MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-8

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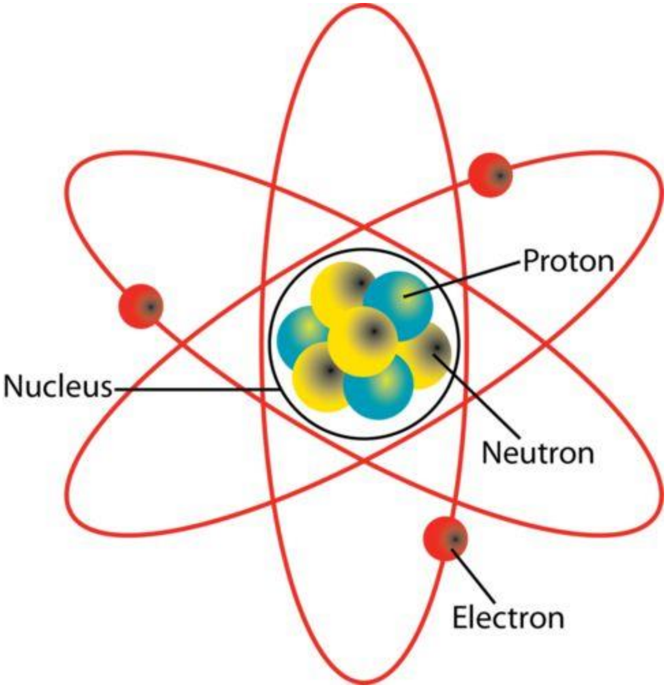

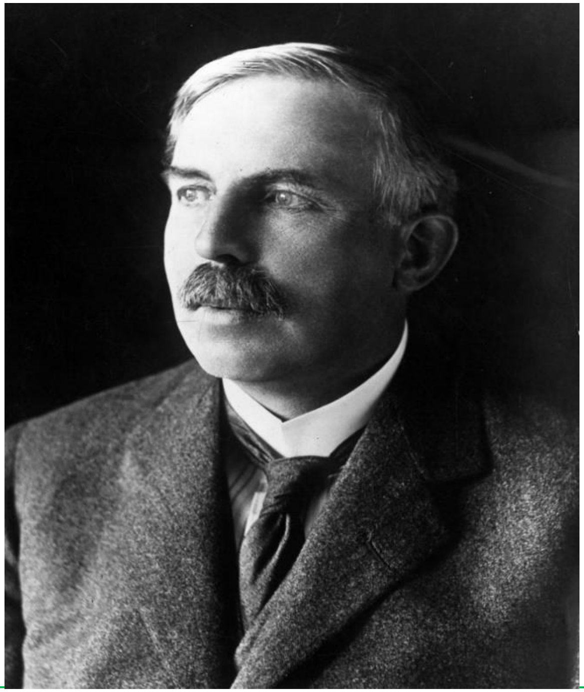

Bethe-Weiszacker mass formula (Semi-Empirical Mass Formula)

- **The liquid drop model**
- **Stability of Nucleus**
- **Semi Empirical Mass Formula**
- **Volume Energy Correction (B1)**
- **Surface Energy (B2)**
- **Coulomb Energy (B3)**
- **Asymmetry Energy (B4)**
- **Pairing Energy (B5)**
- **The SEMF**

Introduction to the SEMF

- Aim: phenomenological understanding of nuclear binding energies as function of A, Z and N.
- Assumptions:
  - Nuclear density is constant (see lecture 1).
  - We can model effect of short range attraction due to strong interaction by a liquid drop model.
  - Coulomb corrections can be computed using electro magnetism (even at these small scales)
  - Nucleons are fermions at T=0 in separate wells (Fermi gas model ➔ asymmetry term)
  - QM holds at these small scales ➔ pairing term.
- Compare with experiment: success & failure!

Comparison of Liquid drop Model With Nucleus

- Phenomenological model to understand binding energies.
- Consider a liquid drop
  - Ignore gravity and assume no rotation
  - Intermolecular force repulsive at short distances, attractive at intermediate distances and negligible at large distances ➔ constant density.
  - *n*=number of molecules, *T*=surface tension, *B*=binding energy

    E=total energy of the drop, α, β=free constants

$$E=-\alpha n + 4\pi R^2 T \quad \Rightarrow \quad B=\alpha n-\beta n^{2/3}$$

Analogy with nucleus

Nucleus has constant density

From nucleon-nucleon scattering experiments we know:

Nuclear force has short range repulsion and is attractive at intermediate distances.

Assume charge independence of nuclear force, neutrons and protons have same strong interactions

Stability of Nucleus

- Binding energy of the nucleus is defined as the as the difference between the energy of constituent particles and the whole nucleus.
- Let us consider the case of the nucleus ${}^{A}_{Z}M$, the binding energy is given as
- B = [ZM_P + N M_N - ${}^{A}_{Z}M$] c^2
- Where M_p = Mass of Proton , Z = Mass of proton, M_n = Mass of Neutrons
- N= Number of Neutrons = (A-Z)
- ${}^{A}_{Z}M$ = Measured mass of neutral atom [also written as M(Z,A)]

Stability of Nucleus

- The above expression is expressed as
- B = [ZM_H + N M_N - ${}^{A}_{Z}M$] c^2
- Where M_H represents the mass of the neutral hydrogen atom. Since there are A nucleons in the nucleus, the binding energy per nucleon is also given as
- $\frac{B}{A} = \frac{c^2}{A}$ [ZM_H + N M_N - ${}^{A}_{Z}M$]
- When binding energy fraction B/A is plotted against A we find B/A remains constant between A=30 and A=100 and decreases for small and large values of A.

Semi Empirical Mass Formula

- We have seen that the nucleus is taken as spherical with radius $R=R_0A^{1/3}$ on the basis of this and some other classical concepts such as surface tension, electrostatic repulsion etc.
- A formula for atomic mass of a nuclide in terms of binding energy correction terms was set by Weizsaker in 1935 was later modified by Bethe and others.
- This formula can be used to predict the stability of nuclei against particle emission, energy release stability for fission.
- The mass of nucleus is given by the formula
- $M_{(Z,A)} = ZM_H + N\,M_N - \frac{B}{c^2}$

Semi Empirical Mass Formula Contd…

- From the mass formula given on the last slide if it is possible to calculate B from a general formula, all the nuclear masses can be evaluated theoretically.
- Weizsaker and others make an attempt to develop the formula.
- They Assume liquid drop model of the nucleus regarding B as similar to the latent heat of the condensation.
- Some of the properties of the nuclear forces (saturation short range etc,) which have been deduced from the approximate linear dependence of the binding energy on the nucleus of particles in the nucleus are analogous to the properties of the forces which hold a liquid drop together.

Semi Empirical Mass Formula Contd…

- Hence there is ample justification in considering the nucleus to the analogous to the drop of incompressible fluid of very high density 10^{17} Kg/ m^3 .

- The value of B was calculated empirically as made up of number of corrections terms as

- B= B_1+ B_2 + B_3 + B_4 + B_5

Volume Energy Correction (B1)

- Binding energy/ Nucleon approximately =8MeV

- Binding energy is proportional to total Number of the nucleons (Mass No. of A)

- Mass No. A **α** Volume of Nucleus
- B_1 **α** A

- $B_1 = a_v$A

- The Major Contribution to B_1 comes from the mutual interactions of the nucleons under the influence of nuclear forces.

Surface Energy (B2)

- In the last slide proportionality between B_1 and A implicitly assumes overall constancy in the strength of the interaction of each nucleon with its immediate surrounding.
- However those nucleons which are situated in the surface regim of the nucleus are necessarily more weakly bound than those in the nuclear interior because they have fewer immediate neighbours.
- No. of such nucleons is proportional to the surface area of the nucleus and therefore proportional to R^2or $A^{2/3}$
- Thus,
- $B_2 \,\boldsymbol{\alpha}\, -R^2$
- $B_2 \,\boldsymbol{\alpha}\, -(R_0 A^{1/3})^2$
- $B_2 = -a_s(A^{2/3})$ Where a_s stands for surface energy constant and a_s = R_0

Coulomb Energy (B3)

- Proton and proton in the nucleus have strong force of repulsion.
- Thus the effect of coulomb self energy on the binding energy is diminutive
- Assuming that the nuclear charge Ze is uniformly distributed throughout the nuclear volume, the coulomb energy of the nucleus is given as
- $E_c = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R} = \frac{1}{4\pi\epsilon_0}\frac{3}{5}\frac{Z^2e^2}{R_0A^{1/3}}$
- Since The force between proton is repulsive so
- $B_3 = - a_c \frac{Z^2}{A^{1/3}}$
- The negative Sign indicates the diminution of energy due to repulsive effect.
- In some cases particularly in case of light nuclei having comparatively small values of Z, the above formula is slightly modified as
- $B_3 = - a_c \frac{Z(Z-1)}{A^{1/3}}$

Asymmetry Energy (B4)

- In case of the condition for N=Z binding energy is highest.
- This is called the symmetry effect.
- Any deviation from N = Z, reduces the stability of nuclei and hence reduces the binding energy.
- When A increases number of neutrons increases compared to the proton then stability of the nucleus decreases.
- The deficit in binding energy depends on the neutron excess {N-Z or (A-Z-Z)} and is proportional to $\frac{(A-2Z)^2}{A}$

Asymmetry Energy (B4)

- This symmetry effect is purely a quantum mechanical effect in contrast to the surface effect and coulomb energy effect.

- The correction term is given as $B_4 = -\, a_a \frac{(A-2Z)^2}{A}$ where a_a is the positive constant.

- The Minus sign represents the weakening in binding caused by asymmetry in N and Z. Since beyond a certain stage the neutron ceases to act as binding agent within the nucleus.

Pairing Energy (B5)

- The stability of nucleus is closely related to the odd and even numbers of neutron and proton in the nucleus.

- Nuclei with even Z and even N (even-even nuclei) are most stable, even- odd and odd-even nuclei are less stable and odd-odd nuclei are most unstable.

- To take account of this pairing effect an additional term is incorporated into the mass formula

- B_5 = +δ for e-e nuclei, 0 for e-o nuclei and o-e nuclei and -δ for o-o nuclei.

- Where δ is empirically found to as
- $\delta = a_p \ (A)^{-3/4}$

Semi Empirical Mass Formula

- Assembling all the above correction terms, the semi-empirical mass formula is given as
- M(Z,A) = ZM_H + N $M_N +\ a_v$A $-a_s(A^{2/3})$ - $a_c \frac{Z(Z-1)}{A^{1/3}}$ - $a_a \frac{(A-2Z)^2}{A} \pm \delta$
- Where the binding energy correction terms are taken in mass units.
- The empirical formula for binding energy is given as
- B = a_vA $-a_s(A^{2/3})$ - $a_c \frac{Z(Z-1)}{A^{1/3}}$ - $a_a \frac{(A-2Z)^2}{A} \pm \delta$
- Now the binding energy per nucleon is given as
- $\frac{B}{A} = a_v - \frac{a_s}{A^{1/3}} a_s$ - $a_c \frac{Z(Z-1)}{A^{4/3}}$ - $a_a \frac{(A-2Z)^2}{A^2} \pm \frac{\delta}{A}$

Semi Empirical Mass Formula Contd…

- The empirical values of the coefficients evaluated by comparison of the above equation with the mass of stable nuclides and energetics of the nuclear reactions are listed below---

- a_v = 14.1 MeV, a_s = 13.0 MeV, a_c = 0.595 MeV, a_a = 33. 5 MeV

- From the SEMF it is clear that the mass M(Z,A) is a quadratic function of Z for a given mass number A.

- Thus the graph of M(Z,A) versus Z will be a parabola, the minimum vertex of which will represent the most stable isobar.

Semi Empirical Mass Formula Contd…

- Experimentally it has been found that for odd-A nuclides there is only one stable isobar.
- For e-A nuclides, there are often two and sometimes three stable isobars.
- It is not possible to find an empirical formula which could fit for all A.
- The formula over a limited range of A > 15 is quite successful and could be used to predict the most stable isobars.

Nucleus	O^{16}	Cr^{52}	Mo^{98}	Au^{197}	U^{238}
Experimental Mass	16.0000	51.956	97.943	197.04	238.12
From Mass Formula	15. 9615	51.959	97. 946	197.04	238.12