Hydraulic Machines BME-51

Unit-1

(Lecture 1)

Lecture contains

- Introduction
- Impulse Momentum Principle
- Application of momentum and momentum

Introduction

Analysis and Design of Hydraulic Machines (Turbines and Pumps) is essentially based on the knowledge of forces exerted on or by the moving fluids.

Learning Objective:

Evaluation of force, both in magnitude and direction, by free jets (constant pressure throughout) when they impinge upon stationary or moving objects such as flat plates and vanes of different shapes and orientation.

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Unit-1(Lecture 1)

IMPULSE-MOMENTUM EQUATIONS:-

The impulse-momentum equations are derived from the impulse-momentum principle (or simply momentum principle) which states that the impulse exerted on any body is equal to the resulting change in momentum of the body. In other words, this principle is a modified form of Newton's second law of motion. Newton's second law of motion states that the resultant external force acting on any body in any direction is equal to the rate of change of momentum of the body in that direction. Thus for any arbitrarily chosen direction x, it may be expressed as,

$$F_x = d(M_x)/dt \qquad \text{... (1)}$$

In which F_x represents the resultant external force in the x-direction and M_x represents the momentum in the x-direction. Equation 1 may also be written as

$$F_x(dt) = d(M_x) \qquad \text{...(2)}$$

In Eq. 2 the term $F_x(dt)$ is impulse and the term $d(M_x)$ is the resulting change of momentum. Equation 2 is thus known as impulse momentum equation. The impulse-momentum relationship in the form as indicated by Eq. 2 is, however, applicable to finite or discrete bodies, for which the action of any force may take place and be completed in a finite period of time. On the other hand, steady flow of fluid involves a motion which is continuous and it is not completed in a finite period of time. Therefore the momentum equation has to be expressed in a form particularly suited to the solution of fluid flow Problems as explained below.

IMPULSE-MOMENTUM EQUATIONS cont….

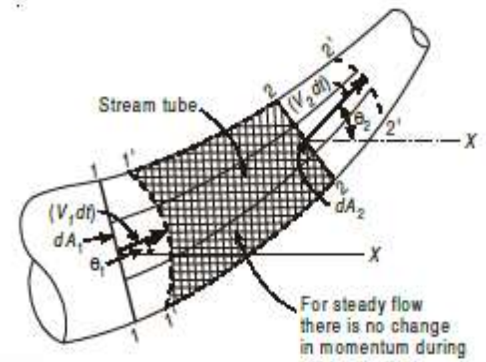

Figure 1 *Change of momentum of fluid mass in flow passage*

Consider as a free-body the fluid mass included between sections 1–1 and 2–2 within a certain flow passage as shown in Fig. 1. The fluid mass of the free body 1–1 and 2–2 at time t moves to a new position 1′–1′ and 2′–2′ at time $(t + dt)$. Section 1′–1′ and 2′–2′ are curved because the velocities of flow at these two sections are non-uniform.

For steady flow the following continuity equation holds:

$$\begin{bmatrix} \text{Fluid mass with in} \\ \text{sections 1–1 and 1'–1'} \end{bmatrix} = \begin{bmatrix} \text{Fluid mass with in} \\ \text{sections 2 – 2 and 2'–2'} \end{bmatrix}$$

Further for an arbitrary direction x the change in momentum of this mass of fluid during a time interval dt is considered, which may be represented as follows:

$$\begin{bmatrix} \text{Change in momentum} \\ \text{of fluid mass} \\ \text{during } dt \end{bmatrix}_x = \begin{bmatrix} \text{Momentum at} \\ (t+dt) \text{ of fluid} \\ \text{1'–1' and 2'–2'} \end{bmatrix}_x - \begin{bmatrix} \text{Momentum at} \\ t \text{ of fluid} \\ \text{1 – 1 and 2–2} \end{bmatrix}_x$$

But

$$\begin{bmatrix} \text{Momentum at} \\ (t+dt) \text{ of fluid} \\ \text{1'–1' and 2'–2'} \end{bmatrix}_x = \begin{bmatrix} \text{Momentum at} \\ (t+dt) \text{ of fluid} \\ \text{1'–1' and 2–2} \end{bmatrix}_x + \begin{bmatrix} \text{Momentum at} \\ (t+dt) \text{ of fluid} \\ \text{2 – 2 and 2'– 2'} \end{bmatrix}_x$$

and

$$\begin{bmatrix} \text{Momentum at} \\ t \text{ of fluid} \\ \text{1–1 and 2 – 2} \end{bmatrix}_x = \begin{bmatrix} \text{Momentum at} \\ t \text{ of fluid} \\ \text{1–1 and 1'– 1'} \end{bmatrix}_x + \begin{bmatrix} \text{Momentum at} \\ t \text{ of fluid} \\ \text{1'– 1' and 2–2} \end{bmatrix}_x$$

Moreover, when the flow is steady, the state of the flowing fluid in the flow passage within sections 1′–1′ and 2–2 remains unchanged at all times.

IMPULSE-MOMENTUM EQUATIONS cont….

Therefore

$$\begin{bmatrix}\text{Momentum at}\\ (t+dt)\text{ of fluid}\\ 1'-1' \text{ and } 2-2\end{bmatrix}_x = \begin{bmatrix}\text{Momentum at}\\ t\text{ of fluid}\\ 1'-1' \text{ and } 2-2\end{bmatrix}_x$$

From which it follows that

$$\begin{bmatrix}\text{Change in momentum}\\ \text{of fluid mass}\\ \text{during } dt\end{bmatrix}_x = \begin{bmatrix}\text{Momentum at}\\ (t+dt)\text{ of fluid}\\ 2-2 \text{ and } 2'-2'\end{bmatrix}_x - \begin{bmatrix}\text{Momentum at}\\ t\text{ of fluid}\\ 1-1 \text{ and } 1'-1'\end{bmatrix}_x$$

The above relationship when expressed in terms of mathematical symbols, it becomes

$$\sum d(mv_x) = \int_{A_2} \rho_2 v_2 dt\, dA_2 (v_2)_x - \int_{A_1} \rho_1 v_1\, dt\, dA_1 (v_1)_x$$

where $(\rho_2 v_2\ dt\ dA_2)$ and $(\rho_1 v_1\ dt\ dA_1)$ represent the mass of flow of fluid during the time interval dt in a stream tube across sections 2–2 and 1–1 respectively as shown in Fig. 1.

Further according to Newton's second law of motion (Eq. 1), the relationship between the resultant external force and the time rate of change of momentum of the fluid flow in the passage may be written in the following form:

$$\Sigma F_x = \frac{\Sigma d(mv_x)}{dt}$$

$$= \int_{A_2} \rho_2 v_2\, dA_2 (v_2 \cos\theta_2) - \int_{A_1} \rho_1 v_1\, dA_1 (v_1 \cos\theta_1) \quad \text{...(3)}$$

in which $v_2 \cos\theta_2 = (v_2)_x$ and $v_1 \cos\theta_1 = (v_1)_x$ (see Fig. 1). Equation 3 may be integrated if the velocity distributions of fluid flow at both sections are known. Since in most of the problems of fluid flow we have to deal with only the mean velocity of flow at each section, it is preferable to express the impulse-momentum equation in terms of the mean velocities. Thus if V_1 and V_2 are the mean velocities at sections 1–1 and 2–2 respectively, then the impulse-momentum Eq. 3 may be written as

$$\Sigma F_x = \rho_2 A_2 \cos\theta_2 V_2^2 - \rho_1 A_1 \cos\theta_1 V_1^2 \quad \text{...(4)}$$

IMPULSE-MOMENTUM EQUATIONS cont….

For a steady flow of incompressible fluid, the impulse-momentum equation for fluid flow may be simplified to the form noted below. The continuity equation for such a flow may be expressed as $Q = A_1V_1 = A_2V_2$ and $\rho_1 = \rho_2 = \rho$. Thus introducing these expressions in Eq. 4 it becomes

or

$$\left.\begin{aligned}\sum F_x &= \rho Q(V_2\cos\theta_2 - V_1\cos\theta_1) \\ \sum F_x &= \rho Q[(V_2)_x - (V_1)_x]\end{aligned}\right\} \qquad \ldots(5)$$

in which suffix x is introduced to represent the components of the velocities in the x-direction. The term ΣF_x should include all the external forces acting on the free-body of the fluid under consideration.

If D'Alembert's principle is applied to the flow system, the system is brought into relative static equilibrium with the inclusion of inertia forces. The resulting impulse-momentum equation takes the following form :

$$\Sigma Fx - \rho Q\ (V_2)_x + \rho Q(V_1)_x = 0 \qquad \ldots(6)$$

The inertia force (ρQV) in fluid flow is usually called the momentum flux.

It may however be noted that the impulse-momentum equation given above has been derived for one direction only, but the same method may be extended to derive the corresponding equations for the other directions of reference as well. Accordingly the impulse-momentum equations for y and z directions may be written as

$$\Sigma F_y = \rho Q\,[(V_2)_y - (V_1)_y] \qquad \ldots(7)$$

$$\Sigma F_z = \rho Q\,[(V_2)_z - (V_1)_z] \qquad \ldots(8)$$

Further the general impulse-momentum equation for steady flow of fluid may be written in a vector form as

$$\Sigma F \rightarrow \rho QV_2 + \rightarrow \rho QV_1 = 0 \qquad \ldots(9)$$

The impulse-momentum equations are often called simply momentum equations. From these equations it may be noted that if the resultant external force that acts on the fluid mass is zero, the momentum of the fluid mass remains constant. This principle is known as the *law of conservation of momentum.*

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Unit-1(Lecture 1)

Application of momentum and momentum:-

- The impulse-momentum equation, together with the energy equation and the continuity equation provides the basic mathematical relationships for solving various engineering problems in fluid mechanics.
- Since the impulse-momentum equation relates the resultant external forces on a chosen free-body of fluid or control volume in a flow passage, to the change of momentum flux at the two end sections.
- It is especially valuable in solving those problems in fluid mechanics in which detailed information about the flow process within the control volume may be either not available or rather difficult to evaluate.
- Thus in order to apply the impulse-momentum equation, a control volume is first chosen which includes the portion of the flow passage which is to be studied. The boundaries of the control volume are usually extended upto such an extent that its end sections lie in the region of uniform flow.
- In general the impulse-momentum equation is used to determine the resultant forces exerted on the boundaries of a flow passage by a stream of flowing fluid as the flow changes its direction or the magnitude of velocity or both. The problems of this type include the pipe bend, jet propulsion, propellers and stationary and moving plates or vanes.

(Lecture 2)

Lecture contains

- Introduction to hydro electric power plants
- Major components
- Efficiencies

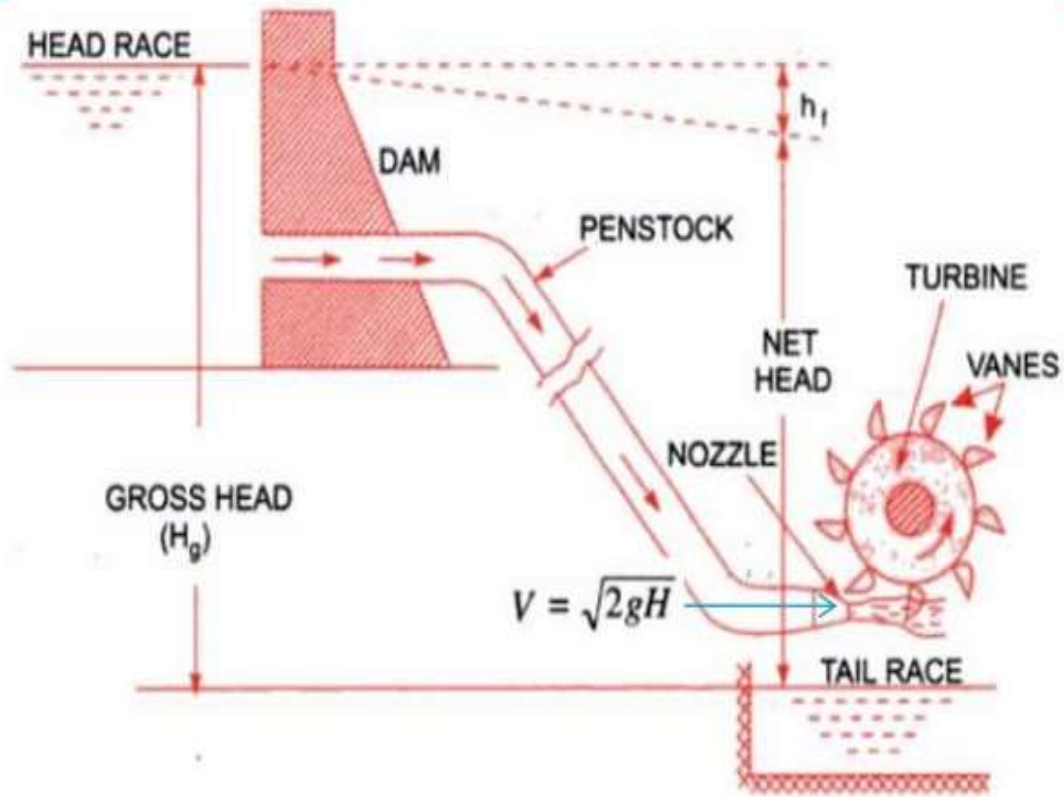
Layout of Hydropower Installation

H_g = Gross Head
h_f = Head Loss due to Friction

$$= \frac{4 \times f \times L \times V^2}{D \times 2g}$$

Where
V = Velocity of Flow in Penstock
L = Length of Penstock
D = Dia. of Penstock

H = Net Head
$= H_g - h_f$



Layout of a hydro-eletric power plant.

Efficiencies of Turbine

1. Hydraulic Efficiency $$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

2. Mechanical Efficiency $$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}}$$

3. Volumetric Efficiency $$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

4. Overall Efficiency $$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}} = \frac{\text{S.P.}}{\text{W.P.}} \times \frac{\text{R.P.}}{\text{R.P.}} = \frac{\text{S.P.}}{\text{R.P.}} \times \frac{\text{R.P.}}{\text{W.P.}} \qquad \left(\because \quad \frac{\text{S.P.}}{\text{R.P.}} = \eta_m \quad \text{and} \quad \frac{\text{R.P.}}{\text{W.P.}} = \eta_h \right)$$

$$= \eta_m \times \eta_h$$

(Lecture 3)

Lecture contains

Impact of Free Jets

Force exerted by the jet on a stationary plate

Impact of Jets

The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the wheel

1. **Force exerted by the jet on a stationary plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved
2. **Force exerted by the jet on a moving plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved

Impulse-Momentum Principle

From Newton's 2nd Law:

$$F = m\,a = m\,(V_1 - V_2)\,/\,t$$

Impulse of a force is given by the change in momentum caused by the force on the body.

$$Ft = mV_1 - mV_2 = \text{Initial Momentum} - \text{Final Momentum}$$

Force exerted by jet on the plate in the direction of jet, $F = m\,(V_1 - V_2)\,/\,t$

$$= (\text{Mass / Time})\,(\text{Initial Velocity} - \text{Final Velocity})$$

$$= (\rho Q)\,(V_1 - V_2) = (\rho a V)\,(V_1 - V_2)$$

Impact of Free Jets cont…

1 FORCE EXERTED BY THE JET ON A STATIONARY VERTICAL PLATE

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in Fig. 1

Let V = velocity of the jet, d = diameter of the jet,

a = area of cross-section of the jet = $\frac{\pi}{4} d^2$.

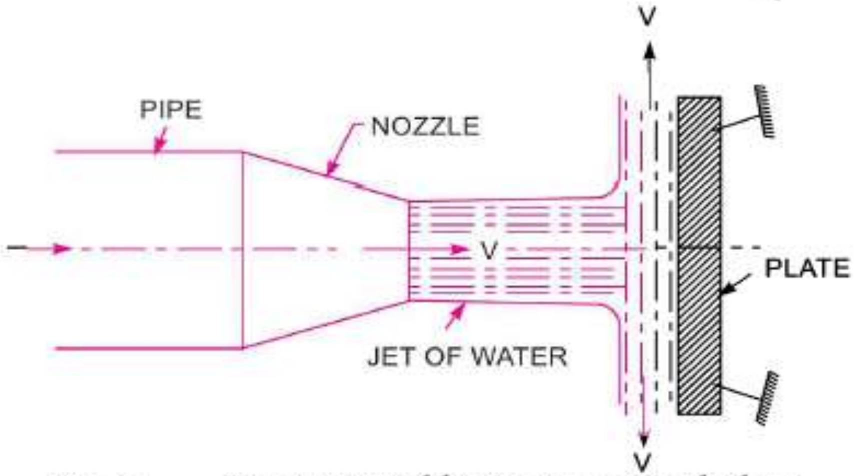


Fig. 1 *Force exerted by jet on vertical plate.*

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking, will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

Impact of Free Jets cont…

The force exerted by the jet on the plate in the direction of jet,

$$F_x = \text{Rate of change of momentum in the direction of force}$$

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{Mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a V[V - 0] \qquad (\because \quad \text{mass/sec} = \rho \times a\, V)$$

$$= \rho a V^2$$

For deriving above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet is to be calculated then final minus initial velocity is taken. But if the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is taken.

2. Force Exerted by a Jet on Stationary Inclined Flat Plate. Let a jet of water, coming out from the nozzle, strikes an inclined flat plate as shown in Fig. 2.

Let V = Velocity of jet in the direction of x,
θ = Angle between the jet and plate,
a = Area of cross-section of the jet.

Then mass of water per sec striking the plate = $\rho \times aV$.

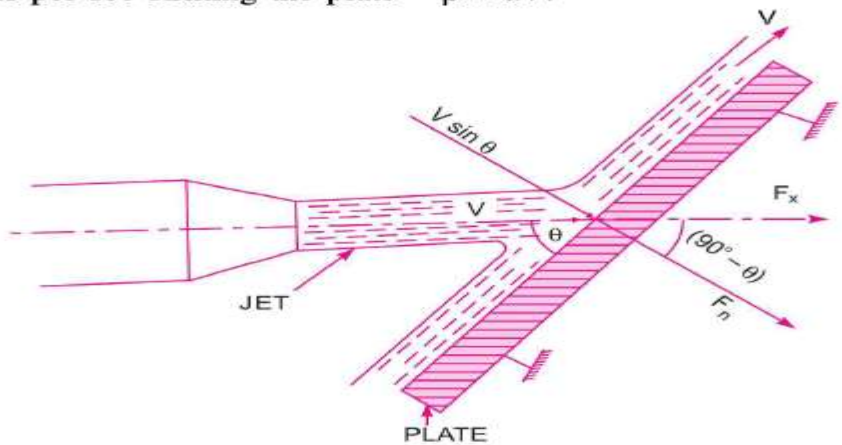


Fig. 2 *Jet striking stationary inclined plate.*

If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, then jet will move over the plate after striking with a velocity equal to initial velocity *i.e.*, with a velocity V. Let us find the force exerted by the jet on the plate in the direction normal to the plate. Let this force is represented by F_n

Then F_n = mass of jet striking per second
× [Initial velocity of jet before striking in the direction of n
– Final velocity of jet after striking in the direction of n]

$$= \rho aV\,[V \sin\theta - 0] = \rho a V^2 \sin\theta$$

This force can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of flow. Then we have,

F_x = component of F_n in the direction of flow

$$= F_n \cos(90° - \theta) = F_n \sin\theta = \rho A V^2 \sin\theta \times \sin\theta \quad (\therefore F_n = \rho a V^2 \sin\theta)$$

$$= \rho A V^2 \sin^2\theta$$

And, F_y = component of F_n, perpendicular to flow

$$= F_n \sin(90° - \theta) = F_n \cos\theta = \rho A V^2 \sin\theta \cos\theta.$$

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(Lecture 4)

Lecture contains

Impact of Free Jets

3 Force Exerted by a Jet on Stationary Curved Plate

(A) Jet strikes the curved plate at the centre. Let a jet of water strikes a fixed curved plate at the centre as shown in Fig. 3. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of the jet, in the tangential direction of the curved plate. The velocity at outlet of the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of the jet.

Component of velocity in the direction of jet = $-V\cos\theta$.

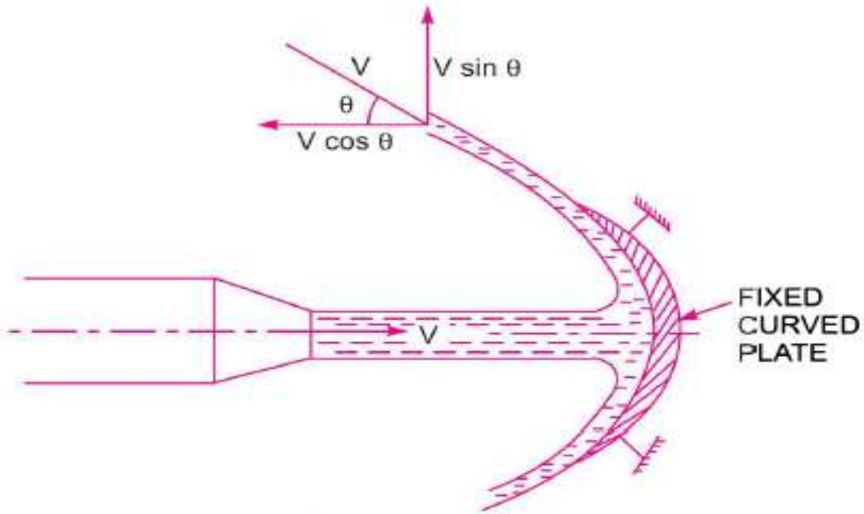

Fig. 3 *Jet striking a fixed curved plate at centre.*

(–ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle).

Component of velocity perpendicular to the jet = $V \sin\theta$

Force exerted by the jet in the direction of jet,

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity in the direction of jet = V

V_{2x} = Final velocity in the direction of jet = $-V\cos\theta$

Impact of Free Jets cont…

$\therefore$ $$F_x = \rho a V[V - (-V\cos\theta)] = \rho a V[V + V\cos\theta]$$
$$= \rho a V^2[1 + \cos\theta]$$

Similarly, $$F_y = \text{Mass per sec} \times [V_{1y} - V_{2y}]$$

where V_{1y} = Initial velocity in the direction of $y = 0$

V_{2y} = Final velocity in the direction of $y = V\sin\theta$

$\therefore$ $$F_y = \rho a V[0 - V\sin\theta] = -\rho a V^2 \sin\theta$$

–ve sign means that force is acting in the downward direction. In this case the angle of deflection of the jet $= (180° - \theta)$

Impact of Free Jets cont…

(B) Jet strikes the curved plate at one end tangentially when the plate is symmetrical. Let the jet strikes the curved fixed plate at one end tangentially as shown in Fig. 4. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = Velocity of jet of water,

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the directions of x and y are

$$F_x = (\text{mass/sec}) \times [V_{1x} - V_{2x}]$$
$$= \rho a V[V\cos\theta - (-V\cos\theta)]$$
$$= \rho a V[V\cos\theta + V\cos\theta]$$
$$= 2\rho a V^2 \cos\theta$$
$$F_y = \rho a V[V_{1y} - V_{2y}]$$
$$= \rho a V[V\sin\theta - V\sin\theta] = 0$$

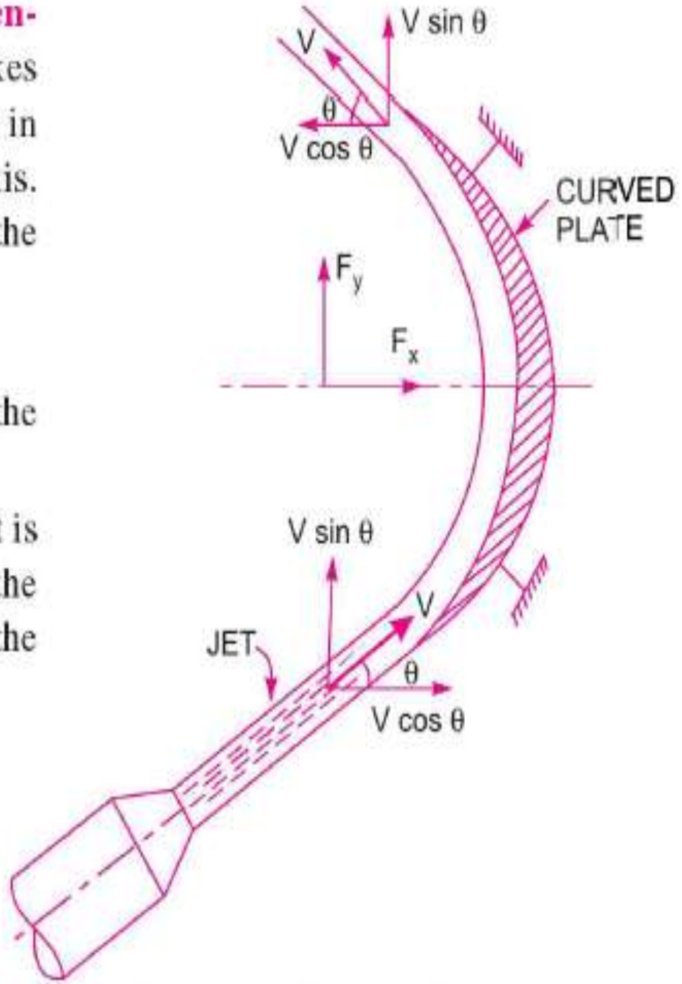

Fig. 4 *Jet striking curved fixed plate at one end.*

Impact of Free Jets cont…

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical. When the curved plate is unsymmetrical about x-axis, then angle made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis,

ϕ = angle made by tangent at outlet tip with x-axis.

The two components of the velocity at inlet are

$$V_{1x} = V \cos\theta \text{ and } V_{1y} = V \sin\theta$$

The two components of the velocity at outlet are

$$V_{2x} = -V \cos\phi \text{ and } V_{2y} = V \sin\phi$$

$\therefore$ The forces exerted by the jet of water in the directions of x and y are

$$F_x = \rho a V[V_{1x} - V_{2x}] = \rho a V[V\cos\theta - (-V\cos\phi)]$$

$$= \rho a V[V\cos\theta + V\cos\phi] = \rho a V^2 [\cos\theta + \cos\phi]$$

$$F_y = \rho a V[V_{1y} - V_{2y}] = \rho a V[V\sin\theta - V\sin\phi]$$

$$= \rho a V^2 [\sin\theta - \sin\phi].$$

Impact of Free Jets cont…

Problem 1 *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Problem *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Problem *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

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Unit-1

(Lecture 5)

Lecture contains

Impact of Free Jets

4 FORCE EXERTED BY A JET ON A HINGED PLATE

Consider a jet of water striking a vertical plate at the centre which is hinged at O. Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge as shown in Fig.

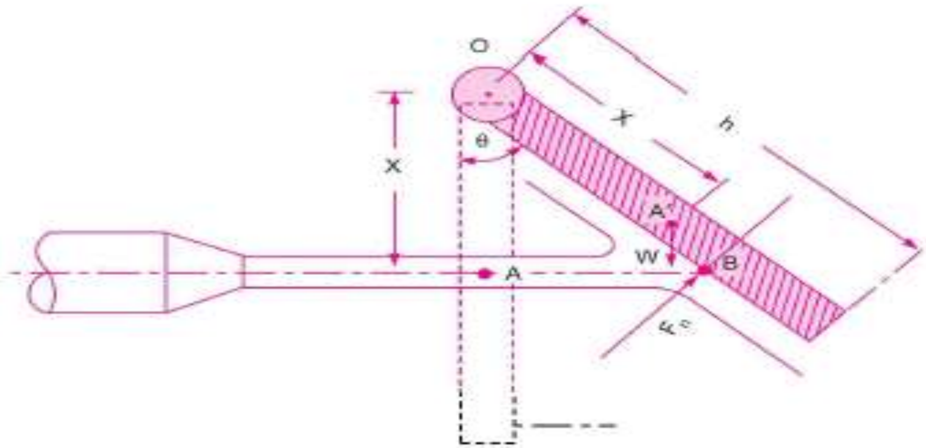


Fig. *Force on a hinged plate.*

Let x = distance of the centre of jet from hinge O,
θ = angle of swing about hinge,
W = weight of plate acting at C.G. of the plate.

The dotted lines show the position of the plate, before the jet strikes the plate. The point A on the plate will be at A' after the jet strikes the plate. The distance $OA = OA' = x$. Let the weight of the plate is acting at A'. When the plate is in equilibrium after the jet strikes the plate, the moment of all the forces about the hinge must be zero. Two forces are acting on the plate. They are :

1. Force due to jet of water, normal to the plate,

$$F_n = \rho a V^2 \sin \theta'$$

where θ' = Angle between jet and plate = $(90° - \theta)$

2. Weight of the plate, W

Moment of force F_n about hinge $= F_n \times OB = \rho a V^2 \sin(90° - \theta) \times OB = \rho a V^2 \cos\theta \times OB$

$$= \rho a V^2 \cos\theta \times \frac{OA}{\cos\theta} = \rho a V^2 \times OA = \rho a V^2 \times x$$

Moment of weight W about hinge $= W \times OA' \sin\theta = W \times x \times \sin\theta$

For equilibrium of the plate, $\rho a V^2 \times x = W \times x \times \sin\theta$

$$\therefore \quad \sin\theta = \frac{\rho a V^2}{W}$$

From equation , the angle of swing of the plate about hinge can be calculated.

Impact of Free Jets cont…

5 FORCE EXERTED BY A JET ON MOVING PLATES

The following cases of the moving plates will be considered :

1. Flat vertical plate moving in the direction of the jet and away from the jet,
2. Inclined plate moving in the direction of the jet, and
3. Curved plate moving in the direction of the jet or in the horizontal direction.

5.1 Force on Flat Vertical Plate Moving in the Direction of Jet.

Fig. shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the jet (absolute),

a = Area of cross-section of the jet,

u = Velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of the plate.

Hence relative velocity of the jet with respect to plate

$= (V - u)$

Mass of water striking the plate per sec

= ρ × Area of jet × Velocity with which jet strikes the plate

$= \rho a \times [V - u]$

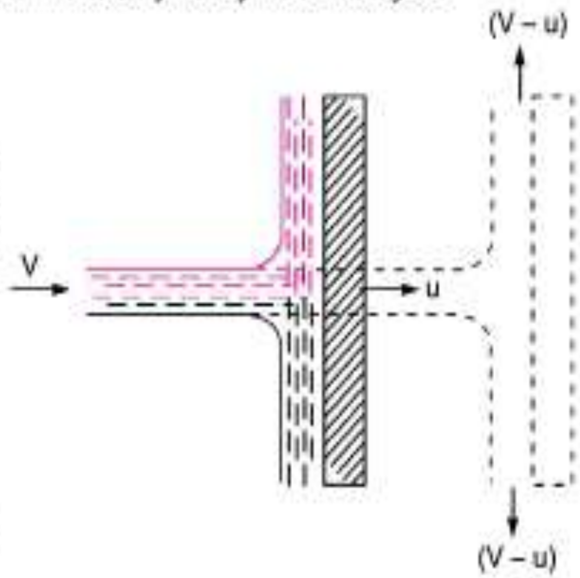


Fig. *Jet striking a flat vertical moving plate.*

∴ Force exerted by the jet on the moving plate in the direction of the jet,

$$F_x = \text{Mass of water striking per sec} \times [\text{Initial velocity with which water strikes} - \text{Final velocity}]$$

$$= \rho a(V-u)[(V-u)-0] \quad (\because \text{ Final velocity in the direction of jet is zero})$$

$$= \rho a(V-u)^2 \quad \ldots(1)$$

In this case, the work will be done by the jet on the plate, as plate is moving. For the stationary plates, the work done is zero.

∴ Work done per second by the jet on the plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force.}}{\text{Time}}$$

$$= F_x \times u = \rho a(V-u)^2 \times u \quad \ldots(2)$$

In equation (2), if the value of ρ for water is taken in S.I. units (*i.e.*, 1000 kg/m^3), the work done will be in N m/s. The term $\frac{\text{'Nm'}}{\text{s}}$ is equal to W (watt).

5.2 Force on the Inclined Plate Moving in the Direction of the Jet. Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in Fig.

Let V = Absolute velocity of jet of water,

u = Velocity of the plate in the direction of jet,

a = Cross-sectional area of jet, and

θ = Angle between jet and plate.

Relative velocity of jet of water = $(V-u)$

∴ The velocity with which jet strikes = $(V-u)$

Mass of water striking per second

$$= \rho \times a \times (V-u)$$

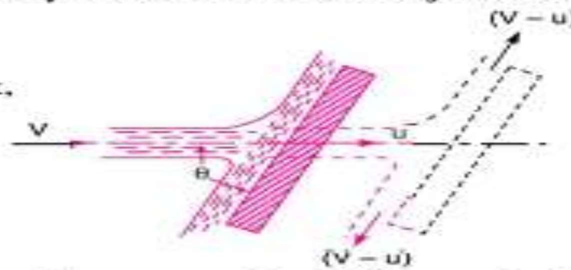


Fig. *Jet striking an inclined moving plate.*

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V-u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

$$F_n = \text{Mass striking per second} \times [\text{Initial velocity in the normal direction with which jet strikes} - \text{Final velocity}]$$

$$= \rho a(V-u)[(V-u)\sin\theta - 0] = \rho a(V-u)^2 \sin\theta \quad \ldots(3)$$

This normal force F_n is resolved into two components namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$\therefore \quad F_x = F_n \sin\theta = \rho a(V-u)^2 \sin^2\theta \quad \ldots(4)$$

$$F_y = F_n \cos\theta = \rho a(V-u)^2 \sin\theta \cos\theta \quad \ldots(5)$$

∴ Work done per second by the jet on the plate

$$= F_x \times \text{Distance per second in the direction of } x$$

$$= F_x \times u = \rho a(V-u)^2 \sin^2\theta \times u = \rho a(V-u)^2 u \sin^2\theta \text{ N m/s.} \quad \ldots(6)$$

5.3 Force on the Curved Plate when the Plate is Moving in the Direction of Jet. Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig.

Let V = Absolute velocity of jet,
a = Area of jet,
u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate = $(V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane = $(V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of the velocity in the direction of jet

$$= -(V - u)\cos\theta$$

(–ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet = $(V - u)\sin\theta$.

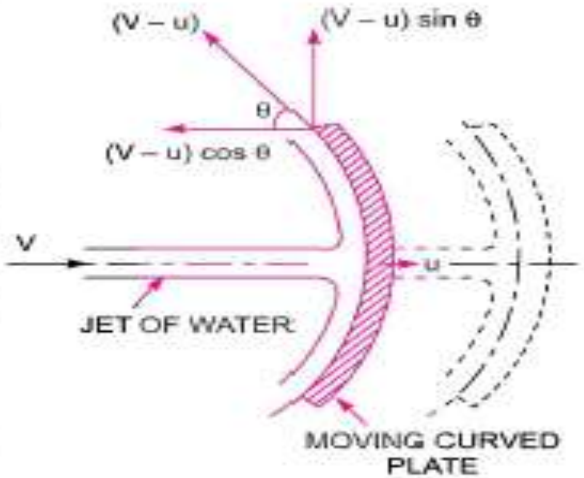


Fig. *Jet striking a curved moving plate.*

Mass of the water striking the plate = $\rho \times a \times$ Velocity with which jet strikes the plate

$$= \rho a(V - u)$$

∴ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$= \rho a(V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$
$$= \rho a(V - u)\,[(V - u) + (V - u)\cos\theta]$$
$$= \rho a(V - u)^2\,[1 + \cos\theta]$$

Work done by the jet on the plate per second

$$= F_x \times \text{Distance travelled per second in the direction of } x$$
$$= F_x \times u = \rho a(V - u)^2\,[1 + \cos\theta] \times u$$
$$= \rho a(V - u)^2 \times u\,[1 + \cos\theta]$$

Impact of Free Jets cont…

Problem *A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find :*

(i) the force on the plate,

(ii) the work done, and

(iii) the efficiency of jet.

Problem *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Problem *A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary , and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.*

Hydraulic Machines BME-51
Unit-1

(Lecture 6)

Lecture contains

Impact of Free Jets

5.4 Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the Tips. Fig. shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

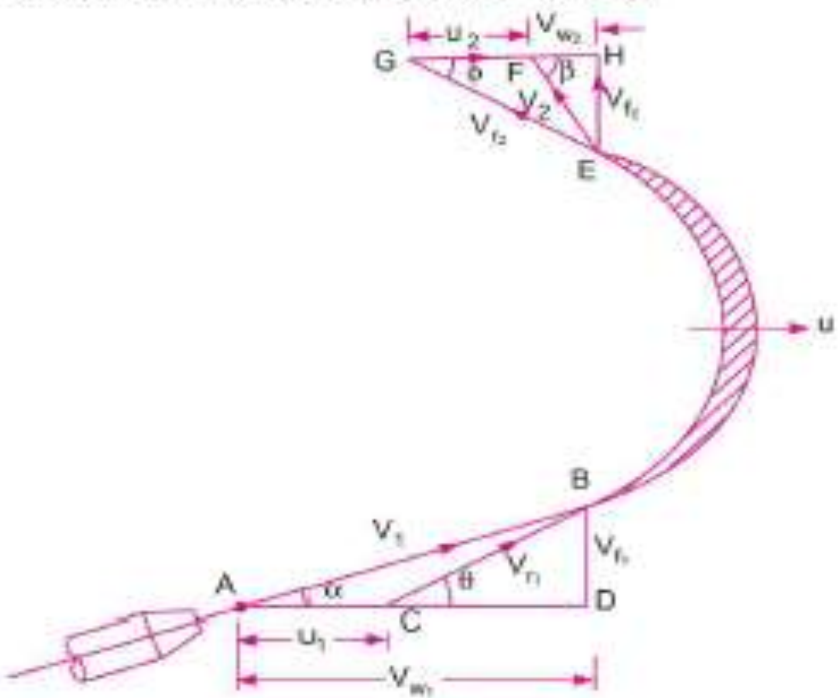


Fig. *Jet striking a moving curved vane at one of the tips.*

Impact of Free Jets cont…

Let V_1 = Velocity of the jet at inlet.

u_1 = Velocity of the plate (vane) at inlet.

V_{r_1} = Relative velocity of jet and plate at inlet.

α = Angle between the direction of the jet and direction of motion of the plate, also called guide blade angle.

θ = Angle made by the relative velocity (V_{r_1}) with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.

V_{w_1} = It is also known as velocity of whirl at inlet.

V_{f_1} = It is also known as velocity of flow at inlet.

V_2 = Velocity of the jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = Velocity of the vane at outlet.

V_{r_2} = Relative velocity of the jet with respect to the vane at outlet.

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet.

ϕ = Angle made by the relative velocity V_{r_2} with the direction of motion of the vane at outlet and also called vane angle at outlet.

V_{w_2} and V_{f_2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet.

V_{w_2} = It is also called the velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

Impact of Free Jets cont…

The triangles *ABD* and *EGH* are called the velocity triangles at inlet and outlet. These velocity triangles are drawn as given below :

1. Velocity Triangle at Inlet. Take any point *A* and draw a line $AB = V_1$ in magnitude and direction which means line *AB* makes an angle α with the horizontal line *AD*. Next draw a line $AC = u_1$ in magnitude. Join *C* to *B*. Then *CB* represents the relative velocity of the jet at inlet. If the loss of energy at inlet due to impact is zero, then *CB* must be in the tangential direction to the vane at inlet. From *B* draw a vertical line *BD* in the downward direction to meet the horizontal line *AC* produced at *D*.

Then BD = Represents the velocity of flow at inlet = V_{f_1}

AD = Represents the velocity of whirl at inlet = V_{w_1}

$\angle BCD$ = Vane angle at inlet = θ.

2. Velocity Triangle at Outlet. If the vane surface is assumed to be very smooth, the loss of energy due to friction will be zero. The water will be gliding over the surface of the vane with a relative velocity equal to V_{r_1} and will come out of the vane with a relative relocity V_{r_2}. This means that the relative velocity at outlet $V_{r_2} = V_{r_1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw *EG* in the tangential direction of the vane at outlet and cut $EG = V_{r_2}$. From *G*, draw a line *GF* in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet. Join *EF*. Then *EF* represents the absolute velocity of the jet at outlet in magnitude and direction. From *E* draw a vertical line *EH* to meet the line *GF* produced at *H*. Then

EH = Velocity of flow at outlet = V_{f_2}

FH = Velocity of whirl at outlet = V_{w_2}

ϕ = Angle of vane at outlet.

β = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and is having velocity in the direction of motion at inlet and outlet equal then we have

$u_1 = u_2 = u$ = Velocity of vane in the direction of motion and

$V_{r_1} = V_{r_2}$.

Now mass of water striking vane per sec = $\rho a V_{r_1}$ ...(*i*)

where a = Area of jet of water, V_{r_1} = Relative velocity at inlet.

$\therefore$ Force exerted by the jet in the direction of motion

F_x = Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion – Final velocity of jet in the direction of motion] ...(*ii*)

But initial velocity with which jet strikes the vane = V_{r_1}

The component of this velocity in the direction of motion

$$= V_{r_1} \cos\theta = (V_{w_1} - u_1) \qquad \text{(See Fig. \qquad)}$$

Similarly, the component of the relative velocity at outlet in the direction of motion = $-V_{r_2}\cos\phi$

$$= -[u_2 + V_{w_2}]$$

–ve sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Substituting the equation (*i*) and all above values of the velocities in equation (*ii*), we get

$$F_x = \rho a V_{r_1}[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}] = \rho a V_{r_1}[V_{w_1} - u_1 + u_2 + V_{w_2}]$$

$$= \rho a V_{r_1}[V_{w_1} + V_{w_2}] \qquad (\because u_1 = u_2) \quad ...(iii)$$

Equation (*iii*) is true only when angle β shown in Fig. 17.15 is an acute angle. If $\beta = 90°$, the $V_{w_2} = 0$, then equation (*iii*) becomes as,

$$F_x = \rho a V_{r_1}[V_{w_1}]$$

If β is an obtuse angle, the expression for F_x will become

$$F_x = \rho a V_{r_1}[V_{w_1} - V_{w_2}]$$

Thus in general, F_x is written as $F_x = \rho a V_{r_1}[V_{w_1} \pm V_{w_2}]$

Work done per second on the vane by the jet

$$= \text{Force} \times \text{Distance per second in the direction of force}$$

$$= F_x \times u = \rho a V_{r_1} \; [V_{w_1} \pm V_{w_2}] \times u$$

$\therefore$ Work done per second per unit weight of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Weight of fluid striking/s}} \frac{\text{Nm/s}}{\text{N/s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{g \times \rho a V_{r_1}} = \text{Nm/N}$$

$$= \frac{1}{g} \; [V_{w_1} \pm V_{w_2}] \times u \text{ Nm/N}$$

Work done/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\text{Mass of fluid striking / s}} \frac{\text{Nm / s}}{\text{kg / s}} = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\rho a V_{r_1}} \text{ Nm/kg}$$

$$= (V_{w_1} \pm V_{w_2}) \times u \text{ Nm/kg}$$

Note. Equation gives the work done per unit weight whereas equation gives the work done per unit mass.

3. Efficiency of Jet. The work done by the jet on the vane given by equation (17.20), is the output of the jet whereas the initial kinetic energy of the jet is the input. Hence, the efficiency (η) of the jet is expressed as

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second on the vane}}{\text{Initial K. E. per second of the jet}} = \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\frac{1}{2} m V_1^2}$$

where m = mass of the fluid per second in the jet = $\rho a V_1$

V_1 = initial velocity of jet

$$\therefore \qquad \eta = \frac{\rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2}.$$

Impact of Free Jets cont…

Problem *A jet of water of diameter 50 mm, having a velocity of 20 m/s strikes a curved vane which is moving with a velocity of 10 m/s in the direction of the jet. The jet leaves the vane at an angle of 60° to the direction of motion of vane at outlet. Determine :*

(i) The force exerted by the jet on the vane in the direction of motion.

(ii) Work done per second by the jet.

Problem *A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane an outlet. Calculate :*

(i) Vane angles, so that the water enters and leaves the vane without shock.

(ii) Work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.

(Lecture 7)

Lecture contains

Jet Propulsion

Jet Propulsion of ships

Jet Propulsion:-

Jet propulsion means the propulsion or movement of the bodies such as ships, aircrafts, rocket etc., with the help of jet. The reaction of the jet coming out from the orifice provided in the bodies is used to move the bodies.

The following cases are important where this principle is used :

(a) Jet propulsion of a tank with an orifice

(b) Jet propulsion of ships

Jet Propulsion of Ships. By the application of the jet propulsion principle, a ship is driven through water. A jet of water which is discharged at the back (also called stern) of the ship, exerts a propulsive force on the ship. The ship carries centrifugal pumps which draw water from the surrounding sea. This water is discharged through the orifice provided at the back of the ship in the form of a jet. The reaction of the jet coming out at the back of the ship propels the ship in the opposite direction of the jet. The water from the surrounding sea by the centrifugal pump is taken by the following two ways :

1. Through inlet orifices which are at right angles to the direction of the motion of the ship, and
2. Through the inlet orifices, which are facing the direction of motion of the ship.

1st Case. Jet propulsion of the ship when the inlet orifices are at right angles to the direction of the motion of the ship.

Fig. shows a ship which is having the inlet orifices at right angles to its direction.

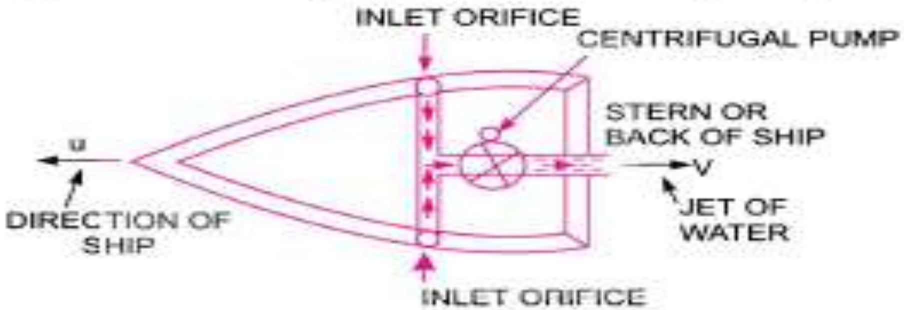


Fig. *Inlet orifices are at right angles.*

Let V = Absolute velocity of jet of water coming at the back of the ship,

u = Velocity of the ship,

V_r = Relative velocity of jet with respect to ship $= (V + u)$.

As the velocity V and u are in opposite direction and hence relative velocity will be equal to the sum of these two velocities.

Mass of water issuing from the orifice at the back of the ship $= \rho a V_r = \rho a(V + u)$,

where a = Area of the jet of water

$\therefore$ Propulsive force exerted on the ship

$$F = \text{Mass of water issuing per sec} \times \text{Change of velocity*}$$

$$= \rho a(V + u)\,[V_r - u] = \rho a(V + u)\,[(V + u) - u] = \rho a(V + u) \times V$$

Work done per second $= F \times u = \rho a\,(V + u) \times V \times u$

Note. (*i*) When the inlet orifices are at right angles to the direction of motion of the ship, then this case is also known as water is drawn **AMID SHIP** which means the water is drawn at the middle of the ship.

(*ii*) The centrifugal pump draws the water from the surrounding sea and discharges through orifice. The kinetic energy of the issuing jet is $\frac{1}{2} \times \text{mass} \times \text{velocity}^2$ *i.e.*, $\frac{1}{2}\,[\rho a(V + u)] \times [V + u]^2 = \frac{1}{2}\,\rho a(V + u)^3$. This energy is provided by centrifugal pump *i.e.*, work is done by pump to provide this energy.

2nd Case. Jet propulsion of ship when the inlet orifices face the direction of motion of the ship. Fig. shows a ship which is having the inlet orifices facing the direction of the motion of the ship. In this case the expression for propelling force and work done per second will be same as in the 1st case in which inlet orifices are at right angles to the ship. But the energy supplied by the jet will be different,

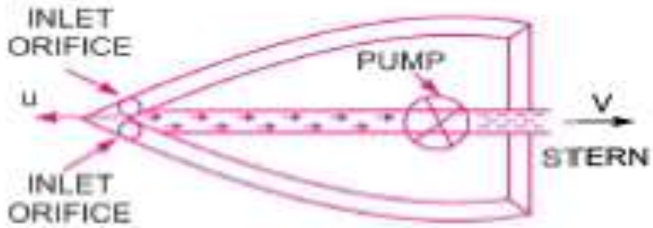


Fig. *Inlet orifices facing the direction of ship.*

as in this case the water enters with a velocity equal to the velocity of the ship, *i.e.*, with a velocity u. Hence the expression for the energy supplied by the jet.

$$= \frac{1}{2}\text{ (Mass of water supplied per sec)} \times [V_r^2 - u^2]$$

$$= \frac{1}{2}(\rho a V_r) \times [V_r^2 - u^2]$$

where $V_r = (V + u)$ as in the previous case

$\therefore$ K.E. supplied by jet $= \frac{1}{2}\rho a(V + u)\,[(V + u)^2 - u^2]$

$\therefore$ Efficiency of propulsion, $\eta = \dfrac{\text{Work done per sec by jet}}{\text{Energy supplied by jet}}$

$$= \frac{\rho a(V+u) \times V \times u}{\frac{1}{2}\rho a(V+u)\left[(V+u)^2 - u^2\right]}$$

$\{\because$ From equation work done $= \rho a\,(V + u)\,V \times u\}$

$$= \frac{2V \times u}{(V+u)^2 - u^2} = \frac{2Vu}{V^2 + u^2 + 2Vu - u^2} = \frac{2Vu}{V^2 + 2Vu} = \frac{2u}{V + 2u}$$

Hydraulic Machines BME-51
Unit-1

(Lecture 8)

Lecture contains

- Introduction
- Classification of turbines
- Impulse turbines
- Pelton wheel
- Constructional details,

Introduction

What is Hydraulic Machines?

- Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (P.E+K.E) or mechanical energy into hydraulic energy.

What is Turbine?

- The hydraulic machines, which converts the hydraulic energy into mechanical energy, are called turbines.
- This mechanical energy is used to in running an electric generator which is directly coupled to the shaft of the turbine.

Types of turbines

1. Steam Turbines
2. Gas Turbines (Combustion Turbines)
3. Hydraulic Turbines (Water Turbines)

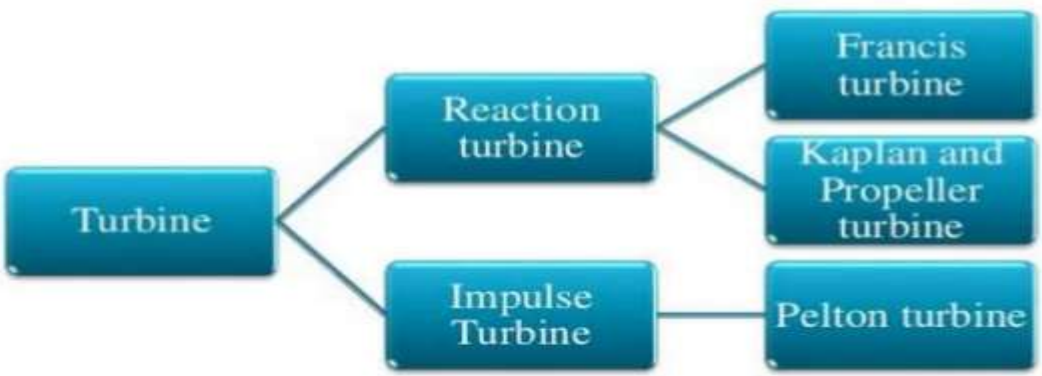
Hydraulic Turbine

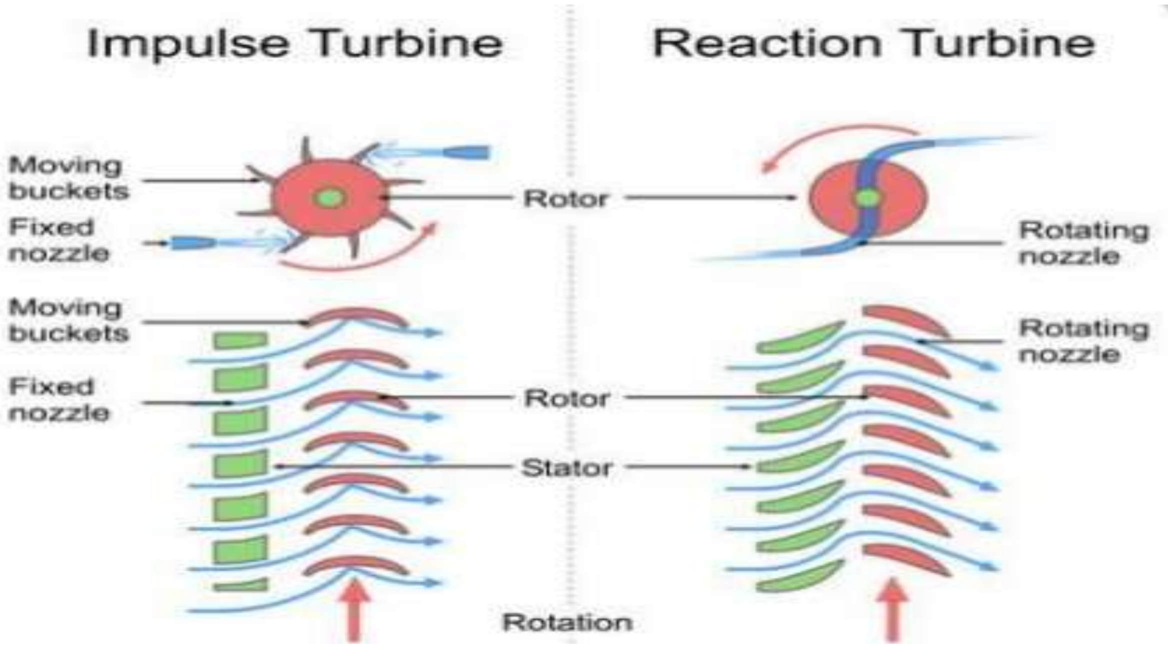
- The hydraulic machines, which converts hydro power (energy of water) into mechanical energy, are called Hydraulic Turbines.
- Mechanical energy is used in running an electric generator which is coupled to the turbine shaft.

Hydraulic Machines BME-51
Unit-1(Lecture 8)

Classification of Turbines

1. According to type of energy at Inlet
   a) Impulse Turbine - Pelton Wheel
      Requires High Head and Low Rate of Flow
   a) Reaction Turbine - Fancis, Kaplan
      Requires Low Head and High Rate of Flow
2. According to direction of flow through runner
   a) Tangential Flow Turbine - Pelton Wheel
   b) Radial Flow Turbine - Francis Turbine
   c) Axial Flow Turbine - Kaplan Turbine
   d) Mixed Flow Turbine - Modern Francis Turbine





Classification of Turbines

3. According to Head at Inlet of turbine
   a) High Head Turbine - Pelton Wheel
   b) Medium Head Turbine - Fancis Turbine
   c) Low Head Turbine - Kaplan Turbine
4. According to Specific Speed of Turbine
   a) Low Specific Speed Turbine - Pelton Wheel
   b) Medium Specific Speed Turbine - Fancis Turbine
   c) High Specific Speed Turbine - Kaplan Turbine

Classification according to Specific Speed of Turbines

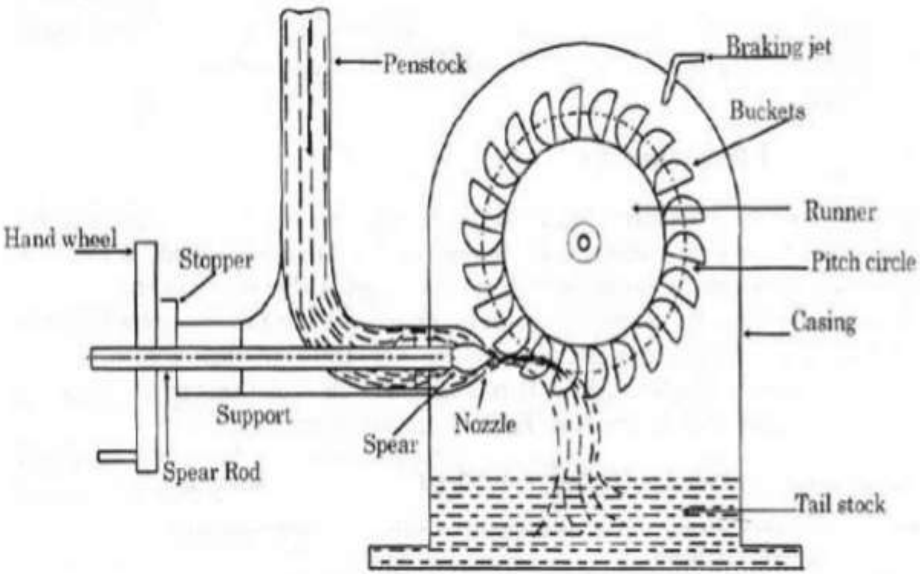
Type of turbine	Type of runner	Specific speed
Pelton	Slow Normal Fast	10 to 20 20 to 28 28 to 35
Francis	Slow Normal Fast	60 to 120 120 to 180 180 to 300
Kaplan	–	300 to 1000

Classification of Turbines

5. According to Disposition of Turbine Shaft
   a) Horizontal Shaft - Pelton Wheel
   b) Vertical Shaft - Fancis & Kaplan Turbines

Hydraulic Machines BME-51

Unit-1(Lecture 8) Pelton Wheel



PELTON WHEEL

PELTON WHEEL WITH MULTILE JETS

Hydraulic Machines BME-51
Unit-1

(Lecture 9)

Lecture contains

- **Velocity triangle and work done for Pelton Wheel**

- Design aspects

Velocity triangle and work done for Pelton Wheel:-

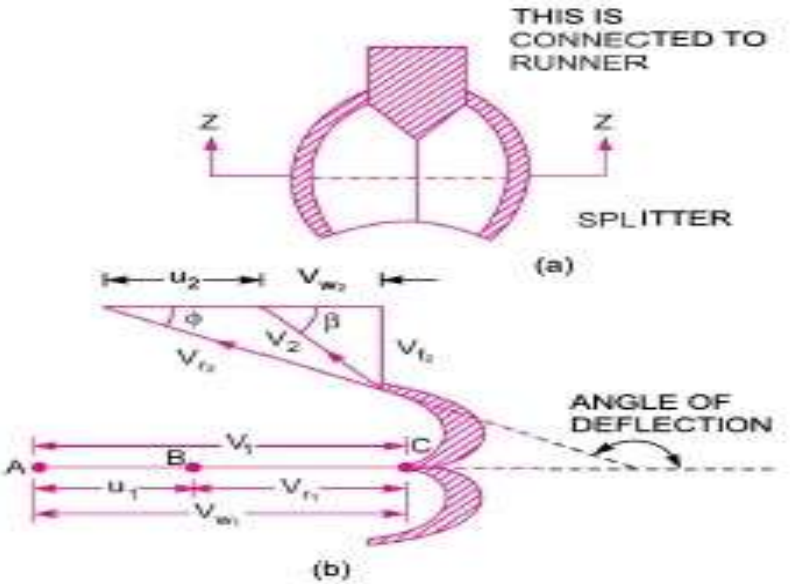


Fig. *Shape of bucket.*

Let H = Net head acting on the Pelton wheel

$$= H_g - h_f$$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,

D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet = $\sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0^\circ \text{ and } \theta = 0^\circ$$

Velocity triangle and work done for Pelton Wheel cont….

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}]$$

As the angle β is an acute angle, +ve sign should be taken. Also this is the case of series of vanes, the mass of water striking is $\rho a V_1$ and not $\rho a V_{r_1}$. In equation 'a' is the area of the jet which is given as

$$a = \text{Area of jet} = \frac{\pi}{4} d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 [V_{w_1} + V_{w_2}] \times u \text{ Nm/s}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2} m V^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency,** $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2}$$

Design aspects:-

Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \;(= 12 \text{ for most cases})$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5\text{ m}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Velocity of Wheel, $u = u_1 = u_2 = \frac{\pi DN}{60}$

Overall Efficiency , $\eta_0 = \eta_m \times \eta_h$ OR $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$

Water Power, W.P. = ½mV² = ρgQH

Shaft Power, S.P. = $\rho a V_1[V_{w_1} + V_{w_2}] \times u = \rho Q[V_{w_1} + V_{w_2}] \times u$

Problem A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

Problem A Pelton wheel is to be designed for the following specifications :
Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :

(i) The wheel diameter, (ii) The number of jets required, and
(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ and $K_{u_1} = 0.45$

Problem The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m^3/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and $C_v = 1.0$.