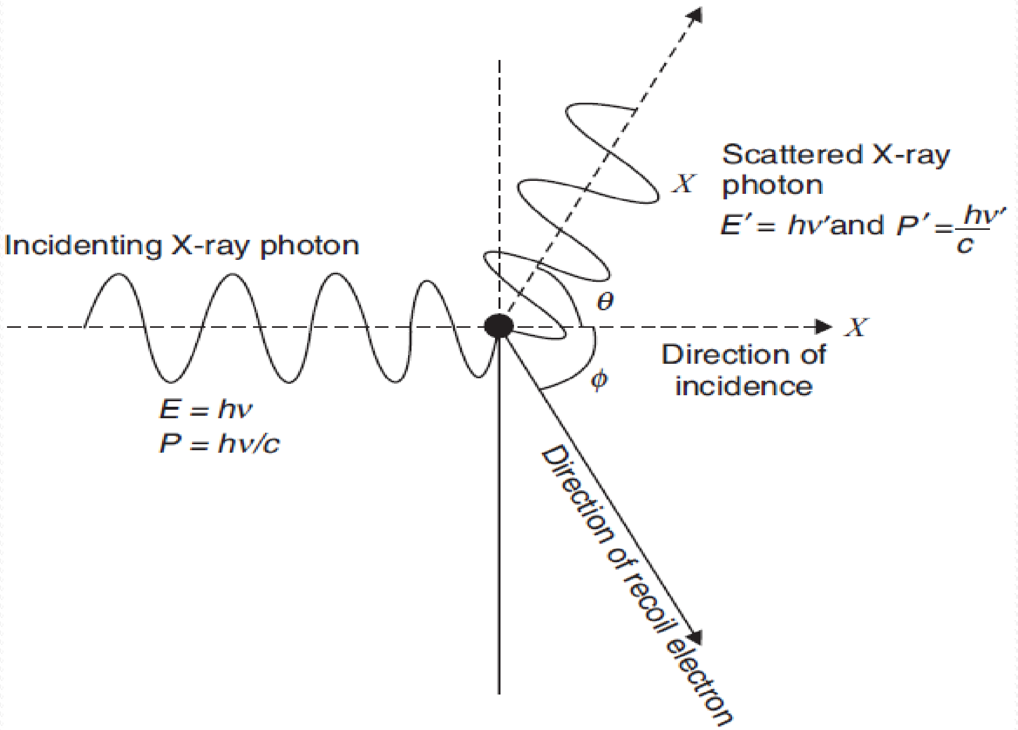


Fig 5: Scattering of X-ray photon with the electron of scattering material

- When an X-ray photon of energy $h\nu$ and momentum $h\nu/c$ collides elastically with a target electron of rest mass m_0, it transfers some of its energy to the electron so that its frequency reduces to ν'.

- Let us also consider that target electron recoils at angle Φ with momentum mv and energy mc^2, while scattered photon makes angle θ with the direction of incidence.

- Now, according to the principle of law of conservation of energy

$$h\nu + m_0c^2 = h\nu' + mc^2$$

From the principle of conservation of momentum, we have

Momentum before the collision = Momentum after the collision

Now, applying the principle of conservation of momentum in the direction of incidence, we get

$$m\upsilon \cos \phi = \frac{h}{c} \ (\nu - \nu' \cos \theta)$$

We know that

$$m = \frac{m_0}{\sqrt{1 - \upsilon^2 / c^2}}$$

By the above equations we get

$$(\lambda' - \lambda) = \frac{h}{m_0 c} \ (1 - \cos \theta)$$

Or,

$$\Delta\lambda = \frac{h}{m_0 c} \ (1 - \cos \theta) = \frac{2h}{m_0 c} \sin^2 \frac{\theta}{2}$$

It is the expression for the Crompton shift $\Delta\lambda$.

DISCUSSION ON COMPTON EFFECT

Compton shift can be discussed under the following points:

(i) From the expression of Compton shift , it is clear that the increase in wavelength or the Compton shift (Δλ) *is independent of the wavelength of the incident radiation as well as the nature of the scattering material. It depends only on the angle of scattering,θ.*

(ii) The constant *h/m_0c has dimensions of length and is called the Compton wavelength (λc) of the target particle. For an electron, the Compton wavelength is*

$$\lambda_c = \frac{h}{m_0 c} = \frac{6.63 \times 10^{-34}\ \text{Js}}{9.1 \times 10^{-31}\ \text{kg} \times 3 \times 10^{8}\ \text{m/s}}$$

$$= 2.43 \times 10^{-12}\ \text{m}$$

$$= 0.0243\ \text{Å}$$

(iii) From Eq. (7.14), it is clear that Compton shift increases from 0 to $2\lambda_c$ *for* $\theta=0$ *to* $\theta=\pi$. If $\theta=\pi$, *then*

$$\Delta\lambda = \lambda_c (1 - \cos\pi)$$

$$= \lambda_c (1+1) = 2\lambda_c$$

This indicates that a head-on collision between the incident photon and the target electron leads to the backscattering of the photon.

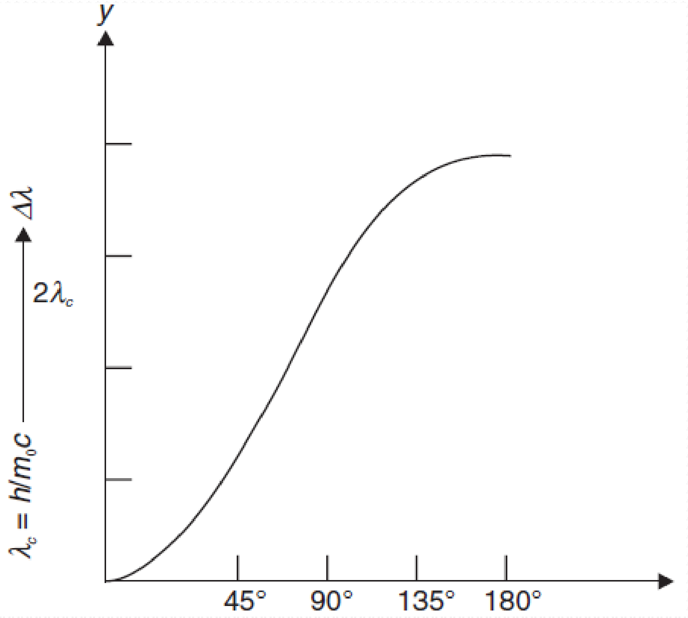


Fig 6: Variation of Δλ with scattering angle θ

IMPORTANCE OF COMPTON EFFECT

Compton effect is an important milestone in the development of modern physics. Compton effect proves the following:

(i) It proves the particle nature of electromagnetic radiations.
(ii) This verifies Planck's quantum hypothesis.
(iii) It provides an indirect verification of the relations

$$m = \frac{m_0}{\sqrt{\left[1 - (\upsilon^2 / c^2)\right]}} \text{ and } E = mc^2$$

as these relations are used in deriving the expression for Compton effect.

EXPERIMENTAL VERIFICATION OF COMPTON EFFECT

- The experimental arrangement used for observing Compton scattering is shown in Figure.

- In this arrangement, X-ray tube is used as the source of X-rays. Collimated beam of monochromatic X-rays of wavelength $0.707A^0$ is allowed to be incident on the graphite target.

- The intensity of the scattered X-rays is measured as a function of their wavelength for different angles of scattering.

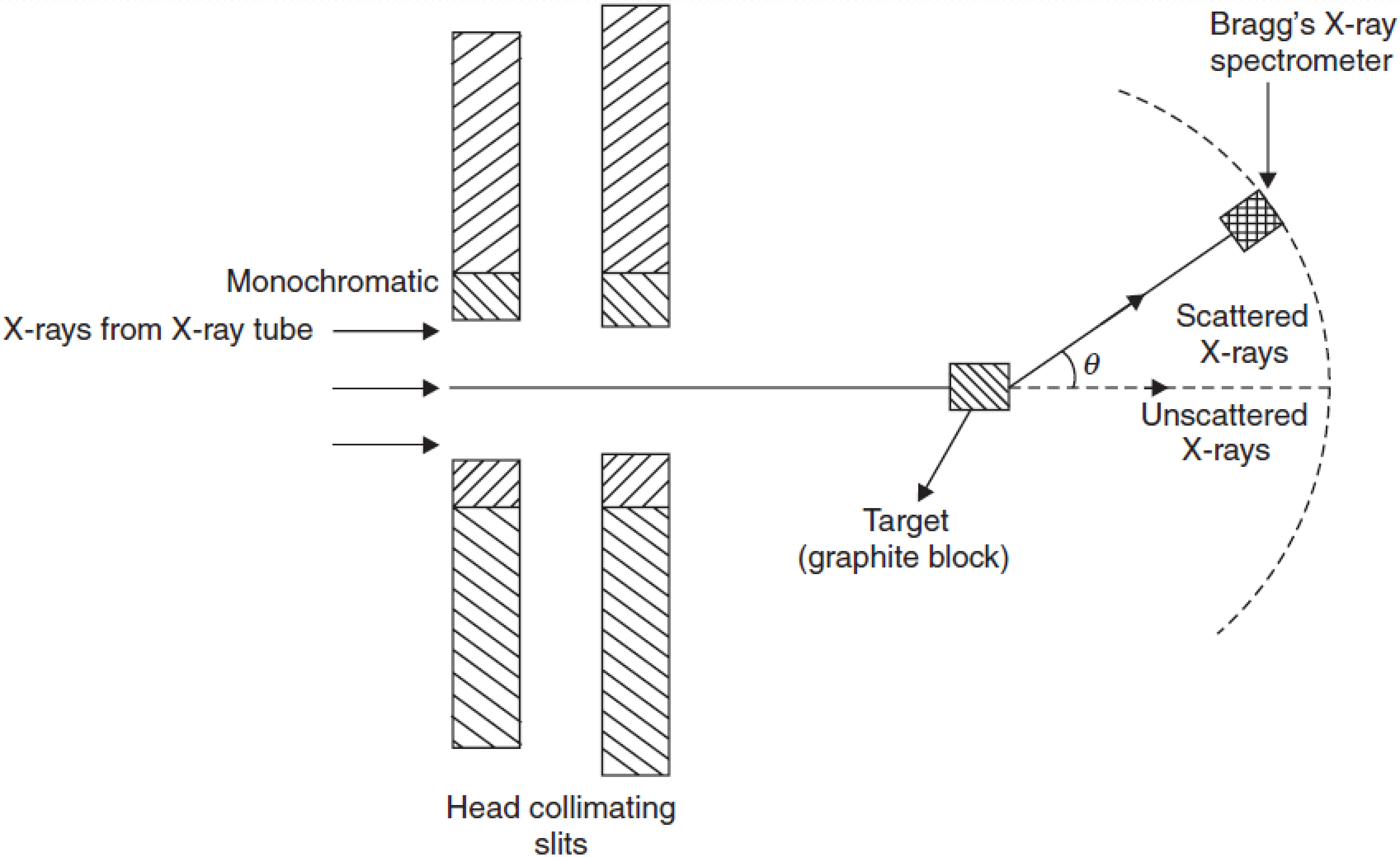


Fig7. Experimental verification of Compton effect

On the basis of the variation of intensity with wavelength, we can draw the following conclusions:

(i) Compton shift increases with increase in scattering angle θ.

(ii) At $\theta = 0$, *there is a single peak which indicates that there is no Compton shift at* $\theta = 0$.

(iii) Spreading is observed in the wavelength of Compton shift due to the movement of electrons in the atom (i.e., due to the momentum distribution of electron).

(iv) For the same scattering angle, the intensity of unmodified peak is greater for heavier elements. It is due to this reason that most of the electrons are tightly bound in heavier atoms, whereas in light atoms electrons are relatively loosely bound.

DIRECTION OF RECOILED COMPTON ELECTRON

In order to determine the direction of recoiled electron, let us use Eqs.

$$m\upsilon c \cos\phi = h(\nu - \nu' \cos\theta)$$

and

$$m\upsilon c \sin\phi = h\,\nu' \sin\theta$$

From the above Eqs. we can write

$$\tan\phi = \frac{\nu' \sin\theta}{\nu - \nu' \cos\theta}$$

We also know that

$$(\lambda' - \lambda) = \frac{h}{m_0 c}(1 - \cos\theta)$$

Hence

$$\tan\phi = \frac{\cot\theta/2}{\left[1 + (h\nu/m_0 c^2)\right]}$$

Above equation gives the direction of recoiled Compton electron.

KINETIC ENERGY OF RECOILED ELECTRON

To derive an expression for the kinetic energy of a recoiled electron, let us start with equation,

$$h\nu + m_0c^2 = h\nu' + mc^2$$

We know that

$$\lambda' - \lambda = \frac{h}{m_0 c}\ (1 - \cos\ \theta)$$

$$\text{Kinetic energy of recoiled electron} = h\nu\left[\frac{2\alpha \sin^2 \theta / 2)}{1 + 2\alpha \sin^2 \theta/2)}\right]$$

Where

$$\alpha = h\nu/m_0c^2,$$

The maximum value of kinetic energy of a recoiled electron (KE_{max}) can be given as

$$KE_{max} = \frac{2h^2\nu^2}{m_0c^2\left[1 + (2h\nu / m_0c^2)\right]}$$

CRYSTAL STRUCTURE OF NaCl

- Sodium chloride forms crystals with face-centered cubic symmetry.
- In these, the larger chloride ions are arranged in a cubic close-packing, while the smaller sodium ions fill all the cubic gaps between them.
- Each ion is surrounded by six ions of the other kind; the surrounding ions are located at the vertices of a regular octahedron.

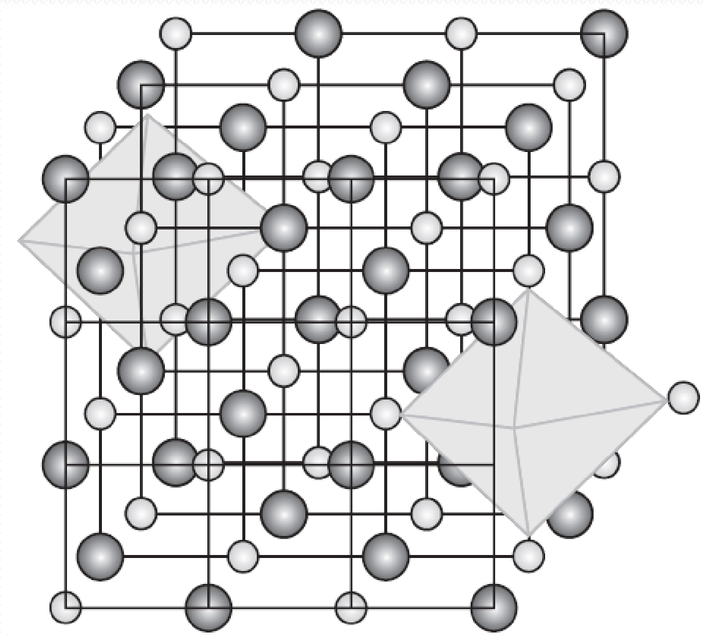


Fig.8 The crystal structure of sodium chloride

CRYSTAL STRUCTURE OF DIAMOND

- Diamond cubic is in the Fd3m space group, which follows the face-centered cubic Bravais lattice.
- The atomic packing factor of the diamond cubic structure is $\pi\sqrt{3}/16$ with eight atoms per unit cell. Mathematically, the points of the diamond cubic structure can be given coordinates as a subset of a three-dimensional integer lattice by using a cubical unit cell, four units across.

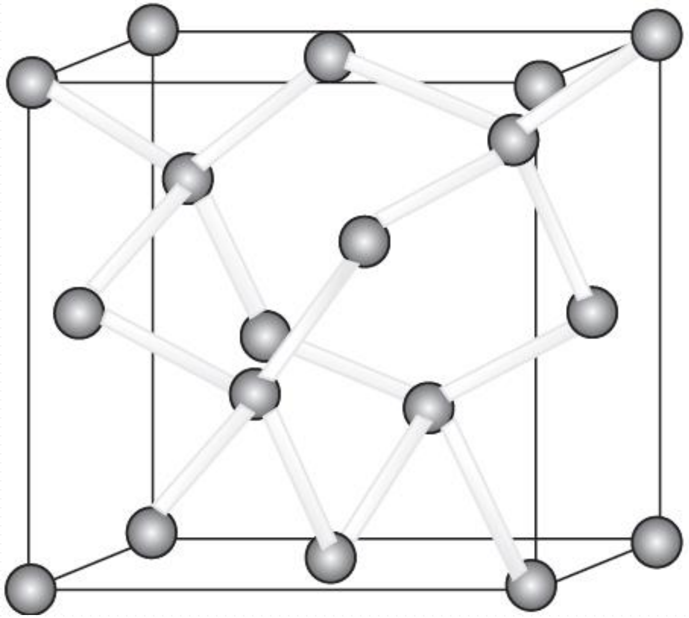


Fig. 9 The crystal structure of diamond

APPLICATIONS OF X-RAYS

Some important applications of X-rays are as follows:

(i) Medicine: Three-dimensional scanners use X-rays to get sharp and clear pictures of heart, lungs, kidneys, and other organs. X-rays are frequently used in the radiography of bones, tissues, peptic ulcers, ruptures of the internal organs of human body.

(ii) Engineering and industry: X-rays are frequently used in identifying the manufacturing defects in industrial products.

(iii) Research: X-rays are used to study the structures of crystalline solids, atoms, and compounds.