Control Systems

Subject Code: BEC-26

Third Year ECE

Unit-III

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Lecture 3

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Analysis of second order system for Step input

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

This is the standard form of the closed loop transfer function

These poles of transfer function are given by;

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\therefore\ s = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4(\omega_n)^2}}{2}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

Analysis of second order system for Step input

The poles are;

(i) Real and Unequal if $\sqrt{\xi^2 - 1} > 0$

i.e. $\xi > 1$ They lie on real axis and distinct

(ii) Real and equal if $\sqrt{\xi^2 - 1} = 0$

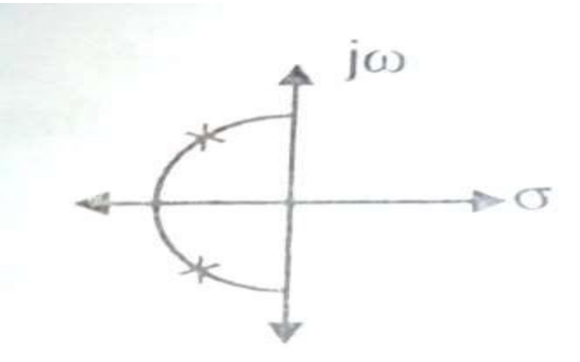
i.e. $\xi = 1$ They are repeated on real axis

(iii) Complex if $\sqrt{\xi^2 - 1} < 0$

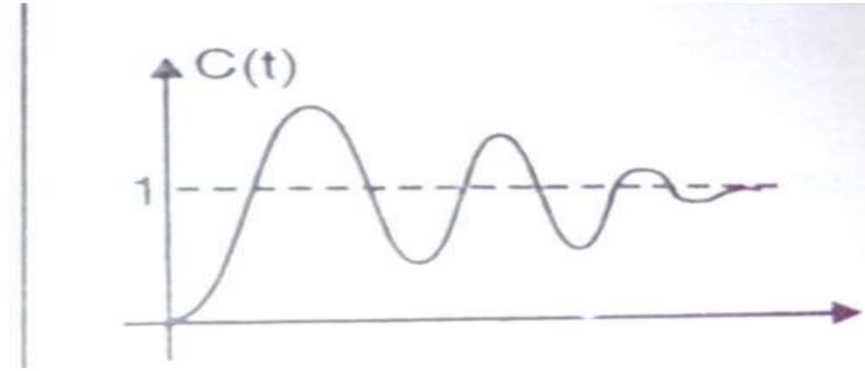
i.e. $\xi < 1$ Poles are in second and third quadrant

Relation between ξ and pole locations

(i) $0 < \xi < 1$ Under damped

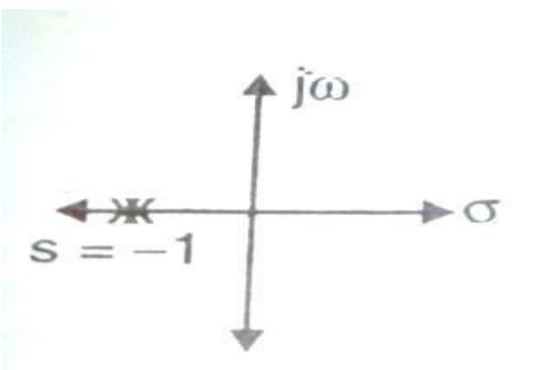


Pole Location

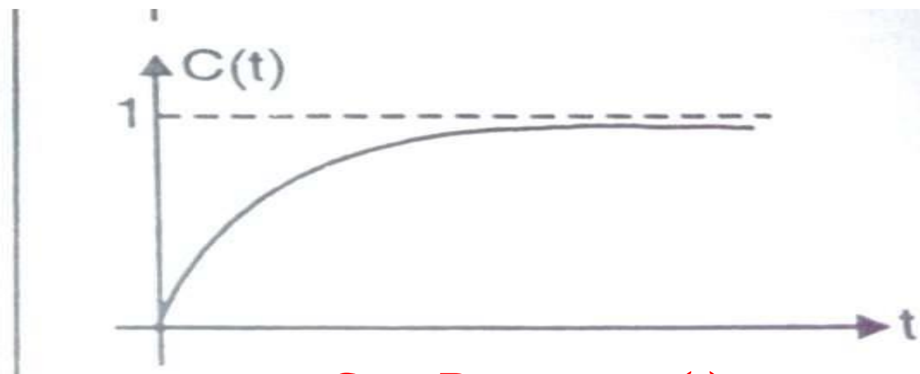


Step Response c(t)

(ii) $\xi = 1$ Critically damped



Pole Location



Step Response c(t)

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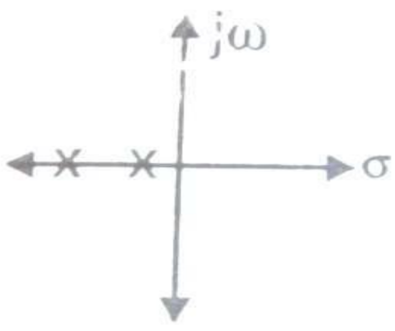
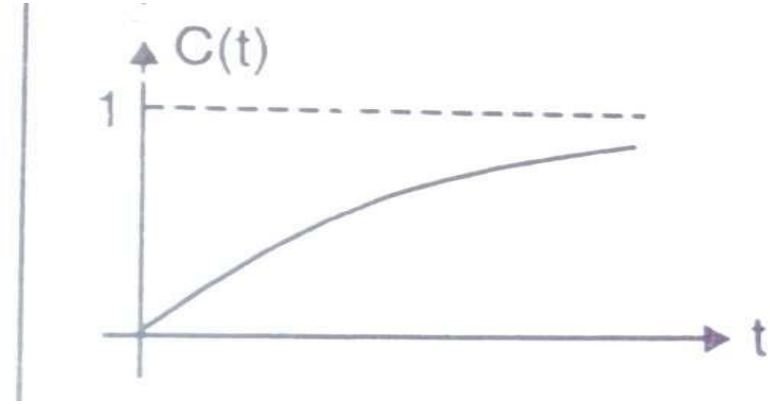
Relation between ξ and pole locations

(iii) $\xi > 1$ over damped

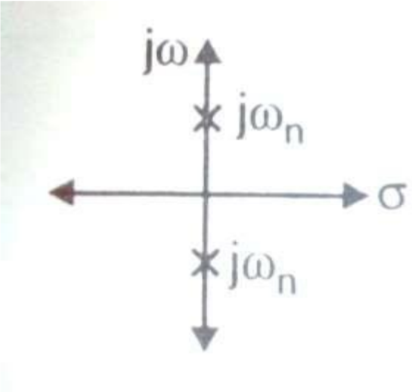


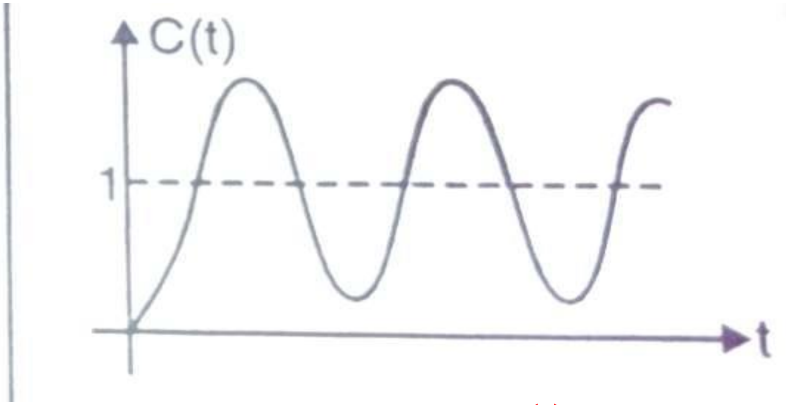


Pole Location

Step Response c(t)

(iv) $\xi = 0$



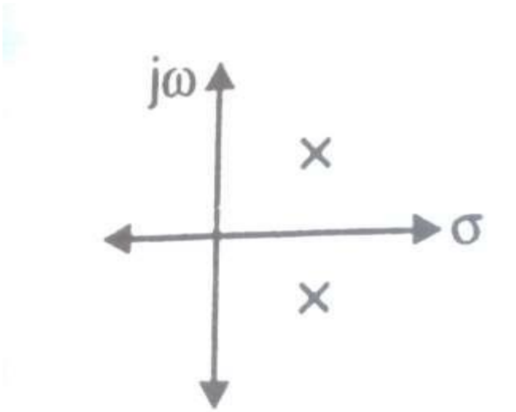


Pole Location

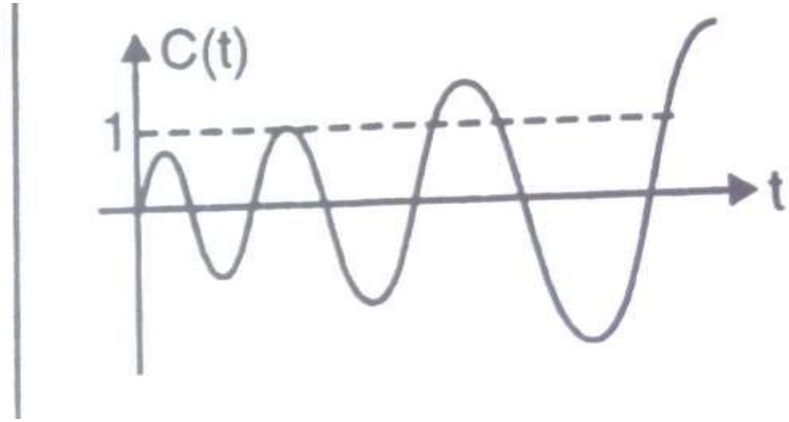
Step Response c(t)

Relation between ξ and pole locations

(v) $0 > \xi > -1$

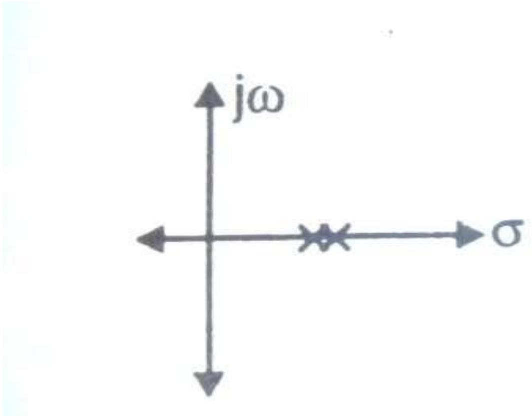


Pole Location

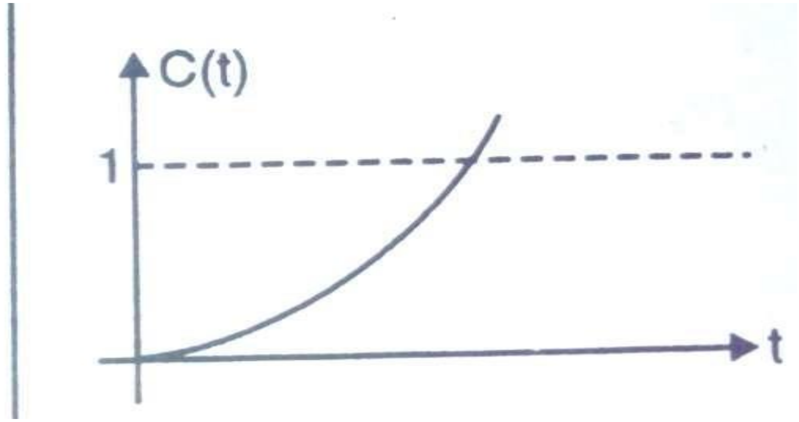


Step Response c(t)

(vi) $\xi = -1$



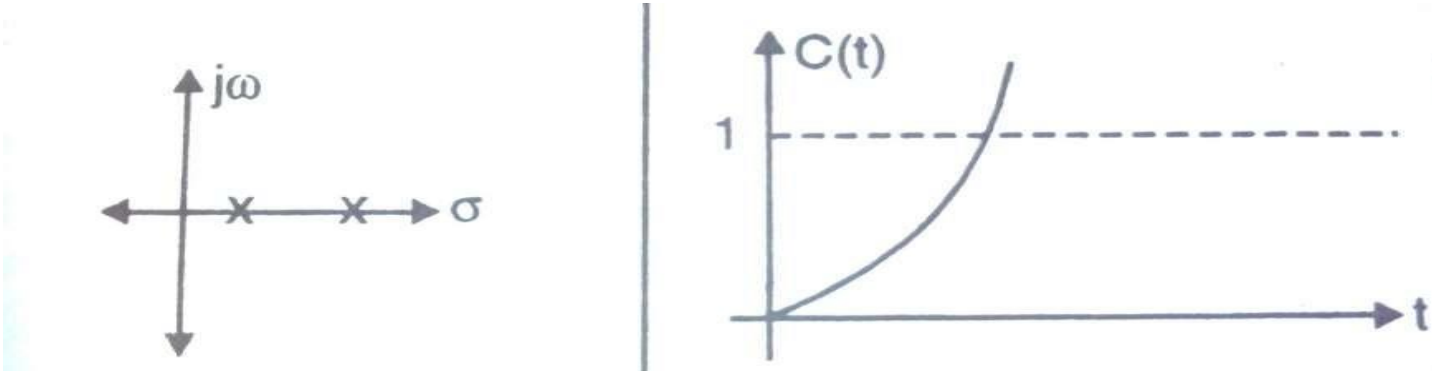
Pole Location



Step Response c(t)

Relation between ξ and pole locations

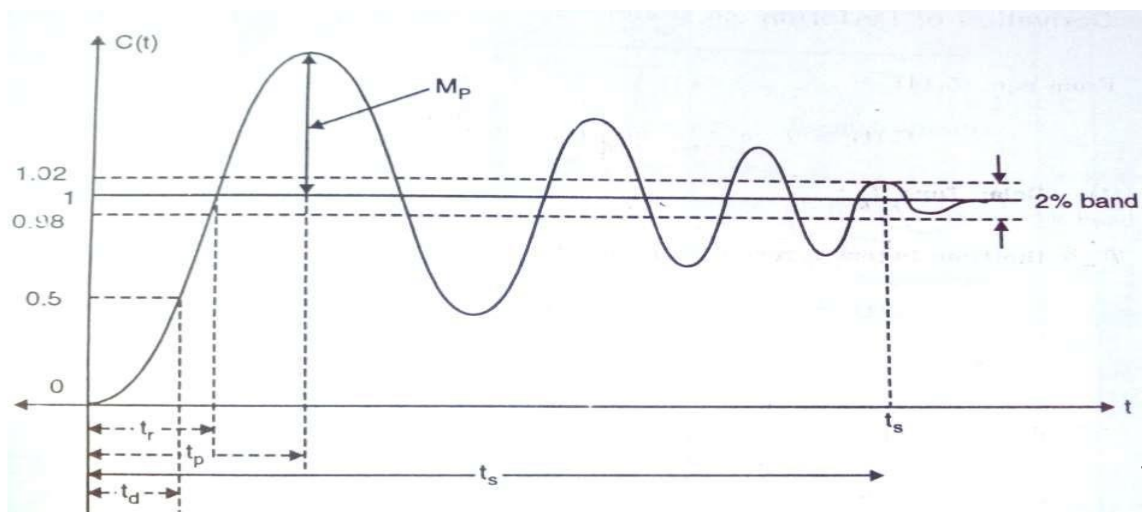
(vii) $\xi < -1$



Pole Location

Step Response c(t)

Time Response Specifications:



Time Response Specifications

- ✓ **Delay Time (t_d)**: It is time required for the response to reach 50% of the final value in the first attempt.

$$t_d = \frac{1 + 0.7\xi}{\omega_n}$$

- ✓ **Rise Time (tr)**: It is time required for the response to rise from 10% to 90% of the final value for overdamped systems. (It is 0 to 100% for under damped systems)

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where, $$\beta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

and $$\omega_d = \omega_n \sqrt{1-\xi^2}$$

Time Response Specifications

- ✓ **Peak Overshoot (Mp):** The maximum overshoot is the maximum peak value of the response curve measured from unity. It is therefore largest error between input and output during the transient period.

$$\% M_p = e^{-\left\{ \frac{\xi \pi}{\sqrt{1-\xi^2}} \right\}} \times 100$$

- ✓ **Peak Time (tp):** It is the time required for the response to reach the first peak.

$$t_p = \frac{\pi}{\omega_d}$$

- ✓ **Settling Time (ts):** It is the time required for the response curve to reach and stay within a specified percentage (usually 2% or 5%) of the final value.

$$T_s = 4T = \frac{4}{\xi \omega_n}$$

Example 3

A unity feedback system has

$$G(s) = \frac{16}{s(s+5)}$$

If a step input is given calculate

1. Damping Ratio
2. Overshoot
3. Settling Time

Solution: $G(s) = \dfrac{16}{s(s+5)}$ $\qquad H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{16}{s(s+5)}}{1+\frac{16}{s(s+5)}} = \frac{16}{s^2+5s+16}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{16}{s^2 + 5s + 16}$$

Compare denominators of both

Natural Frequency;

$$\omega_n^2 = 16 \qquad \therefore \omega_n = 4 \quad rad/\sec$$

Damping Ratio;

$$2\xi\omega_n s = 5s \qquad \therefore \xi = \frac{5}{2\times\omega_n} = \frac{5}{2\times 4} = 0.625$$

Settling Time;

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{(0.625)\times(4)} = 1.6 \ \sec$$

Example 3 cont..

Overshoot

$$\%M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\%M_p = e^{-\left\{\frac{(0.625)\pi}{\sqrt{1-(0.625)^2}}\right\}} \times 100$$

$$\%M_p = 8.08\%$$

Example 4

The open loop transfer function of a unity feedback system is,

$$G(s) = \frac{4}{s(s+1)}$$

Determine;

1. Delay Time
2. Rise Time
3. Peak Time
4. Settling Time
5. Maximum Peak Overshoot

Example 4 cont..

Solution: $G(s) = \dfrac{4}{s(s+1)}$ $\quad H(s) = 1$

Determine the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\dfrac{4}{s(s+1)}}{1+\dfrac{4}{s(s+1)}} = \frac{4}{s^2+s+4}$$

Compare closed loop TF with standard form of second order system

$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{4}{s^2+s+4}$$

By comparing denominators of both

Natural Frequency;

$$\omega_n^2 = 4 \qquad \therefore \omega_n = 2 \quad rad/\sec$$

Example 4 cont..

Damping Ratio;

$$2\xi\omega_n s = s \qquad \therefore \xi = \frac{1}{2\times\omega_n} = \frac{1}{2\times 2} = 0.25$$

Damped frequency of oscillations;

$$\omega_d = \omega_n\sqrt{1-\xi^2} \qquad \therefore \omega_d = 2\sqrt{1-(0.25)^2} = 1.936\ rad/\sec$$

Delay Time;

$$T_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.25)}{2} = 0.587\ \sec$$

Example 4

Rise Time;

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \tan^{-1}\frac{\sqrt{1-(0.25)^2}}{(0.25)} = 1.310 \text{ rad}$$

$$T_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-1.310}{(1.936)} = 0.945 \text{ sec}$$

Peak Time;

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1.936} = 1.622 \text{ sec}$$

Settling Time;

$$T_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{(0.25)\times(2)} = 8 \text{ sec}$$

Example 4 cont..

Maximum Peak Overshoot

$$\%M_p = e^{-\left\{\frac{\xi\pi}{\sqrt{1-\xi^2}}\right\}} \times 100$$

$$\%M_p = e^{-\left\{\frac{(0.25)\pi}{\sqrt{1-(0.25)^2}}\right\}} \times 100$$

$$\%M_p = 43.26\%$$