Heat & Mass Transfer (BME-27)

Prashant Saini

Mechanical Engineering

Madan Mohan Malviya University of Technology Gorakhpur ***(UP State Govt. University)***

Email: psme@mmmut.ac.in

Introduction to Heat Transfer

Heat Transfer: It is defined as the science that deals with transfer of heat between two regions of a given medium or between two mediums by virtue of temperature difference between the two in the direction of decreasing temperature until thermal equilibrium is established.

- Heat always moves from a warmer place to a cooler place.
- Hot objects in a cooler room will cool to room temperature.
- Cold objects in a warmer room will heat up to room temperature.

Application: Rubbing of hands, Emulsion water heater, Press of clothing, Steam boiler, Condenser, Cooling tower, air preheater, Refrigerator, Ice making plant, Milk chiller plant, Radiators of IC engine, Fins of IC engine, Solar water heater, Wind mills, Laptops etc.

Modes or Mechanism of Heat Transfer

- Conduction
- Convection
- Radiation

Conduction

Conduction is the mode of heat transfer taking place by virtue of transfer of energy from higher energetic particle to lower energetic particle in account of particulate interaction, without appreciable bulk motion of particle it themself. **Example:**

- All solids with internal temperature gradients (Ag, Cu, Al, C. I., steel etc..)
- All apparently stationary liquid and gases with internal temperature gradients.

The important parameter that decides the intensity of conduction in a body is **particulate interaction** measure in terms of strength and frequency.

In Solids:

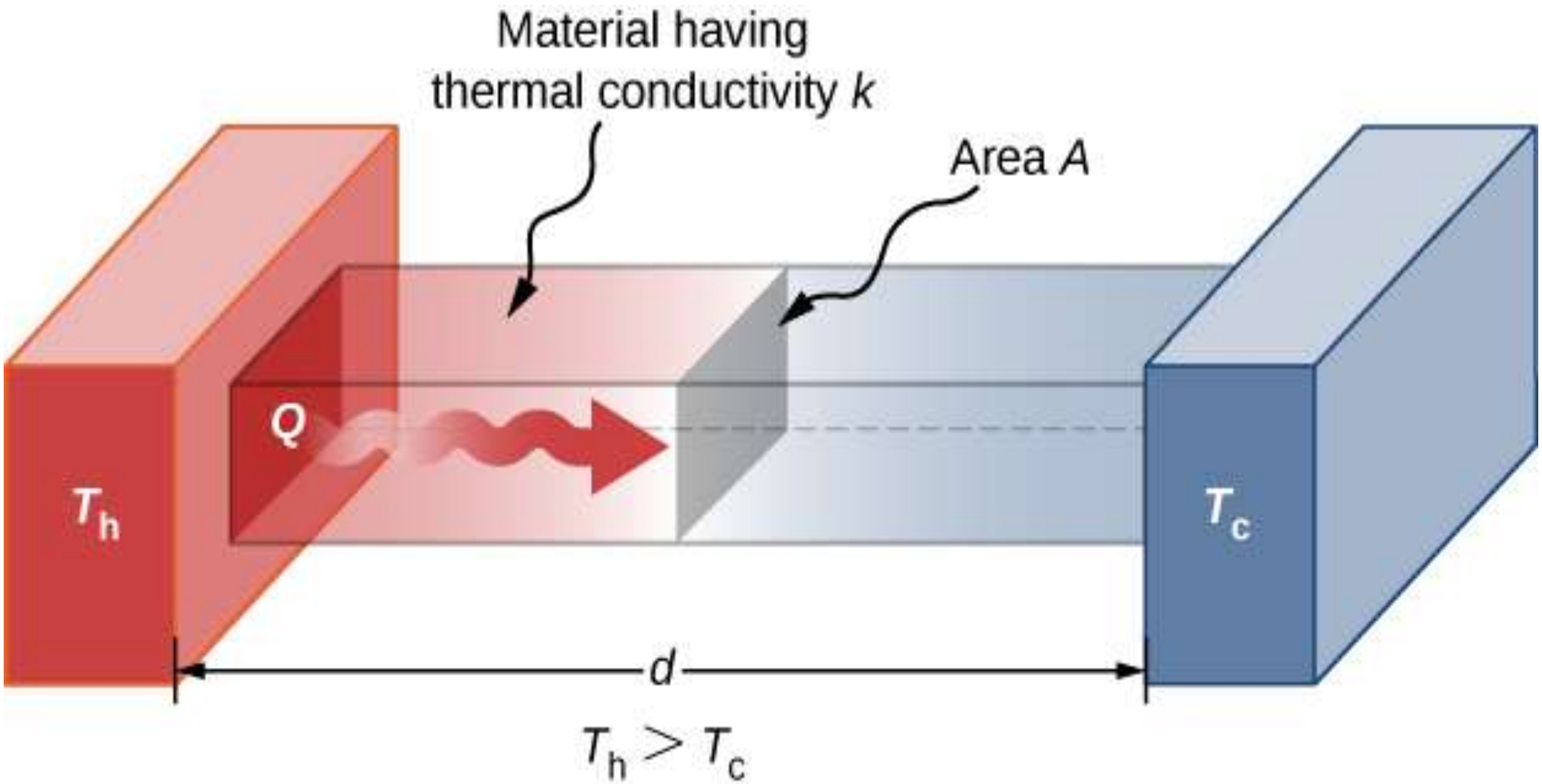
- Metals (transport of free electrons)
- Non Metals (Lattice vibration)

In Liquids and gases:

- Collision
- Diffusion

The order of conduction activity:
Solid > Liquid > Gas

Contd…



Fourier's law of heat conduction

Joseph B. Fourier (1882) French mathematical physicist gave empirical relation, in which rate of heat conduction through depends on geometry, its thickness, material of medium as well as temperature across the medium. Law is derived based on the assumptions:

- Material is homogeneous, isotropic and temperature is varying only in x direction and conduction is at steady state.
- No decrement and increment in stored energy.
- Temperature gradient is constant and temperature profile is linear.
- Boundary surface are isothermal in character.
- $q_{cond,x} = -k_x A \frac{\partial T}{\partial x}$ The rate of heat conduction is directly proportional to temperature gradient in the direction of heat transfer and perpendicular area.
- k_x is the thermal physical property (quality) of the material called thermal conductivity. Its unit is (W/m K).
- Negative sign is because of decreasing direction of temperature.

Contd….

If the parameters are given as per Fig. 1.

Fourier's law can be written as:

$$q_{cond,x} = k_x A \frac{T_1 - T_2}{L} \quad (1)$$

It can also be written as:

$$q_{cond,x} = \frac{T_1 - T_2}{\left({L}/{k_x A}\right)} \quad (2)$$

Electrical analogy:

Ohm's law can be written as:

$$I = \frac{V_1 - V_2}{R} \quad (3)$$

On comparison of Eq. 2 and 3.

$$R_{th} = {L}/{k_x A} \quad \text{(K/W)} \quad (4)$$

Where, R_{th} is internal thermal resistance

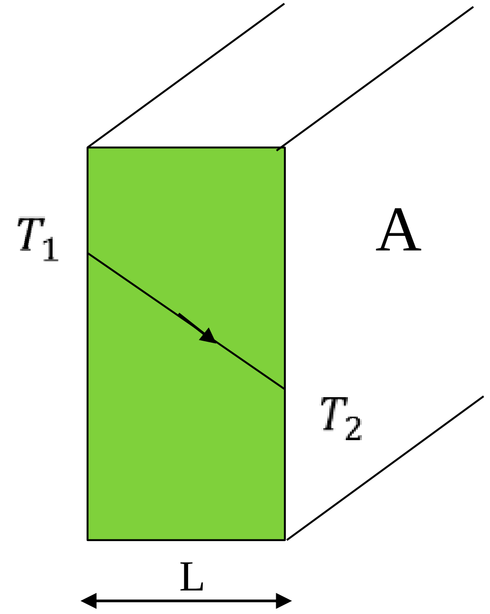


Fig. 1. Plate

Convection, when a fluid is in communication with a solid with a difference of temperature between the two then heat transfer occurs due to bulk macroscopic motion of fluid with reference to solid surface (Advection) and intermolecular diffusion occurring randomly at microscopic level with in the fluid body (Diffusion). The heat transfer due to cumulative effect of the two is called convection.

Free Convection

- A Cup of Coffee
- Condenser of Domestic Refrigerator
- Babcock & Wilcox Boiler
- Emulsion Water Heater
- Ar Gas Within a Light Bulb

Forced Convection (Mixed Convection)

- Automobile Radiator (Fan type)
- Window Air conditioner (Blower type)
- Loeffler Boiler (Pump type)
- Metal Cutting
- Shell and Tube HX

The empirical governing law for convection heat transfer is Newtons law of cooling/heating. This law states that the rate of heat transfer by convection is directly proportional to the surface area of the solid in communication with fluid and the temperature difference between the fluid and solid surface. It can be expressed as based on Fig. 2.

$$q_{conv} = h\,S\,(T_w - T_\infty) \qquad (5)$$

Where, h is convection heat transfer coefficient. Its unit is **W/m^2 K.**

Contd….

Convection heat transfer coefficient (Film coefficient) is neither the property of solid nor the property of fluid. It is a complex parameter depending on a variety of factors like geometry of solid, flow is external or internal, whether convection is free or forced, whether the flow is laminar or turbulent and temperature under thermophysical properties of fluid (Cp, ν, k and μ).

h for liquid > h for gas
h for mixed > h for forced > h for free

Type of Convection	Air h (W/m^2 K)	Water h (W/m^2 K)
Free Convection	5	25
Forced Convection	100	500

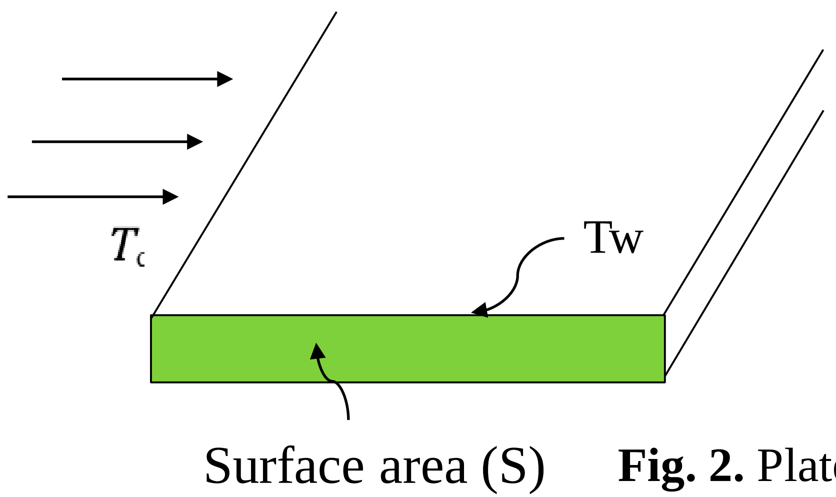


Fig. 2. Plate

Electrical analogy: Eq, (5) can be written as:

$$q_{conv} = \frac{T_w - T_\infty}{(1/\text{h } S)} \quad (6)$$

Now comparing Eq. 6 with Eq. 3. We get:

$$R_{conv} = \frac{1}{h\,S} \quad (7)$$

Where, R_{conv} is convection resistance.

Radiation

All the bodies above absolute zero temperature exhibits a tendency to emit as well as receive energy by virtue of their temperature on account of changes in electro configuration of their constituent atoms or molecules, the energy thus emitted or received is defined as thermal radiation.

- It does nor require intermediate medium unlike conduction and convection.
- It can travel in vacuum.

Example: Sun, Candle and bulb, Heat received from furnace, boilers and I C engines, Combustion chamber etc..

Empirical governing law for radiation heat transfer is given by Steafan-Boltzman.

The law states that the rate of heat transfer is directly proportional to surface area of the body and temperature difference between fourth power of absolute temperature of the body and environment.

$$q_{rad} = \sigma \epsilon S\left(T_{body}^4 - T_{\infty}^4\right) \quad (8)$$

Where, ϵ is the surface emissivity of body and σ is the Steafan-Boltzman coanstant and its values 5.6697*10^-8 $W/m^2\,K^4$

Combined heat transfer mechanism.

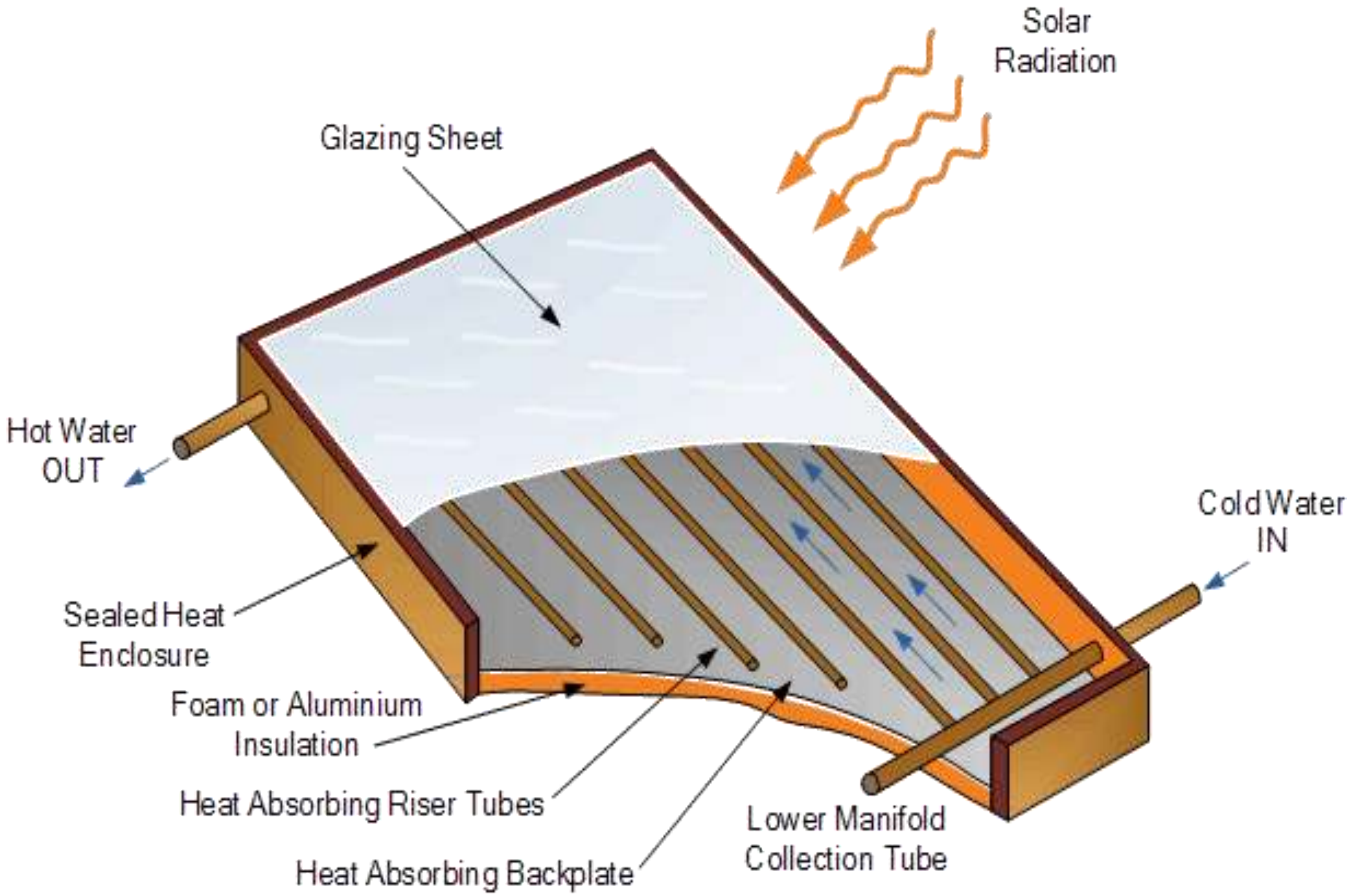


Fig. 3. Flat plate solar collector (Source: Alternate energy tutorial)

Thermal conductivity of some substances

All Values are at 20 °C.	
Substance	**Thermal conductivity (W/m K)**
Pure copper	384
Brass	110
Steel	54
Stainless steel	10
Non Metals:	
Asbestos	0.23
Plastic	0.58
Wood	0.17
Liquids:	
Water	0.60
Light oil	0.14
Gases:	
Dry air	0.026
Steam	0.025

Thermal diffusivity or heat diffusivity (α) m²/s

Thermal diffusivity tells about the degree of penetration of heat through a body whenever the body is subjected to a changed thermal environment. Mathematically it can be defined as the ratio of thermal conductivity and heat storage capacity (ρc_p). It can be expressed as:

$$\alpha = \frac{k}{\rho c_p} \tag{9}$$

Example: heat treatment of materials, precooling of foods and vegetables.

Where, k is thermal conductivity (W/m K), ρ is density of material (kg/m^3) and c_p is specific heat at constant pressure (J/kg K).

Thermal conductivity: how well a material can carry heat.

Heat capacity: heat or thermal storage capacity of a material.

Check your understanding

Que: If a cup of coffee and a red popsickle were left on the table in the room what would happen to them? Why?

Ans: The cup of coffee will cool until it reaches room temperature. The popsickle will melt and then the liquid will warm to room temperature.

Que: Why does metal feel colder than wood? If they are both at same temperature.

Ans: Metal is a conductor, wood is an insulator. Metal conducts the heat away from your hands. Wood does not conduct the heat away from your hands as well as the metal, so the wood feels warmer than the metal.

Radiation travels in straight lines. (True/False)

Radiation can travel through a vacuum. (True/False)

Radiation requires particles to travel. (True/False)

Radiation travels at the speed of light. (True/False)

Numericals

1. The wall of an annealing furnace has inside and outside temperature as 1000 ºC and 200 ºC respectively. While the ambient air is at a temperature of 40 ºC. The wall is constructed using fire bricks of thermal conductivity of 1.28 W/m K and thickness of 25.4 cm. Calculate: i) rate of heat loss per meter square area ii) convective heat transfer coefficient between outer surface and surrounding.
2. In a boiling problem, it is purpose to use electrical resistance 0.22 Ω, diameter 1 mm and 10 cm length, that is submerged in water boiling at 100 ºC. measurement reveals that a current of 10 amp is enough for the job, that maintain the surface 114 ºC. Calculate boiling convection heat transfer coefficient.
3. A hot plate of dimensions 50*75 cm^2 cross-section and 2 cm thickness has one of its surface held 250 ºC. Air is blown pass the surface at 20 ºC by offering convection heat transfer coefficient as 25 W/m^2 K. the same surface of the plate is also loosing heat 300 W by radiation. The material has thermal conductivity 43 W/m K. Calculate: i) net rate of heat transfer from the plate into ambient ii) the surface temperature of plate which is greater than the temperature of the second surface given above.


THANK YOU

Heat & Mass Transfer (BME-27)

Steady State Heat Conduction

Prashant Saini

Mechanical Engineering

Madan Mohan Malviya University of Technology Gorakhpur ***(UP State Govt. University)***

Email: psme@mmmut.ac.in

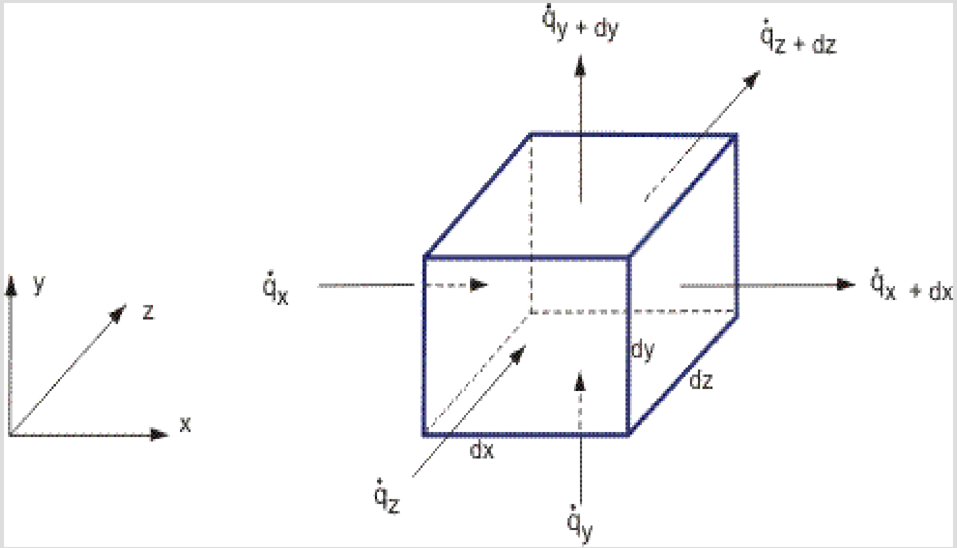
Objectives

- Understand the concept of thermal resistance and its limitations, and develop thermal resistance networks for practical heat conduction problems
- Solve steady conduction problems that involve multilayer rectangular, cylindrical, or spherical geometries
- Develop an intuitive understanding of thermal contact resistance, and circumstances under which it may be significant
- Identify applications in which insulation may actually increase heat transfer
- Solve multidimensional practical heat conduction problems

General heat conduction equation in cartesian coordinate system

Consider an arbitrary solid with sp. Mass ρ (kg/m^3), constant pressure sp. Heat cp (J/kg K). Let qv be the volumetric heat generation in the body (W/m^3). The objective is to find the most general governing equation for temperature as a function of space and time as:
T=f(x, y, z, t)
Consider an infinitesimal volume element of dimensions dx, dy and dz as shown in figure. **Following assumptions are considered:** i) change in K. E. and P. E. are negligible ii) Work transition on account of temperature change in solid is negligible. Hence, first law of thermodynamics can be written as:

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$



Heat conduction equation can be written as:

$$q_x - q_{x+\mathrm{dx}} + q_y - q_{y+\mathrm{dy}} + q_z - q_{z+\mathrm{dz}} + q_{gen} = \mathrm{m\ cp}\,\frac{\partial T}{\partial t}$$

Applying the Taylor series expansion, the previous equation can be evaluated as:

The heat equation in Cartesian coordinates is given by,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}.$$

Simplify the equation for the following cases:

(a) no thermal energy generation and uniform k

(b) steady-state and two-dimensional conduction

(c) one-dimensional conduction

(d) two-dimensional conduction and uniform k

(e) one-dimensional, steady-state conduction, with no thermal energy generation and uniform k.

For homogeneous and isotropic material, above equation is modified as:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_v}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=0$$ Laplace Equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_v}{k}=0$$ Poisson's Equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$ Fourier Equation

General heat conduction equation in cartesian, polar and spherical coordinates respectively can be extracted and written as:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2 sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}+\frac{q_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

The above equations can be written in 1-D form as:

One Dimensional Fourier's Equations

Constant thermal conductivity & No heat generation:

Cartesian coordinates: $\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$

Cylindrical coordinates: $\frac{\partial T}{\partial t}=\alpha\left\{\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right\}$

Spherical coordinates: $\frac{\partial T}{\partial t}=\alpha\left\{\frac{\partial^2 T}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}\right\}$

Boundary conditions

Three types of boundary conditions are used:

First kind of boundary condition (Dirichlet BC): In this temperatures are prescribed at the boundary. See Fig. A.

For:

($0 \leq y \leq w$, x=0 and z=0); T= T_1

($0 \leq y \leq w$, x=L and z=0); T= T_2

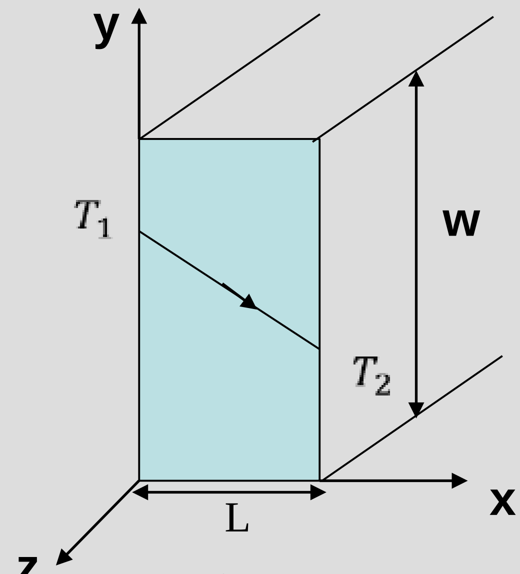


Fig. A.

Second kind of boundary condition (Neumann BC): In this heat flux are prescribed. Also, boundary may be adiabatic/insulated. See Fig. B.

For:

x=0; $q''_{\text{left}} = -k\dfrac{\partial T}{\partial x}$

x=L; $-q''_{\text{right}} = -k\dfrac{\partial T}{\partial x}$

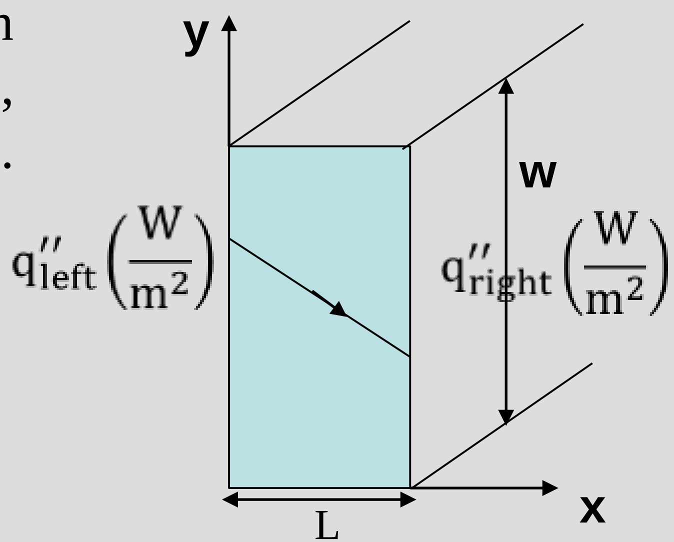


Fig. B.

Third kind of boundary condition (Newtonian BC): Conjugate or combined convection and radiation boundary condition. See Fig. C.

At:

x=0; $$h_1 A\left(T_1 - T_{x,y,z,t}\right) = -kA\frac{\partial T_{x,y,z,t}}{\partial x}$$

x=L; $$-kA\frac{\partial T_{x,y,z,t}}{\partial x} = h_2 A\left(T_{x,y,z,t} - T_2\right)$$

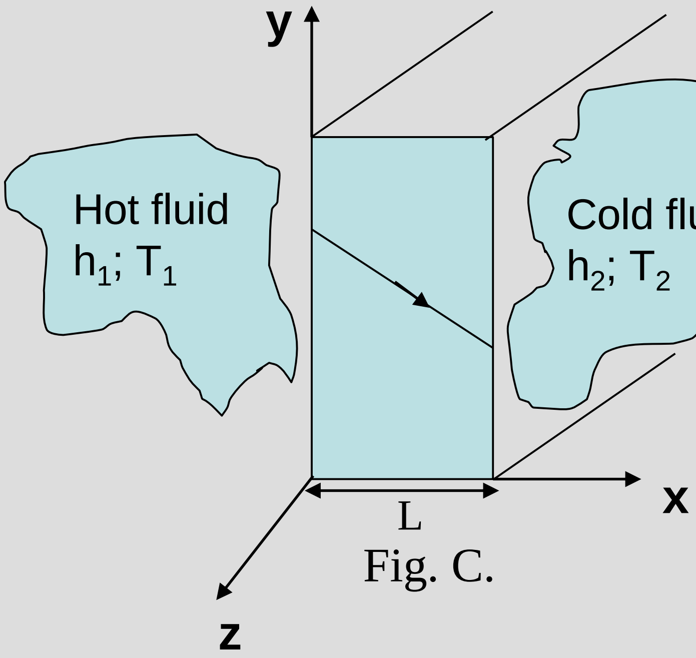


Fig. C.

These three boundary conditions will be applicable to solve the steady state 1-D heat conduction problems in Cartesian, polar and spherical coordinate systems

STEADY HEAT CONDUCTION IN PLANE WALLS

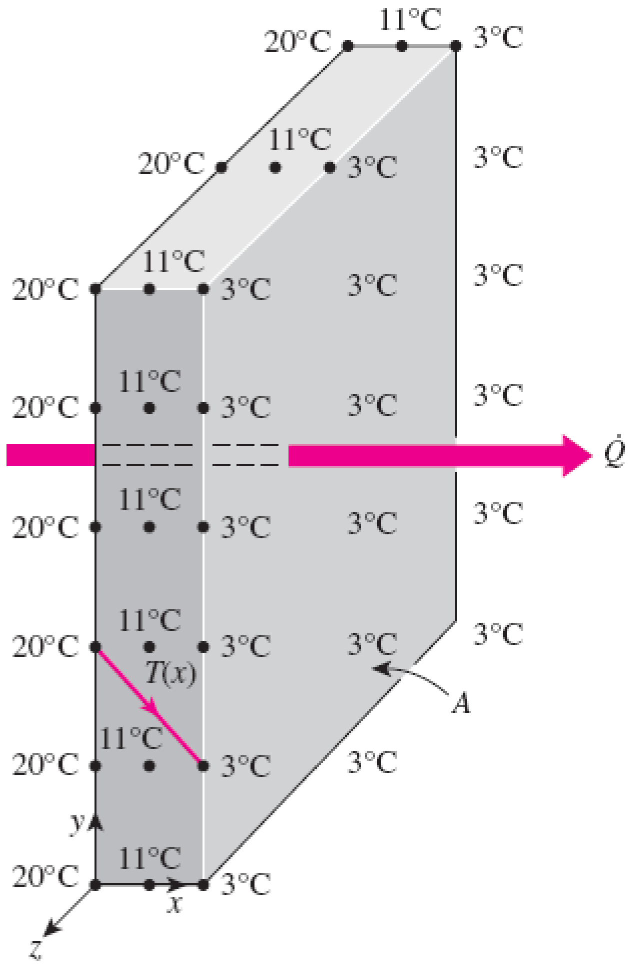


FIGURE 3–1
Heat transfer through a wall is one-dimensional when the temperature of the wall varies in one direction only.

Heat transfer through the wall of a house can be modeled as *steady* and *one-dimensional.*

The temperature of the wall in this case depends on one direction only (say the *x*-direction) and can be expressed as $T(x)$.

$$\left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{array}\right) - \left(\begin{array}{c}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{array}\right) = \left(\begin{array}{c}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{array}\right)$$

$$\dot{Q}_{in} - \dot{Q}_{out} = \frac{dE_{wall}}{dt}$$

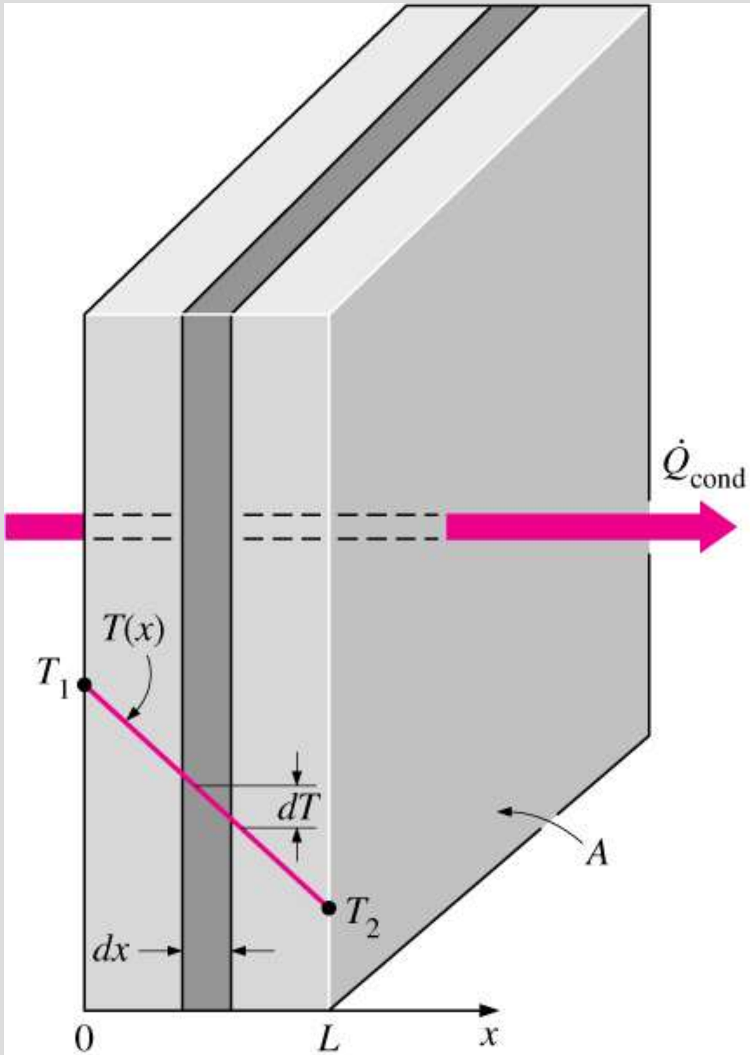
$$dE_{wall}/dt = 0$$

for *steady* operation

In steady operation, the rate of heat transfer through the wall is constant.

$$\dot{Q}_{cond,\ wall} = -kA\frac{dT}{dx} \qquad \text{(W)}$$

Fourier's law of heat conduction



Under steady conditions, the temperature distribution in a plane wall is a straight line: ***dT/dx* = const.**

$$\dot{Q}_{\text{cond, wall}} = -kA\frac{dT}{dx}$$

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)}$$

The rate of heat conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

Once the rate of heat conduction is available, the temperature $T(x)$ at any location x can be determined by replacing T_2 by T, and L by x.

Thermal Resistance Concept

$$\dot{Q}_{\text{cond, wall}} = kA \frac{T_1 - T_2}{L}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)}$$

$$R_{\text{wall}} = \frac{L}{kA} \qquad (°\text{C/W})$$

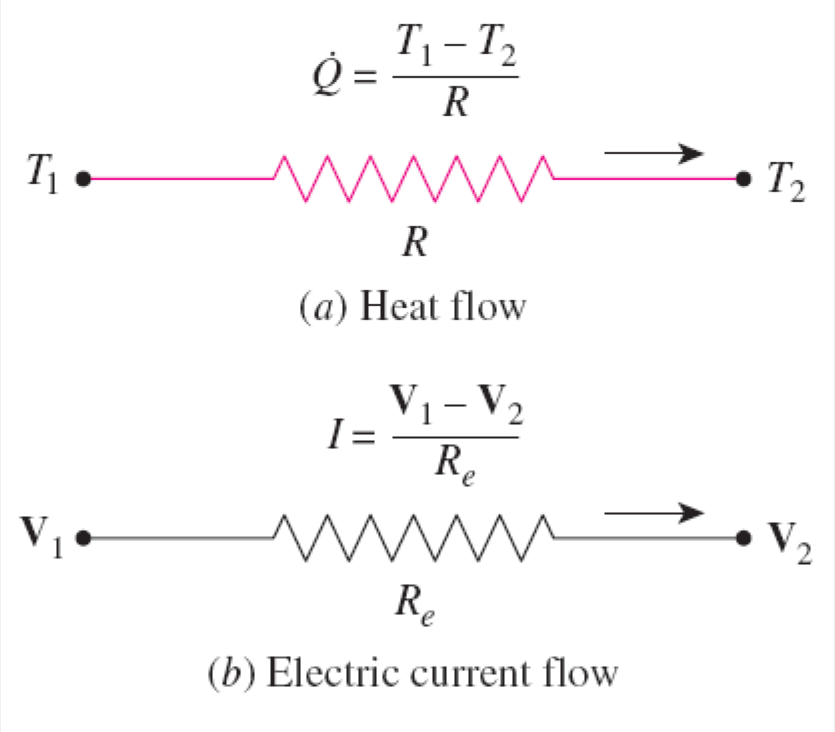
Conduction resistance of the wall: *Thermal resistance* of the wall against heat conduction.

Thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e}$$

$$R_e = L/\sigma_e A$$

Electrical resistance



Analogy between thermal and electrical resistance concepts.

rate of heat transfer → electric current
thermal resistance → electrical resistance
temperature difference → voltage difference

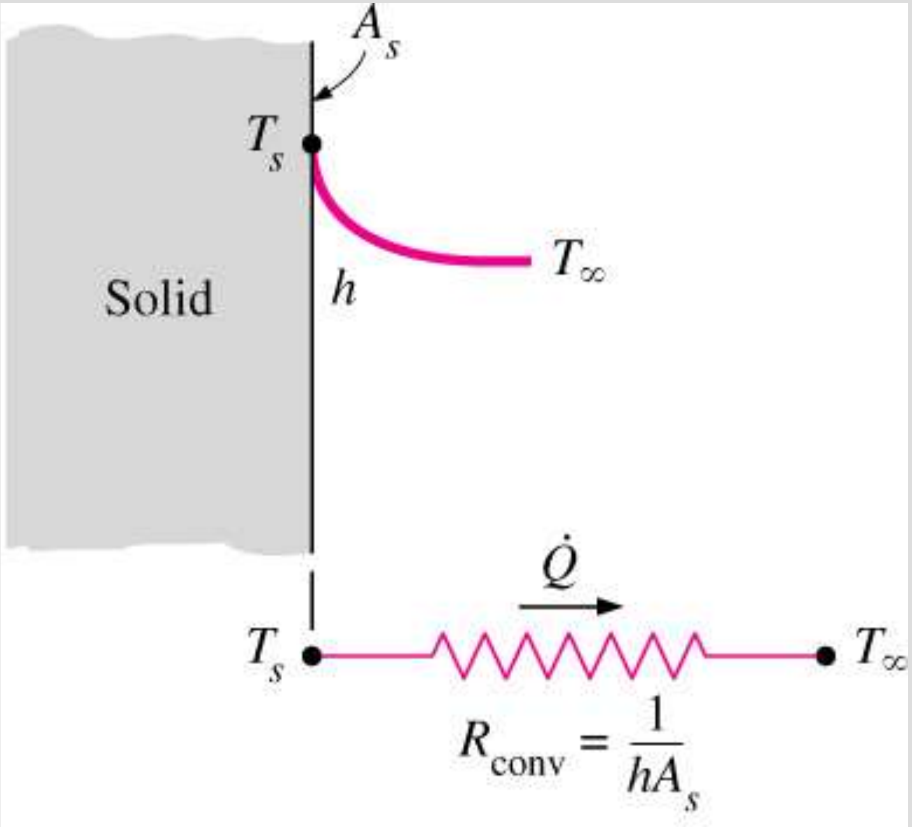
Newton's law of cooling

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad (W)$$

$$R_{conv} = \frac{1}{hA_s} \qquad (°C/W)$$

Convection resistance of the surface: *Thermal resistance* of the surface against heat convection.



Schematic for convection resistance at a surface.

When the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T$.

That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process.

This situation is approached in practice at surfaces where boiling and condensation occur.

$$\dot{Q}_{rad} = \varepsilon\sigma A_s(T_s^4 - T_{surr}^4) = h_{rad} A_s(T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}}$$

$$R_{rad} = \frac{1}{h_{rad} A_s} \qquad (\text{K/W})$$

Radiation resistance of the surface: *Thermal resistance* of the surface against radiation.

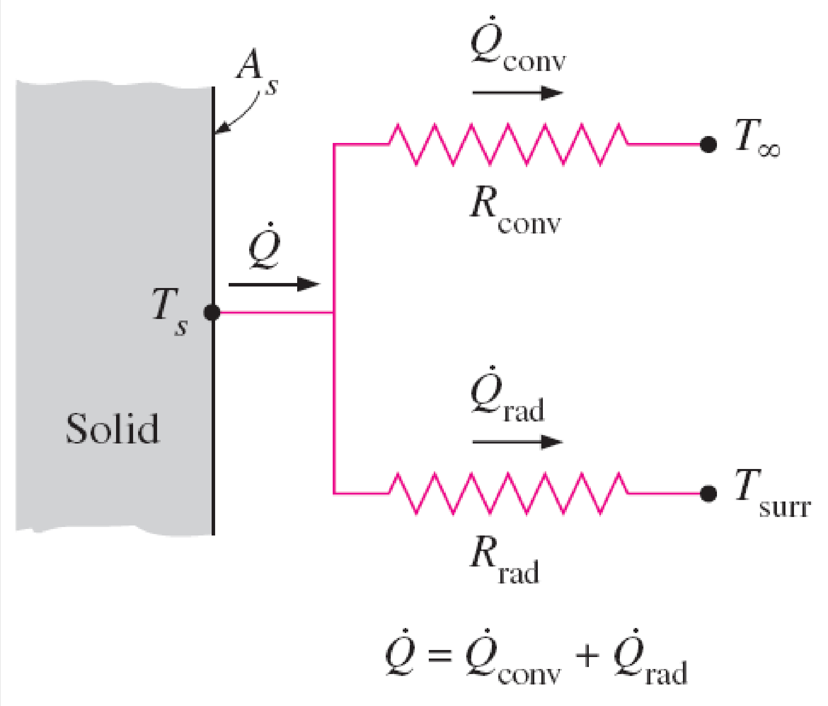
$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (\text{W/m}^2 \cdot \text{K})$$

Radiation heat transfer coefficient

When $T_{surr} \approx T_\infty$

$$h_{combined} = h_{conv} + h_{rad}$$

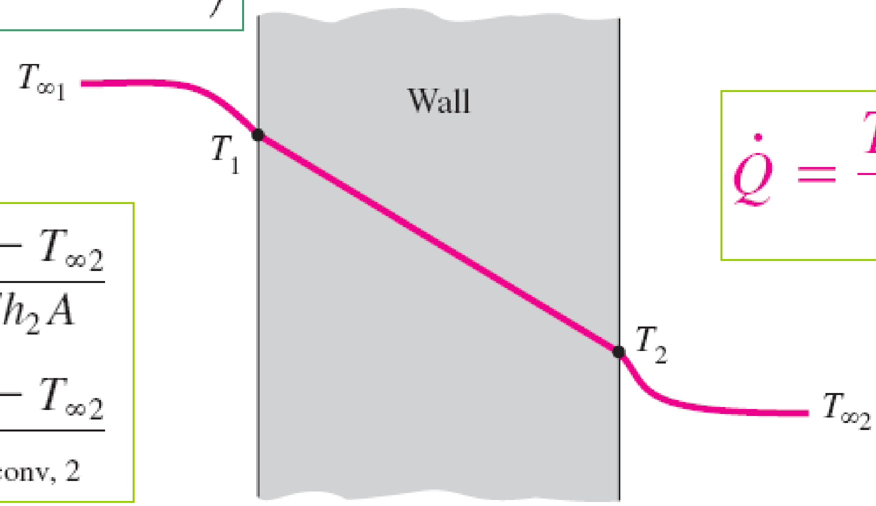
Combined heat transfer coefficient



Schematic for convection and radiation resistances at a surface.

Thermal Resistance Network

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$



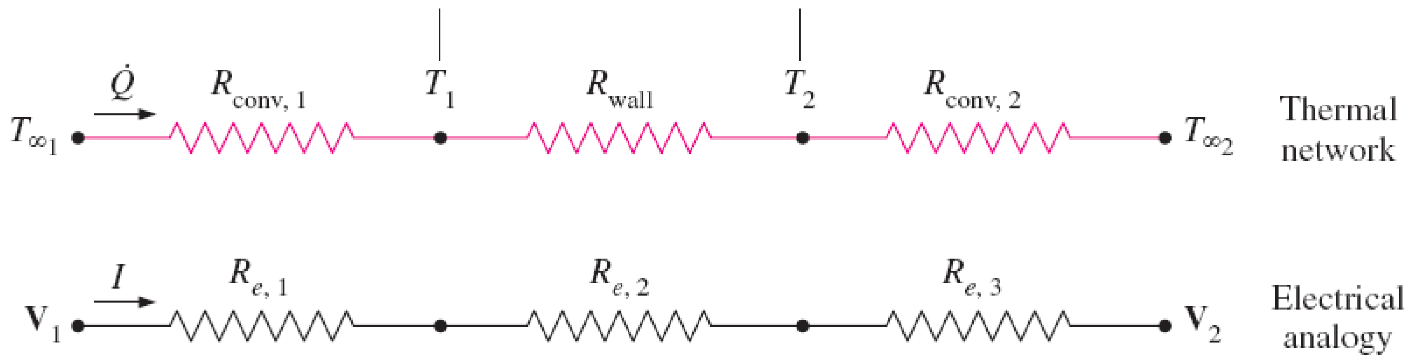
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$

$$I = \frac{V_1 - V_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$



The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W})$$

Temperature drop

$$\Delta T = \dot{Q}R \qquad (°C)$$

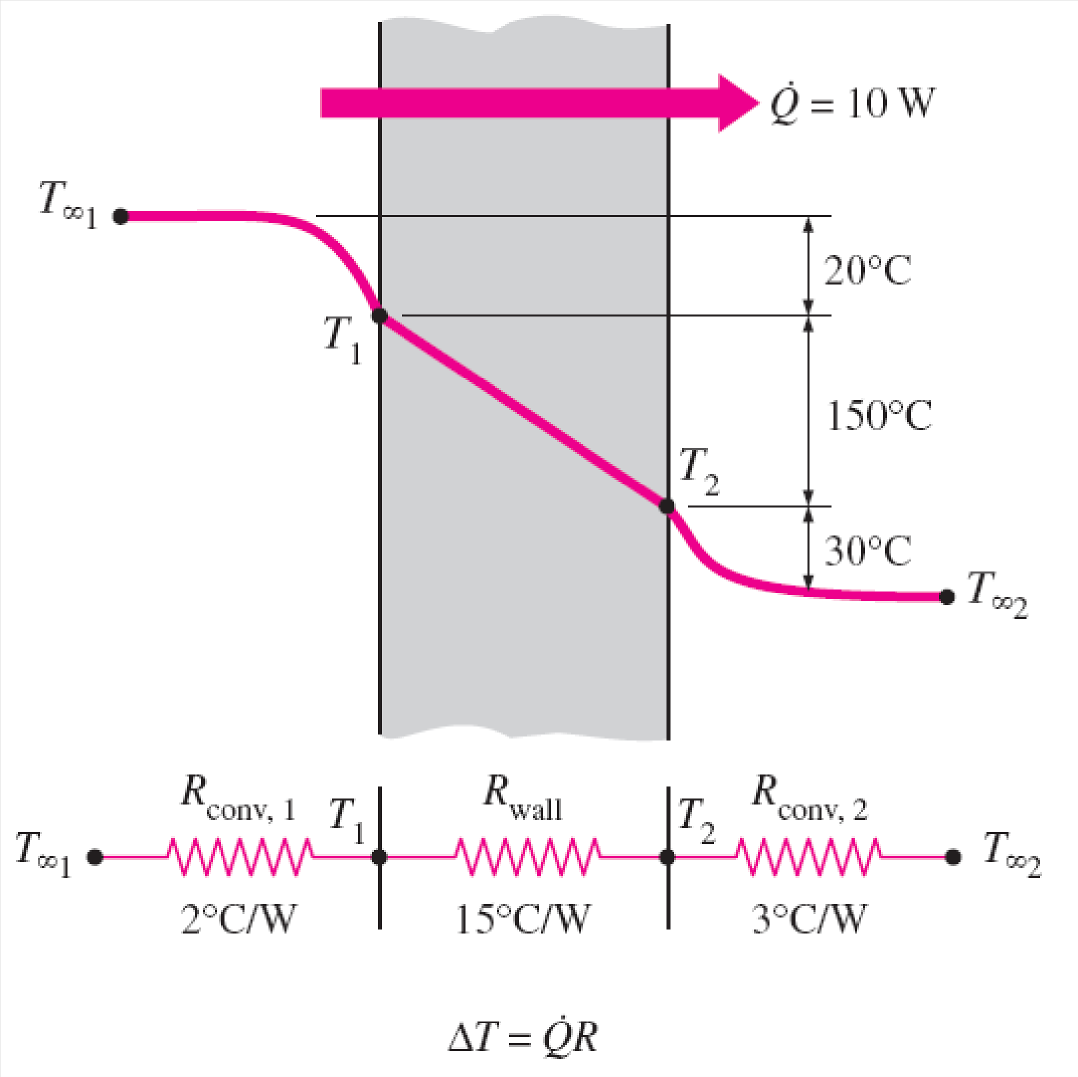
$$\dot{Q} = UA\,\Delta T \qquad (W)$$

$$UA = \frac{1}{R_{total}} \qquad (°C/K)$$

U overall heat transfer coefficient

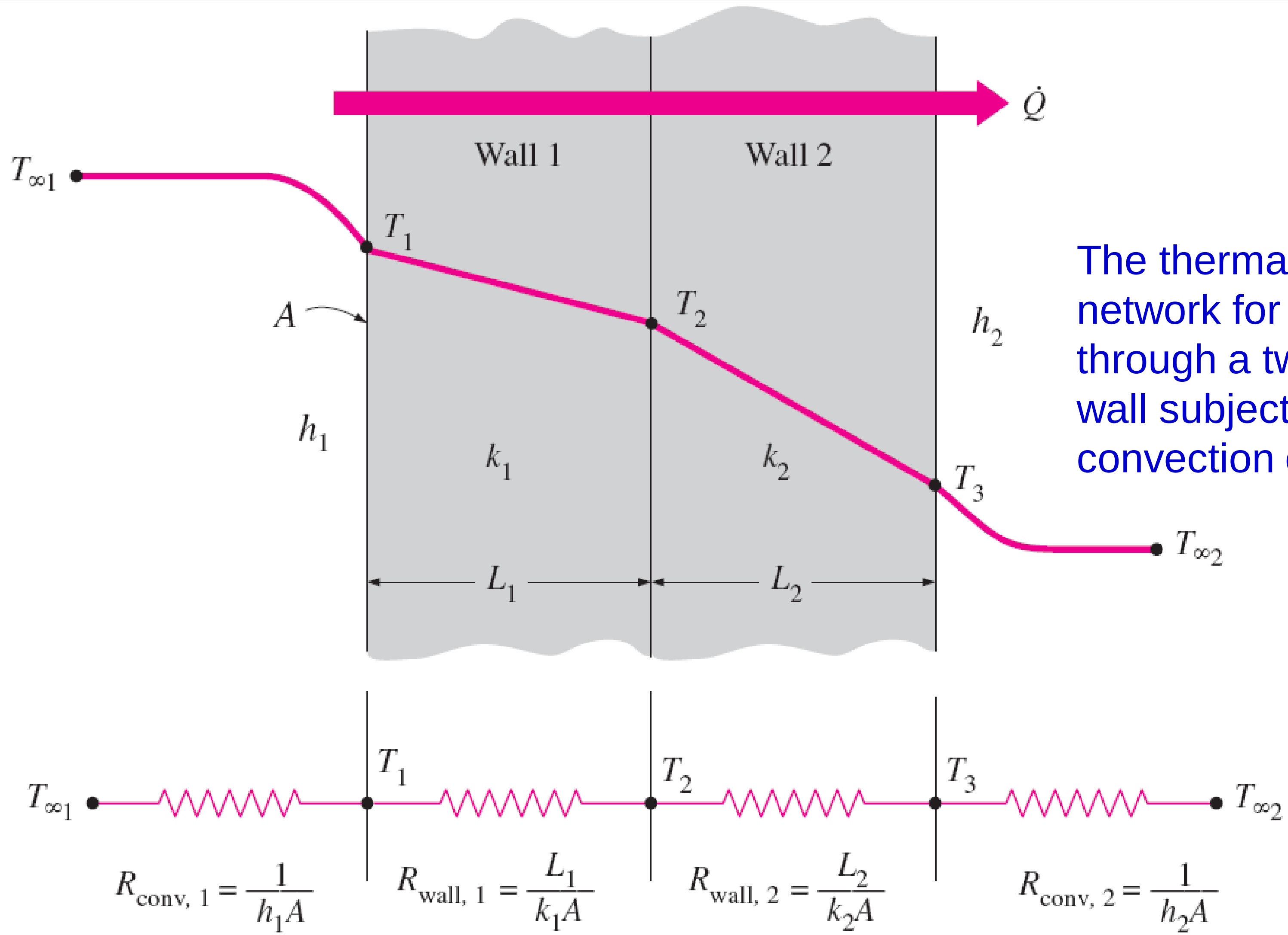
Once Q is evaluated, the surface temperature T_1 can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,\,1}} = \frac{T_{\infty 1} - T_1}{1/h_1 A}$$



The temperature drop across a layer is proportional to its thermal resistance.

Multilayer Plane Walls



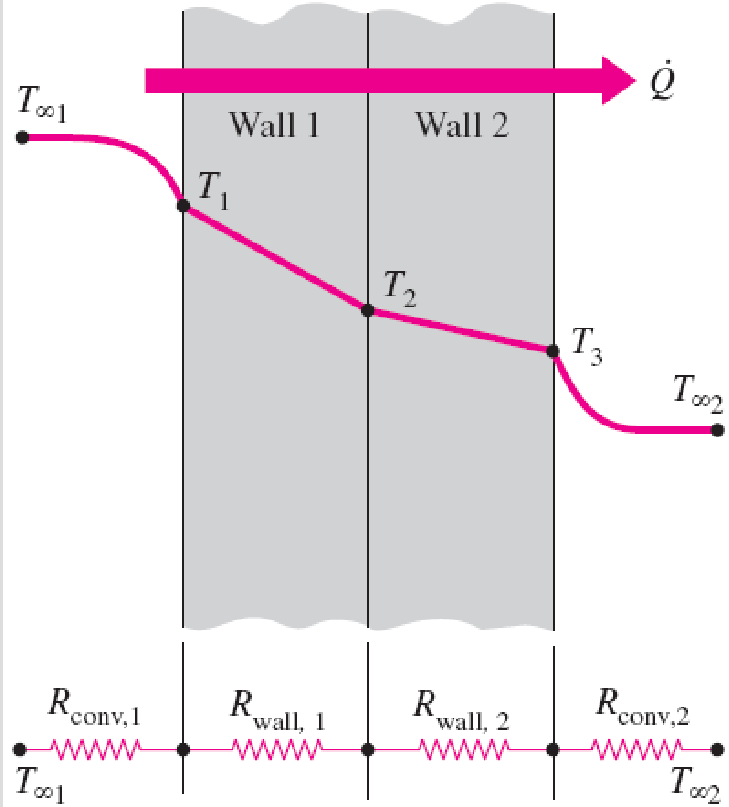
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall, 1}} + R_{\text{wall, 2}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A}$$

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total}, i-j}}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv}, 1} + R_{\text{wall}, 1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}}$$



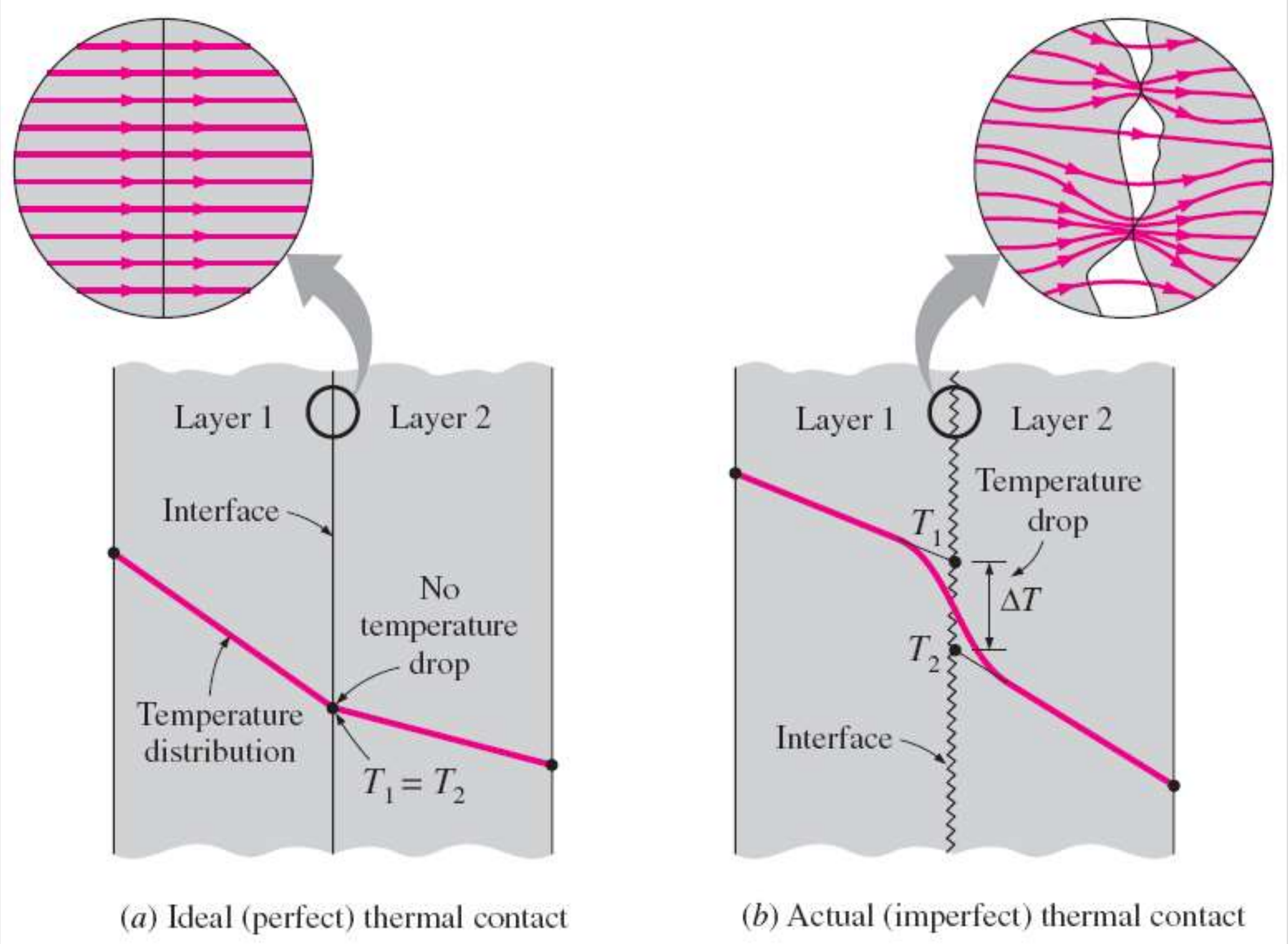
To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{wall},1}}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

THERMAL CONTACT RESISTANCE



Temperature distribution and heat flow lines along two solid plates pressed against each other for the case of perfect and imperfect contact.

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}}$$

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}}$$

h_c thermal contact conductance

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W})$$

$$R_{c,\,\text{insulation}} = \frac{L}{k} = \frac{0.01 \text{ m}}{0.04 \text{ W/m} \cdot {}^\circ\text{C}} = 0.25 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

$$R_{c,\,\text{copper}} = \frac{L}{k} = \frac{0.01 \text{ m}}{386 \text{ W/m} \cdot {}^\circ\text{C}} = 0.000026 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

The value of thermal contact resistance depends on:

- *surface roughness,*
- *material properties,*
- *temperature* and *pressure* at the interface
- *type of fluid* trapped at the interface.

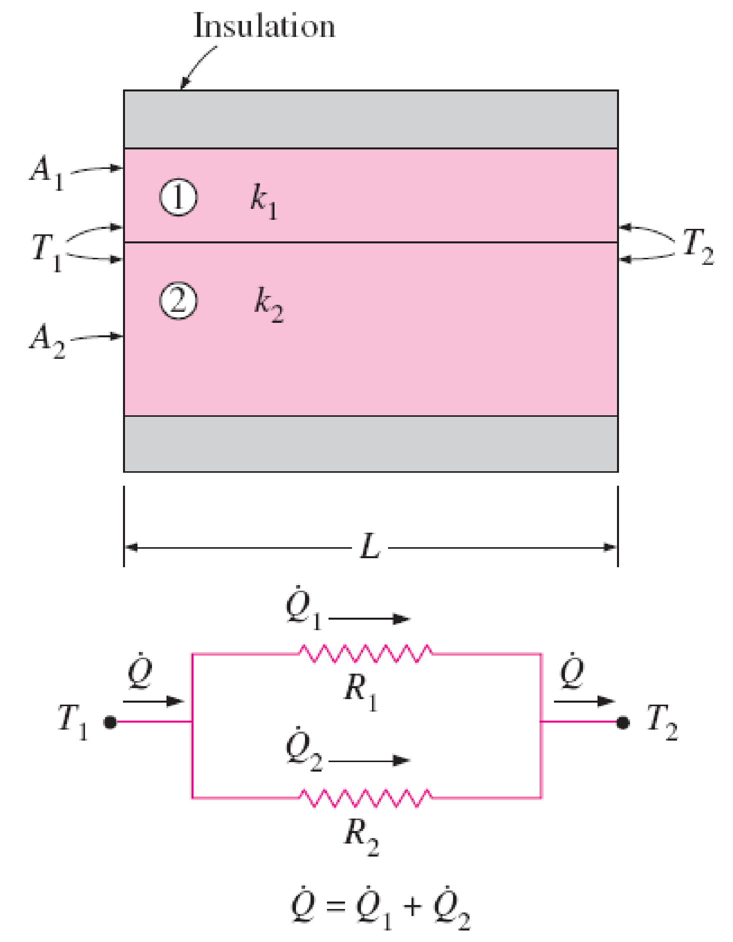
Thermal contact resistance is significant and can even dominate the heat transfer for good heat conductors such as metals, but can be disregarded for poor heat conductors such as insulations.

GENERALIZED THERMAL RESISTANCE NETWORKS

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$



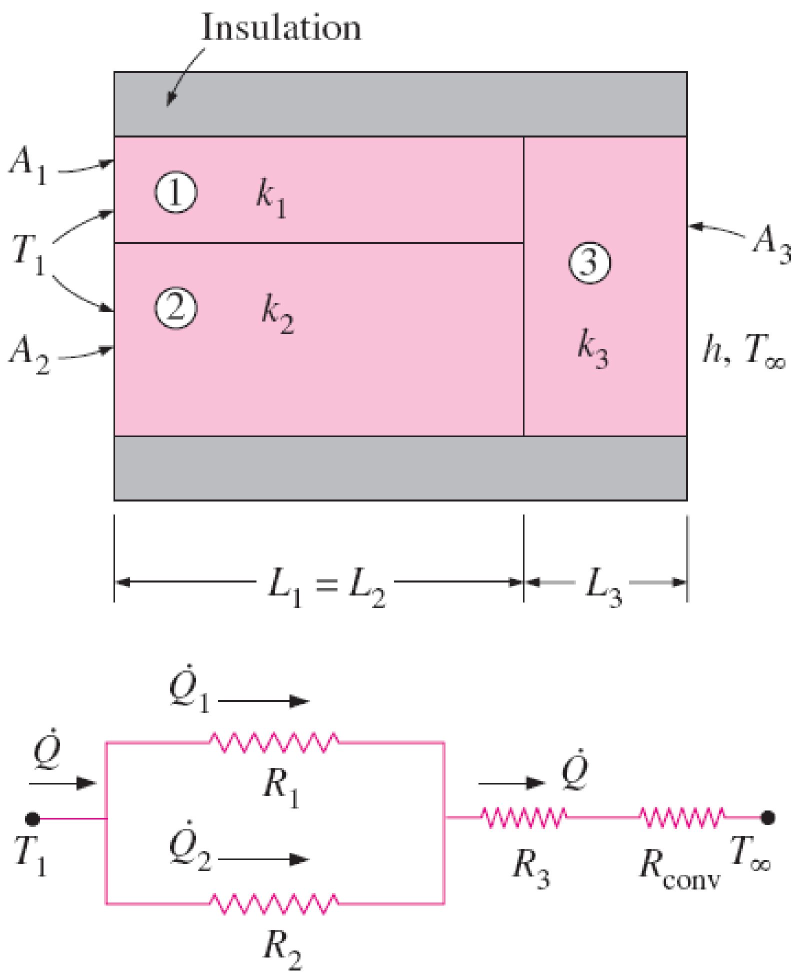
Thermal resistance network for two parallel layers.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{total}}$$

$$R_{total} = R_{12} + R_3 + R_{conv} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{conv}$$

$$R_1 = \frac{L_1}{k_1 A_1} \qquad R_2 = \frac{L_2}{k_2 A_2}$$

$$R_3 = \frac{L_3}{k_3 A_3} \qquad R_{conv} = \frac{1}{hA_3}$$



Thermal resistance network for combined series-parallel arrangement.

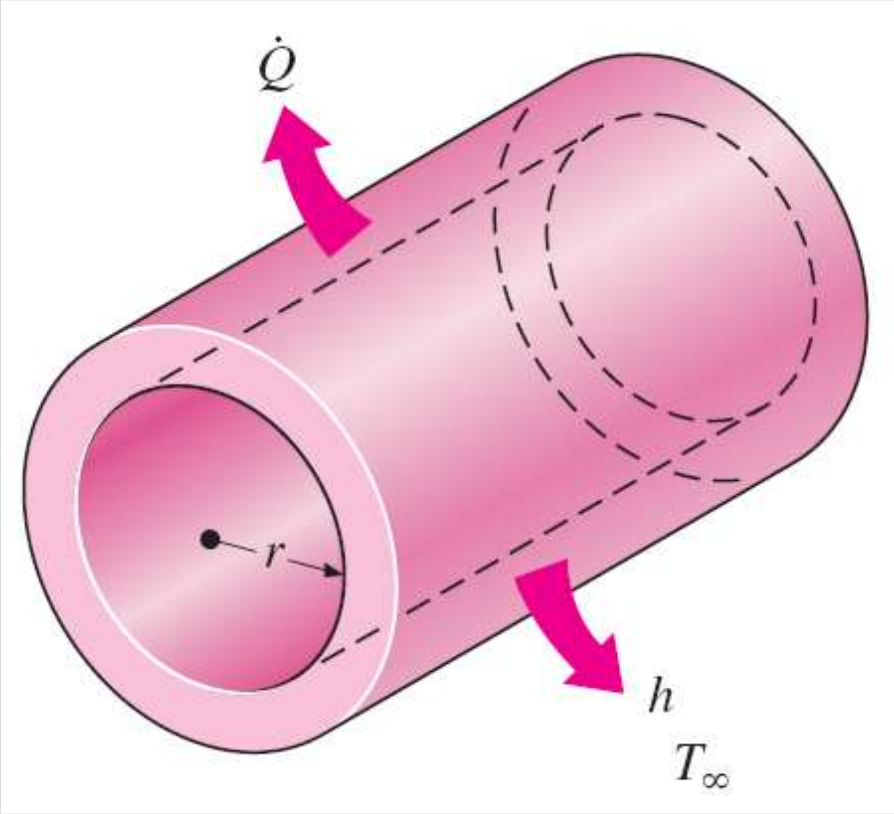
Two assumptions in solving complex multidimensional heat transfer problems by treating them as one-dimensional using the thermal resistance network are

(1) any plane wall normal to the *x*-axis is *isothermal* (i.e., to assume the temperature to vary in the *x*-direction only)

(2) any plane parallel to the *x*-axis is *adiabatic* (i.e., to assume heat transfer to occur in the *x*-direction only)

Do they give the same result?

HEAT CONDUCTION IN CYLINDERS AND SPHERES



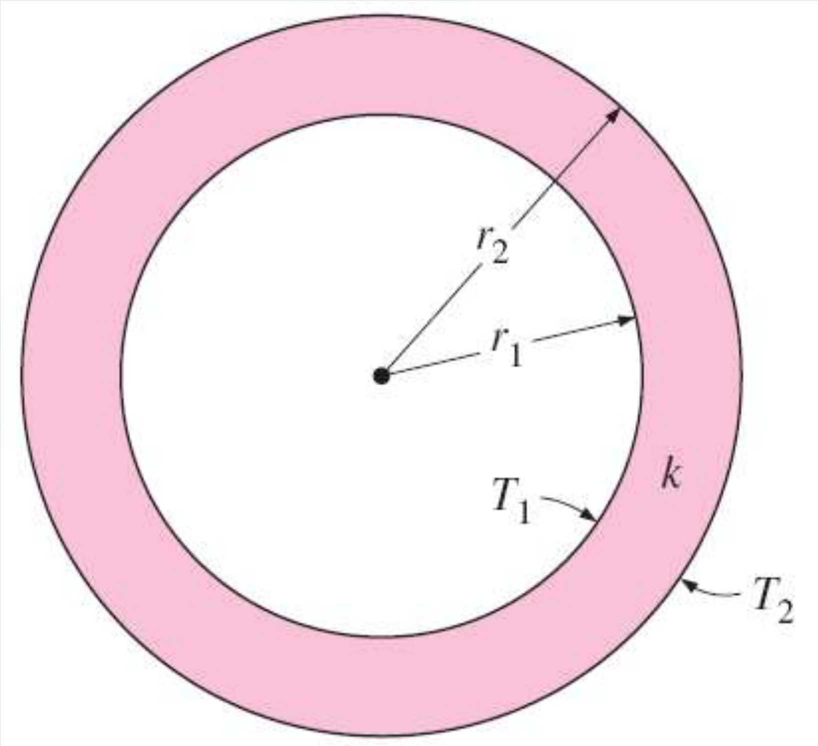
Heat is lost from a hot-water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

Heat transfer through the pipe can be modeled as *steady* and *one-dimensional.*

The temperature of the pipe depends on one direction only (the radial *r*-direction) and can be expressed as $T = T(r)$.

The temperature is independent of the azimuthal angle or the axial distance.

This situation is approximated in practice in long cylindrical pipes and spherical containers.



A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)}$$

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\,dr = -\int_{T=T_1}^{T_2} k\,dT$$

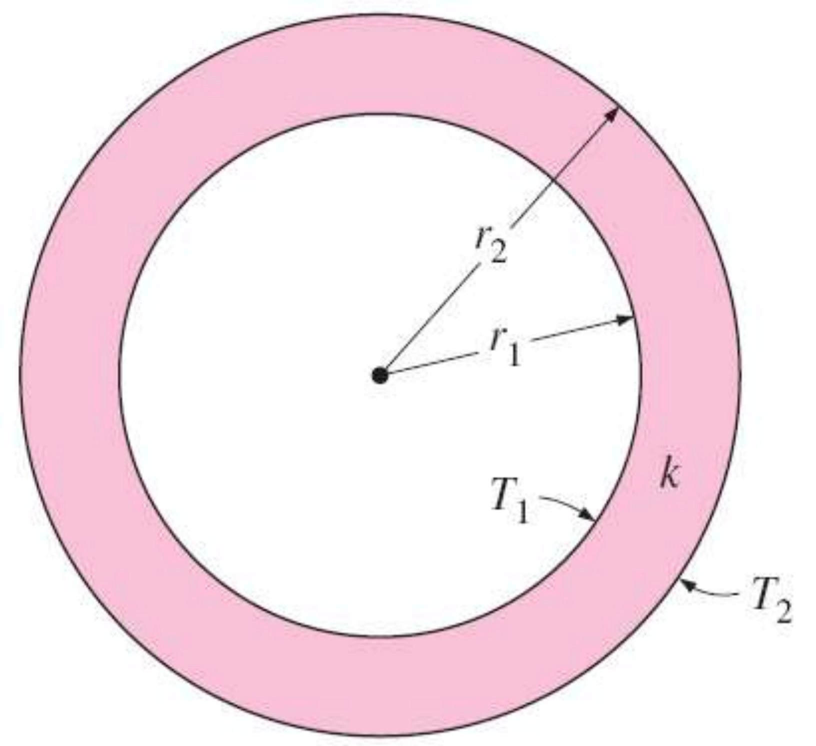
$$A = 2\pi rL$$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)}$$

$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times \text{Length} \times \text{Thermal conductivity}}$$

Conduction resistance of the cylinder layer

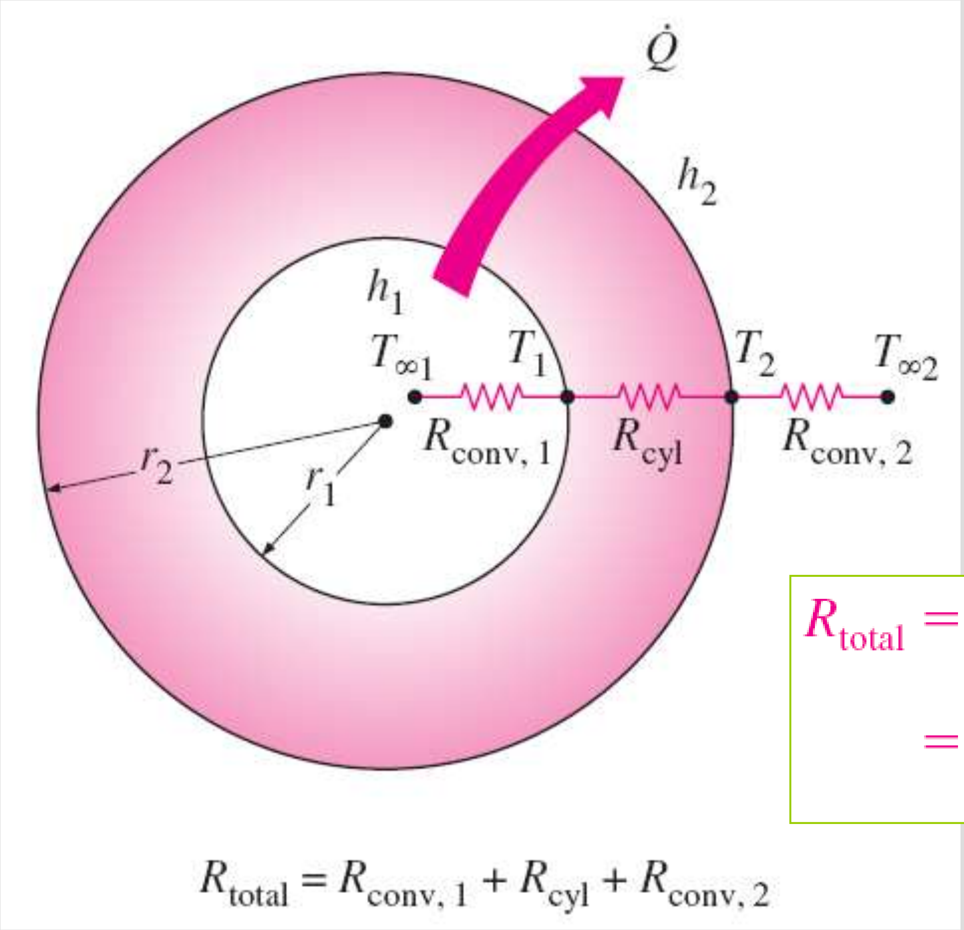


A spherical shell with specified inner and outer surface temperatures T_1 and T_2.

$$\dot{Q}_{\text{cond, sph}} = \frac{T_1 - T_2}{R_{\text{sph}}}$$

$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$

Conduction resistance of the spherical layer



The thermal resistance network for a cylindrical (or spherical) shell subjected to convection from both the inner and the outer sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

for a *cylindrical* layer

$$R_{total} = R_{conv,\,1} + R_{cyl} + R_{conv,\,2} = \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2}$$

for a *spherical* layer

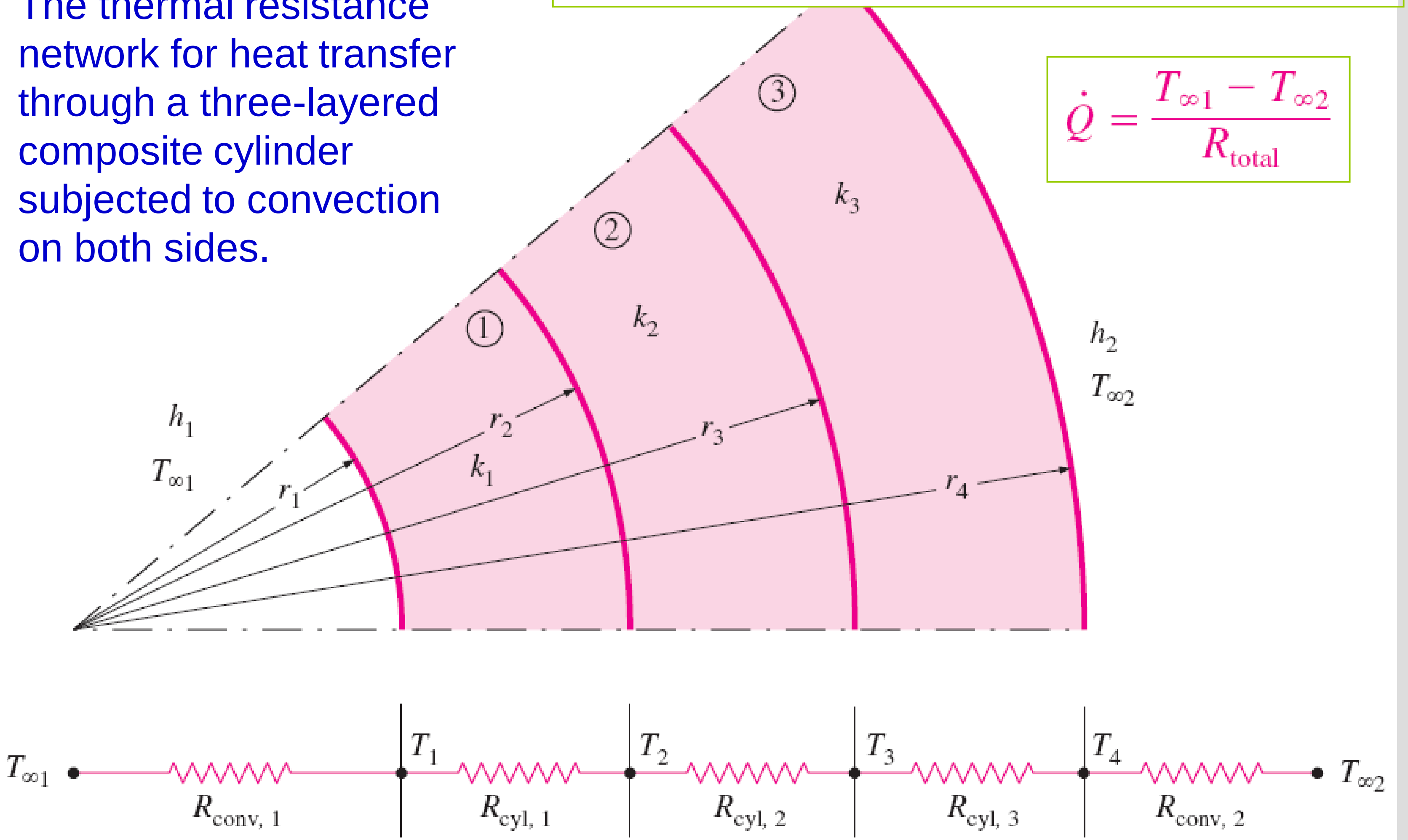
$$R_{total} = R_{conv,\,1} + R_{sph} + R_{conv,\,2} = \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2}$$

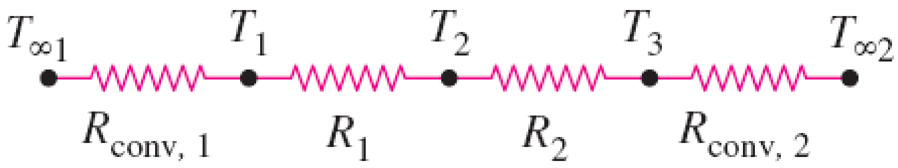
Multilayered Cylinders and Spheres

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$
$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$





The ratio $\Delta T/R$ across any layer is equal to $\dot{Q}$, which remains constant in one-dimensional steady conduction.

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv},1}} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1} = \frac{T_1 - T_3}{R_1 + R_2} = \frac{T_2 - T_3}{R_2} = \frac{T_2 - T_{\infty 2}}{R_2 + R_{\text{conv},2}} = \cdots$$

Once heat transfer rate Q has been calculated, the interface temperature T_2 can be determined from any of the following two relations:

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_{\text{cyl},1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1(2\pi r_1 L)} + \dfrac{\ln(r_2/r_1)}{2\pi L k_1}}$$

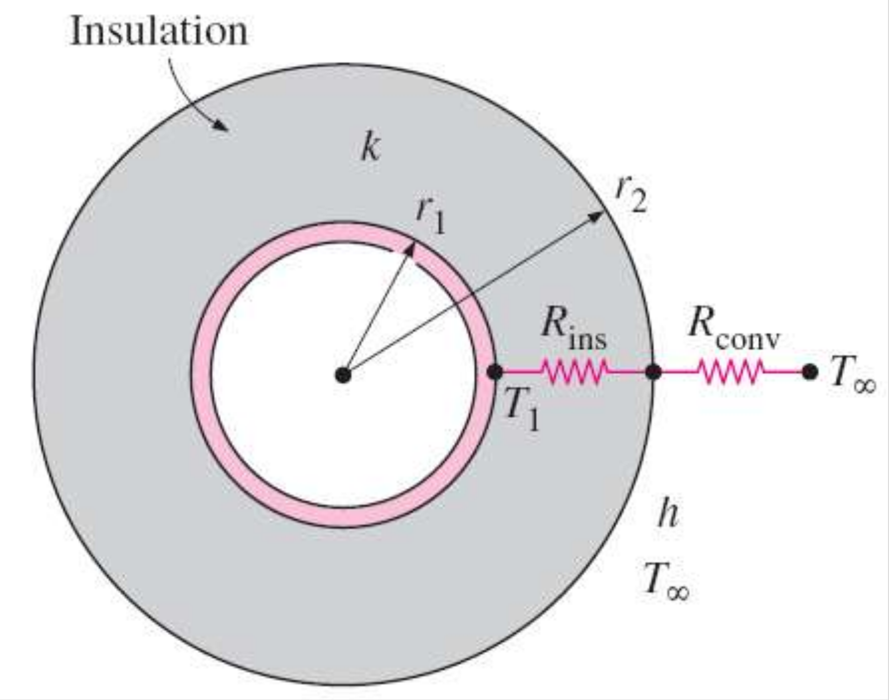
$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_2 + R_3 + R_{\text{conv},2}} = \frac{T_2 - T_{\infty 2}}{\dfrac{\ln(r_3/r_2)}{2\pi L k_2} + \dfrac{\ln(r_4/r_3)}{2\pi L k_3} + \dfrac{1}{h_o(2\pi r_4 L)}}$$

CRITICAL RADIUS OF INSULATION

Adding more insulation to a wall or to the attic always decreases heat transfer since the heat transfer area is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

In a a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

The heat transfer from the pipe may increase or decrease, depending on which effect dominates.



An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi L k} + \dfrac{1}{h(2\pi r_2 L)}}$$

The critical radius of insulation for a cylindrical body:

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)}$$

The critical radius of insulation for a spherical shell:

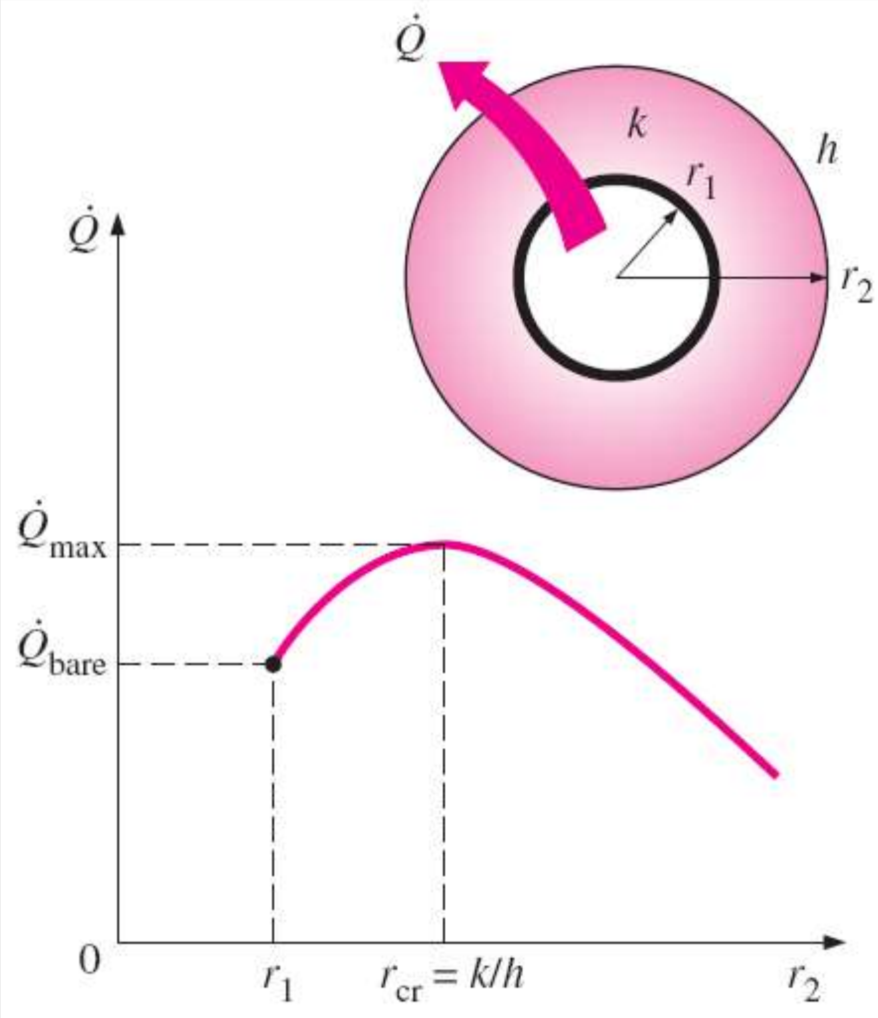
$$r_{\text{cr, sphere}} = \frac{2k}{h}$$

The largest value of the critical radius we are likely to encounter is

$$r_{\text{cr, max}} = \frac{k_{\text{max, insulation}}}{h_{\text{min}}} \approx \frac{0.05\ \text{W/m}\cdot{}^\circ\text{C}}{5\ \text{W/m}^2\cdot{}^\circ\text{C}}$$

$$= 0.01\ \text{m} = 1\ \text{cm}$$

We can insulate hot-water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.



The variation of heat transfer rate with the outer radius of the insulation r_2 when $r_1 < r_{cr}$.

Numerical

- The composite wall of an oven in an application consists of three materials, two of which are of known thermal conductivity 20 W/m K & 50 W/m K and known thickness as 30 cm & 15 cm respectively. The third material is sandwiched between the above two material of thickness 15 cm but unknown thermal conductivity. Measurement reveals that under steady state condition the temperature of outer and inner materials are 20 °C and 600 °C respectively. Further it is noticed that inside oven air temperature is 800 °C offering a convection heat transfer coefficient 25 W/m^2 K. Calculate thermal conductivity of sandwiched material.
- A long hollow cylinder of inner and outer radii r_1 and r_2 and length L has its inner surface subjected to a uniform heat flux at a rate of q_i'' (W/m^2), while the outer surface is in contact with atmospheric pressure, there is no internal heat generation in the cylinder. Derive an expression for steady state radial temperature. Further calculate the temperature of the inner surface of the cylinder. If r_1 = 10 cm, r_2 = 20 cm, k = 50 W/m K and q_i'' = $1.16*10^5$ W/m^2.
- A cable of 10 mm outside diameter is to be laid in an atmosphere of 25°C (h_0 = 12.5W/m^2 °C) and its surface temperature is likely to be 75 °C due to heat generated within it. How would the heat flow from the cable be affected if it is insulated with rubber having thermal conductivity (0.15 W/m °C)?
- A steel pipe with 50 mm OD is covered with a 6.4 mm asbestos insulation (k = 0.166 W/m K) followed by a 25 mm layer of fiber glass insulation (0.0485W/m K). The pipe wall temperature is 393 K and the outside insulation temperature is 38 °C. Calculate the interface temperature between the asbestos and fiber glass.

Assignment

1. Derive the general heat conduction equation in polar coordinate system.

2. Derive the general heat conduction equation in spherical coordinate system.

3. An insulated wall is to be constructed of common brick 20 cm thick and metal lathe with plaster 2.5 cm thick with inter mediate layer of loosely packed rock wool. The outer surfaces are at a temperature of 600 °C and 500 °C respectively calculate the thickness of insulation required in order that the heat loss/square meter shall not exceed 600 watts. Given thermal conductivity of materials: Brick=0.32 W/m K; Metal plastic = 0.04 W/m K; rock wool= 0.046 W/m K.

4. Hot air at temperature of 40 °C is flowing through a steel pipe of 10 cm dia. The pipe is covered with two layers by different insulating materials of thickness 3 cm each their corresponding thermal conductivity. k_1= 0.2 W/m K, k_2=0.32 W/m K inside heat transfer coefficient 50 W/m^2 K outside heat transfer coefficient its 10 W/m^2 K. Assuming atmospheric temperature of 10 °C, find heat lost from 40 m length of pipe.

5. Consider a spherical container of inner radius 8 cm and outer radius 10 cm of thermal conductivity 45 W/m ºC. The inner and outer surfaces of container are maintained at 200 °C and 25 °C respectively. Determine the temperature at a radius 9 cm and rate of heat transfer.

Summary

- Steady Heat Conduction in Plane Walls
  - ✓ Thermal Resistance Concept
  - ✓ Thermal Resistance Network
  - ✓ Multilayer Plane Walls
- Thermal Contact Resistance
- Generalized Thermal Resistance Networks
- Heat Conduction in Cylinders and Spheres
  - ✓ Multilayered Cylinders and Spheres
- Critical Radius of Insulation