ENGINEERING MECHANICS (BME-01)

UNIT-I

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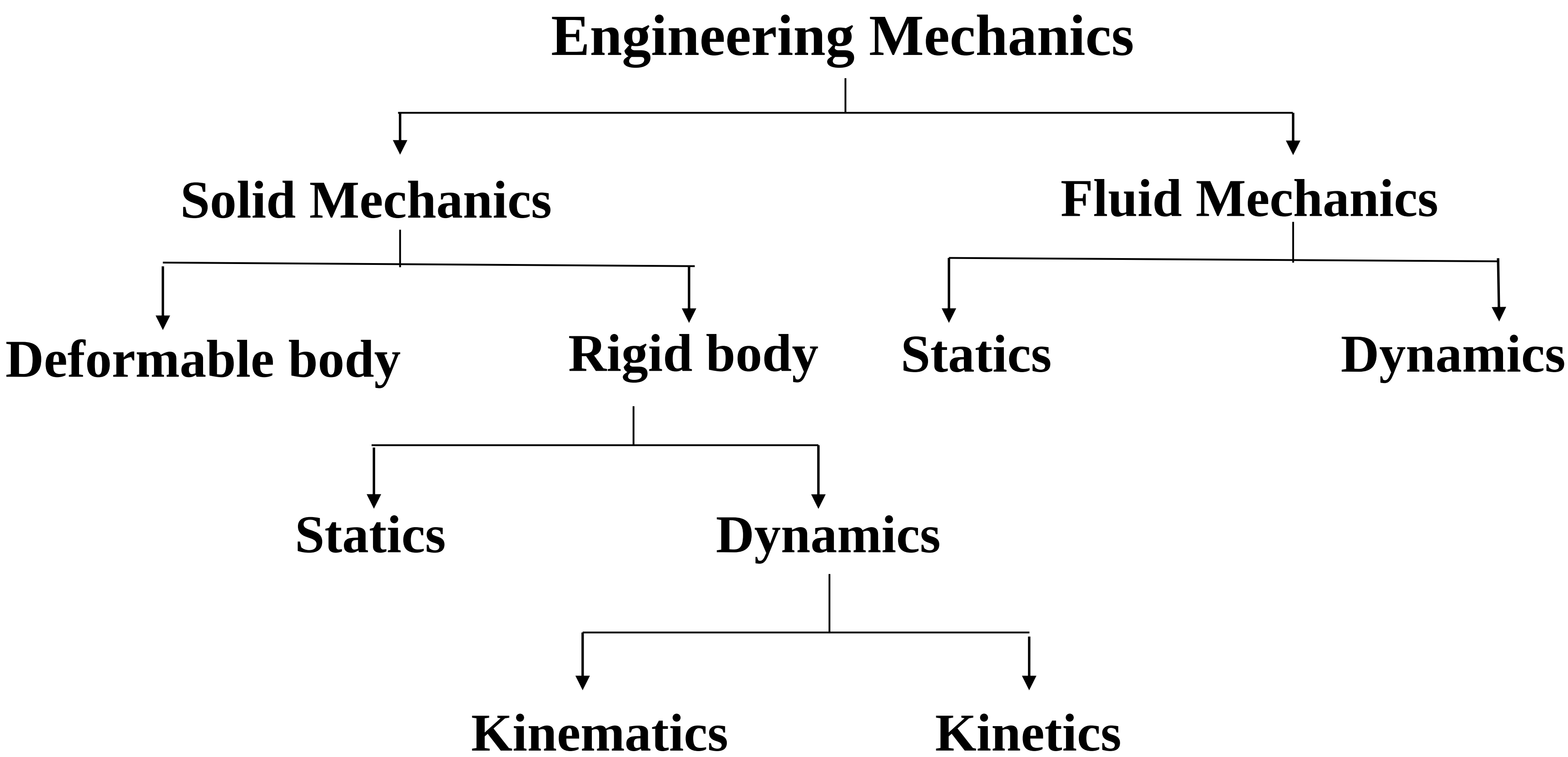
Engineering Mechanics

- Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces.
- The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, engine performance, fluid flow, electrical machines.
- The earliest recorded writings in mechanics are those of Archimedes (287–212 B.C.) on the principle of the lever and the principle of buoyancy.
- Laws of vector combination of forces and principles of statics were discovered by Stevinus (1548–1620).
- The first investigation of a dynamics problem is credited to Galileo (1564–1642) for his experiments with falling stones.
- The accurate formulation of the laws of motion, as well as the law of gravitation, was made by Newton (1642–1727).

Classification of Engineering Mechanics



- Solid mechanics deals with the study of solids at rest or in motion.
- Fluid mechanics deals with the study of liquids and gases at rest or in motion.
- Deformable bodies change their shape or size when acted upon by forces.
- Rigid bodies do not deform acted upon by forces. The rigid bodies may change their orientation or position under the action of force, i.e. the relative position of Particles of rigid body remains unchanged.
- Statics deals with equilibrium of bodies under action of forces (bodies may be either at rest or move with a constant velocity).
- Dynamics deals with motion of bodies (accelerated motion).
- Kinematics is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.
- Kinetics is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

Fundamental Concepts

- Length is used to locate the position of a point in space and thereby describe the size of a physical system.
- Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.
- Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.
- Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body.
- Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its magnitude, direction of its action, point of application and line of action. Thus force is a vector quantity.

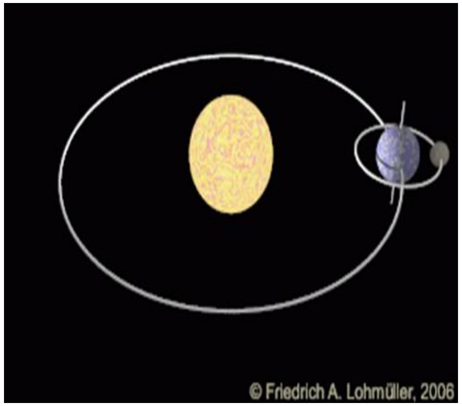
- Newtonian Mechanics
  - Newtonian mechanics is based on application of Newton's Laws of motion which assume that the concepts of distance, time, and mass, are absolute, that is, motion is in an inertial frame.
  - Length, Time, and Mass are absolute concepts and independent to each other
  - Force is a derived concept and not independent of the other fundamental concepts.
- Force can also occur between bodies that are physically separated (For ex: gravitational, electrical, and magnetic forces)
- Weight is defined as the force exerted on a body due to gravitation. It is numerically equal to the multiplication of mass and acceleration due to gravity.

Idealizations

Idealizations are used in mechanics in order to simplify application of the theory.

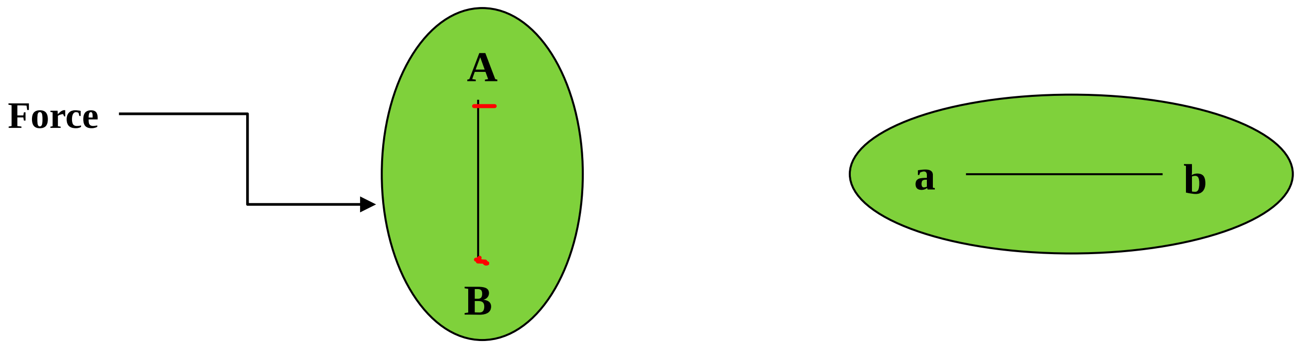
1. Particle

- A particle has a mass, but a size that can be neglected, i.e. negligible dimensions.
- For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modelled as a particle when studying its orbital motion.
- Dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point
- A body can be treated as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it



2. Rigid body

- A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying a load.

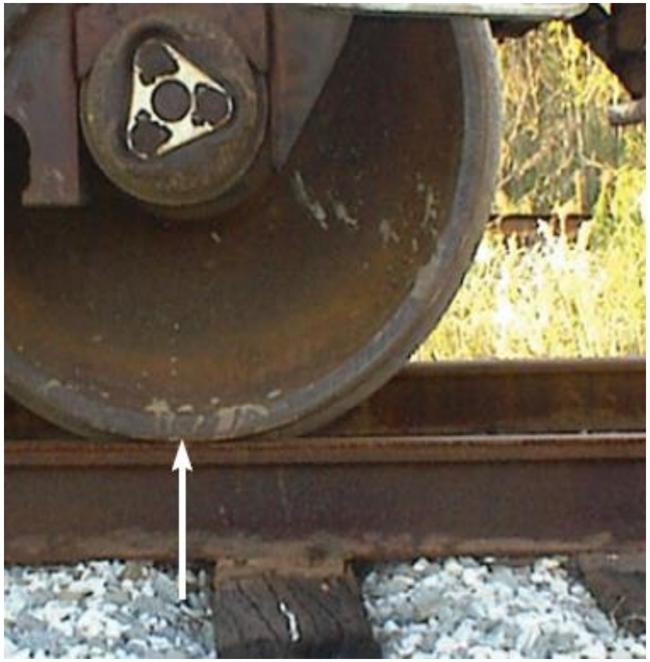


Before applying force **After applying force**

- From figure, body will be called rigid, if AB=ab, i.e. the distance between the particles A & B will be same before and after the application of force.
- Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium
- The actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis

3. **Concentrated Force**

- A concentrated force represents the effect of a loading which is assumed to act at a point on a body.
- A load can be represented by a concentrated force, provided the area over which the load is applied is very small compared to the overall size of the body.
- For example, contact force between a wheel and the ground.

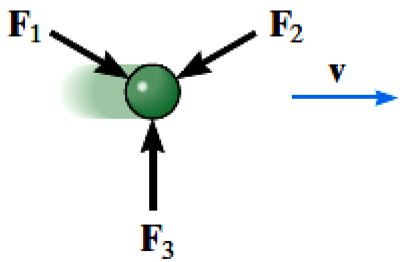


Newton's Laws of Motion

These laws apply to the motion of a particle as measured from a nonaccelerating reference frame.

1. First Law of motion

- A particle originally at rest, or moving in a straight line with constant velocity, tends to remain in this state provided the particle is not subjected to an unbalanced force.

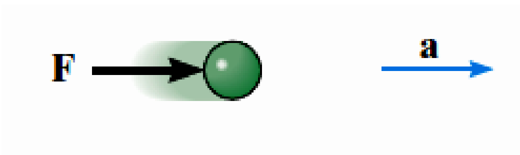


Equilibrium

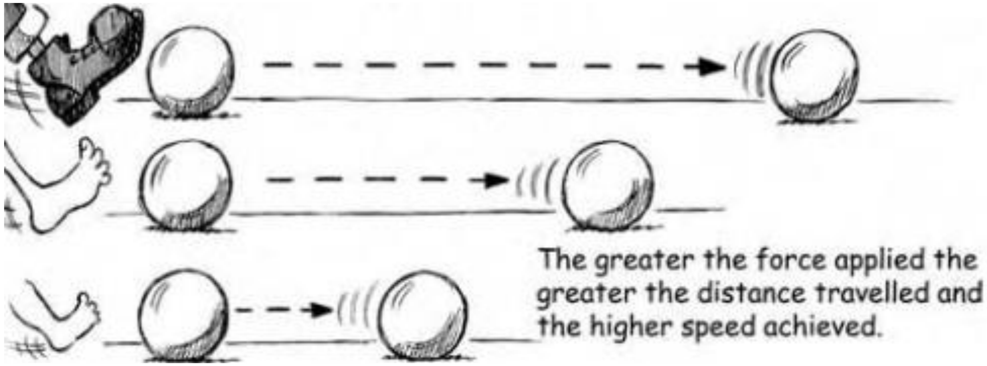
2. Second Law of motion

- A particle acted upon by an unbalanced force F experiences an acceleration a that has the same direction as the force and a magnitude that is directly proportional to the force.
- If F is applied to a particle of mass m, this law may be expressed mathematically as, $\boxed{F = ma}$

- Also, the unbalanced force acting on the particle is proportional to the time rate of change of the particle's linear momentum

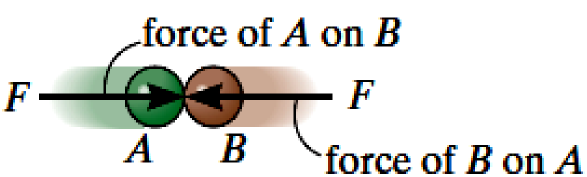


Accelerated motion



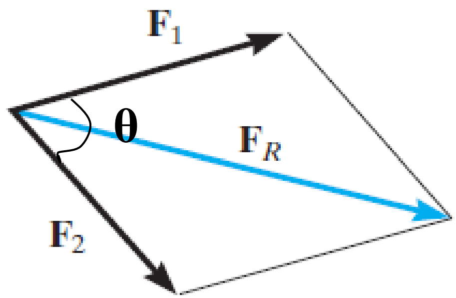
3. Third Law of motion

- The mutual forces of action and reaction between two particles are equal, opposite, and collinear.



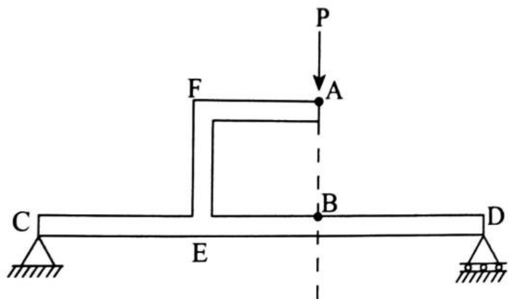
Action – reaction

- **Parallelogram Law:** "If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."
  - Magnitude of Resultant, $F_R = \sqrt{F_1{}^2 + F_2{}^2 + 2F_1F_2\cos\theta}$
  - Direction of Resultant, $tan\phi = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta}$



- **Triangle Law of Forces:** "if three forces acting at a point be represented in magnitude and direction by the three sides of a triangle, taken in order, they will be in equilibrium."

- **Principle of transmissibility of forces:** A force may be applied at any point on its line of action without altering its *external effects* on the *rigid body* on which it acts.
  - Internal effects, such as- stresses do not remain same
  - A deformable body deforms differently, if the point of force is shifted.



$\theta = 60$

$$\tan\alpha = \frac{P\sin\theta}{Q + P\cos\theta}$$

$\theta - \alpha = \alpha$

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

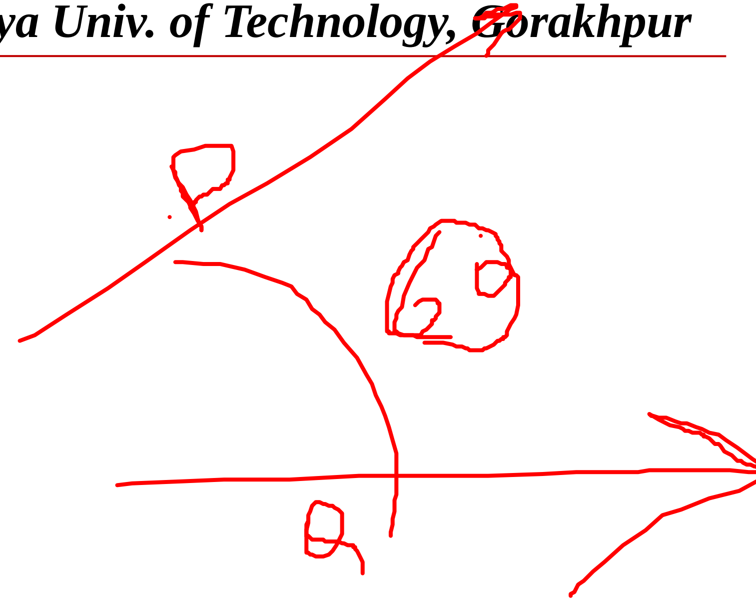
$R = 30.41\ \text{kN}$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

1 cm = 5 kN

15 = 3 cm

20 = 4 cm

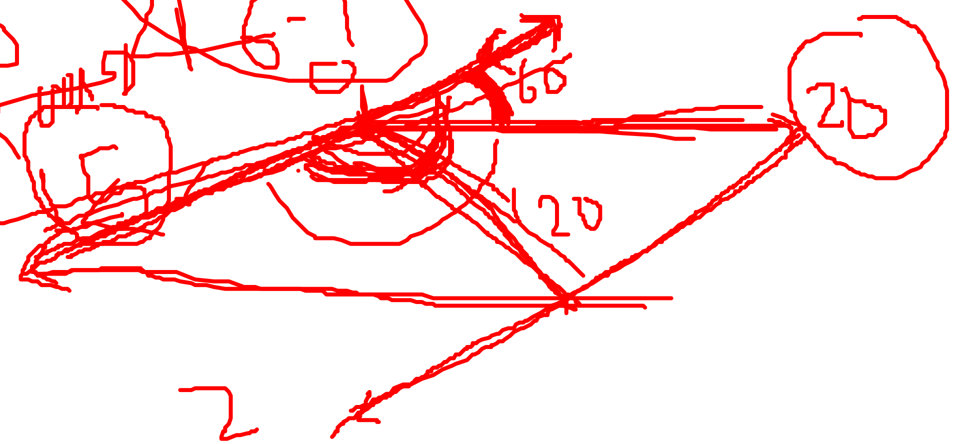


1 cm = 5

4.6 × 5

$\tan\theta =$

R = 18.03 kN



$$R = \sqrt{(20)^2 + 15^2 + 2 \times 20 \times 15 \cos 120^\circ}$$

Systems of Units

- *Kinetic Units*: length [L], time [t], mass [m], and force.
- Three of the kinetic units, referred to as *basic units*, may be defined arbitrarily. The fourth unit, referred to as a *derived unit*, must have a definition compatible with Newton's 2nd Law,

$$F = ma$$

- *International System of Units* (SI): The basic units are length, time, and mass which are arbitrarily defined as the meter (m), second (s), and kilogram (kg). Force is the derived unit,

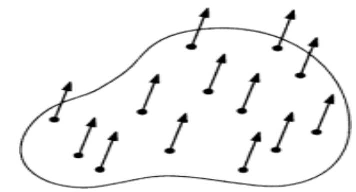
$$F = ma$$

$$1N = (1kg) \times (1\frac{m}{s^2})$$

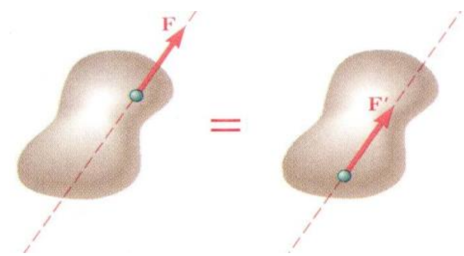
SCALARS AND VECTORS

- Scalar quantities are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, and mass.
- Vector quantities, on the other hand, possess direction as well as magnitude and must obey the parallelogram law of addition. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum.
- Vectors representing physical quantities can be classified as free, sliding, or fixed vector.

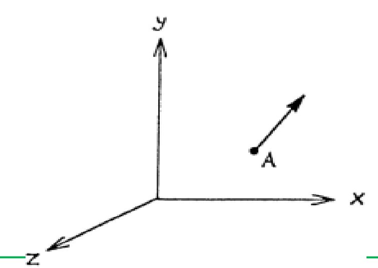
➢ A free vector is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in body may be a free vector.



➢ A sliding vector has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole (Law of transmissibility of forces).

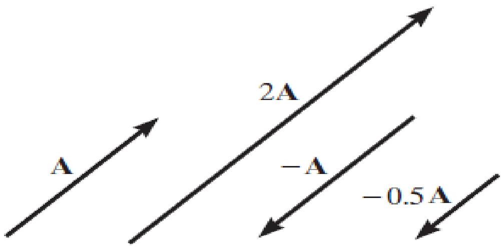


➢ A fixed vector is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force.

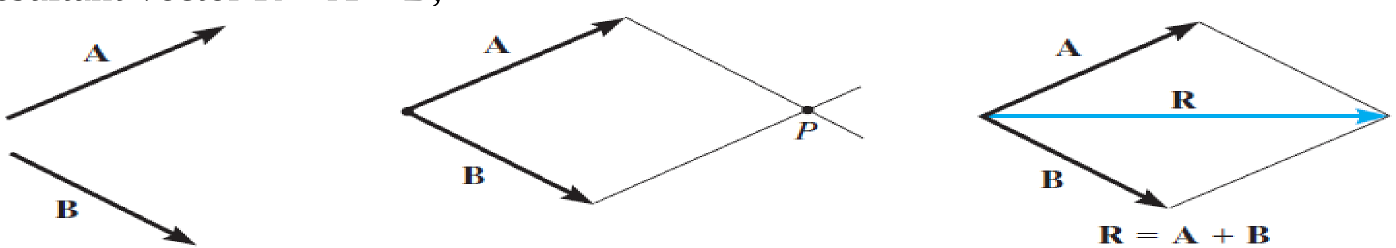


Vector Operations

- **Multiplication and Division of a Vector by a Scalar:** If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. When multiplied by a negative scalar it will also change the directional sense of the vector.

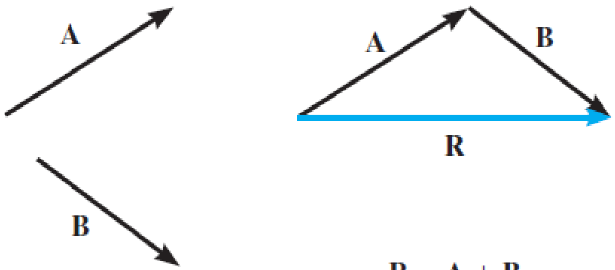


- **Vector Addition:** All vector quantities obey the parallelogram law of addition. Two "component" vectors A and B are added to form a "resultant" vector R =A+B using the following procedure:
    - ➢ First join the tails of the components at a point so that it makes them concurrent,
    - ➢ From the head of B, draw a line parallel to A. Draw another line
    - ➢ From the head of A that is parallel to B. These two lines intersect at point P to form the adjacent sides of a parallelogram.
    - ➢ The diagonal of this parallelogram that extends to P forms R, which then represents the resultant vector R = A + B,



Parallelogram law

- We can also add B to A using the triangle rule, which is a special case of the parallelogram law, whereby vector B is added to vector A in a "head-to-tail" fashion, i.e., by connecting the head of A to the tail of B. The resultant R extends from the tail of A to the head of B.

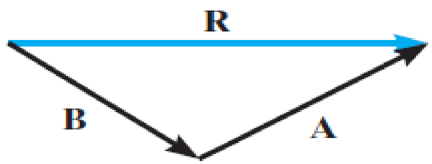


R = A + B

Triangle rule

- In a similar manner, R can also be obtained by adding A to B. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either order, i.e.,

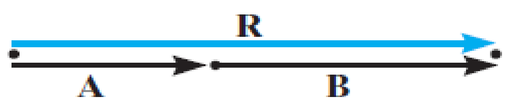
$$\mathbf{R = A + B = B + A}$$



R = B + A

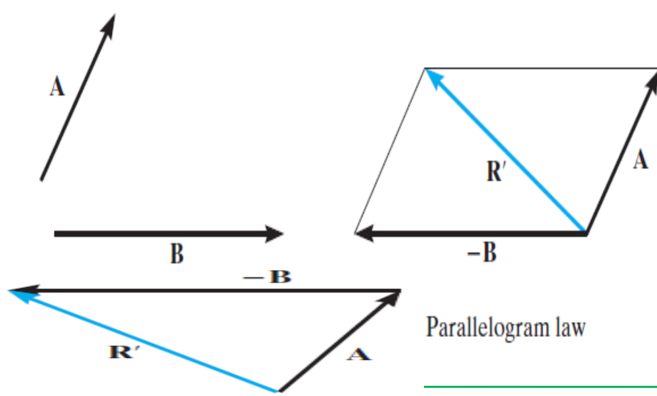
Triangle rule

- As a special case, if the two vectors A and B are collinear, i.e., both have the same line of action, the parallelogram law reduces to an algebraic or scalar addition R = A + B



$R = A + B$

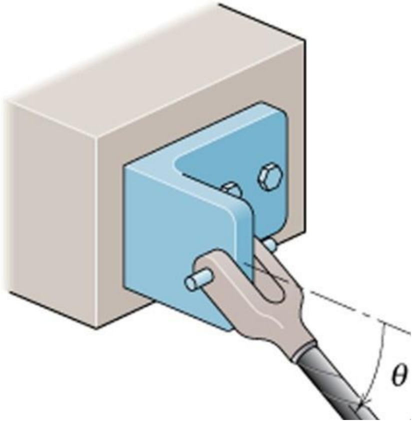
- **Vector Subtraction:** The resultant of the difference between two vectors A and B of the same type may be expressed as, **R' = A - B = A + (–B)**
- Subtraction is therefore defined as a special case of addition, so the rules of vector addition also apply to vector subtraction



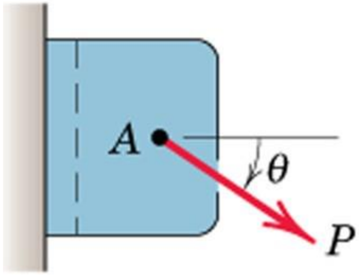
Triangle construction

System of forces

- **Force:** Magnitude (P), direction (arrow) and point of application (point A) is important
- Change in any of the three specifications will alter the effect on the bracket.
- Force is a Fixed Vector
- In case of rigid bodies, line of action of force is important (not its point of application if we are interested in only the resultant external effects of the force), we will treat most forces as
  - **External effect**: Forces applied (applied force); Forces exerted by bracket, bolts, Foundation (reactive force)
  - **Internal effect**: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts, etc



Cable Tension *P*

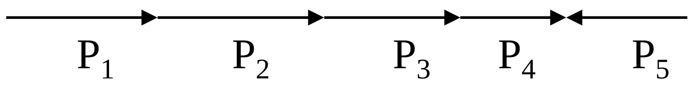


- Considering the plane in which force is applied and depending upon the position of line of action, forces may be classified as,

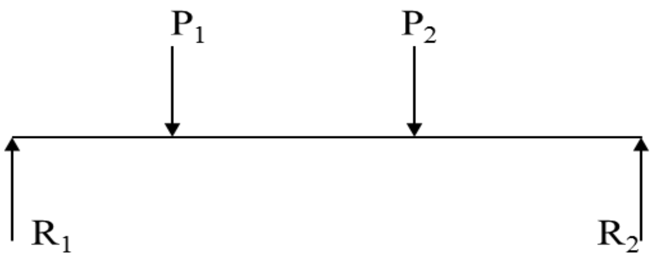
System of forces

- **Coplanar forces**
  - **Concurrent**
  - **Parallel**
    - **Like**
    - **Unlike**
  - **Non-concurrent & Non-parallel**
- **Non-coplanar forces**
  - **Concurrent**
  - **Parallel**
  - **Non-concurrent & Non-parallel**

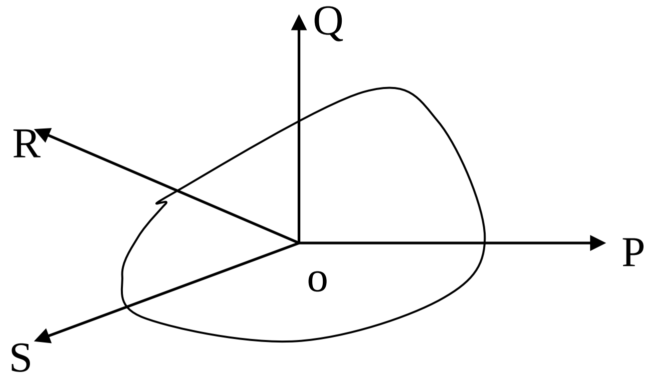
- **Collinear forces**: Line of action of all forces lie in same straight line. Example: Forces on a rope in tug of war.



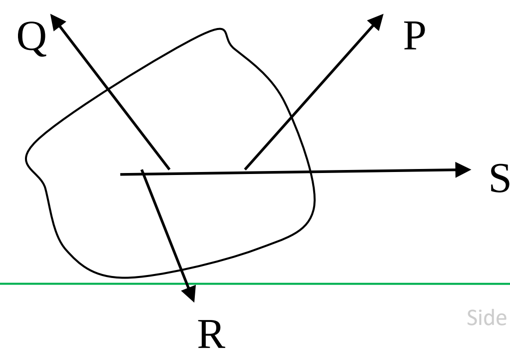
- **Coplanar parallel forces**: Line of action of all forces are parallel and lie on same plane. Example: Vertical loads acting on a beam.



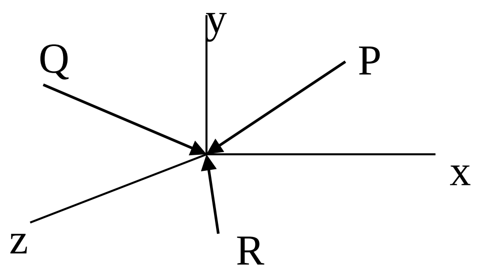
- **Coplanar concurrent forces**: Line of action of all forces, acting on same plane, pass through one point and directions will be different. Example: Forces on a rod resting against a wall.



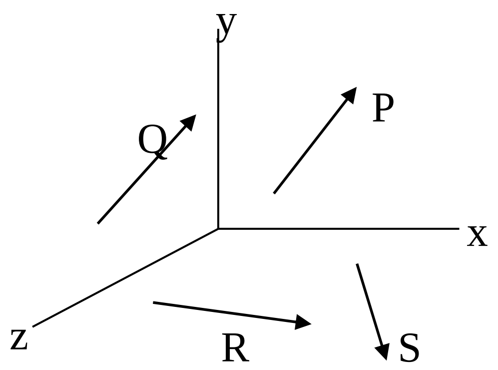
- **Coplanar non-concurrent forces:** Line of action of all forces, acting on same plane, do not pass through one point. Example: Forces on a ladder against a wall and a man standing on a rung which is not its centre of gravity.



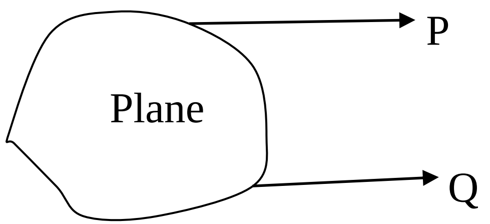
- **Non-coplanar concurrent forces:** All forces do not lie on same plane but their line of action pass through a single point. Example: Forces on a tripod carrying a camera.



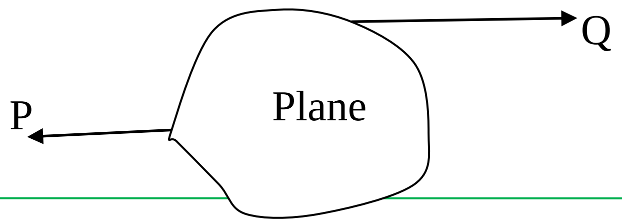
- **Non-coplanar non-concurrent forces:** All forces do not lie on same plane and also their line of action do not pass through a single point. Example: Forces on a moving bus.



- **Coplanar like parallel forces**: Line of action of all forces are parallel, lie on same plane and in the same direction.



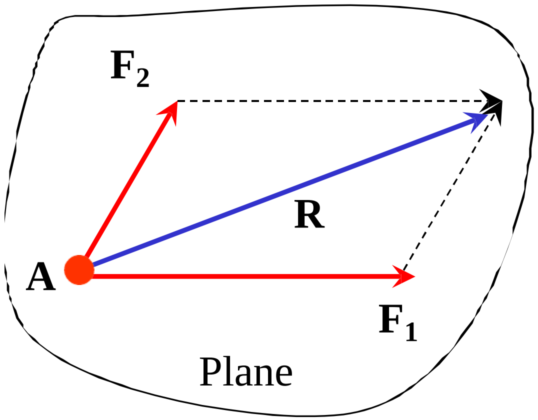
- **Coplanar unlike parallel forces**: Line of action of all forces are parallel, lie on same plane and in the opposite direction.

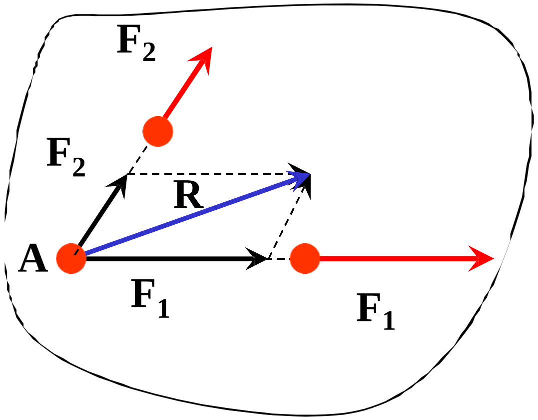


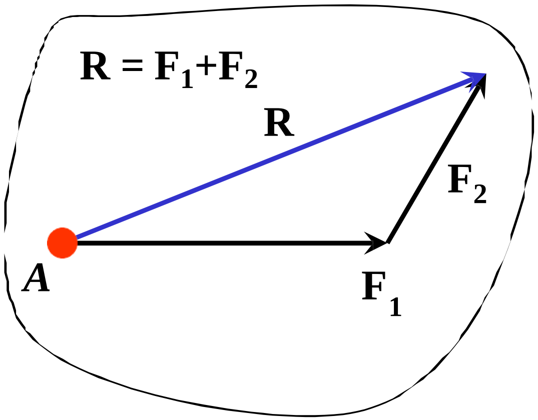
Concurrent force:

Forces are said to be concurrent at a point if their lines of action intersect at that point

F_1, F_2 are concurrent forces; R will be on same plane; $R = F_1 + F_2$







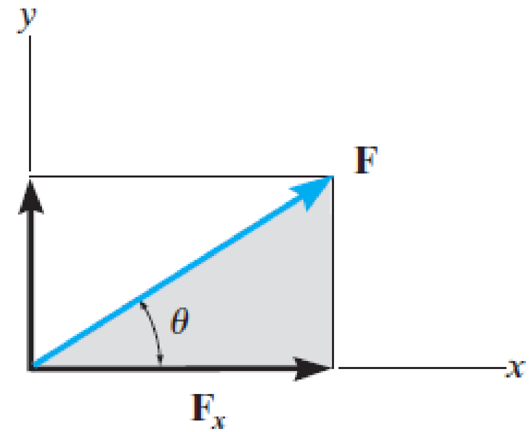
Forces act at same point

Forces act at different point (Principle of Transmissibility)

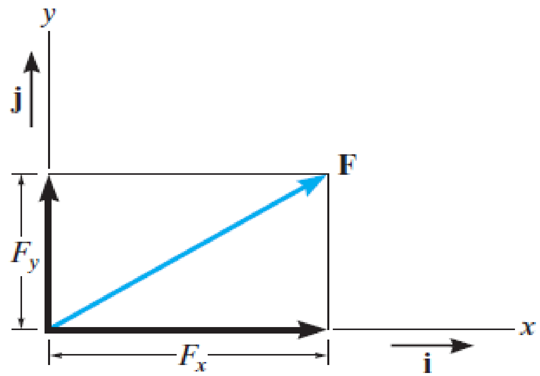
Triangle Law

Addition of a System of Coplanar Forces

- When a force is resolved into two components along the x and y axes, the components are then called rectangular components.
- One can represent these components in one of two ways, using either scalar notation or Cartesian vector notation
- **Scalar notation**: Rectangular components of force F are found using the parallelogram law, so that $F = F_x + F_y$
  - ➢ magnitudes can be determined by $F_x = F\cos\theta$
    $F_y = F\sin\theta$



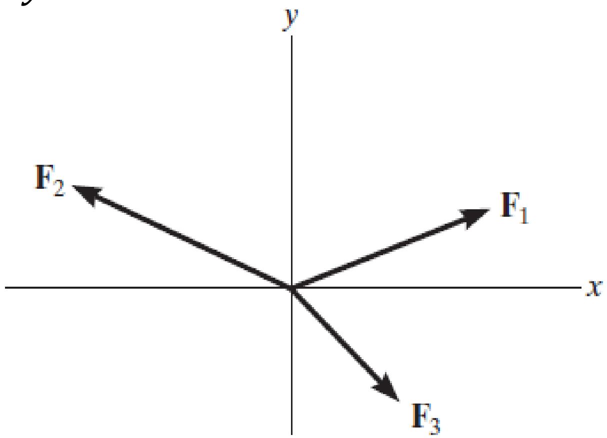
- **Cartesian Vector Notation**: It is also possible to represent the *x* and *y* components of a force in terms of Cartesian unit vectors **i** and **j**.
  - ➢ Since the magnitude of each component of F is always a positive quantity, which is represented by the (positive) scalars Fx and Fy, then we can express F as a Cartesian vector, $$F = F_x i + F_y j$$



Coplanar Force Resultants

- Each force is first resolved into its x and y components, and then respective components are added using scalar algebra since they are collinear.
- The resultant force is then formed by adding the resultant components using the parallelogram law.
- We can represent the components of the resultant force of any number of coplanar forces symbolically by the algebraic sum of the x and y components of all the forces, i.e. $F_{Rx} = \sum F_x$ and $F_{Ry} = \sum F_y$
- Magnitude of resultant, $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$
- Direction of resultant, $\theta = \tan^{-1}(\frac{F_{Ry}}{F_{Rx}})$
- From given problem, $F_1 = F_{1x}i + F_{1y}j$, $F_2 = -F_{2x}i + F_{2y}j$

$F_3 = F_{3x}i - F_{3y}j$



Resultant, $F_R = F_1 + F_2 + F_3$

$F_R = F_{1x}i + F_{1y}j - F_{2x}i + F_{2y}j + F_{3x}i - F_{3y}j$

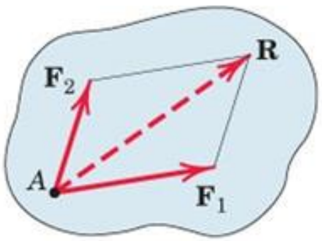
$F_R = (F_{1x} - F_{2x} + F_{3x})i + (F_{1y} + F_{2y} - F_{3y})j$

$F_R = F_{Rx}i + F_{Ry}j$

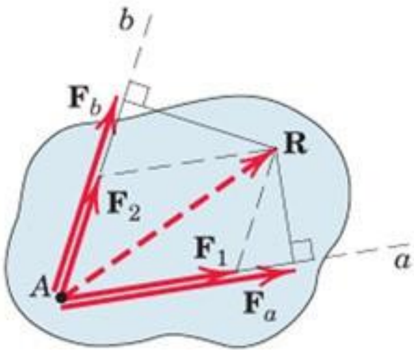
Components of Force

Components of a Force are not necessarily equal to the Projections of the Force unless the axes on which the forces are projected are orthogonal (perpendicular to each other).

F_1 and F_2 are components of R. $R = F_1 + F_2$

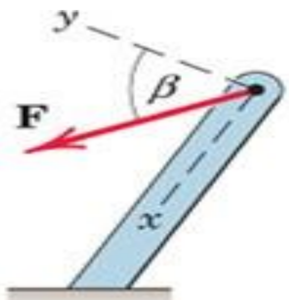


F_a and F_b are perpendicular projections on axes a and b, respectively.



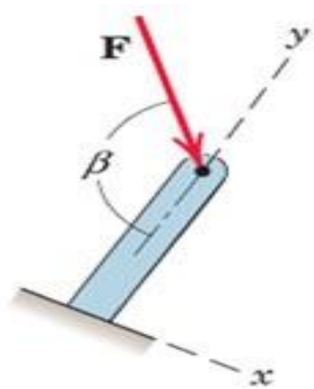
$R \neq F_a + F_b$ unless a and b are perpendicular to each other

Components of Force



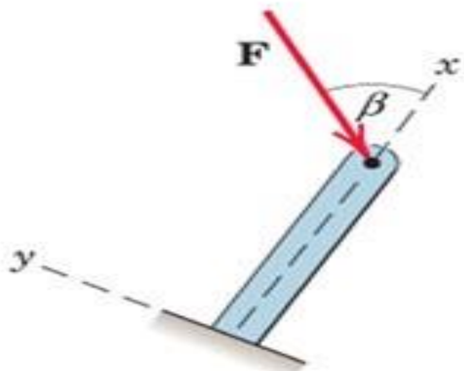
$F_x = F \sin \beta$

$F_y = F \cos \beta$



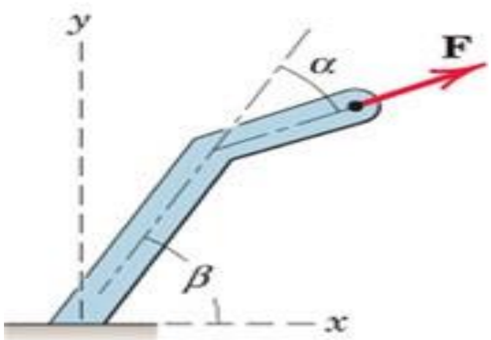
$F_x = F \sin(\pi - \beta)$

$F_y = -F \cos(\pi - \beta)$



$F_x = -F \cos \beta$

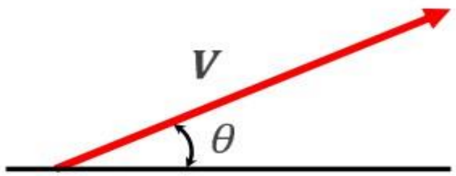
$F_y = -F \sin \beta$



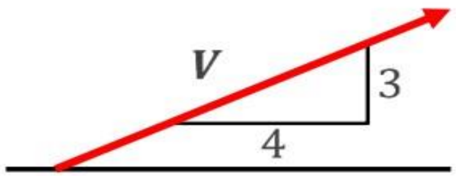
$F_x = F \cos(\beta - \alpha)$

$F_y = F \sin(\beta - \alpha)$

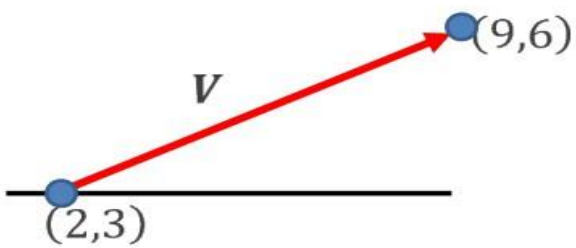
Vector



$$\boldsymbol{V} = V(cos\theta\boldsymbol{i} + sin\theta\boldsymbol{j})$$



$$\boldsymbol{V} = V\left(\frac{4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{4^2 + 3^2}}\right)$$

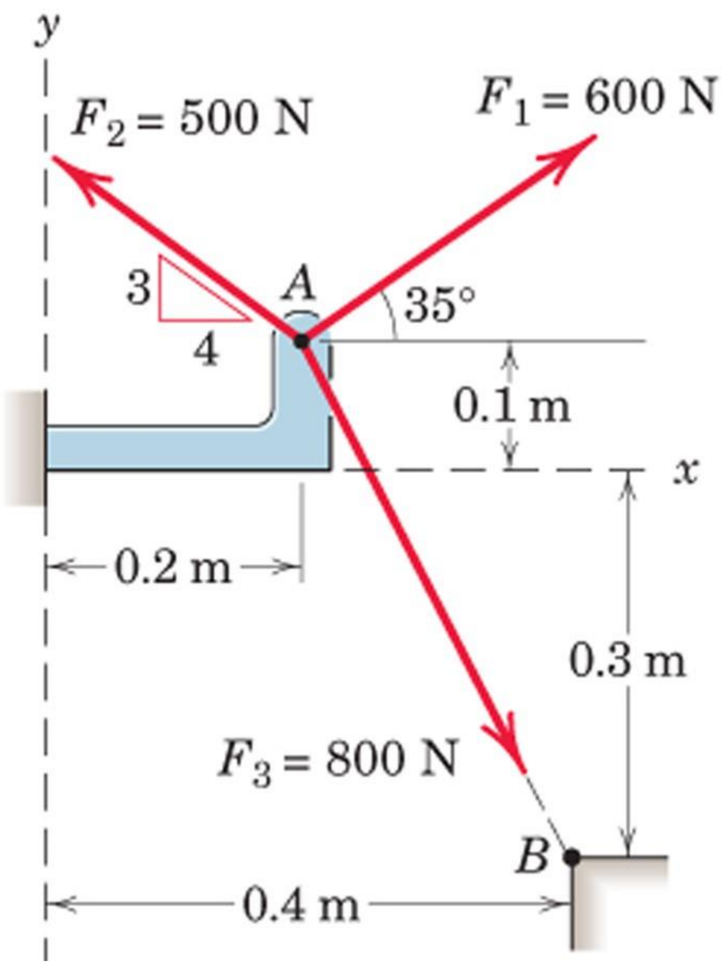


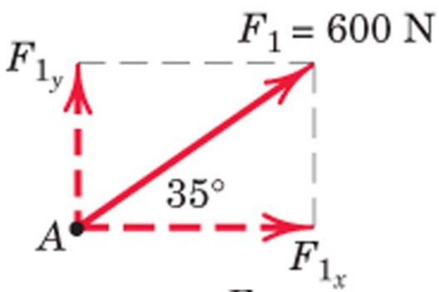
$$\boldsymbol{V} = V\left(\frac{(9-2)\boldsymbol{i} + (6-3)\boldsymbol{j}}{\sqrt{(9-2)^2 + (6-3)^2}}\right)$$

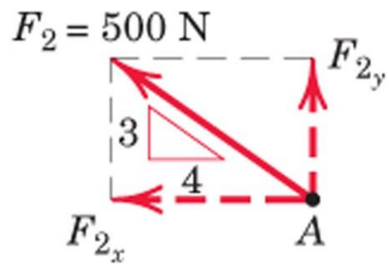
Components of Force

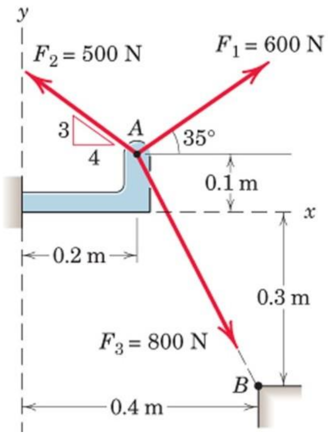
Example 1:

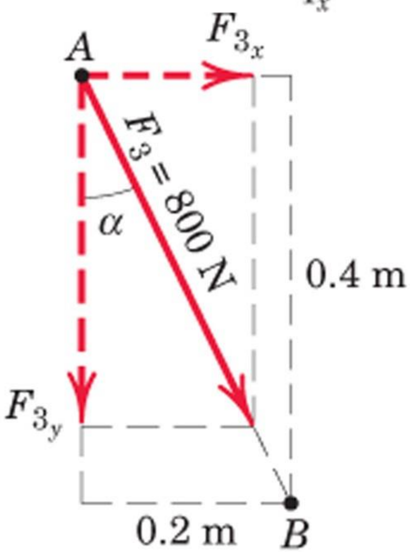
Determine the x and y scalar components of F_1, F_2, and F_3 acting at point A of the bracket











$$F_{1_x} = 600 \cos 35° = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

$$F_{3_x} = F_3 \sin\alpha = 800 \sin 26.6° = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos\alpha = -800 \cos 26.6° = -716 \text{ N}$$

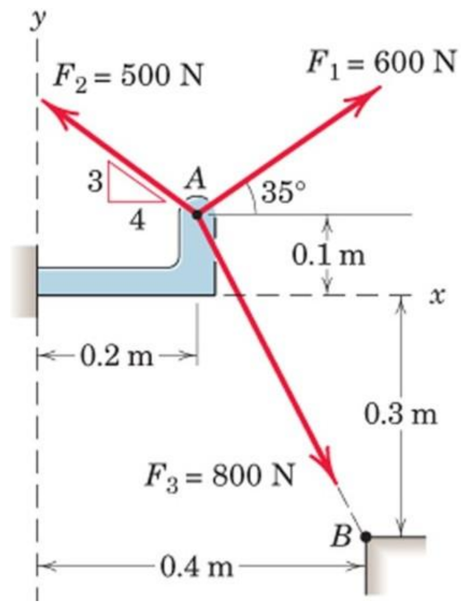
Alternative Solution

$$\boldsymbol{F}_1 = F_1\boldsymbol{n}_1 = F_1\frac{cos(35^o)\boldsymbol{i} + sin(35^o)\boldsymbol{j}}{\sqrt{(cos(35^o))^2 + (sin(35^o))^2}}$$

$$= 600[0.819\boldsymbol{i} + 0.5735\boldsymbol{j}]$$

$$= 491\boldsymbol{i} + 344\boldsymbol{j}$$

$F_{1x} = 491\ N$ $\qquad$ $F_{1y} = 344\ N$

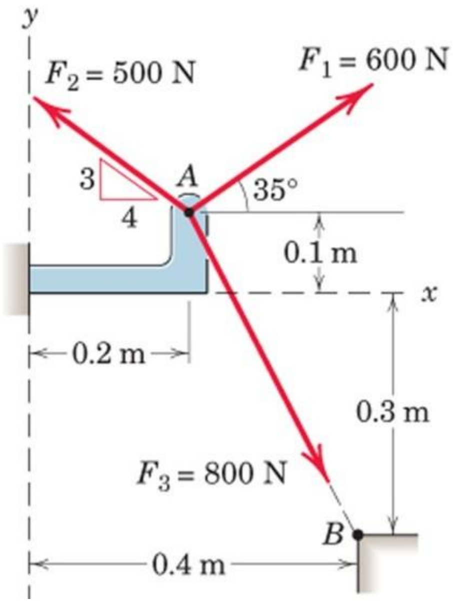


$$\boldsymbol{F}_2 = F_2\boldsymbol{n}_2 = F_2\frac{-4\boldsymbol{i} + 3\boldsymbol{j}}{\sqrt{(-4)^2 + (3)^2}}$$

$$= 500[-0.8\boldsymbol{i} + 0.6\boldsymbol{j}] \quad = -400\boldsymbol{i} + 300\boldsymbol{j}$$

$F_{2x} = -400\ N$ $\qquad$ $F_{2y} = 300\ N$

Alternative Solution



$$\overrightarrow{AB} = 0.2\boldsymbol{i} - 0.4\boldsymbol{j}$$

$$\overline{AB} = \sqrt{(0.2)^2 + (-0.4)^2}$$

$$\boldsymbol{F_3} = F_3\boldsymbol{n}_3 = F_3\frac{\overrightarrow{AB}}{\overline{AB}}$$

$$= 800\frac{0.2\boldsymbol{i} - 0.4\boldsymbol{j}}{\sqrt{(0.2)^2 - (-0.4)^2}}$$

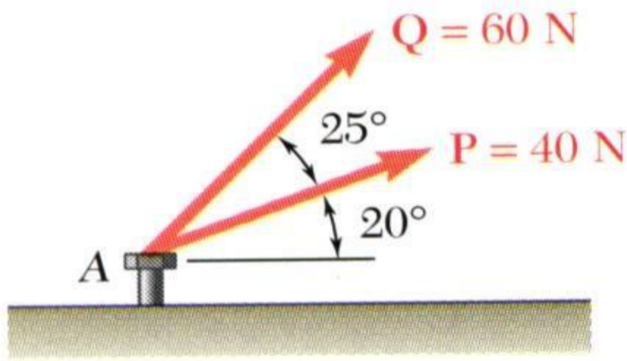
$$= 800[0.447\boldsymbol{i} - 0.894\boldsymbol{j}]$$

$$= 358\boldsymbol{i} - 716\boldsymbol{j}$$

$F_{3x} = 358\ N$ $\qquad$ $F_{3y} = -716\ N$

Components of Force

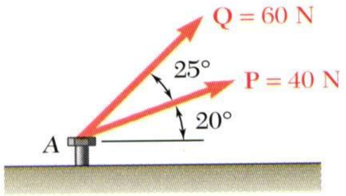
Example 2: The two forces act on a bolt at A. Determine their resultant.

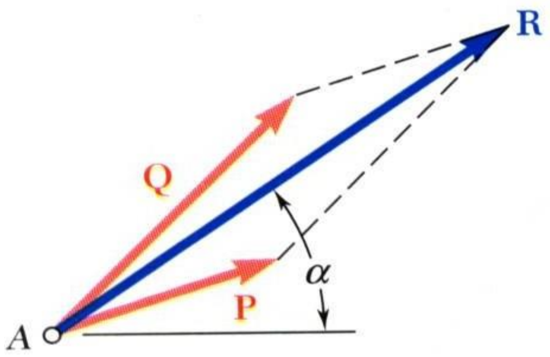


Graphical solution - construct a parallelogram with sides in the same direction as P and Q and lengths in proportion. Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.

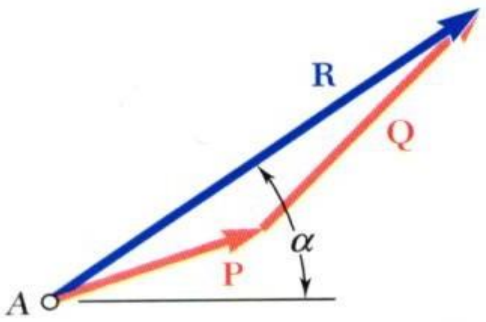
Solution:





- **Graphical solution** - A parallelogram with sides equal to P and Q is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

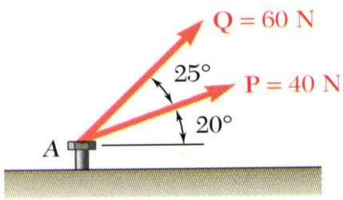
$$R = 98\ \text{N} \qquad \phi = 35^{\circ}$$

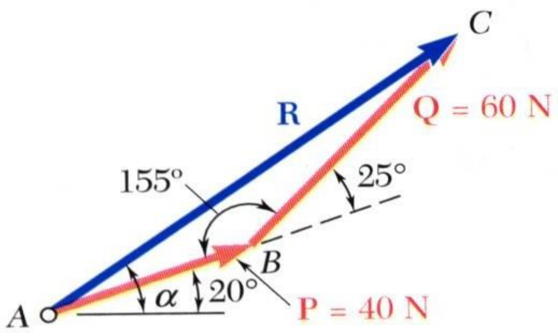


- **Graphical solution-** A triangle is drawn with P and Q head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$R = 98\ \text{N} \qquad \phi = 35^{\circ}$$

Trigonometric Solution: Apply the triangle rule.





From the Law of Cosines,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$= 40^2 + 60^2 + 2*40*60*\cos 155$$

$$R = 97.73 \text{ N}$$

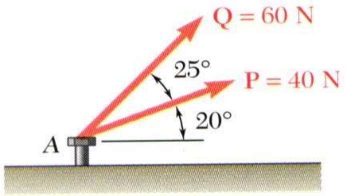
From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$A = 15.04°$$

$$\alpha = 20° + A$$

$$\boxed{\alpha = 35.04°}$$



$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$$

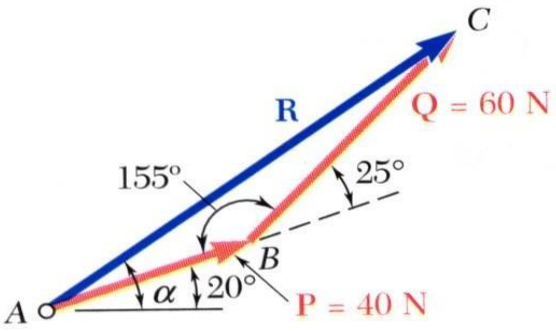
$$\boldsymbol{P} = 40[\cos(20)\boldsymbol{i} + \sin(20)\boldsymbol{j}]$$
$$= 37.58\mathbf{i} + 13.68\mathbf{j}$$

$$\boldsymbol{Q} = 60[\cos(45)\,\boldsymbol{i} + \sin(45)\boldsymbol{j}]$$
$$= 42.43\mathbf{i} + 42.43\mathbf{j}$$

$$\boldsymbol{R} = 80.01\boldsymbol{i} + 56.10\boldsymbol{j}$$

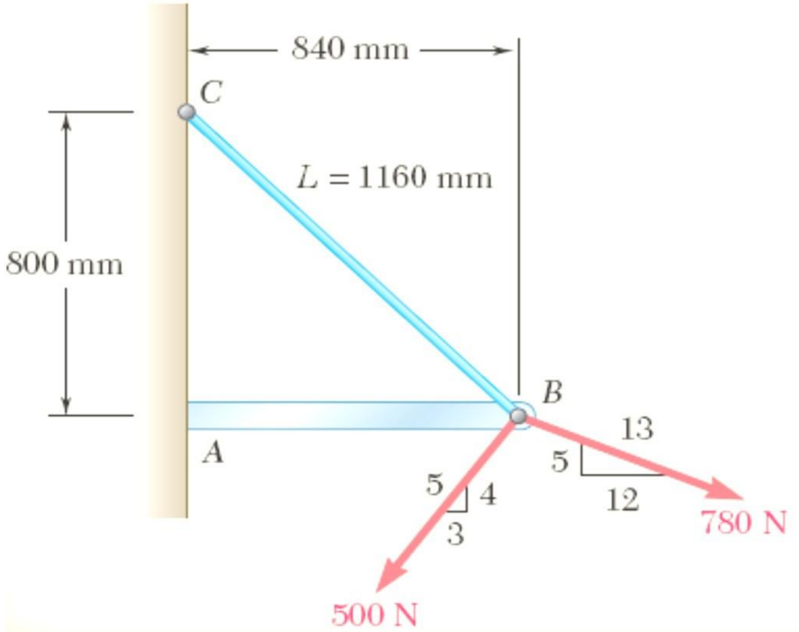
$$R = 97.72$$

$$\alpha = 35.03^{o}$$



Components of Force

Example 3:Tension in cable *BC* is 725-N, determine the resultant of the three forces exerted at point *B* of beam *AB*.

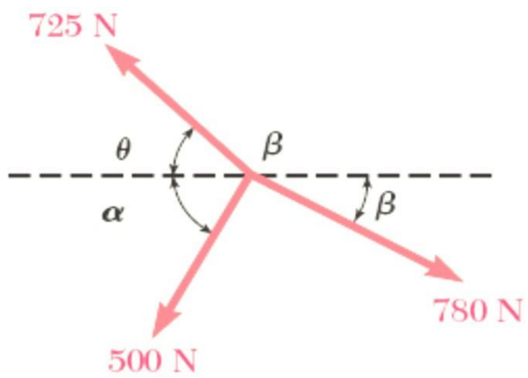


Solution:

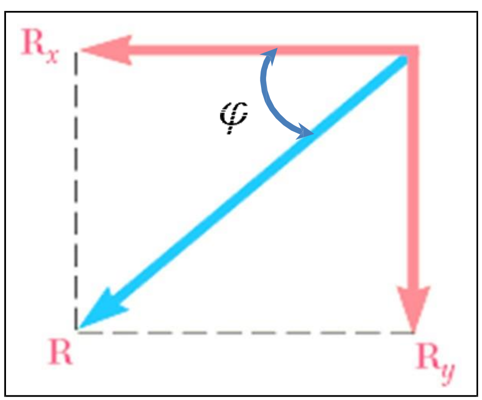
- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Solution:

Resolve each force into rectangular components



Magnitude (N)	X-component (N)	Y-component (N)
725	-525	500
500	-300	-400
780	720	-300
	$R_x = -105$	$R_y = -200$



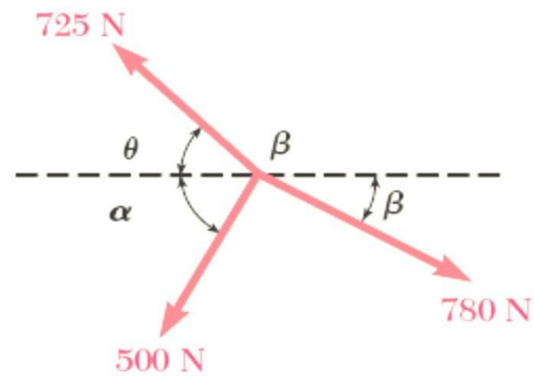
$$R = R_x i + R_y j \qquad \mathbf{R} = (-105)i + (-200)j$$

magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\ 3^o$$

$$R = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Alternate solution



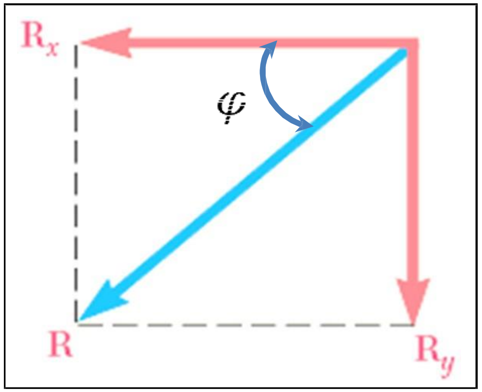
$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

$$\boldsymbol{F}_1 = 725[-0.724\boldsymbol{i} + 0.689\boldsymbol{j}]$$

$$\boldsymbol{F}_2 = 500[-0.6\boldsymbol{i} - 0.8\boldsymbol{j}]$$

$$\boldsymbol{F}_2 = 780[0.923\boldsymbol{i} - 0.384\boldsymbol{j}]$$

$$\boldsymbol{R} = -105\boldsymbol{i} - 200\boldsymbol{j}$$

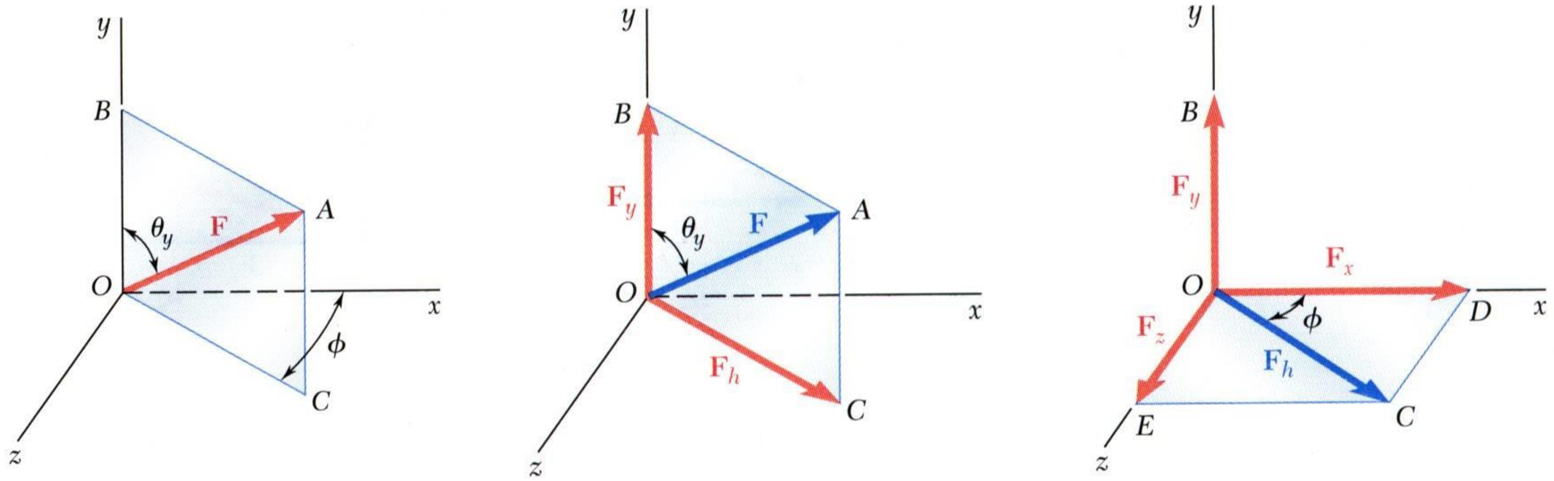


magnitude and direction

$$tan\varphi = \frac{R_x}{R_y} = \frac{105}{200} \qquad \varphi = 62\ 3^o$$

$$R = \sqrt{R_x^2 + R_y^2} = 225.9N$$

Rectangular Components in Space



- The vector F is contained in the plane *OBAC*.
- Resolve $\boldsymbol{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

$$F_h = F\sin\theta_y$$

- Resolve F_h into rectangular components

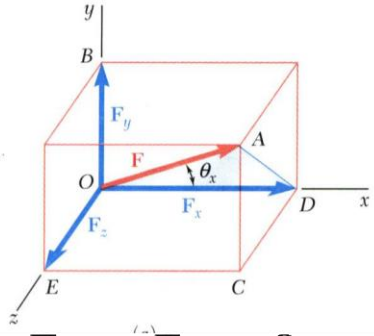
$$F_x = F_h\cos\phi$$

$$= F\sin\theta \quad \cos\phi$$

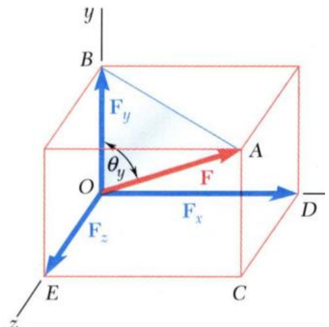
$$F_z = F_h\sin\phi_y$$

$$= F\sin\theta_y\sin\phi$$

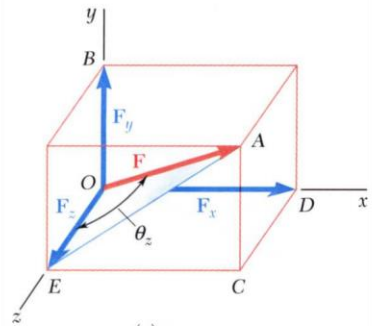
Rectangular Components in Space



$F_x = F\cos\theta_x$



$F_y = F\cos\theta_y$



$F_z = F\cos\theta_z$

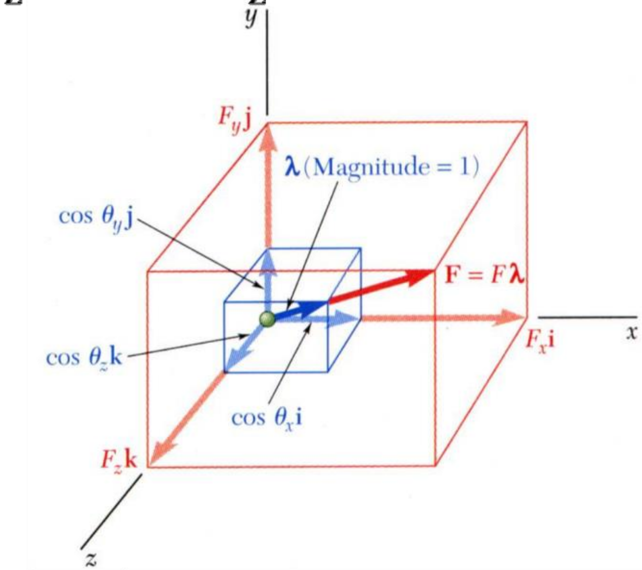
$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\cos\theta_x\boldsymbol{i} + F\cos\theta_y\boldsymbol{j} + F\cos\theta_z\boldsymbol{k}$$

$$\boldsymbol{F} = F\left(\cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}\right)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

Where $\boldsymbol{\lambda} = \cos\theta_x\boldsymbol{i} + \cos\theta_y\boldsymbol{j} + \cos\theta_z\boldsymbol{k}$

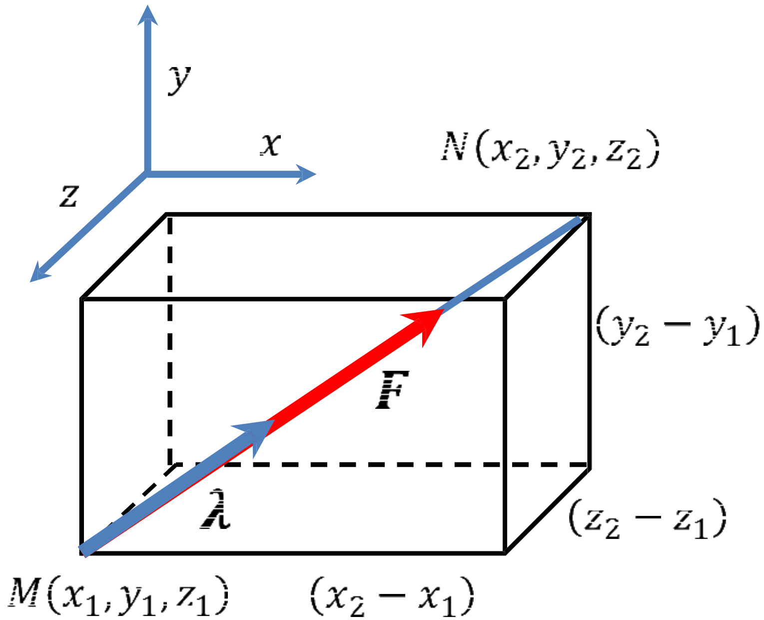


$\boldsymbol{\lambda}$ is a unit vector along the line of action of $\boldsymbol{F}$ and $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ are the direction cosine for $\boldsymbol{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$\boldsymbol{d}$ is the vector joining M and N

$$\boldsymbol{d} = d_x\boldsymbol{i} + d_y\boldsymbol{j} + d_z\boldsymbol{k}$$

$$d_x = (x_2 - x_1) \qquad d_y = (y_2 - y_1)$$

$$d_z = (z_2 - z_1)$$

$$\boldsymbol{F} = F\boldsymbol{\lambda}$$

$$= F\left(\frac{d_x\boldsymbol{i} - d_y\boldsymbol{j} + d_z\boldsymbol{k}}{d}\right)$$

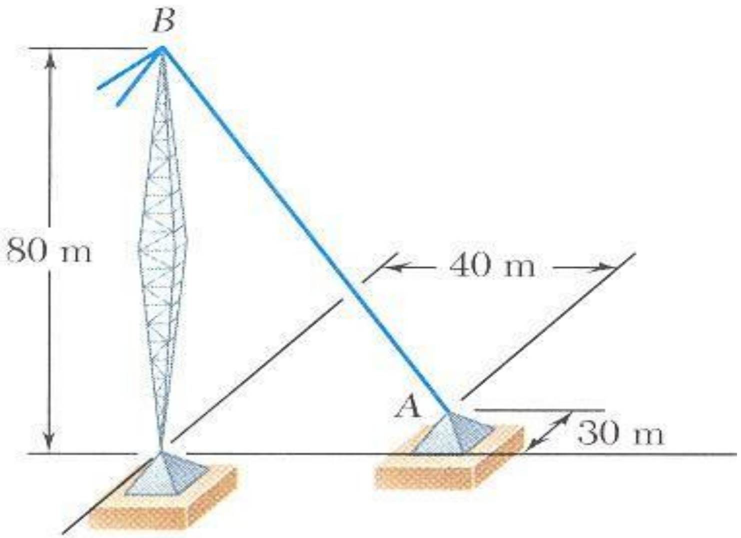
$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

Rectangular Components in Space

Example: The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles q_x, q_y, q_z defining the direction of the force

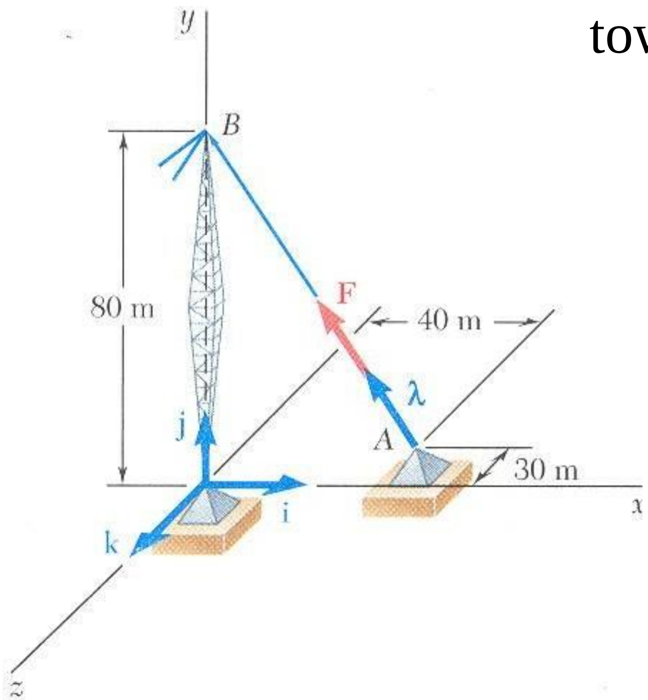


SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force acting on *A*.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Solution:

Determine the unit vector pointing from *A* towards *B*.



$$\boldsymbol{AB} = -40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}$$

$$AB = \sqrt{(-40)^2 - (80)^2 - (30)^2} = 94.3$$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{AB}}{AB} = \frac{-40\boldsymbol{i} - 80\boldsymbol{j} - 30\boldsymbol{k}}{94.3}$$

$$= -0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k}$$

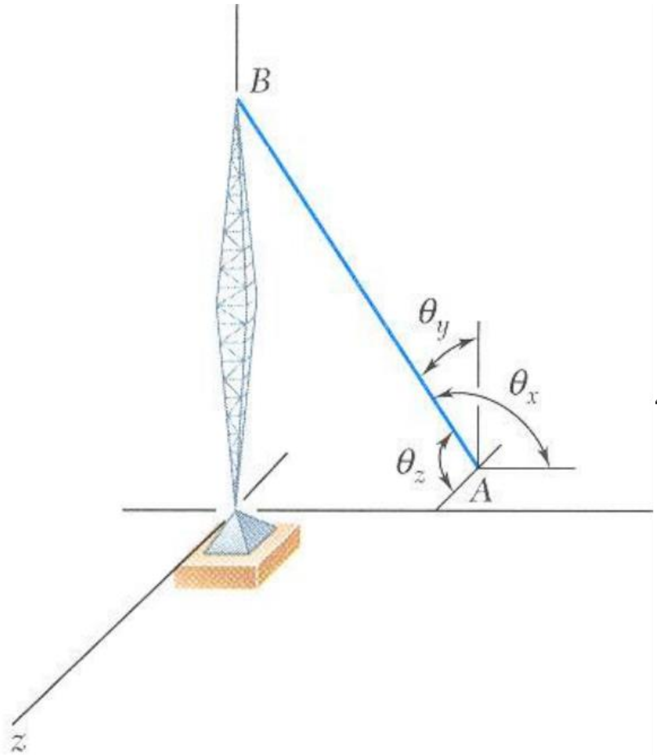
Determine the components of the force.

$$\boldsymbol{F} = F\boldsymbol{\lambda} \qquad = 2500(-0.424\boldsymbol{i} - 0.848\boldsymbol{j} - 0.318\boldsymbol{k})$$

$$= -1060\boldsymbol{i} + 2120\boldsymbol{j} + 795\boldsymbol{k}$$

$$\boxed{\begin{aligned} F_x &= -1060\ N \\ F_y &= 2120\ N \\ F_z &= 795\ N \end{aligned}}$$

Direction of the force:

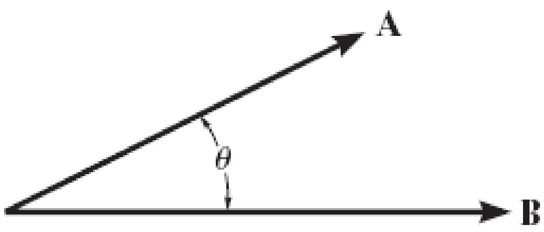


Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\boldsymbol{\lambda} = cos\theta_x \boldsymbol{i} + cos\theta_y \boldsymbol{j} + cos\theta_z \boldsymbol{k}$$
$$= -0.424\boldsymbol{i} + 0.848\boldsymbol{j} + 0.318\boldsymbol{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^0 \\ \theta_y &= 32.0^0 \\ \theta_z &= 71.5^0 \end{aligned}}$$

Vector Products



Dot Product $\boldsymbol{A}.\boldsymbol{B} = AB\cos\theta$

This is used to:

- to determine the angle between two vectors
- to determine the projection of a vector in a specified direction

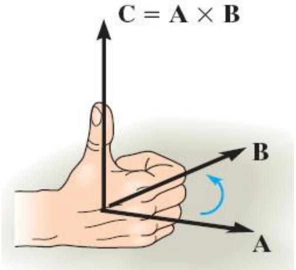
***A.B* = *B.A* (commutative)**
***A.(B+C)* = *A.B+A.C* (distributive operation)**

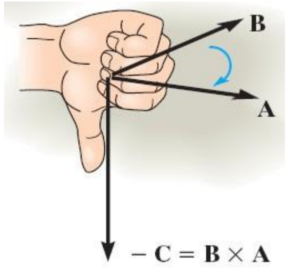
$$\boldsymbol{A}.\boldsymbol{B} = \left(A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}\right).\left(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}\right) \qquad \boldsymbol{i}\ \boldsymbol{i} = 1$$

$$= A_xB_x + A_yB_y + A_zB_z \qquad \boldsymbol{i}.\boldsymbol{j} = 0$$

Cross Product: $\boldsymbol{A} \times \boldsymbol{B} = \boldsymbol{C} = ABsin\theta$

$$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$$

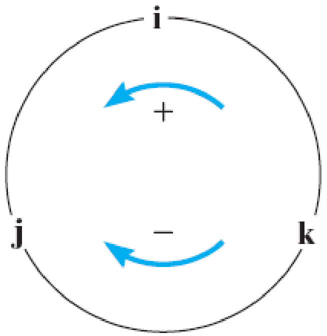


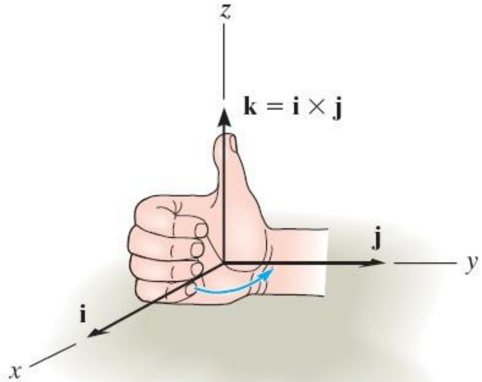


$$\boldsymbol{A} \times \boldsymbol{B} = (A_x\boldsymbol{i} - A_y\boldsymbol{j} - A_z\boldsymbol{k}) \times (B_x\boldsymbol{i} - B_y\boldsymbol{j} - B_z\boldsymbol{k})$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\boldsymbol{i} + (A_zB_x - A_xB_z)\boldsymbol{j} + (A_xB_y - A_yB_x)\boldsymbol{k}$$

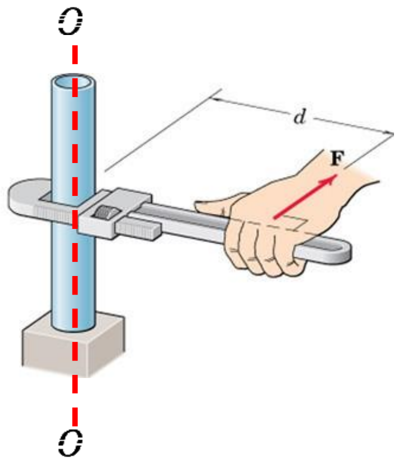
Cartesian Vector





$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = \mathbf{0}$$
$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = \mathbf{0}$$
$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = \mathbf{0}$$

Moment of a Force (Torque)

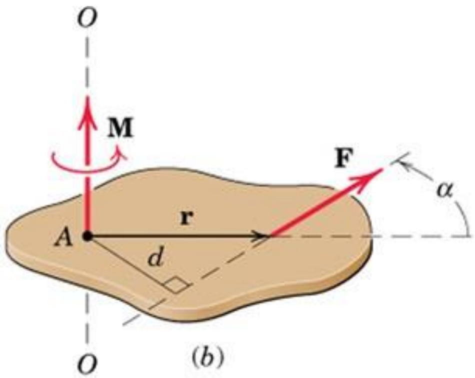


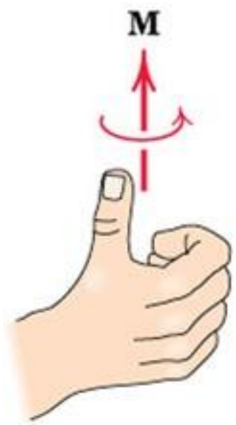
Moment about axis O-O is $M_o = Fd$

Magnitude of M_o measures tendency of **F** to cause rotation of the body about an axis along M_o.

Moment about axis O-O is, $M_o = Frsin\alpha$

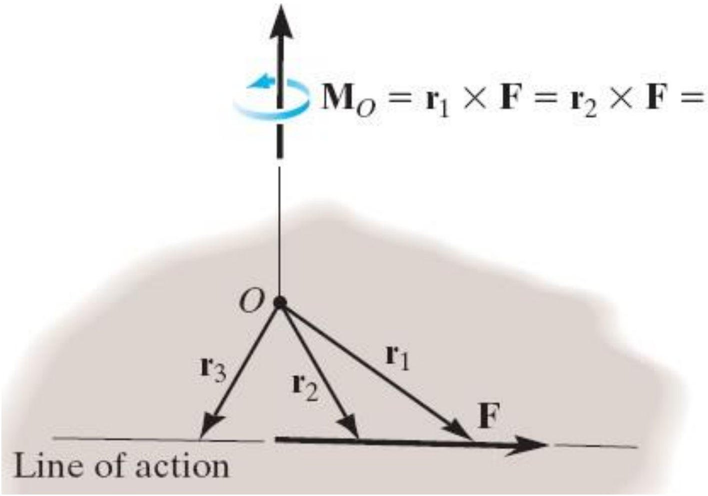
$M_o = r \times F$



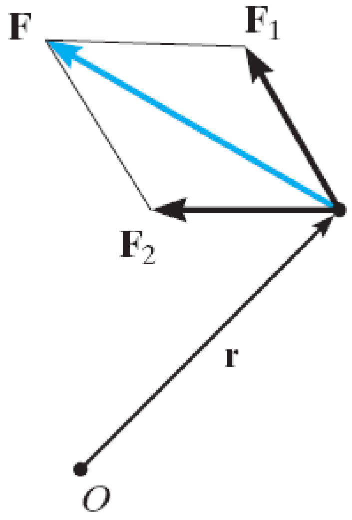


Sense of the moment may be determined by the right-hand rule

Moment of a Force



Principle of Transmissibility: Any force that has the same magnitude and direction as F, is *equivalent* if it also has the same line of action and therefore, produces the same moment.



Varignon's Theorem (Principle of Moments):
Moment of a Force about a point is equal to the sum of the moments of the force's components about the point.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

Rectangular Components of a Moment

The moment of F about O,

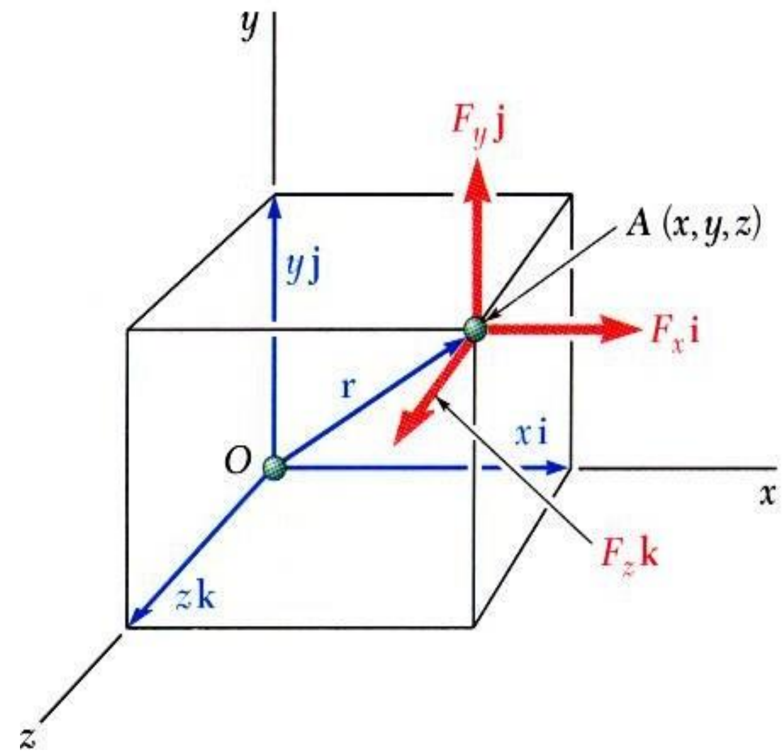
$$\boldsymbol{M}_o = \boldsymbol{r} \times \boldsymbol{F}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

$$\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

$$\boldsymbol{M}_o = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$



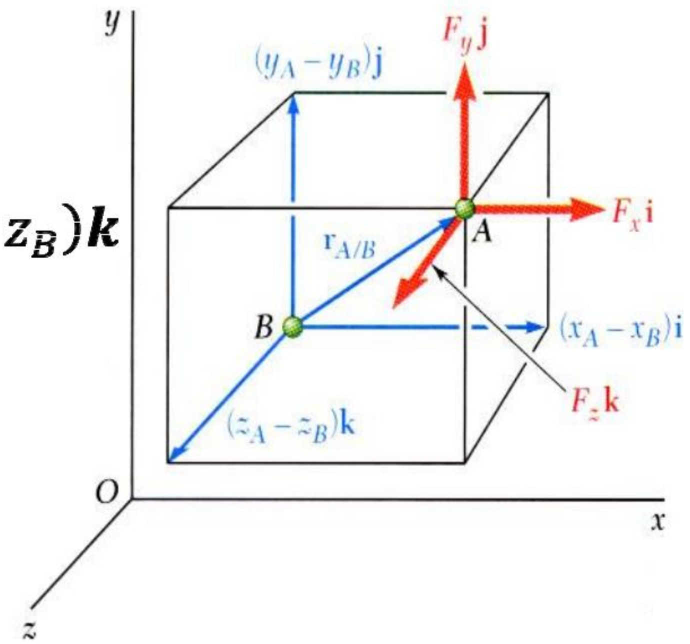
$$= (yF_z - zF_y)\boldsymbol{i} + (zF_x - xF_z)\boldsymbol{j} + (xF_y - yF_x)\boldsymbol{k}$$

The moment of *F* about *B*,

$$\boldsymbol{M}_B = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$\boldsymbol{r}_{AB} = (x_A - x_B)\boldsymbol{i} + (y_A - y_B)\boldsymbol{j} + (z_A - z_B)\boldsymbol{k}$$

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$$

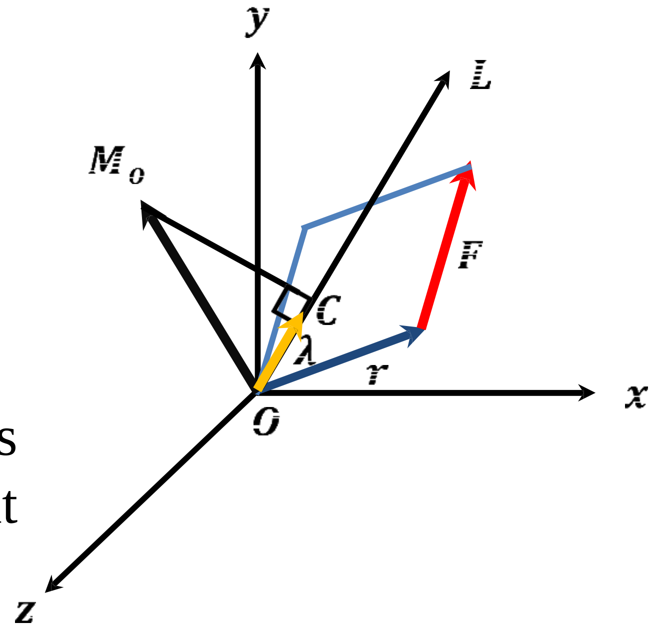


$$\boldsymbol{M}_B = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A - x_B & y_A - y_B & z_A - z_B \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force About a Given Axis

Moment M_O of a force F applied at the point A about a point O



$$M_o = r \times F$$

Scalar moment M_{OL} about an axis OL is the projection of the moment vector M_O onto the axis,

$$M_{OL} = \lambda . M_o = \lambda . (r \times F)$$

Moments of F about the coordinate axes (using previous slide)

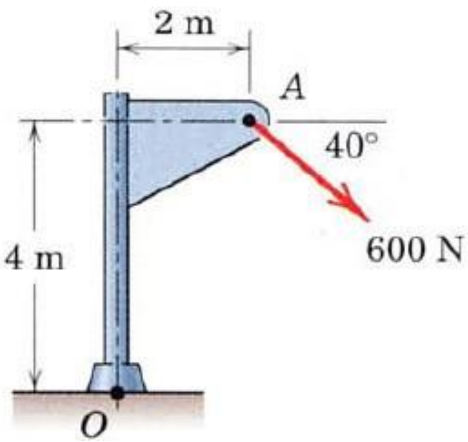
$$M_x = (yF_z - zF_y)$$

$$M_y = (zF_x - xF_z)$$

$$M_z = (xF_y - yF_x)$$

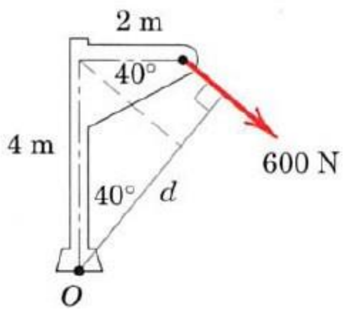
Moment: Example

Calculate the magnitude of the moment about the base point O of the 600 N force in different ways



Solution 1.

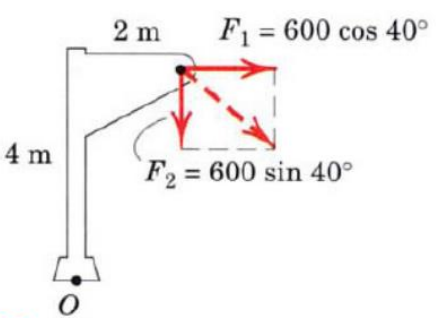
Moment about O is



$$M_o = dF \qquad d = 4\cos40^o + 2\sin40^o = 4.35m$$

$$M_o = 600(4.35) = 2610\ N.m\ \textbf{Ans}$$

Solution 2.

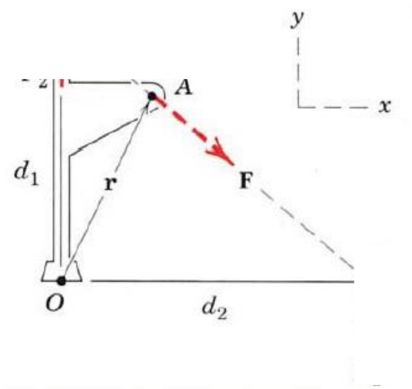


$$F_x = 600\cos40^o = 460\ N$$

$$F_x = 600\sin40^o = 386\ N$$

$$M_o = 460(4.00) + 386(2.00) = 2610\ N.m\ \textbf{Ans}$$

Moment: Example



Solution 3.

$$d_1 = 4 + 2tan40^o = 5.68m$$

$$M_o = 460(5.68) = 2610\ N.m \quad \textbf{Ans}$$

Solution 4.

$$d_2 = 2 + 4cot40^o = 6.77m$$

$$M_o = 386(6.77) = 2610\ N.m \quad \textbf{Ans}$$

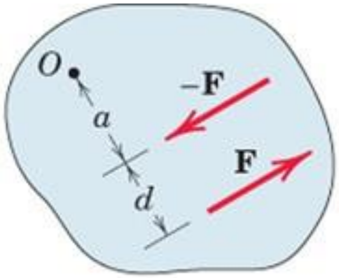
Solution 5.

$$\boldsymbol{M_0} = \boldsymbol{r} \times \boldsymbol{F} = (2\boldsymbol{i} + 4\boldsymbol{j}\,) \times 600(cos40^o\boldsymbol{i} - sin40^o\boldsymbol{j})$$

$$\boldsymbol{M_0} = -2610\ N.m \quad \textbf{Ans}$$

The minus sign indicates that the vector is in the negative z-direction

Moment of a Couple



Moment produced by two equal, opposite and non-collinear forces is called a *couple*.

Magnitude of the combined moment of the two forces about O:

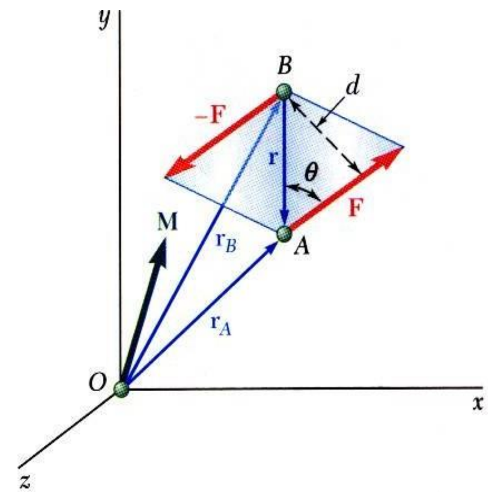
$$M = F(a + d) - Fa = Fd$$

$$\boldsymbol{M} = \boldsymbol{r}_A \times \boldsymbol{F} + \boldsymbol{r}_B \times (-\boldsymbol{F})$$

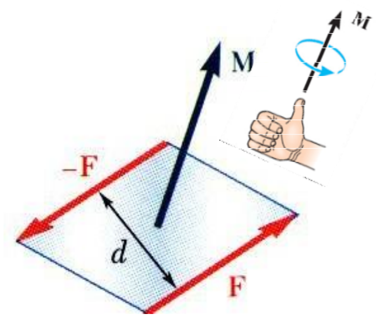
$$= (\boldsymbol{r}_A - \boldsymbol{r}_B) \times \boldsymbol{F}$$

$$= \boldsymbol{r} \times \boldsymbol{F}$$

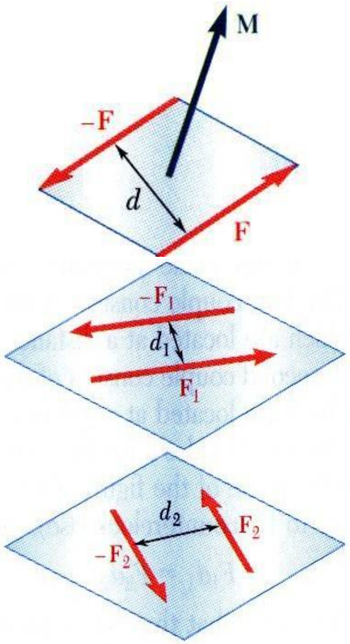
$$M = rF\sin\theta = Fd$$



The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.



Moment of a Couple

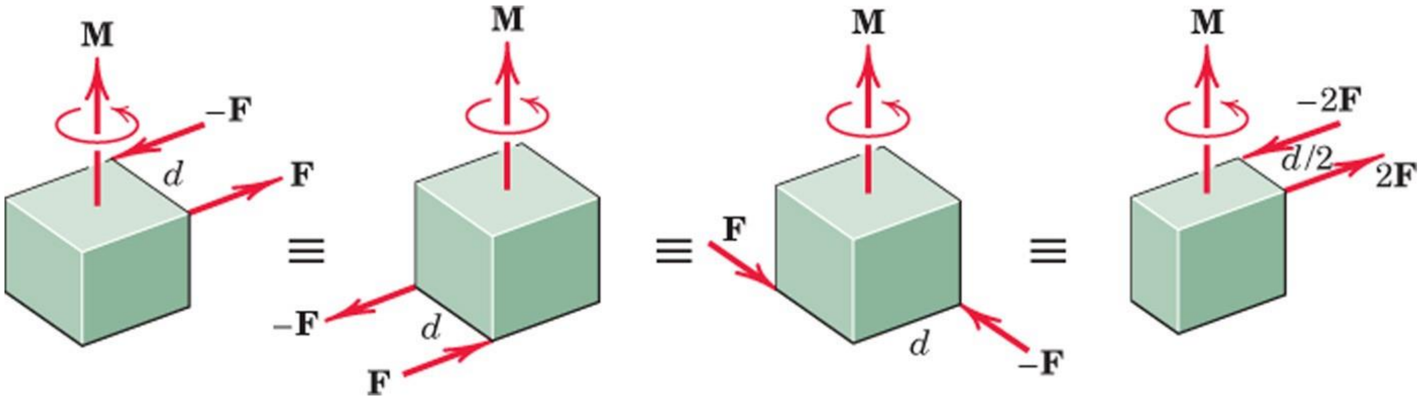


Two couples will have equal moments if $F_1 d_1 = F_2 d_2$

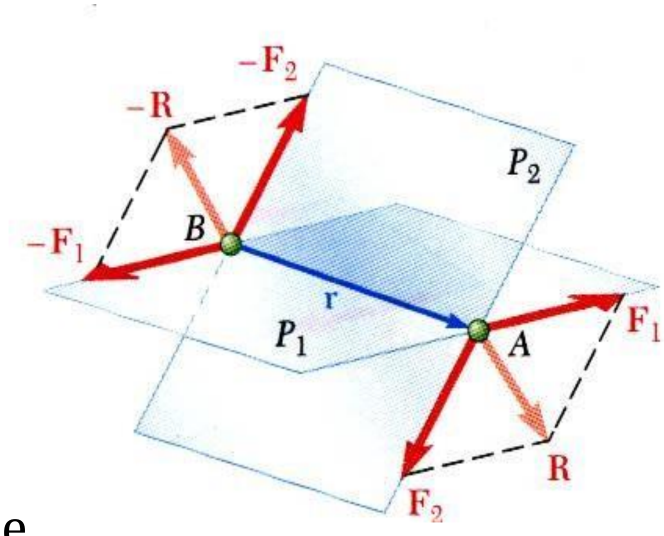
The two couples lie in parallel planes

The two couples have the same sense or the tendency to cause rotation in the same direction.

Examples:



Addition of Couples



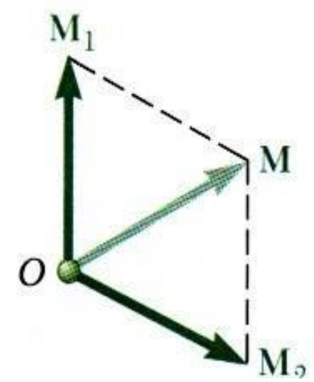
Consider two intersecting planes P_1 and P_2 with each containing a couple

$M_1 = r \times F_1$ in plane P_1

$M_2 = r \times F_2$ in plane P_2

Resultants of the vectors also form a couple

$$M = r \times R = r \times (F_1 + F_2)$$

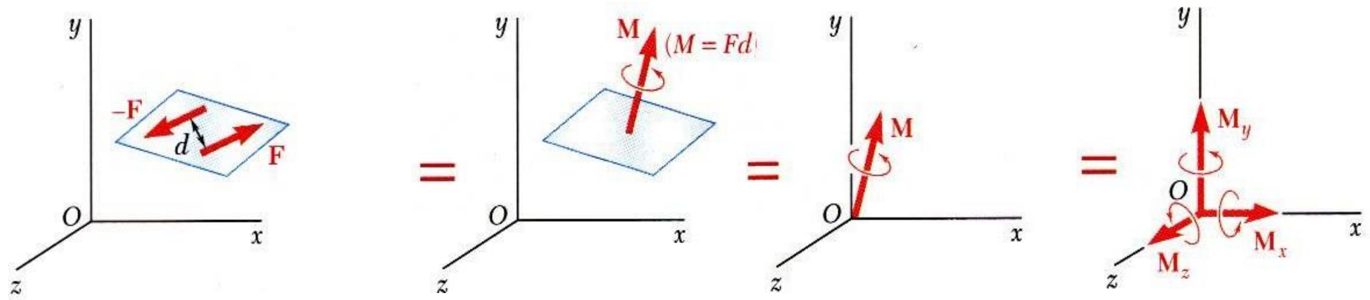


By Varigon's theorem

$$M = r \times F_1 + r \times F_2$$
$$= M_1 + M_2$$

Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Vectors



- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- Couple vectors obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Couple: Example

Moment required to turn the shaft connected at center of the wheel = 12 Nm

Case I: Couple Moment produced by 40 N forces = 12 Nm

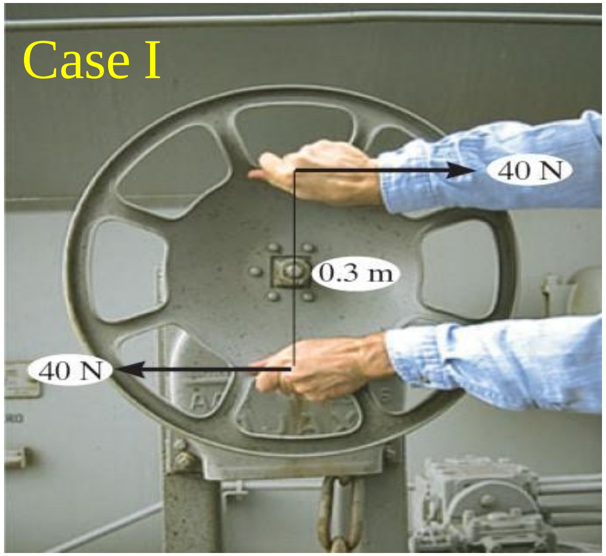
Case II: Couple Moment produced by 30 N forces = 12 Nm

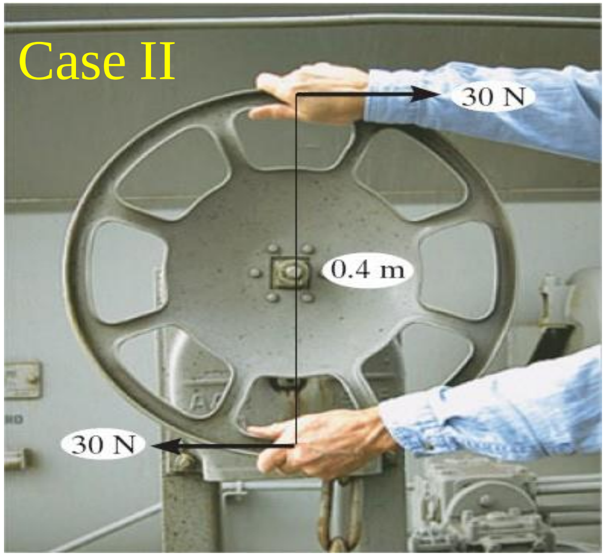
If only one hand is used?

Force required for case I is 80N Force required for case II is 60N

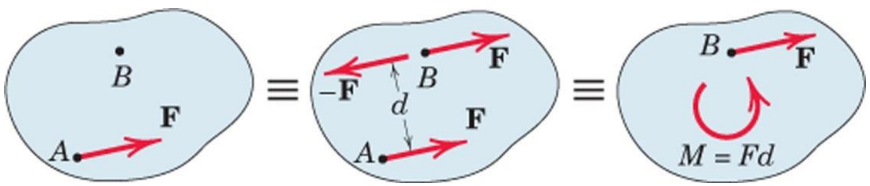
What if the shaft is not connected at the center of the wheel?

Is it a Free Vector?





Equivalent Systems

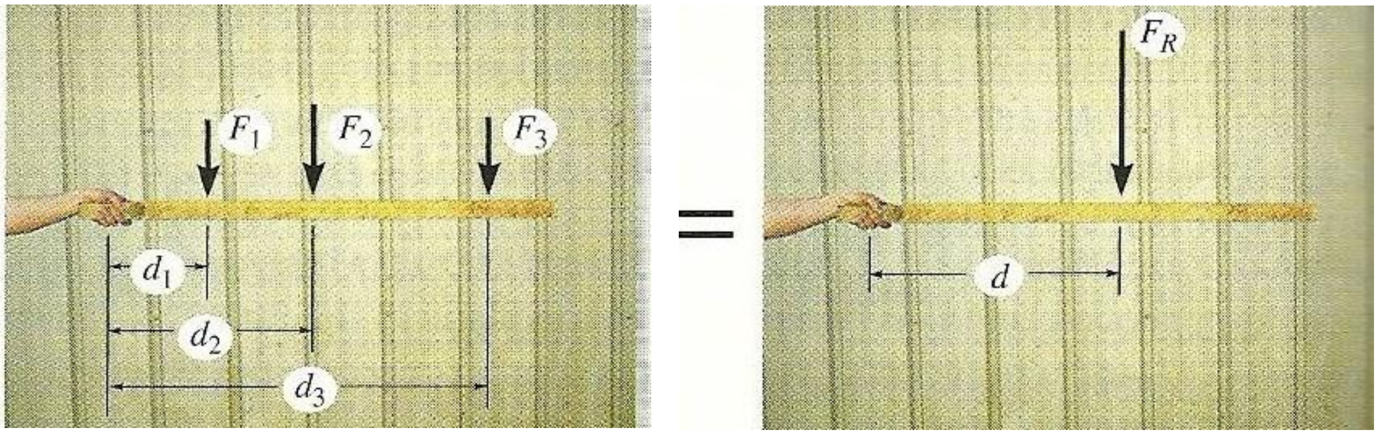


At support O

$$W_r = W_1 + W_2$$

$$M_o = W_1 d_1 + W_2 d_2$$

Equivalent Systems:



$F_R = F_1 + F_2 + F_3$

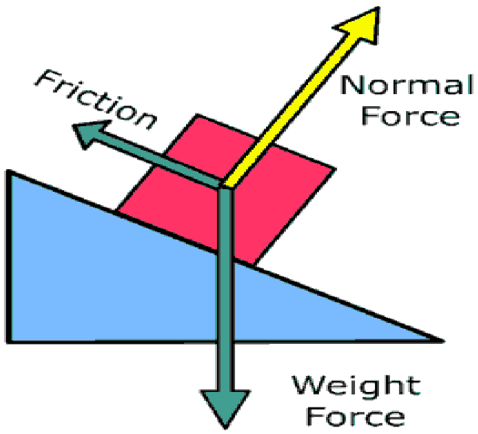
What is the value of d?

Moment of the Resultant force about the grip must be equal to the moment of the forces about the grip

$$\boxed{\mathbf{F_R d = F_1 d_1 + F_2 d_2 + F_3 d_3}}$$

Equilibrium Conditions

Friction



Usual Assumption till now:

Forces of action and reaction between contacting surfaces act normal to the surface

→valid for interaction between smooth surfaces

→in many cases ability of contacting surfaces to support tangential forces is very important (Ex: Figure above)

Frictional Forces

Tangential forces generated between contacting surfaces

- occur in the interaction between all real surfaces
- always act in a direction opposite to the direction of motion

Frictional forces are Not Desired in some cases:

- Bearings, power screws, gears, flow of fluids in pipes, propulsion of aircraft and missiles through the atmosphere, etc.
  - Friction often results in a loss of energy, which is dissipated in the form of heat
  - Friction causes Wear

Frictional forces are Desired in some cases:

- Brakes, clutches, belt drives, wedges
- walking depends on friction between the shoe and the ground

Ideal Machine/Process: Friction small enough to be neglected

Real Machine/Process: Friction must be taken into account

Types of Friction

- Dry Friction (Coulomb Friction)
  - occurs between unlubricated surfaces of two solids
  - Effects of dry friction acting on exterior surfaces of rigid bodies
- Fluid Friction
  - occurs when adjacent layers in a fluid (liquid or gas) move at a different velocities. Fluid friction also depends on viscosity of the fluid. → Fluid Mechanics
- Internal Friction
  - occurs in all solid materials subjected to cyclic loading, especially in those materials, which have low limits of elasticity → Material Science

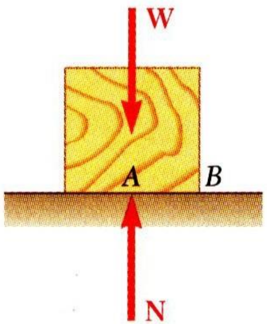
Mechanism of Dry Friction

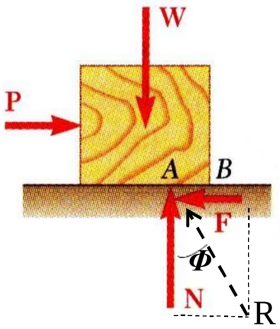
- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.
- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. *F is a Static-Friction force.*
- As P increases, static-friction force F increases as well until it reaches a maximum value F_m.

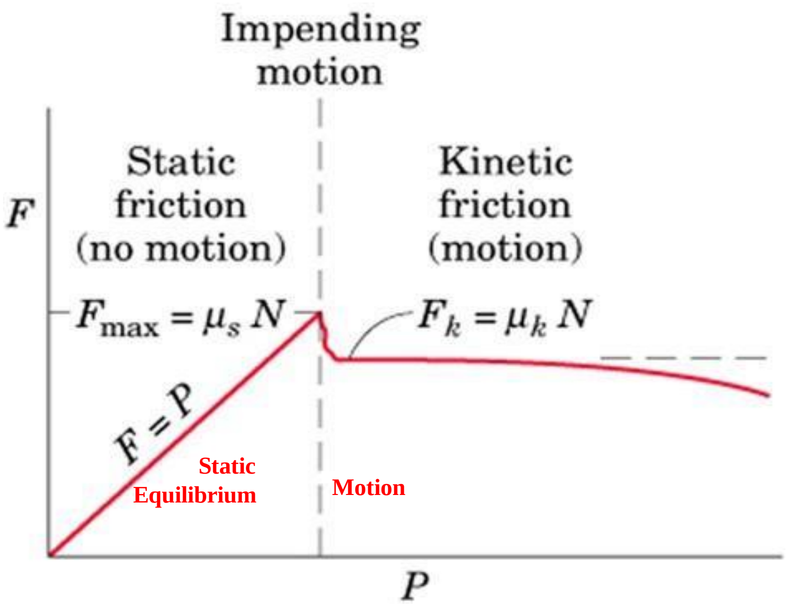
$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller *Kinetic-Friction force F_k*.

$$F_k = \mu_k N$$







μ_s is the Coefficient of Static Friction

μ_k is the Coefficient of Kinetic Friction

Mechanism of Dry Friction

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

$$F_k = \mu_k N$$

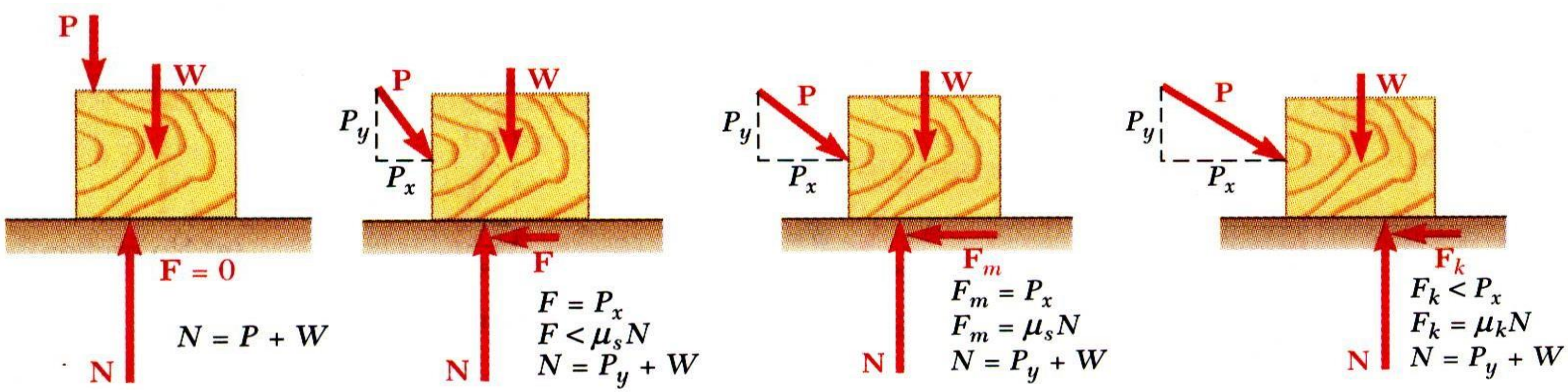
$$\mu_k \cong 0.75 \mu_s$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

- **Force necessary to maintain motion is generally less than that required to start the block when the surface irregularities are more nearly in mesh** $\rightarrow \boldsymbol{F_m > F_k}$

Mechanism of Dry Friction

- Four situations can occur when a rigid body is in contact with a horizontal surface:



No friction, ($P_x = 0$)

Equations of Equilibrium Valid

No motion, ($P_x < F_m$)

Equations of Equilibrium Valid

Motion impending, ($P_x = F_m$)

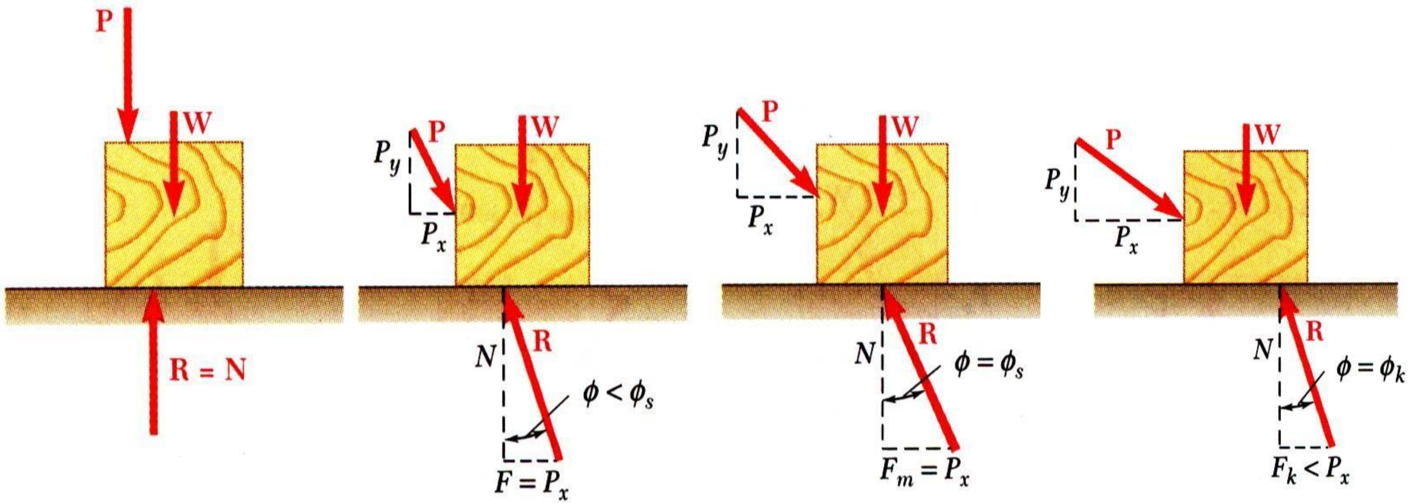
Equations of Equilibrium

Valid

Motion, ($P_x > F_m$)

Equations of Equilibrium Not Valid

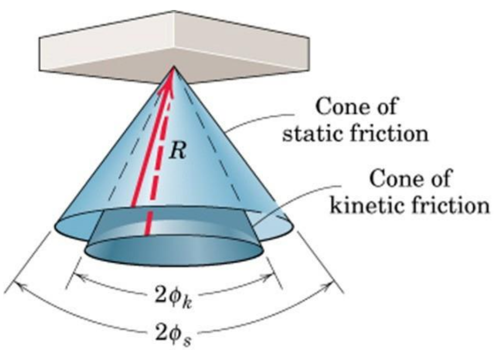
Sometimes it is convenient to replace normal force N & friction force F by their resultant R:



No friction | No motion | Motion impending | Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N} \qquad \tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\,\phi_s = \mu_s \qquad \tan\,\phi_k = \mu_k$$

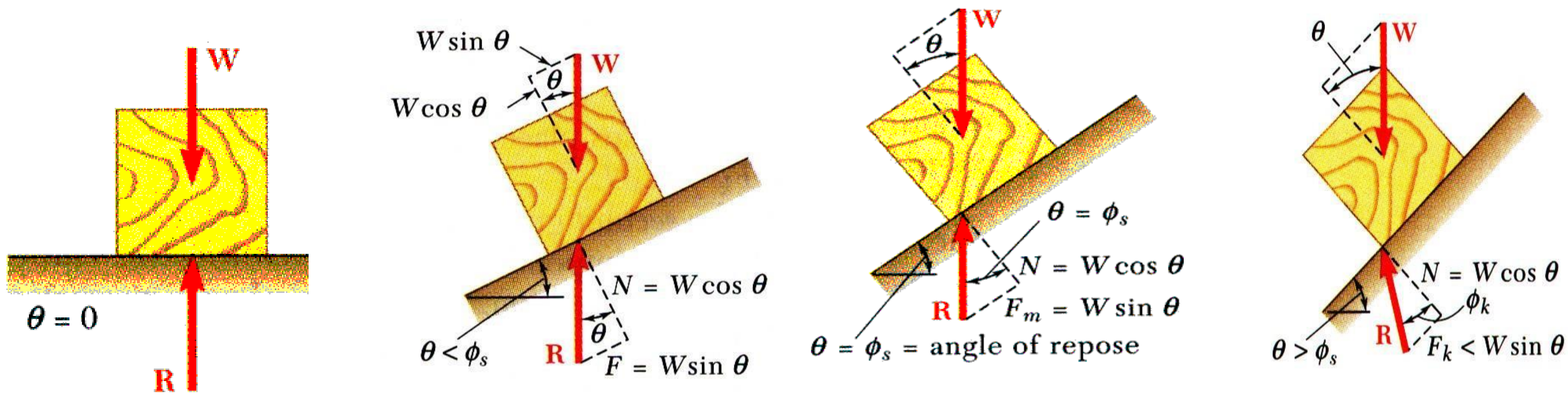


vertex angle

Friction Angles

ϕ_s = angle of static friction, ϕ_k = angle of kinetic friction

- Consider block of weight W resting on board with variable inclination angle θ



No friction

No motion

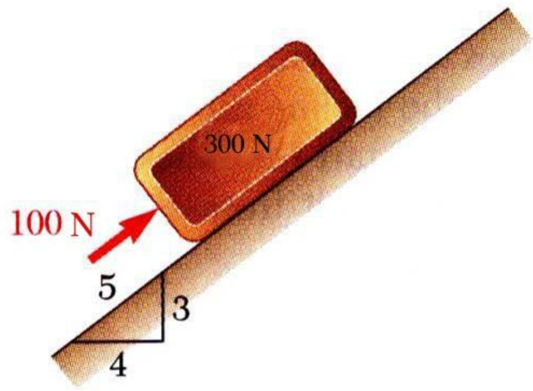
Impending motion

Angle of Repose =
Angle of Static Friction

Motion

The reaction R is not vertical anymore, and the forces acting on the block are unbalanced

Example

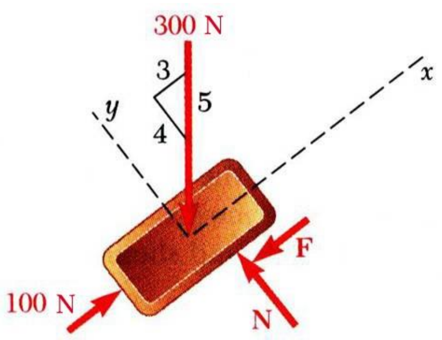


A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are μ_σ= 0.25 and μ_κ= 0.20.
Determine whether the block is in equilibrium and find the value of the friction force.

<u>SOLUTION</u>:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic- friction force.

Dry Friction



SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \quad 100\ \text{N} - \tfrac{3}{5}(300\ \text{N}) - F = 0$$

$$F = -80\ \text{N} \quad \rightarrow F \text{ acting upwards}$$

$$\sum F_y = 0: \quad N - \tfrac{4}{5}(300\ \text{N}) = 0$$

$$N = 240\ \text{N}$$

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

$$F_m = \mu_s N \qquad F_m = 0.25(240\ \text{N}) = 60\ \text{N}$$

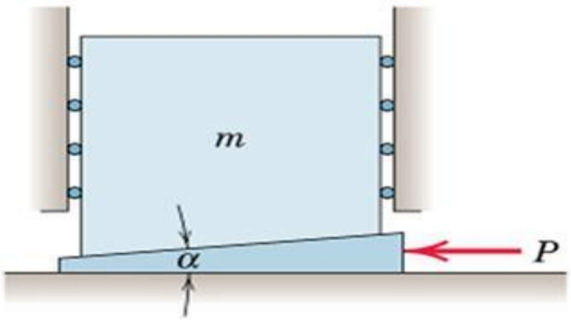
The block will slide down the plane along F.

- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k \text{N}$$
$$= 0.20(240\ \text{N})$$
$$F_{actual} = 48\ \text{N}$$

Wedges

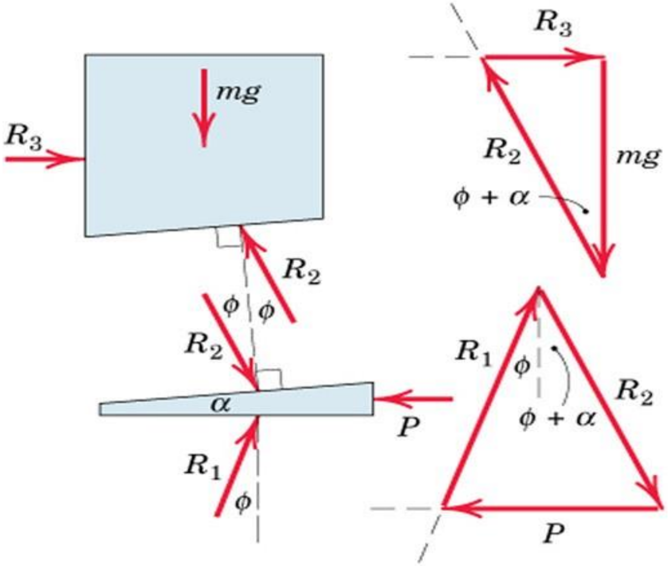
- Simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force *P* to raise block.



Coefficient of Friction for each pair of surfaces $\mu = \tan\phi$ (Static/Kinetic)

FBDs:

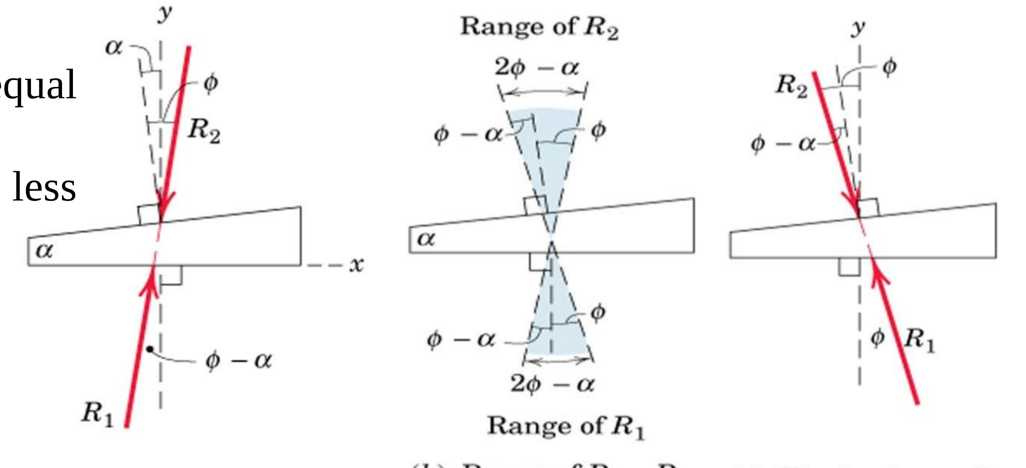
- Reactions are inclined at an angle Φ from their respective normals and are in the direction opposite to the motion.
- Force vectors acting on each body can also be shown.
- R_2 is first found from upper diagram since
- *mg* is known.
- Then *P* can be found out from the lower diagram since R_2 is known.



Forces to raise load

P is removed and wedge remains in place

→ Equilibrium of wedge requires that the equal reactions R_1 and R_2 be collinear
→ In the figure, wedge angle α is taken to be less than ϕ



(a) Slipping impending at upper surface
(b) Range of $R_1 = R_2$ for no slip
(c) Slipping impending at lower surface

Impending slippage at the upper surface

Impending slippage at the lower surface

Slippage must occur at both surfaces simultaneously

In order for the wedge to slide out of its space

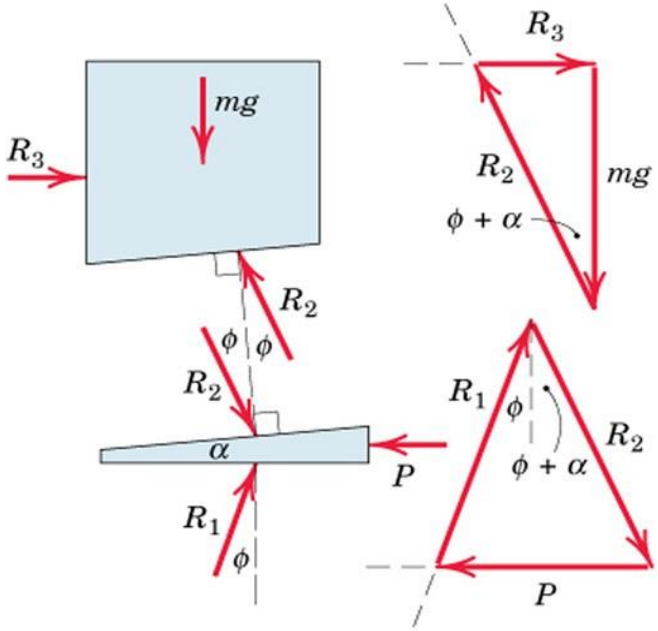
→ Else, the wedge is Self-Locking

Range of angular positions of R1 and R2 for which the wedge will remain in place is shown in figure (b)

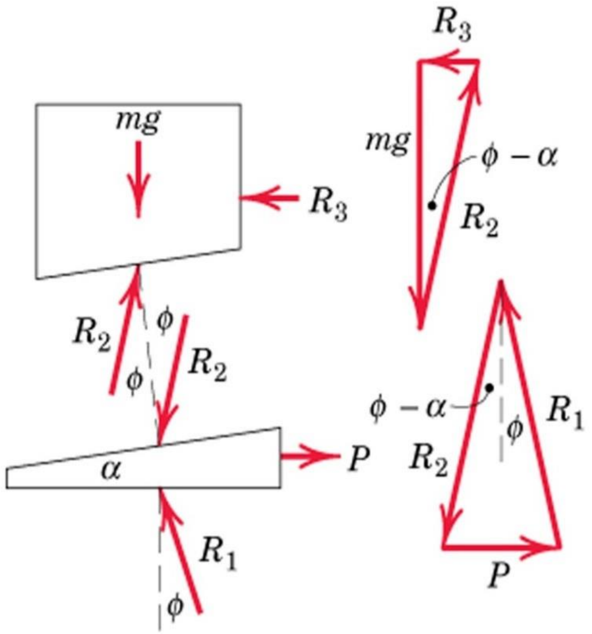
Simultaneous slippage is not possible if $\alpha < 2\phi$

A pull P is required on the wedge for withdrawal of the wedge

- The reactions R_1 and R_2 must act on the opposite sides of their normal from those when the wedge was inserted
- Solution by drawing FBDs and vector polygons
- Graphical solution
- Algebraic solutions from trigonometry



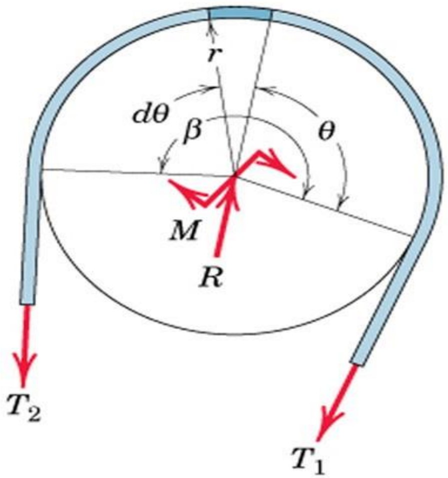
Forces to raise load



Forces to lower load

Belt Friction

- Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums
- It is necessary to estimate the frictional forces developed between the belt and its contacting surface.



- Consider a drum subjected to two belt tensions (T_1 and T_2) M is the torque necessary to prevent rotation of the drum R is the bearing reaction, r is the radius of the drum, β is the total contact angle between belt and surface (β in radians)

- $T_2 > T_1$ since M is clockwise