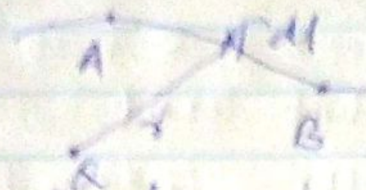
Electric field:-

let a point charge 'Q' is moved from point A to B in an Electric field E

from coulomb's law,
force on charge, $F = QE$



∴ work done in displacing charge by dl,

$$W = -\vec{F}\cdot dl = -QE\,dl$$

-ve sign shows that work is done by external field.

Total work done in moving charge from 'A' to 'B' $\quad W = -Q\int_A^B E\cdot dl$

potential diff $V_{AB} = \dfrac{W}{Q} = -\int_A^B E\cdot dl$

→ If V_{AB} is -ve, there is loss in P.E.
i.e. work done by field.

→ If V_{AB} is +ve, there is gain in P.E.
i.e. work done by external agent.

→ V_{AB} is independent of path.

If field is produced by charge Q located at origin, $\quad E = \dfrac{Q}{4\pi\epsilon_0 r^2}\,a_r$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2}\,a_r\cdot dr\,a_r$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

$$V_{AB} = V_B - V_A$$

* vectors whose line integral does not depend on path of integratn are called conservative.

Potential at distance r from origin is work done per unit charge by an external agent to bringing a test charge from infinity to that point.

$$V = -\int_{\infty}^{r} E \cdot dl \quad \text{(when E given)}$$

let $Q_1, Q_2, \dots, Q_n$ are located at $r_1, r_2, \dots, r_n$

potential at r, $V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{Q_k}{|r - r_k|}$ (when Q given)

Relatn Between E & V - Maxwell's eqn:-

$$V_{BA} = -V_{AB} \Rightarrow V_{BA} + V_{AB} = 0$$

$$\oint E \cdot dl = 0$$

→ Line integral of E along closed path must be zero. Applying strokes theorem

$$\oint E \cdot dl = \int_S (\nabla \times E) \cdot ds = 0$$

$\nabla \times E = 0 \rightarrow 2^{nd}$ Maxwell eqn

⇓

such field are known as conservative or irrotational.

Now, $dV = -E \cdot dl = -E_x dx - E_y dy - E_z dz$

$$dV = \frac{\partial V}{\partial x} \times dx + \frac{\partial V}{\partial y} \times dy + \frac{\partial V}{\partial z} \times dz$$

Comparing, $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$

$E_z = -\frac{\partial V}{\partial z}$

$$\therefore \boxed{E = -\nabla V}$$

Ques:- $E = (3x^2 + y) a_x + x a_y$ kV/m

$Q = -2\mu C$

$(0,5,0) \rightarrow (2,-1,0)$

$(0,5,0) \rightarrow (2,5,0) \rightarrow (2,-1,0)$

$$\frac{W}{Q} = -\int_L E \cdot dl \Rightarrow -\frac{W}{Q} = \int_L E \cdot dl$$

A 0,5,0 $\downarrow$ $dl = dx\, a_x$

B 2,5,0 $\xrightarrow{dl = dy\, a_y}$ C (2,-1,0)

$$W_{AC} = W_{AB} + W_{BC}$$

$$\therefore -\frac{W}{Q} = \left(\int_{AB} + \int_{BC}\right) E \cdot dl$$

$$= \int_0^2 (3x^2 + y) dx \Big|_{y=5, z=0} + \int_5^{-1} x\, dy \Big|_{x=2}$$

$$= \frac{3x^3}{3} + 5x \Big|_0^2 + 2y \Big|_5^{-1}$$

$$= 8 + 10 + (-2 - 10)$$

$$= 18 - 12 = 6$$

$W = -6Q = 12\,mJ$

(b) $y = 5 - 3x$

$dy = -3dx$

$E = (3x^2 - y)a_x + x a_y$

$\bar{dl} = dx\, a_x + dy\, \bar{a}_y$

$E \cdot dl = (3x^2 - y)dx + x\,dy$

$= (3x^2 + 5 - 3x)dx + \frac{5-y}{3}dy$

$-\frac{W}{Q} = \int_0^2 (3x^2 + 5 - 3x)dx + \int_5^{-1} \frac{5-y}{3}dy$

$= x^3 + 5x - \frac{3x^2}{2}\Big]_0^2 +$

$\frac{5}{3}y - \frac{y^2}{6}\Big]_5^{-1}$

$= 8 + 10 - \frac{3}{2}\times 4 - 0 +$

$-\frac{5}{3} - \frac{1}{6} - \frac{25}{3} + \frac{25}{6}$

$= 18 - 6 - \frac{30}{3} + 4$

$= 12 - 10 + 4$

$= 6$

$W = 12\,\mu J$

material

Conductor — Non-Conductor

$\sigma \gg 1$ — $(\sigma \ll 1)$

(Huge free e) — insulator/dielectric (few free e)

Electrical properties:- conduct, electric current, polarization

Material prop - susceptibility, permittivity, Linearity, Isotropy, Homogeneity, dielectric strength, Relaxation time

conductivity:- unit → mho/m S/m

$\sigma\uparrow, T\downarrow$

at T = 0K, some metal superconductors $\sigma = \infty$

Ex:- Lead, Al

Convection current:-

electric charge....

$I = \frac{dQ}{dt}$ C/s or A

current density (J): if ΔI current passes through ΔS surface.

$J = \frac{\Delta I}{\Delta S} \Rightarrow \Delta I = J\Delta S$

current density is ⊥ to surface

If not, $\Delta I = \bar{J}\cdot\Delta \bar{S}$

Total current, $I = \int_S J \cdot ds$

Current density → Convect

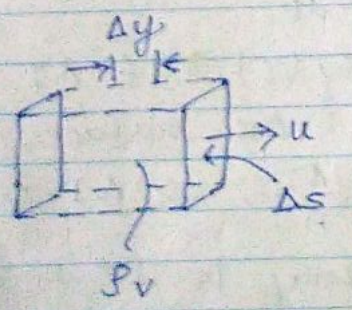
- → Convection current density
- → Conduction current ,,
- → Displacement ,,

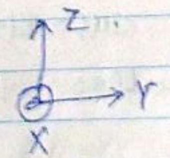
→ Convection current does not involve conductor and does not satisfy ohm's law.

→ It occurs when flow through insulating medium such as liquid, rarefied gas, or vaccum.

Consider a filament of charge density ρ_v, velocity of flow of charge $u = u_y a_y$

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \frac{\Delta S \, \Delta y}{\Delta t} = \rho_v \Delta S \cdot u_y$$





$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$$

$$\boxed{J = \rho_v \cdot u}$$

I is convection current and J is convection current density.

Conduction current :-

It required as conductor. When 'E' field is applied, the force on electron of charge $-e$:-

$$F = -eE$$

Electron will experience collise with atomic lattice. let 'm' is the mass moving in E-field with velocity u.

Rate of change in momentum $\frac{mu}{\tau} = -eE$

$$\rightarrow u = \frac{-eE\tau}{m}$$

where τ is the average time interval between collision. u is the drift velocity of e^-. let charge conc. is n.

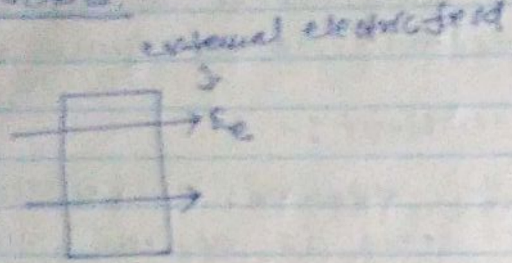
then $\rho_v = -ne$

conduction current density,

$$J = \rho_v \cdot u = \frac{ne^2\tau}{m} E = \sigma E$$

$\boxed{J = \sigma E}$ → ohm's law in point form

Conductors:-



Isolated conductor.

Due E_e, +ve charged are forced along E_e and -ve free charge in opposite direc^n.

(i) Free charge accumulate on the surface of conductor ~~...~~ forming surface induced charge. (ii) Induced charged produce internal E-field, which cancel E_e.

$$E = -\nabla V = 0$$

∴ conductor is called equipotential body.

Another way:

$$J = \sigma E$$

for perfect conductor, $\sigma = \infty$

$$\therefore E = 0$$

→ If some charge is introduce inside conductor it redistribute on surface such that E field inside is zero.

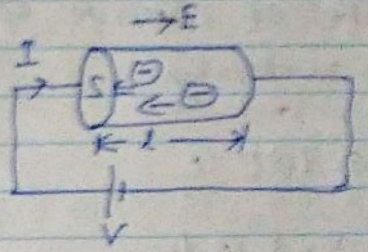
From Gauss Law;

E0 then $\rho_v = 0$.

Hence under static condit^n:-

$$\boxed{E = 0 \;;\; \rho_v = 0 \;:\; V_{ab} = 0}$$

↳ inside conductor



Taking a conductor whose ends are maintained potential difference V.

Here, $E \neq 0$; inside conductor, since there is no static equilibrium i.e. conductor is not isolated and wired to a source of electromotive force, which prevents establishment of electrostatic equilibrium.

→ Direc^n of E is same direc^n of flow of +ve charge.

$$E = \frac{V}{l}$$

$$J = \frac{I}{S}$$

$$\rightarrow \sigma E = \frac{I}{S}$$

$$\sigma \frac{V}{l} = \frac{I}{S} \Rightarrow \frac{V}{I} = \frac{l}{\sigma S} = \frac{\rho_c l}{S}$$

ρ_c → Resistivity.

$\boxed{R = \frac{\rho_c l}{S}}$ → Applicable when uniform cross section.

For any, $R = \frac{V}{I} = \frac{\int E \cdot dl}{\int J \cdot ds}$

Power (P) ⇒ Rate of change of energy

or, Force × velocity.

$= \int \rho_v \, dv \, E \cdot u$

$= \int E \cdot \rho_v u \, dv$

$\boxed{P = \int E \cdot J \, dV}$,

Joule's Law.

Power density $W_P = \frac{dP}{dV} = E \cdot J$

$= \sigma E^2$

$dv = ds \cdot dl$

$P = \int E \cdot dl \int J \cdot ds$

$= VI$

$\boxed{P = I^2 R}$ → Joule's Law.

Ques: If $J = \frac{1}{r^3}(2\cos\theta \, a_r + \sin\theta \, a_\theta)$ A/m²

calculate current passing through:

(a) Hemispherical shell of radius 20 cm.

$0 < \theta < \pi/2$; $0 < \phi < 2\pi$

(b) spherical shell of radius 10 cm.

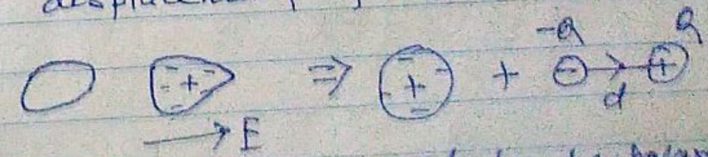
Soln: $I = \int J \cdot ds$

$ds = r^2 \sin\theta \, d\theta \, d\phi \, a_r$

Ques: For the current density $J = 10 z \sin^2\phi \, a_\rho$ A/m². Find current through the cylindrical surface $\rho = 2$, $1 \le z \le 5$ m.

Polarization in Dielectrics:-

In dielectric materials nuclei in molecules taken as point charge and electronic structure as single cloud of negative charges similar to the atom. When E field is applied force experienced by dielectric is $F = QE$, and a dipole results from the displacement of charges.



Dielectric is said to be polarised

Dipole moment, $P = Qd$

Let there are N dipoles in volume Δv of dielectric.

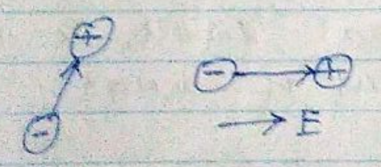
Total dipole moment $= \sum_{k=1}^{N} Q_k d_k$

Polarization 'P' is dipole moment per unit volume.

$$P = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} Q_k d_k}{\Delta v}$$

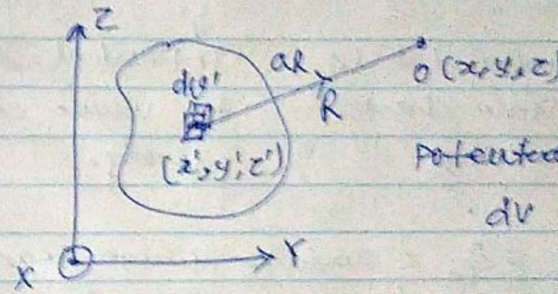
This type of dipole is called non-polar where ~~dielectric~~ there is no dipole before applying E-field. In presence of E all dipoles aligned in the direction of E-field. Ex:- H_2, O_2, N_2, Rare gas.

→ Polarized dielectrics are those having built-in dipoles and aligned in the direction of E-field when E is applied.



Electric field due to polarized dielectric:

P = Dipole moment per unit volume.



Potential due to dv'

$$dV = \frac{P \cdot a_R \, dv'}{4\pi\epsilon_0 R^2}$$

where, $R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2$

since $\nabla\left(\frac{1}{R}\right)' = \frac{a_R}{R^2}$
gradient

$$\therefore dV = \frac{1}{4\pi\epsilon_0} P \cdot \nabla\left(\frac{1}{R}\right) dv'$$

since, $\nabla \cdot f\bar{A} = f\nabla \cdot \bar{A} + \bar{A}\nabla f$

$$\rightarrow \bar{A} \cdot \Delta f = \nabla \cdot f\bar{A} - f\nabla \cdot \bar{A}$$

$$\therefore dV = \frac{1}{4\pi\epsilon_0}\left[\nabla \cdot \frac{1}{R} P - \frac{1}{R}\nabla \cdot P\right]$$

Total potential :-

$$V = \frac{1}{4\pi\epsilon_0}\int_{v'}\left[\nabla \cdot \frac{P}{R} - \frac{\nabla \cdot P}{R}\right] dv'$$

$$\int \nabla \cdot \frac{P}{R} dv' = \int_{s'} \frac{P \cdot a_n}{R} ds \quad \text{(Divergence theorem)}$$

a_n is the unit normal to surface ds'.

$$\therefore V = \int_{s'} \frac{P \cdot a_n}{4\pi\epsilon_0 R} ds' + \int_{v'} \frac{-\nabla \cdot P}{4\pi\epsilon_0 R} dv'$$

↓ Potential due to surface charge density

↓ Potential due to volume charge density

$P \cdot a_n = \rho_{ps}$ = surface charge density over surface of dielectric

$-\nabla \cdot P = \rho_{pv}$ volume ——— throughout the dielectric.

These are bound charges.

Total bound charge on surface,

$$Q_b = \oint \bar{P} \cdot d\bar{s} = \int \rho_{ps} ds$$

outside volume

$$-Q_b = \int_v \rho_{pv} dv = -\int_v \nabla \cdot P \, dv$$

If ρ_v is the free charge density to the dielectric.

total volume charge density,

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \epsilon_0 E$$

$$\therefore \rho_v = \nabla \cdot \epsilon_0 E - \rho_{pv}$$

$$= \nabla \cdot (\epsilon_0 E + P) = \nabla \cdot D$$

$$\therefore \boxed{D = \epsilon_0 E + P}$$

P can also be written as-

$$P = \chi_e \epsilon_0 E$$

χ_e = Electric susceptibility of material.

$$D = \epsilon_0 E + \chi_e \epsilon_0 E$$

$$= \epsilon_0 [1 + \chi_e] E$$

$$= \epsilon_0 \epsilon_r E$$

$$D = \epsilon E$$

$$\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$

The dielectric constant (or relative permittivity) is the ratio of the permittivity of dielectric to that of free space.

→ The dielectric strength is the maximum electric field that a dielectric can tolerate ~~or~~ without electrical breakdown.

__Continuity Eq^n :-__

Charge conservation - Rate of decrease of charge within a given volume must be equal to net outward current flow through the surface of volume.

∴ current coming out of surface,

$$I_{out} = \oint J \cdot ds = -\frac{dQ_{in}}{dt}$$

from divergence theorem,

$$\oint J \cdot ds = \int_V \nabla \cdot J \, dv$$

$$-\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int \rho_v \, dv = -\int \frac{d\rho_v}{dt} \cdot dv$$

$$\therefore \int_V \nabla \cdot J \, dv = -\int \frac{d\rho_v}{dt} dv$$

$$\boxed{\nabla \cdot J = -\frac{d\rho_v}{dt}} \rightarrow \text{continuity eq}^n$$

For steady current, $\frac{d\rho_v}{dt} = 0$

$$\nabla \cdot J = 0$$

total charge leaving = Total charge entering

(KCL)

Now, ohm's law ⇒ $J = \sigma E$

Gauss' law ⇒ $\nabla \cdot E = \frac{\rho_v}{\epsilon}$

from continuity eq^n:-

$$\nabla \cdot \sigma E = \frac{\sigma \rho_v}{\epsilon} = -\frac{\partial \rho_v}{dt}$$

$$\rightarrow \frac{d\rho_v}{dt} + \frac{\sigma \rho_v}{\epsilon} = 0.$$

$$\rightarrow \frac{d\rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} dt$$

integrating, $\ln \rho_v = -\frac{\sigma t}{\epsilon} + \ln \rho_{v_0}$ ← const.

$$\rho_v = \rho_{v_0} e^{-t/\tau}$$

where, $\tau = \frac{\epsilon}{\sigma}$

ρ_{v_0} is initial charge at $t=0$.

Time const 'τ' is relaxation time.

Relaxation time : It is the time taken so that a charge placed in interior of material to drop to e^{-1} (36.8%) of its initial value.

Boundary Condition:-

Interface.

Dielectric (ϵ_{r1}) & Dielectric (ϵ_{r2})

Conductor & "

" & free space.

∴ current coming out of surface,

$$I_{out} = \oint J \cdot ds = -\frac{dQ_{in}}{dt}$$

from divergence theorem,

$$\oint J \cdot ds = \int_V \nabla \cdot J \, dv$$

$$-\frac{dQ_{in}}{dt} = -\frac{d}{dt}\int \rho_v \, dv = -\int \frac{d\rho_v}{dt} \cdot dv$$

$$\therefore \int_V \nabla \cdot J \, dv = -\int \frac{d\rho_v}{dt} dv$$

$$\boxed{\nabla \cdot J = -\frac{d\rho_v}{dt}} \rightarrow \text{continuity eq}^n$$

For steady current, $\frac{d\rho_v}{dt} = 0$

$$\nabla \cdot J = 0$$

total charge leaving = total charge entering.

✓(KCL)

Now,

ohm's law ⇒ $J = \sigma E$

Gauss's law ⇒ $\nabla \cdot E = \frac{\rho_v}{\epsilon}$

from continuity eq:-

$$\nabla \cdot \sigma E = \frac{\sigma \rho_v}{\epsilon} = -\frac{d\rho_v}{dt}$$

$$\rightarrow \frac{d\rho_v}{dt} + \frac{\sigma \rho_v}{\epsilon} = 0$$

$$\rightarrow \frac{d\rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} dt$$

Integrating, $\ln \rho_v = -\frac{\sigma t}{\epsilon} + \underset{\text{const}}{\ln \rho_{v0}}$

$$\rho_v = \rho_{v0} e^{-t/\tau}$$

where, $\tau = \frac{\epsilon}{\sigma}$

ρ_{v0} is initial charge at $t=0$

Time const 'τ' is relaxation time

Relaxation time: It is the time taken so that a charge placed in interior of material to drop to e^{-1} (36.8%) of its initial value.

Boundary Condition:-

Interface.

Dielectric (ϵ_{r1}) & Dielectric (ϵ_{r2})

Conductor & "

" & free space.

Maxwell's eqn:

$\oint E \cdot dl = 0$

$\oint_S D \cdot ds = Q_{enclosed}$

$E = E_t + E_n \rightarrow$ normal comp

E_t — Tangential comp

Dielectric - Dielectric Boundary conditions:-

let two different medium are characterised by dielectric coeff $\epsilon_1 = \epsilon_0 \epsilon_{r_1}$, $\epsilon_2 = \epsilon_0 \epsilon_{r_2}$

E_1 and E_2 are the E-fields in media 1 & 2, respectively.

$E_1 = E_{t_1} + E_{n_1}$

$E_2 = E_{t_2} + E_{n_2}$

$\oint E \cdot dl = 0$

$\rightarrow E_t \Delta w - E_{n_1} \frac{\Delta h}{2} + E_{n_1} \frac{\Delta h}{2}$

$- E_{t_2} \Delta w - E_{2n} \frac{\Delta h}{2} + E_{n_2} \frac{\Delta h}{2}$

$E_{t_1} \Delta w - E_{t_2} \Delta w = 0$

as $\Delta h \to 0$

$\rightarrow E_{t_1} = E_{t_2}$

$\rightarrow$ Tangential components of E-fields are continuous across the boundary.

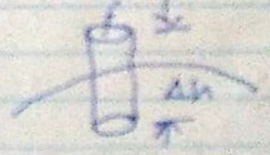
$D = \epsilon E$

$= D_t + D_n$

$\therefore \boxed{\frac{D_{t_1}}{\epsilon_1} = \frac{D_{t_2}}{\epsilon_2}}$

D_t undergoes some change across the interface. Hence D is discontinuous across the boundary.

Taking cylindrical Gaussian surface.



$\Delta h \to 0$

$\Delta Q = \rho_s \cdot \Delta S$

$= D_{n_1} \Delta S - D_{n_2} \Delta S$

$\therefore \boxed{D_{n_1} - D_{n_2} = \rho_s}$

ρ_s is the free charge density at the surface of boundary.

If no free charge exist then

$D_{n1} = D_{n2}$

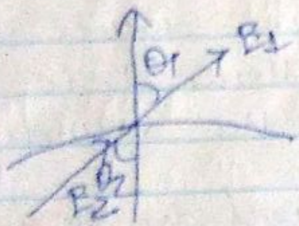
→ Normal component of D is continuous across the surface.

$D = \epsilon E$

$\therefore \epsilon_1 E_{n1} = \epsilon_2 E_{n2}$

Normal components of E is discontinuous at boundary.

Refraction of E-field at boundary:



$E_{t1} = E_{t2}$

$E_1 \sin\theta_1 = E_2 \sin\theta_2$

$D_{n1} = D_{n2}$

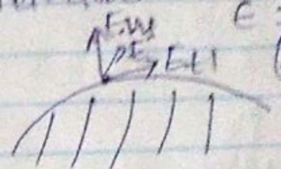
$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$

dividing

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}}}$$

Bim: Two extensive homogeneous & Conductor - Dielectric Boundary Condition

Dielectric $\epsilon = \epsilon_r \epsilon_0$ ①



conductor ②
$E = 0$

$E_{t1} = E_{t2}$

$\rightarrow E_{t1} = 0.$

$\Delta Q = D_n \Delta S - 0 \cdot \Delta S$

$D_n = \frac{\Delta Q}{\Delta S} = \rho_s$

$\boxed{D_n = \rho_s}$

→ No electric field exist inside conductor

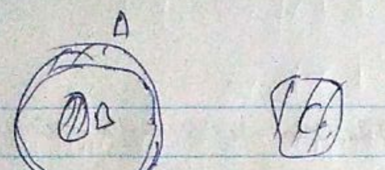
→ No P.D. within conductor

$E = -\nabla V = 0.$

→ An Electric field E must be external to the conductor and must be normal to its surface.

i.e. $D_t = \epsilon_0 \epsilon_r E_t = 0$

$D_n = \epsilon_0 \epsilon_r E_n = \rho_s.$



Conductor - free space Boundary condition

$D_t = \epsilon_0 E_t = 0$

$D_n = \epsilon_0 E_n = \rho_s$.

Ques: $z = 0$.

$z > 0$; $\epsilon_{r_1} = 4$

$z < 0$; $\epsilon_{r_2} = 3$

$E_1 = 5a_x - 2a_y + 3a_z$ KV/m ; $z \geq 0$.

Normal comp. $\rightarrow a_z$

$E_{n_1} = E_1 \cdot a_n = E_1 \cdot a_z = 3$

$E_{n_1} = 3a_z$

$E_{n_2} = (E_2 \cdot a_z) a_z$

$E = E_n + E_t$

$E_{t_1} = E_1 - E_{n_1} = 5a_x - 2a_y$

$E_{t_2} = E_{t_1} = 5a_x - 2a_y$.

$D_{n_2} = D_{n_1}$

$\epsilon_{r_2} E_{n_2} = \epsilon_{r_1} E_{n_1}$

$\rightarrow E_{n_2} = \dfrac{E_{n_1} \epsilon_{r_1}}{\epsilon_{r_2}} = 4a_z$

$\therefore E_2 = 5a_x - 2a_y + 4a_z$ KV/m

$E_{n_1} = 3$

$E_{t_1} = \sqrt{25+4} = \sqrt{29}$

$\tan\theta_1 = \dfrac{E_{n_1}}{E_{t_1}} \quad \dfrac{E_{t_1}}{E_{n_1}} = \dfrac{\sqrt{29}}{3}$.

$\tan\theta_2 = \dfrac{E_{t_2}}{E_{n_2}}$

Laplace and Poisson's eqn :-

→ Most common charge distribution is a volume charge ''.

→ Laplace and Poisson's eqn are 2nd order differential eqn relating the voltage developed due to a volume charge.

let ρ_v -- for volume.

$E = -\nabla V$

$D = \epsilon E$

$\nabla \cdot D = \rho_v$

$\nabla \cdot (\epsilon E) = \rho_v$

$\nabla \cdot (\epsilon(-\nabla V)) = \rho_v$

when ϵ is const.

$\nabla \cdot \nabla V = \dfrac{-\rho_v}{\epsilon}$

$\boxed{\nabla^2 V = \dfrac{-\rho_v}{\epsilon}}$ ⇒ Poisson's eqn

when $\rho_v = 0$ i.e. charge free region

$\nabla^2 V = 0$

→ This is called Laplace eqn.

→ Laplace and Poisson's eqn are 2nd order differential eqn they always have a single soln individually because voltage is always unique at a point. This is called uniqueness theorem.

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3 h_1}{h_2}\frac{\partial V}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial w}\right)\right]$$

x	y	z	1	1	1
ρ	ϕ	z	1	ρ	1
r	θ	ϕ	1	r	$r\sin\theta$
u	v	w	h_1	h_2	h_3

capacitor:-

Capacitance is the ability to hold E field conforming it to a small region.

special geometry:-

parallel plate

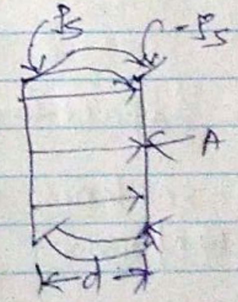
concentric cylinders

concentric spheres

$$C = Q/V = \frac{\oint D \cdot ds}{\int E \cdot dl} = \frac{\epsilon \oint E \cdot ds}{\int E \cdot dl}$$

→ capacitance is the ratio of charge utilized and the potential developed due to the charge.

Parallel plate capacitor:-



$A >> d$

→ Electric fields are non-uniform at the edge of plate known as fringing effect.

$\rho_s = \dfrac{Q}{S}$

If we ignore fringing at the edge of the plate then $|D| = \rho_s$

$$E = \frac{\rho_s}{\epsilon} = \frac{Q}{A\epsilon}$$

$$V = \frac{Qd}{A\epsilon}$$

$$\therefore C = \frac{Q}{V} = \frac{A\epsilon}{d}$$

Energy stored,

W_E = Total energy of E-field

$$= \left(\frac{1}{2}\epsilon E^2\right)(Ad)$$

$$= \frac{1}{2}\frac{\epsilon A}{d}(Ed)^2$$

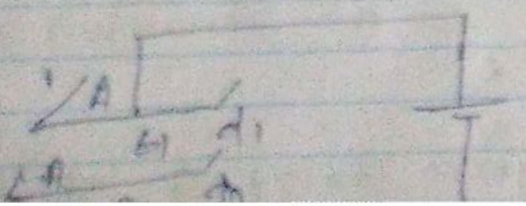
$$= \frac{1}{2}CV^2$$

$$= \frac{Q^2}{2C}$$

$$= \frac{1}{2}QV$$

Multiple dielectrics in capacitor plates

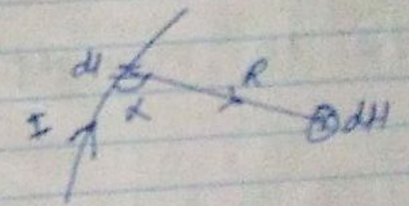
Equal area of cross-section and unequal width of dielectrics



Here capacitor are in series due to voltage division

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{\epsilon_1 A}{d_1}\cdot\frac{\epsilon_2 A}{d_2}}{\frac{\epsilon_1 A}{d_1}+\frac{\epsilon_2 A}{d_2}}$$

Biot - savart's Law :-



$$dH \propto \frac{Idl\sin\alpha}{R^2}$$

$$dH = \frac{KIdl\sin\alpha}{R^2}$$

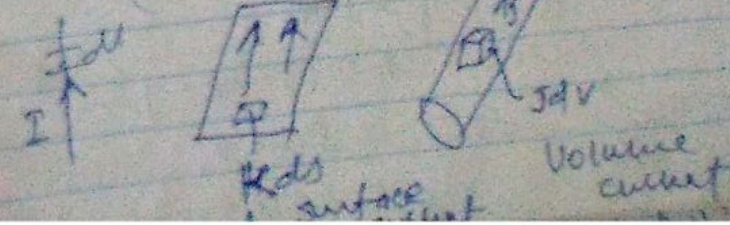
Idl = differential current element

$$= \frac{1}{4\pi}\frac{Idl\sin\alpha}{R^2}$$

$$dH = \frac{1}{4\pi}\frac{Idl \times a_R}{R^2}$$

$$= \frac{1}{4\pi}\frac{Idl \times R}{|R|^3}$$

Current distribution :-



$K \rightarrow$ surface current density A/m

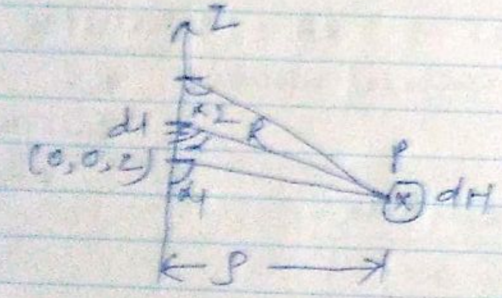
$J \rightarrow$ volume ———————— A/m²

$$H = \int \frac{I\,d\bar{l} \times a_R}{4\pi R^2}$$ line current

$$H = \int \frac{K\,ds \times a_R}{4\pi R^2}$$ surface ,,

$$H = \int \frac{J\,ds \times a_R}{4\pi R^2}$$ volume current.

Magnetic field due to straight line current :-



$$\bar{R} = \rho a_\rho - z a_z$$

$$dl = dz\, a_z$$

$$dH = \frac{I\,d\bar{l} \times \vec{R}}{4\pi R^3}$$

$$dl \times R = \rho\, dz\, a_\phi$$

Total magnetic field

$$H = \int \frac{I \rho\, dz}{4\pi [\rho^2 + z^2]^{3/2}}\, a_\phi$$

$\tan\alpha = \frac{\rho}{z} \Rightarrow z = \rho \cot\alpha$

$dz = -\rho \operatorname{cosec}^2\alpha \cdot d\alpha$

$$H = -\frac{I}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2\alpha\, d\alpha}{\rho^3 \operatorname{cosec}^3\alpha}\, a_\phi$$

$$= -\frac{I}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \sin\alpha \cdot d\alpha$$

$$H = \frac{I}{4\pi\rho} (\cos\alpha_2 - \cos\alpha_1)\, a_\phi$$

→ when the conductor is semi-infinite

i.e. (0,0,0) to (0,0,∞) $\alpha_1 = 90°, \alpha_2 = 0°$

$$H = \frac{I}{4\pi\rho} a_\phi$$

when conductor is of infinite length

(0,0,−∞) to (0,0,∞)

$\alpha_1 = 180°, \alpha_2 = 0°$

$$H = \frac{I}{2\pi\rho} a_\phi$$

H field is along concentric circular path.

<u>Ampere's circuit law:. Maxwell eq:.</u>

→ Line integral of magnetic field H around a close path is same as the net current I enclosed by path

$$\boxed{\oint H\cdot dl = I}$$

Applying stokes theorem -

$$\oint H\cdot dl = \int_S (\nabla\times H)\cdot ds$$

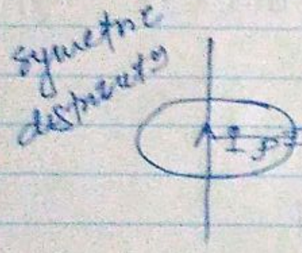
$$I = \int_S J\cdot ds$$

$$\therefore \int_S J\cdot ds = \int_S (\nabla\times H)\, ds$$

$$\rightarrow \boxed{\nabla\times H = J}$$ Maxwell 3rd

3rd Maxwell's eqn.

<u>Application</u>:- Infinite line current -



$dl = \rho\, d\phi$

$$I = \oint \bar{H}\cdot \bar{dl}$$

$$= \oint H a_\phi \cdot \rho\, d\phi\, a_\phi$$

$$= H\int_0^{2\pi} \rho\, d\phi$$

$$= H 2\pi\rho$$

$$H = \frac{I}{2\pi\rho}$$

Magnetic flux density:-

$$B = \mu_0 H$$

μ_0 is the permeability of free space.

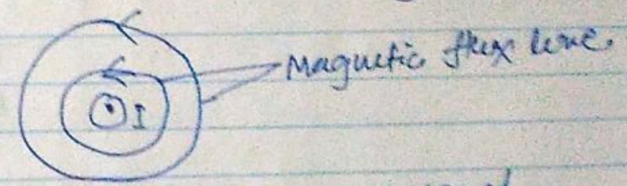
Unit $\rightarrow$ H/m. $\mu_0 = 4\pi\times10^{-7}$ H/m

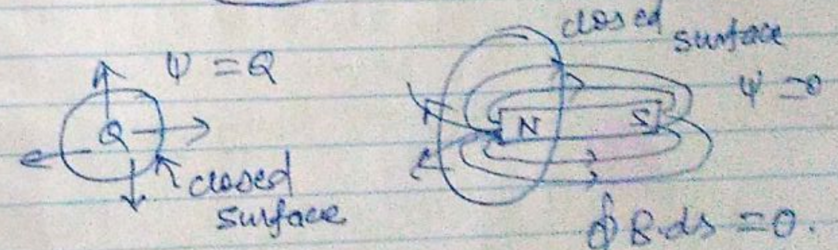
Magnetic flux through a surface,

$$\Psi = \int_S B\cdot ds$$

$\Psi \rightarrow$ wb $\qquad B \rightarrow$ wb/m^2

$\rightarrow$ magnetic flux line is path to which B is tangential at every point on line.





$$\oint B\cdot ds = 0.$$

$\rightarrow$ An isolated magnetic charge does not exist.

$$\oint B\cdot ds = 0$$

$\Downarrow$ $\int(\nabla\cdot B)\,dV = 0 \Rightarrow \nabla\cdot B = 0$ Maxwell fourth

Maxwell eqn for static field:-

Differential form	Integral	Remark.
$\nabla \cdot D = \rho_v$	$\oint D \cdot ds = Q$	Gauss law
$\cancel{\nabla \cdot B = 0}$		
$\nabla \times E = 0$	$\oint E \cdot dl = 0$	Conservative nature of E-field
$\nabla \times H = J$	$\oint H \cdot dl = I$	Ampere's Law
$\nabla \cdot B = 0$	$\oint B \cdot ds = 0$	Non-existance of monopole.

Magnetic scalar and vector potential

Magnetic potential may be vector A or scalar V_m.

since, $\nabla \times (\nabla V) = 0$

$\nabla \cdot (\nabla \times A) = 0$

for scalar V and vector A.

similar to $E = -\nabla V$

$H = -\nabla V_m$ if $J = 0$.

$J = \nabla \times H = \nabla \times (-\nabla \cdot V_m) = 0$

$\nabla^2 V_m = 0$

vector magnetic potential,

$\nabla \cdot B = 0$

$B = \nabla \times A$

so that $\nabla \cdot (\nabla \times A) = 0$.

Now, similar to, $V = \int \frac{dQ}{4\pi\epsilon_0 r}$

$A = \int \frac{\mu_0 I \, dl}{4\pi R}$ for line current

$= \int \frac{\mu_0 K \, ds}{4\pi R}$ surface "

" volume "

magnetic flux, $\psi = \int B \cdot ds$

$= \int (\nabla \times A) \cdot ds$

Applying stroke's theorem.

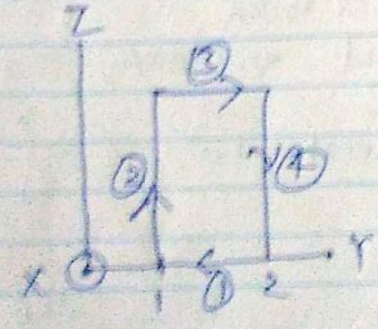
$\psi = \oint A \cdot dl$

Que: Given the magnetic vector potential $A = \frac{-\rho^2}{4} a_z$ wb/m, calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{2}$; $1 \le \rho \le 2$ m.

$0 \le z \le 5$ m.

Solⁿ:

$\psi = \oint A \cdot dl = \int_1 A \cdot dl + \int_2 A \cdot dl + \int_3 A \cdot dl + \int_4 A \cdot dl$



$\int (A)(-d\rho a_\rho) = 0.$

$\int_2 A \cdot dz\, a_z = \int_0^5 \frac{-\rho^2}{4} dz = \frac{-\rho^2}{4} \times 5$

$= \frac{-5}{4}$

$\int_3 A \cdot d\rho a_\rho = 0 \cdot 0$

$\Rightarrow \int_5^0 \frac{-\rho^2}{4} dz = (-1)[-5] = 5$

$\int_4 \frac{-\rho^2}{4}(-dz) = \frac{\rho^2}{4}[\;]_5^0$

$\psi = \frac{-5}{4} + 5 = \frac{-5+20}{4} = 15/4$ wb

magnetic force:-

Force due to magnetic field can be experienced in three ways

1. Due to moving charged particle in B field
2. on a current element in an external " "
3. Between two current elements.

Force on a charged particle:-

From coulomb's law

$F_e = QE$

→ magnetic field apply force only on moving charged particle.

$F_m = Q\, u \times B$

$\because F_m \perp u, B$, Hence there is no work done by magnetic field and no change ... B-field.

when both fields are present.

$F = F_e + F_m = Q[E + u \times B]$

↳ Lorentz force eqn.

Force on a current Element :-

current element → $I dl$

Convection current → $J = \rho_v u$

$I dl = K ds = J dv$

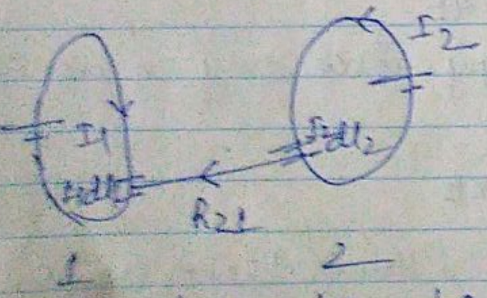
$\therefore I dl = J dv = \rho_v dv\, u = dQ u$

$\therefore dF_m = dQ u \times B = I dl \times B$

$\therefore$ Total force $(F) = \oint_L I dl \times B$

Also, $F = \int_S K ds \times B$ or, $F = \int_V J dv \times B$

Force between two current elements :-



Force on $I_1 dl_1$ due to magnetic field B_2 produced by coil 2.

Let dB_2 be the magnetic field

$\therefore dF_1 = I_1 dl_1 \times dB_2$

from Biot savart's law

$dB_2 = \dfrac{\mu_0 I_2 dl_2 \times a_{R21}}{4\pi R_{21}^2}$

$\therefore dF_1 = \dfrac{I_1 dl_1 \times \mu_0 I_2 dl_2 \times a_{R21}}{4\pi R_{21}^2}$

$= \dfrac{\mu_0 I_1 dl_1 \times (I_2 dl_2 \times a_{R21})}{4\pi R_{21}^2}$

$\therefore$ Total force $= \oint_{L_1}\oint_{L_2} \left(\dfrac{\mu_0 I_1 I_2}{4\pi}\right) \dfrac{dl_1 \times (dl_2 \times a_{R21})}{R_{21}^2}$

Ques:- A charged particle of mass 2 kg and charge 3C starts at point (1, -2, 0) with velocity $4a_x + 3a_z$ m/s in an E-field $12a_x + 10a_y$ V/m. At time $t = 1$ s find

(a) Acceleration

Ques:-

Ques:-

Ques:-

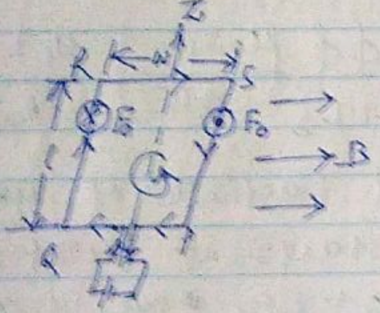
Ques:-

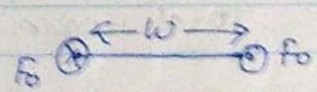
Magnetic torque and moment :-

when a current conducting loop is placed parallel to a magnetic field, it experience a force that tends to rotate it.

Torque T is mechanical moment of force applied on the loop and defined as vector product of force and moment arm r.

$T = r \times F$; unit $\to$ N-m





let rectangular loop of length 'l' and width 'w' is placed in uniform magnetic field

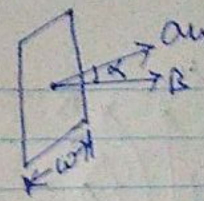
$$F = I\int_Q^R dl \times B + I\int_S^P dl \times B + 0 + 0$$

$$= I\int_0^l dz\, a_z \times B + I\int_l^0 dz\, a_z \times B$$

$F = F_0 - F_0 = 0 \Rightarrow$ overall force on the loop is zero.

$|F_0| = IlB$

But F_0 & $-F_0$ occurs at different point on the loop



a_n (Normal of loop)

$T = |F_0|$ ($\perp^r$ distance)

$= |F_0| \left[\frac{w}{2}\sin\alpha + \frac{w}{2}\sin\alpha\right]$

$= |F_0|\, w\sin\alpha$

$\to T = (IlB)\, w\sin\alpha$

$= IBS \cdot \sin\alpha$

$= (B)(IS\sin\alpha) = ISa_n \times B$

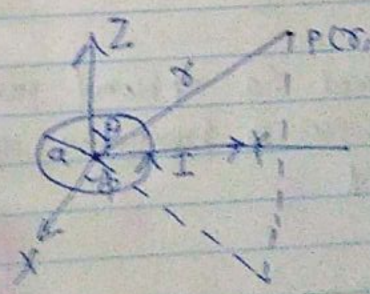
ISa_n = Magnetic dipole moment (m)

unit $\to$ A·m²

$\therefore \boxed{T = m \times B}$

Magnetic dipole :-

Bar magnet or elementary current loop is known as magnetic dipole.



$A = \oint \frac{\mu_0 I\, dl}{4\pi r}$

$\cancel{\frac{\mu_0 a \sin\theta\, d\phi}{4\pi r}}$

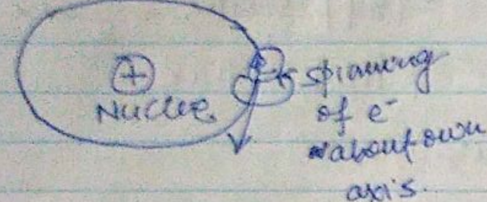
$dl = a\sin\theta\, d\phi$

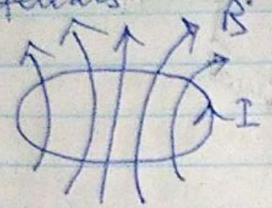
$A = \frac{\mu_0 m \times a_r}{4\pi r^2}$

$B = \nabla \times A = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\, a_r + \sin\theta\, a_\theta)$

$\to$ Assignment

Magnetization in materials.

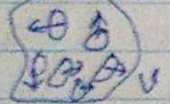




Due to spinning of e^- about their own axis, an internal magnetic field is created ~~around~~ and around the nucleus ~~the nucleus~~. Similar to the magnetic field created by current loop.

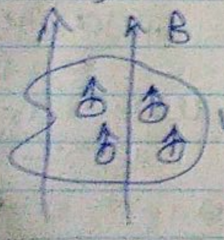
Magnetic moment of current loop, $m = IS a_n$

→ In the absence of external magnetic field algebric sum of all moments is zero due to the random orientation of loop.



$B = 0 ; M = 0.$

→ When external B-field is applied portion of loops get aligned with B producing non-zero magnetic moment.



$M \neq 0$

→ Net magnetic moment per unit volume is known as magnetization (M)

$$M = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} m_k}{\Delta v}$$

Above magnetization leads to some current given by $J_b = \nabla \times M$

$K_b = M \times a_n$

J_b → bound volume current density or, magnetization volume current density and K_b is bound surface current density.

→ M is analogus to polarization P in dielectrics

→ M is also known as magnetic polarization density.

$$B = \mu_0 (H + M)$$

$$M = \chi_m H$$

$$B = \mu_0 (1 + \chi_m) H$$

$$= \mu_0 \mu_r H$$

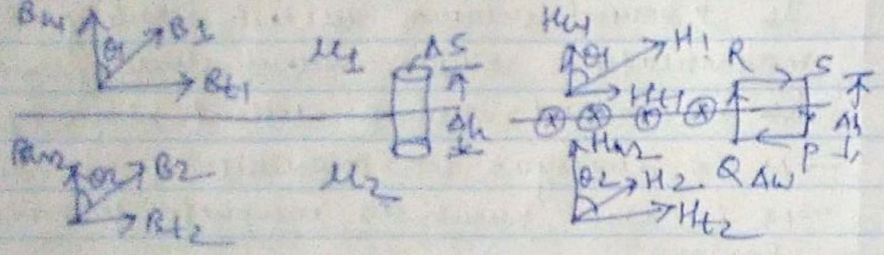
where $\mu_r = 1 + \chi_m$

Magnetic Boundary Conditions:-

Gauss Law for magnetic field:-

$\nabla \cdot B = 0 \Rightarrow \oint_S B \cdot ds = 0.$

Ampere's ckt law, $\oint H \cdot dl = I$



Consider the boundary betn two media characterised by μ_1 and μ_2, respectively.

When $\Delta h \xrightarrow{lt} 0$ $\oint B \cdot ds = 0$

$\rightarrow B_{n1} \Delta S - B_{n2} \Delta S = 0.$

$\rightarrow B_{n1} = B_{n2}$

$\mu_1 H_{n1} = \mu_2 H_{n2}$

→ Normal Component of B is Continuous across the boundary.

→ Normal component of H is discontinuous.

let K is surface current on boundary.

Applying $\oint H \cdot dl = I$

$(K) \times \Delta w \Rightarrow H_{t1} \Delta w - H_{t2} \Delta w + H_{n1} \frac{\Delta h}{2} + H_{n2} \frac{\Delta h}{2}$

$- H_{n2} \frac{\Delta h}{2} - H_{n1} \frac{\Delta h}{2} = (K) \Delta w$

When $\Delta h \xrightarrow{lt} 0$

$\boxed{H_{t1} - H_{t2} = K} \Rightarrow (H_1 - H_2) \times a_{n12}$

When surface current exist then tangential component of H is also discontinuous.

$\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K.$

When free current density, $K = 0$

$H_{t1} = H_{t2}$ $\Rightarrow \frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$

Now, $B_{n1} = B_{n2}$

$B_1 \cos\theta_1 = B_2 \cos\theta_2 \rightarrow (1)$

$H_{t1} = H_{t2}$

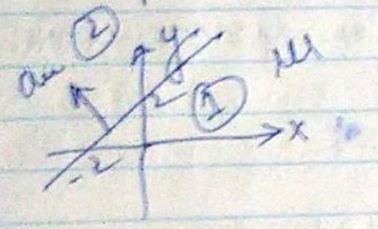
$\frac{B_1}{\mu_1} \sin\theta_1 = \frac{B_2}{\mu_2} \sin\theta_2 \rightarrow (2)$

Dividing:- $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}.$

Ques: Given $H_1 = -2a_x + 6a_y + 4a_z$ A/m in region $y - x \leq 2$; where $\mu_1 = 5\mu_0$

(a) find M_1 and B_1

(b) H_2 and B_2 in region $y - x \geq 2$ where $\mu_2 = 2\mu_0$.



Unit normal vector for any func. f

$$a_n = \frac{\nabla f}{|\nabla f|}$$

$$f = y - x - 2$$

$$\nabla f = \frac{\partial}{\partial x} f \, a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

$$= -a_x + a_y$$

$$\therefore a_n = \frac{-a_x + a_y}{\sqrt{2}}$$

Now, $M_1 = \chi_{m_1} H_1 = (\mu_{r_1} - 1) H_1$

$$= -8a_x + 24a_y + 16a_z.$$

$B_1 = \mu_1 H_1$

(b) $H_{n1} = (H_1 \cdot a_n) a_n$

$$= \frac{1}{\sqrt{2}}(\cancel{2a_x} + 6a_y)\left(\frac{-a_x + a_y}{\sqrt{2}}\right)$$

$$= \left[(-2a_x + 6a_y) \cdot \left(\frac{-a_x + a_y}{\sqrt{2}}\right)\right]\left[\frac{-a_x + a_y}{\sqrt{2}}\right]$$

$$= \left[\frac{2 + 6}{2}\right](-a_x + a_y)$$

$$= -4a_x + 4a_y$$

$\therefore H_{t1} = H_1 - H_{n1} = 2a_x + 2a_y + 4a_z$

Now, $H_{t2} = H_{t1} = 2a_x + 2a_y + 4a_z$

$$H_{n2} = \frac{\mu_1}{\mu_2} H_{n1} = -10a_x + 10a_y$$

$\therefore H_2 = H_{n2} + H_{t2} = -8a_x + 12a_y + 4a_z$

$B_2 = \mu_2 H_2$.

→ static Electric and magnetic fields are independent of each other.

→ whereas in dynamic EM these two are interdependent.

→ Time varying fields are represented by $E(x,y,z,t)$ and $H(x,y,z,t)$.

static charge → Electrostatic field
constant current → Magnetostatic "
Time-varying current → EM field (waves)
⇓
time variable fields

→ Faraday's Law ⇒ Electromotive force
→ Displacement current

Ae^{jt}

Faraday's Law:-

→ static magnetic field produce no flow of current.

→ But a time varying field produces an induced voltage (electromotive force or emf in a closed ckt, which causes flow of current.

is equal to rate of change of magnetic flux. linkage by the ckt.

$$V_{emf} = -\frac{d\phi}{dt}$$

If there is 'N' no. of turns in ckt.

$$V_{emf} = -N\frac{d\psi}{dt}$$

ψ is the flux through a single turn

→ -ve sign shows that induced voltage acts in such a way that it oppose the flux producing it.

→ Direcⁿ of current flow in the ckt is such that the induced magnetic field produced by the induced current will oppose the chan in original magnetic field.

let $N = 1$.

$$V_{emf} = -\frac{d\psi}{dt}$$

$$\rightarrow \oint_L E\cdot dl = -\frac{d}{dt}\int_S B\cdot ds$$

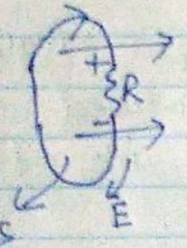
where, B is magnetic field and S surface area of ckt bounded by L.

→ Here E and B are inter related.

Time varying flux may exist, when-

1. stationary loop & time varying B.
2. Time varying " & static B.
3. ____________ time varying ".

(1) stationary loop in time varying (B) :-
(Transformer action)



$V_{emf} = \oint_L E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot ds$

Applying stokes theorem

$\int_S (\nabla \times E) \cdot ds = -\int \frac{\partial B}{\partial t} ds$

Maxwell's eqn for $\leftarrow$ $\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$
time varying field.

* Is Energy conservatn violated →?
since $\nabla \times E \neq 0$.

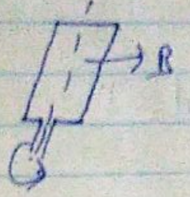
soln: No since, work done on charge was been done by ~~B~~ B(t).

(2) Moving loop in static B-field:-
(Motional emf) → generator action.

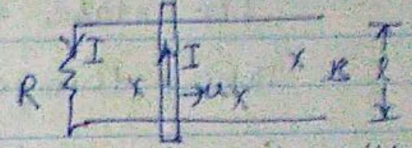
charge moving with velocity u.

$F_m = q\, u \times B$

$\therefore V_{emf} = \oint_L F_m \cdot dl = \oint_L (u \times B) dl$



Next:-



Rod is moving betn pair of rails with velocity B.

$F_m = I l \times B = Il B$.

$V_{emf} = \oint_L (\underbrace{u \times B}_{E_m}) \cdot dl$

$= \cancel{(u B \cdot dl)}$

$= \iint_S [\nabla \times (u \times B)] \cdot ds$

$\nabla \times E_m = \nabla \times (u \times B)$.

Moving loop in time varying field

$V_{emf} = \oint_L E \cdot dl$

$= -\int \frac{\partial B}{\partial t} ds + \oint_L (u \times B) dl$

$\rightarrow \nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (u \times B)$

Displacement current:-

For static field, $\nabla \times H = J$

Taking divergence,

$$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J \rightarrow (1)$$

from the continuity eqn

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0 \rightarrow (2)$$

Hence for time varying field (1) and (2) are not compatible.

There should be some modification

$$\nabla \times H = J + J_d$$

Taking divergence,

$$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J + \nabla \cdot J_d$$

$$\therefore \nabla \cdot J = -\nabla \cdot J_d$$

$$\rightarrow \nabla \cdot J_d = \frac{\partial \rho_v}{\partial t} = \frac{d}{dt}(\nabla \cdot D)$$

$$\rightarrow \nabla \cdot J_d = \nabla \cdot \frac{d}{dt} D$$

$$\rightarrow \boxed{J_d = \frac{\partial D}{\partial t}} \rightarrow \text{Displacement current.}$$

$$\therefore \boxed{\nabla \times H = J + \frac{dD}{dt}}$$

conduction current density

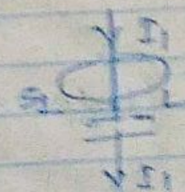
Maxwell eqn for time varying field.

Displacement current.

$$I_d = \int J_d \cdot ds = \int \frac{dD}{dt} \cdot ds$$

→ Displacement current exists when there is time varying E-field.

Ex:- current capacitor through capacitor when alternating voltage source is applied.



→ Applying Ampere's Law:-

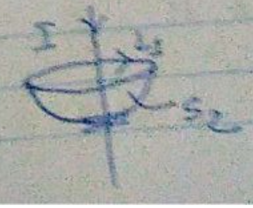
$$\oint_L H \cdot dl = \int_{S_1} J \cdot ds = I_1$$

I_1 is current through conductor.

S_1 is the surface area of loop L.

Taking a curved surface that passes ~~through~~ between the capacitor plates.

Ampere's Law:-



$$\oint_L H \cdot dl = \int_{S_2} J \cdot ds = I_{enc} = 0$$

Because there is no conduction current J through S_2.

→ There are the conflicting situations.

so to avoid this, some another current concept should be present. i.e. displacement current J_d.

Total current density $= J + J_d$

for case ①

Now, ~~Here~~ $J_d = 0$.

for case ②:-

$$\oint_{L_1} H \cdot dl = \int_{S_2} (\underset{0}{J} + J_d) \cdot ds$$

$$= \int_{S_2} J_d \cdot ds = \frac{d}{dt} \int_{S_2} D \cdot ds = \frac{dQ}{dt}$$

$$= I$$

→ Hence in case ① we have conduction current but in case ②, we have displacement current.

Ques: In free space $E = 20\cos(\omega t - 50z)\, a_y$ calculated J_d & H.

Soln: $J_d = \frac{d}{dt}(D)$.

$$\nabla \times E = -\frac{dB}{dt}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -(j\omega)B$$

$$= -j\omega\mu H$$

Maxwell's eqn:-

$\nabla \cdot D = \rho_v \qquad \oint D \cdot ds = Q$

$\nabla \cdot B = 0 \qquad \oint B \cdot ds = 0$

$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \oint E \cdot dl = -\frac{\partial}{\partial t} \int B\, ds$ Faraday's

$\nabla \times H = J + \frac{dD}{dt} \qquad \oint H \cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) ds$ Ampere Law.

Time Harmonic fields:-

→ Time harmonic field varies periodically or sinusoidally with time.

Phasor:-

for any complex no. $z = x + jy$. phasor is given by

$z = r\angle\phi$

$z = re^{j\phi} = r(\cos\phi + j\sin\phi)$

$r = \sqrt{x^2+y^2}$; $\phi = \tan^{-1}(y/x)$

$z_1 = x_1 + jy_1 = r_1\angle\phi_1$

$z_2 = x_2 + jy_2 = r_2\angle\phi_2$

$z_1 z_2 = r_1 r_2 \angle\phi_1+\phi_2$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1-\phi_2$

$\sqrt{z} = \sqrt{r}\angle\phi/2$

$z^* = r\angle-\phi$

For time element, $\phi = \omega t + \theta$

$z = re^{j\phi} = re^{j\theta}e^{j\omega t}$

$Re(re^{j\phi}) = r\cos(\omega t+\theta)$

$Im(z) = r\sin(\omega t+\theta)$

complex term $re^{j\theta}$ is called phasor term.

instantaneous term ~~z(t)~~ is $Re(z(t))$

$z(t)$

→ phasor can be scalar or vector.

let $A(x,y,z,t)$ is time harmonic field.

~~phasor form is~~, $A = Re(\underset{\text{phasor term}}{A_s} e^{j\omega t})$

$A = A_0\cos(\omega t - \beta x)a_y$

$A = Re(A_0 e^{-j\beta x} a_y e^{j\omega t})$

phasor form is:- $A_s = A_0 e^{-j\beta x} a_y$

$\frac{\partial A}{\partial t} = \frac{\partial}{\partial t} Re(A_0 e^{-j\beta x} a_y e^{j\omega t})$

$= Re(j\omega A_s e^{j\omega t})$

$\frac{\partial A}{\partial t} \rightarrow j\omega A_s$

$\int A\,\partial t \rightarrow \frac{A_s}{j\omega}$

EM waves :- Radio waves, TV signal, radar signal, light

characteristics :-

(1) Travel with high velocity
(2) Follow the properties of waves
(3) Radiate outward from the source.

Medium characteristics :- $[\sigma, \mu, \epsilon]$

Free space :- $\sigma = 0$; $\epsilon = \epsilon_0$; $\mu = \mu_0$

Lossless dielectric :- $\sigma = 0$; $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$
$\hookrightarrow \sigma << \omega\epsilon$

Lossy dielectrics :- $\sigma \neq 0$; $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0 \mu_r$

Good conductors :- $\sigma = \infty$; $\epsilon = \epsilon_0$; $\mu = \mu_0 \mu_r$
$\hookrightarrow \sigma >> \omega\epsilon$

ω = Angular frequency of wave.

<u>Wave Propagation in lossy dielectrics</u> :-

→ EM waves propagating in lossy dielectric losses power due to imperfect dielectric (partially conducting medium)

→ consider a linear, isotropic, homogeneous, lossy dielectric medium which is charge free ($\rho_v = 0$)

from maxwell's eqn :-

$\nabla \cdot E_s = 0$ [since, $\rho_v = 0$] → (1)

$\nabla \cdot H_s = 0$ → (2)

$\nabla \times E = -\frac{dB}{dt} = -j\omega\mu H_s$ → (3)

$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E_s + j\omega\epsilon E_s$

→ $\nabla \times H = [\sigma + j\omega\epsilon] E_s$ → (4)

Taking curl of eqn (3) :-

$\nabla \times \nabla \times E_s = -j\omega\mu (\nabla \times H_s)$

since, $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$\therefore \nabla(\nabla \cdot E_s) - \nabla^2 E_s = -j\omega\mu (\nabla \times H_s)$

using eqn (1) :- and (4)

$\nabla^2 E_s = j\omega\mu (\sigma + j\omega\epsilon) E_s$

$\nabla^2 E_s - \gamma^2 E_s = 0$

where, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

γ is called propagation constant of medium.

similarly, $\boxed{\nabla^2 H_s - \gamma^2 H_s = 0}$

→ These equations are known as Helmholtz eqn / vector wave equations.

$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$

$\gamma = \alpha + j\beta$

$Re(\gamma^2) = Re[-\omega^2\mu\epsilon + j\omega\mu\sigma]$

$= -\omega^2\mu\epsilon$

$|\gamma|^2 = \alpha^2 + \beta^2 = \left[\sqrt{\sigma^2 + (\omega\epsilon)^2}\right]\omega\mu$

$Re(\gamma^2) = Re(\alpha^2 + j2\alpha\beta - \beta^2)$

$= \alpha^2 - \beta^2$

$\therefore \alpha^2 - \beta^2 = -\omega^2\mu\epsilon$

and, $\alpha^2 + \beta^2 = \omega\mu\sqrt{\alpha^2 + (\omega\epsilon)^2}$

solving, $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$

$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$

let wave is propagating in z-direcn and E has only x-component

then $E = E_x(z)a_x$

$\nabla^2 E - \gamma^2 E = 0$.

$(\nabla^2 - \gamma^2)E_x(z) = 0$

$\underbrace{\frac{\partial^2}{\partial x^2}E_x(z)}_{0} + \underbrace{\frac{\partial^2}{\partial y^2}E_x(z)}_{0} + \frac{\partial^2}{\partial z^2}E_x(z) - \gamma^2 E_x = 0$

$\therefore \left[\frac{\partial^2}{\partial z^2} - \gamma^2\right]E_x(z) = 0$.

$\therefore E_x = E_0 e^{-\gamma z} + E_0' e^{\gamma z}$

E_0 and E_0' are constants.

since field should be finite at ∞.

$\therefore E_0' = 0$.

$\rightarrow e^{\gamma z}$ indicates that wave propagating along $-a_z$, but assumption is that, it is along a_z

$\therefore E_0' = 0$.

Introducing time factor $e^{j\omega t}$

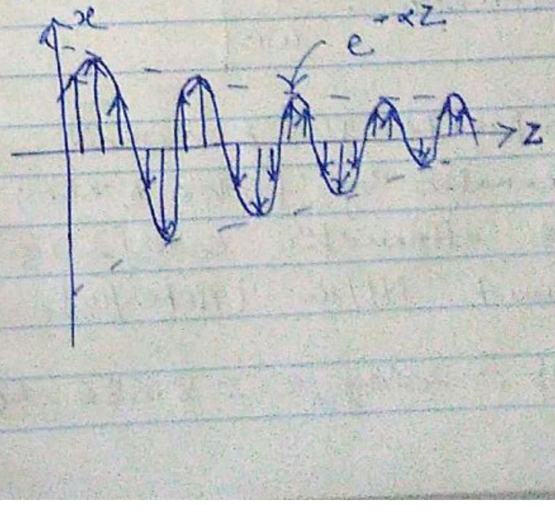
$E_x = E_0 e^{-(\alpha + j\beta)} e^{j\omega t}$

Instantaneous value,

~~$E_x(z,t) = E(z,t)a_x$~~

$E(z,t) = Re\left[E_0 e^{-(\alpha + j\beta)z} e^{j\omega t} a_x\right]$

$$\boxed{E(z,t) = E_0 e^{-\alpha z}\cos(\omega t - j\beta z)a_x}$$



$\nabla \times E = -\mu \frac{dH}{dt} \rightarrow H = -\frac{1}{\mu}\int(\nabla \times E)dt$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$$

$$= a_x(0-0) + a_y\left[\frac{\partial}{\partial z}E_x - 0\right] - 0 \cdot a_z$$

$$= \frac{\partial}{\partial z}E_x\, a_y$$

since, $E_x(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)$

$\therefore H(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z - \theta_\eta)\, a_y$

& $\frac{E_0}{\eta} e^{-\alpha z}\cos(\omega t - \beta z)\, a_y$

where $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = |\eta|\angle\theta_\eta$

$= |\eta| e^{j\theta_\eta}$

$$\boxed{\tan 2\theta_\eta = \frac{\sigma}{\omega\epsilon}}$$

→ wave propagates in z-direcⁿ.

It attenuates by factor $e^{-\alpha z}$;

$\therefore \alpha$ is attenuatⁿ coeff.

unit Np/m (Neper/m).

1 Np = $20\log_{10} e = 8.686$ dB.

→ 1 neper indicates reducⁿ to e^{-1} of original value.

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

when, $\sigma = 0$, $\alpha = 0$

i.e when lossless medium or free space, there is no attenuation / or wave no

→ β is phase coeff which gives measure of phase shift per unit length. unit → Radian/m.

$v = \frac{\omega}{\beta}$; $\beta = \frac{2\pi}{\lambda}$

→ E & H are out of phase by θ_η at any instant of time due to complex intrinsic impedance of medium.

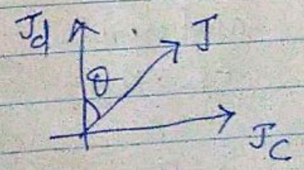
→ E leads H by θ_η.

for lossy medium,

$\frac{|J_c|}{|J_d|} = \frac{\sigma E}{\omega\epsilon E} = \frac{\sigma}{\omega\epsilon} = \tan\theta$

→ $\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$

→ $\tan\theta$ is loss tangent



θ is the loss angle.

(15)

Loss tangent, $\tan\theta = \frac{\sigma}{\omega\epsilon}$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\epsilon}\right]^2}+1\right]}$$

Given that, $\mu = 5\mu_0$

$\epsilon = \epsilon_0$, $f = 5$ MHz

$\frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3\times10^8$

$v_p = \frac{1}{\sqrt{\mu\epsilon}}$

$\frac{\sigma}{\omega\epsilon} = v$

$\rightarrow \sigma = (v)\,\omega\epsilon$

(2)

Reflection and transmission of EM waves:-

Dielectric - dielectric ; Di-free:

(a) Normal incidence:-

material 1 | material 2
ϵ_1, η_1 | ϵ_2, η_2

incident wave $\rightarrow E_t, H_t$

E_i, H_i

$E = \eta H$

$\therefore E_i = \eta_1 H_i$

$E_t = \eta_2 H_t$

$E_r = -\eta_1 H_r$

→ when wave reflected, propagation direction changes and one of the field component also change its direction either E or H.

	E_x	H_y	
Incident	a_x	a_y	$\rightarrow a_z$
	$-a_x$	a_y	$-a_z$
	a_x	$-a_y$	$-a_z$

→ when normal incidence of wave mean E and H fields are tangential to boundary

$E_{t1} = E_{t2}$; $H_{t1} = H_{t2}$

when no surface bound charge

0 current

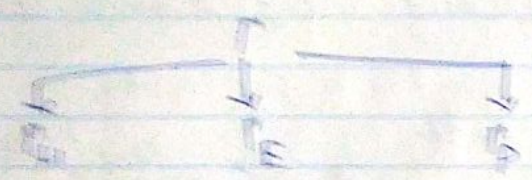
$E_i + E_r = E_t$ → fraction of trans. wave

$1 + \frac{E_r}{E_i} = \frac{E_t}{E_i}$ $\tau = \frac{E_t}{E_i}$

↳ fraction of reflected wave $= \Gamma$ = Reflection coeff.

$1+\Gamma = \tau$; $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$

since $\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$ and $\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$; $\Gamma = \frac{\sqrt{\epsilon_1}-\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}}$



$\Gamma_H = \frac{\eta_1-\eta_2}{\eta_2+\eta_1}$

$\Gamma_P = \Gamma_E \cdot \Gamma_H = -\Gamma_H^2 = -\Gamma_E^2$

when $\epsilon_1 > \epsilon_2$

$\Gamma_E = +ve$

$\tau_E > 1$

$E_t > E_i$

$\Gamma_H = -ve$

$\tau_H < 1$

$\Gamma_P = -ve$

$\therefore 1+\Gamma_P = \tau_P \Rightarrow \tau_P < 1$

(5) $\Gamma_E = \frac{\sqrt{\epsilon_1}-\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}+\sqrt{\epsilon_2}}$

$\tau_E = 1+\Gamma_E \rightarrow$ conductor [illegible]

$\frac{E_t}{E_i} = \tau_E$

$\Rightarrow E_t = \checkmark$ for (1) boundary.

Again for 2nd boundary

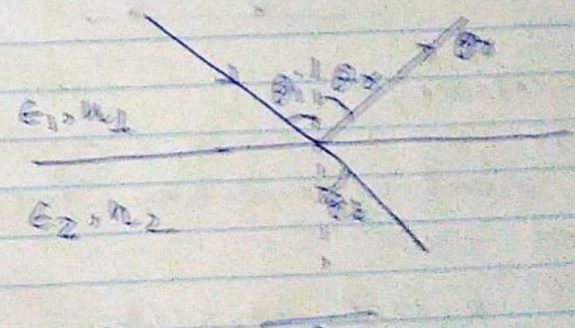
$E_i = E_t$

$\Gamma_E = \checkmark$

$\tau_E = \checkmark$

$\frac{E_t}{E_i} = \checkmark \Rightarrow E_t = \checkmark$

(6) oblique incidence (internal).



(1) $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

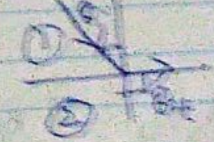
(2) $\theta_i = \theta_r$

for critical angle :- when $\theta_t = 90°$

$\Downarrow$

$\theta_i = \theta_c$

$\sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$



for $\theta_i > \theta_c \Rightarrow$ zero transmission and total reflection.

Polarization and reflecⁿ :-

horizontal polarizatⁿ
s-polarization

vertical polarizatⁿ
p-polarizatⁿ

(normal Incidence)

E-field boundary condition for s-polarized wave

p-polarizatⁿ

$$\Gamma_s = \frac{\eta_2 \sec\theta_t - \eta_1 \sec\theta_i}{\eta_2 \sec\theta_t + \eta_1 \sec\theta_i}$$

$$\Gamma_p = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}} \; ; \; \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

$$\Gamma_s = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

$$\Gamma_p = \frac{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}{\frac{\epsilon_2}{\epsilon_1}\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i}}$$

Now, if $\Gamma_p = 0$; complete transmission, zero reflecⁿ.

$$\left(\frac{\epsilon_2}{\epsilon_1}\right)^2 (1 - \sin^2\theta_i) = \frac{\epsilon_2}{\epsilon_1} - \sin^2\theta_i$$

$$\rightarrow \sin^2\theta_i = \frac{\epsilon_2}{\epsilon_1 + \epsilon_2} \Rightarrow \sin\theta_i = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\boxed{\tan\theta_i = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}}$$

$\theta_i \rightarrow$ Brewster's angle of incidence.

if $\Gamma_s = 0$
$\Downarrow$
$\epsilon_1 = \epsilon_2 \rightarrow$ meaningless.

∴ Brewster's angle for zero reflecⁿ for s-polarized wave for different media not exist.

→ $\tan\theta_B = $ ✓

→ Any two media can have θ_B.

→ exactly at $\theta_i = \theta_B$.

→ only P-polarized wave can have θ_B

(13) $\eta = 240 \angle 30^\circ$

$\hookrightarrow \theta_n = 30^\circ$

loss tangent $= \frac{\sigma}{\omega\epsilon} = \tan 2\theta_n$

$= \tan 60^\circ = \sqrt{3}$.

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$f = 1$ MHz ; $\omega = $ ✓

$$\eta = \sqrt{\frac{\mu_0}{\epsilon}}$$

$\rightarrow \epsilon = $ ✓

$$\epsilon_c = \epsilon\left[1 - j\frac{\sigma}{\omega\epsilon}\right]$$