UNIT-II
Part-1

Design against static load

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Unit-II Syllabus

UNIT-I

Introduction

- Definition, Design requirements of machine elements, General design procedure, Introduction to Design for Manufacturing, Interchangeability, Limits, Fits and Tolerances, Standards in design, Selection of preferred sizes.

Engineering materials and their properties

- Classification, Mechanical properties, Ferrous and non-ferrous metals, Non metallic materials, Indian Standards designation of carbon & alloy steels, Selection criteria of materials.

UNIT-II

Design under Static Load

- Modes of failure, Factor of safety and basis of determination, Principal stresses, Torsional and bending stresses, Principal stresses in design of machine element, Theory of failure, Eccentric loading.

Design under Variable Loads

- Cyclic stresses, Fatigue and endurance limit, Factors affecting endurance limit, Stress concentration factor, Stress concentration factor for machine components, Notch sensitivity, Design for finite and infinite life, Soderberg, Goodman & Gerber criteria.

Learning Outcomes

At the end of this lesson, the students should be able to understand

- Difference between strength and stress.
- Meaning of static load.
- Modes of failure.
- Factor of Safety.
- Understanding the types of loading on machine elements. and allowable stresses.
- Different theories of elastic failure.
- Construction of yield surfaces for failure theories.
- Design of simple parts.
- Optimize a design comparing different failure theories.

Stress and Strength

- **Strength** is a property of a material or of a mechanical element. The strength of an element depends on the choice, the processing of the material.
- **Stress** is a state property at a specific point within a body, which is a function of load, geometry, temperature, and manufacturing processing.
- The survival of many products depends on how the designer adjusts the maximum stresses in a component to be less than the component's strength at specific locations of interest.
- Determination of stresses in structural or machine components would be meaningless unless they are compared with the material strength.
- If the induced stress is less than or equal to the limiting material strength then the designed component may be considered to be safe and an indication about the size of the component is obtained.
- We shall use the capital letter S to denote strength, the Greek letters σ (sigma) and τ (tau) to designate normal and shear stresses, respectively.

Design for Static Load

Static Load: A static load may be defined as an external force,

- Which is gradually applied to a mechanical element and
- Which does not change its magnitude or direction with respect to time.

Strength and Modes of Failure

A primary consideration in designing any component is to make it strong enough so that it will not fail in service. Actual breaking of the component is no doubt a failure; however, in designing we consider a broader perspective that an element is considered failed as and when it is unable to perform its function satisfactorily, due to any one of the following modes:

- Failure by elastic elongation or deflection. (columns, shafts)
- Failure by general yielding (ductile materials)
- Failure by actual breaking or fracture (brittle material, ductile material under certain conditions)
- Failure by wear (gears, cams, rolling element bearings)

Factor of safety

$$(fs) = \frac{failure\ load}{working\ load\ or\ service\ load}$$

$$(fs) = \frac{failure\ stress}{design\ stress\ \ or\ allowable\ stress}$$

$$\text{Design stress} = \frac{failure\ stress}{(fs)}$$

$$\text{Design stress}\ \sigma = \frac{S_{ut}}{(fs)}$$ (static load, brittle material) and

$$\text{Design stress}\ \sigma = \frac{S_{yt}}{(fs)}$$ (static load, ductile material)

Selection of factor of safety

Selection of an appropriate numerical value of factor of safety is perhaps one of the most important decisions in the design process, because the selection of an arbitrarily large value of factor of safety leads to an overdesigned and expensive component. On the other hand, selection of a too small value for factor of safety may cause failure and disaster . We need to design a strong component, but not too strong. There are several guidelines to select a suitable value for factor of safety .

- Codes
- Published Literature

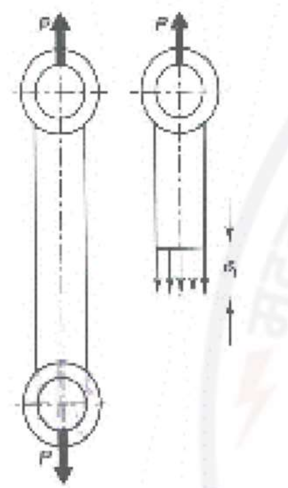
Selection of Factor of Safety

- How well do we know about the magnitude and kind of load?
- How well do we know about the material properties: percentage elongation, ultimate strength and yield strength?
- Degree of reliability of material: homogeneity, heat treatment, possibility of initial or residual stresses during heat treatment or manufacturing.
- Are we able to calculate stresses? The extent to which the properties may change during use.
- The kind of environment in which machine has to work; normal, high or low temperatures, corrosive, etc. Any inbuilt safety mechanism?
- Have we taken care of localized stresses?
- Cost of failure when in operation; is human life and property endangered? What will be the production loss?
- Cost of machine being designed. Is it for mass manufacture or only a few machines?

Decision Matrix for the Selection of Factor of Safety

Parameter	Very well known or known			One parameter not known			
Loads	✓✓	✓✓	✓	✓	✓	✓	?
Stresses	✓✓	✓✓	✓	✓	✓	?	✓
Material Properties	✓✓	✓✓	✓	✓	?	✓	✓
Environment	✓✓	✓✓	✓	?	✓	✓	✓
Low Weight required	Yes	No	No	No	No	No	No
(fs) for ductile materials	1.0 to 1.5	1.5 to 2	2 to 2.5	3 to 4			
(fs) for brittle materials	--	2 to 4	4 to 5	5 to 8			
✓✓Very well known; ✓ known; ? somewhat uncertain							

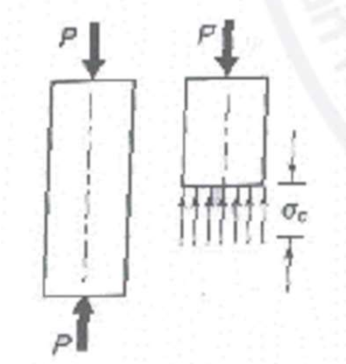
Stresses and Strains due to Simple Loads



$$\sigma_t = \frac{P}{A}$$

$$\delta = \frac{PL}{AE}$$

Tensile Stress

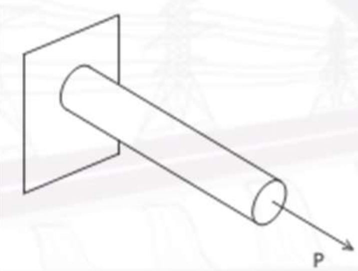


$$\sigma_c = \frac{P}{A}$$

$$\delta_c = \frac{AL}{AE}$$

Compressive Stress

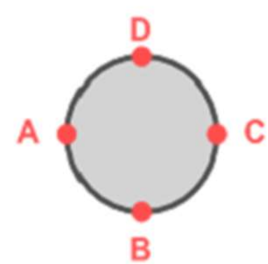
Axial Load (P)



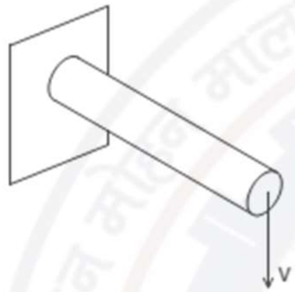
Axial Stress $\sigma = \frac{P}{A}$

A: cross sectional area

Stresses on element

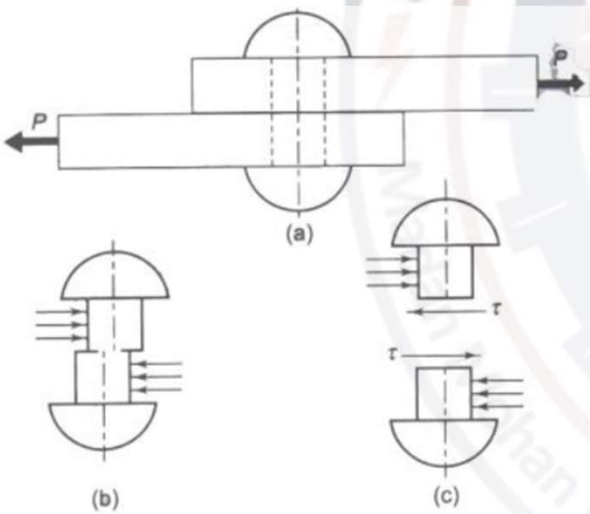


Shear Stress

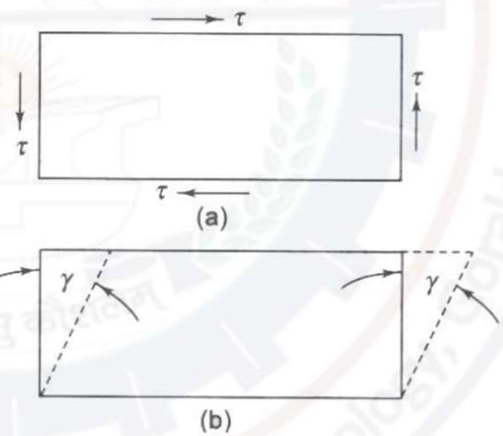


Transverse shear stress $\tau = \frac{V}{A}$

A: cross sectional area



$$\tau = \frac{P}{A}$$



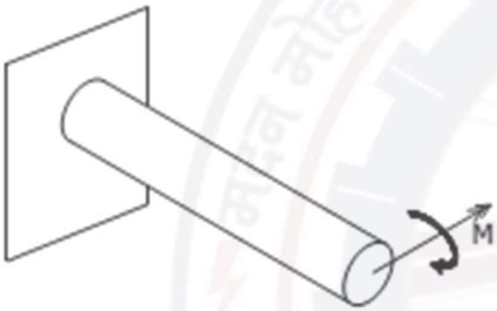
$$\gamma = \frac{\tau}{G}$$

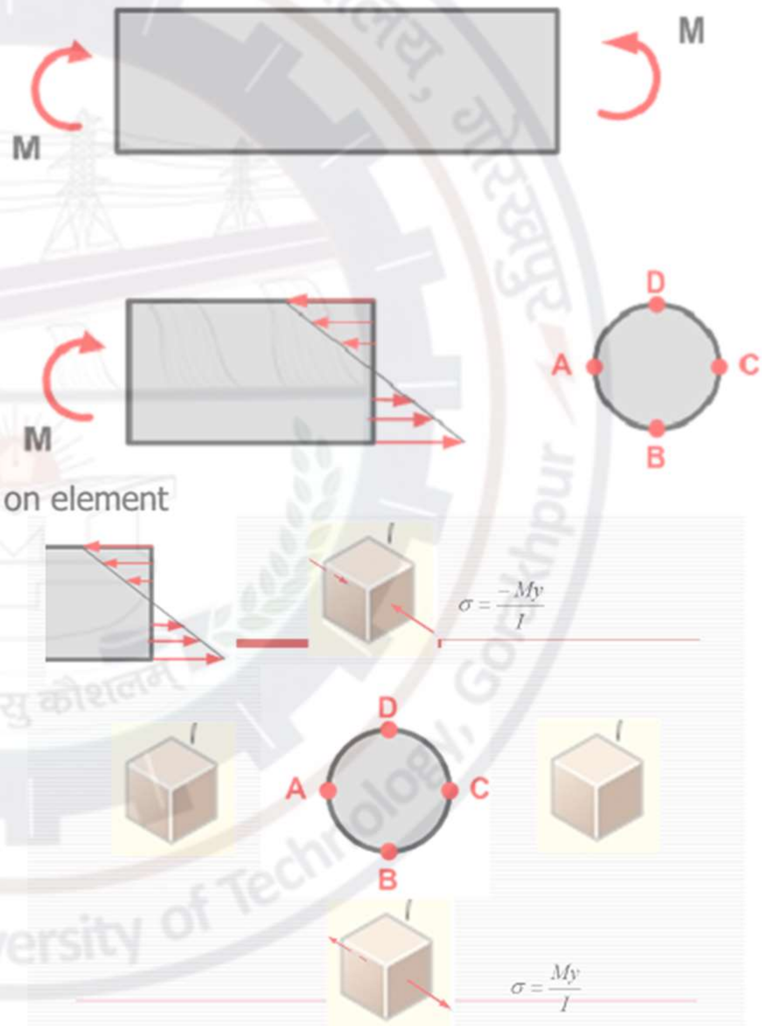
$$E = 2G\,(1 + \mu)$$

Shear Stress

Shear Strain

Bending Stress

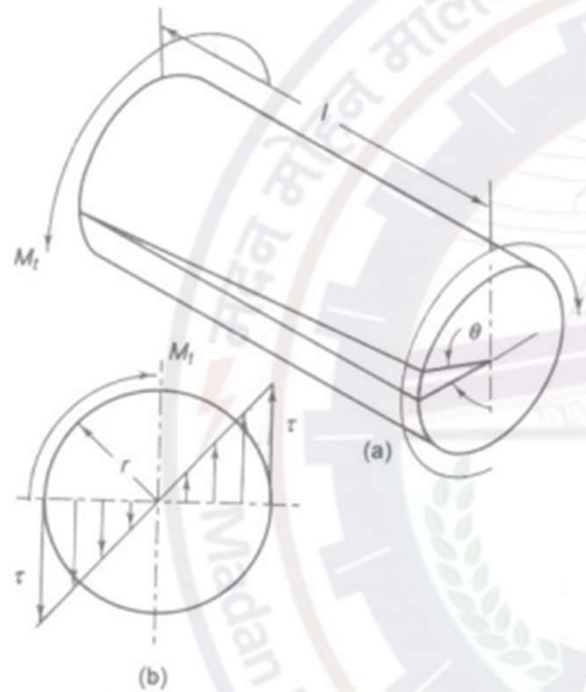


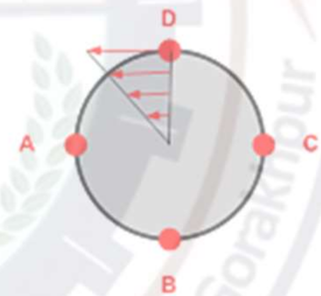


The bending stress $\sigma_b = \dfrac{M_b y}{I}$

$$= \frac{M}{Z}$$

Torsional Stress

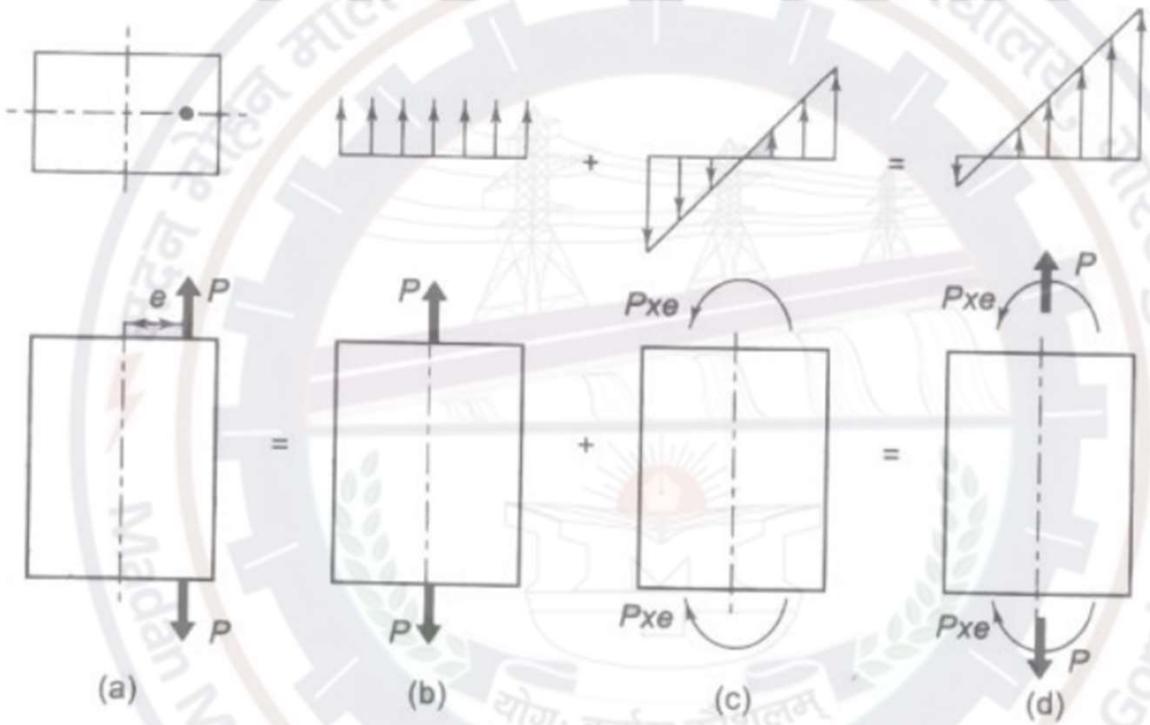




The torsional shear stress $\tau = \frac{M_t r}{J}$ N/mm^2

and angle of twist $\theta = \frac{M_t L}{JG}$ radian

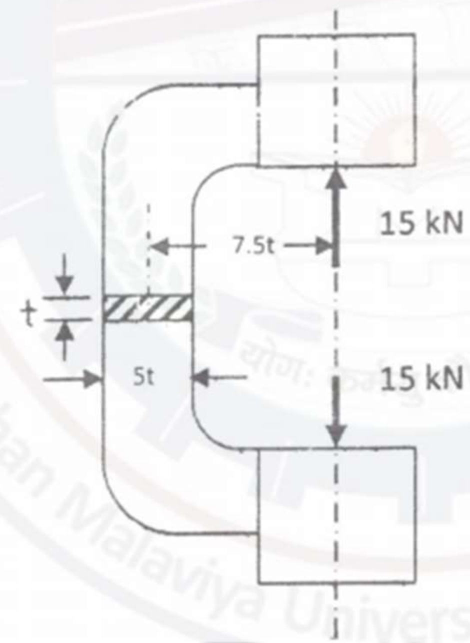
Eccentric Loading



The direct stress $\sigma = \frac{P}{A} \pm \frac{\mathrm{P}_{\mathrm{e}}\mathrm{y}}{I}$

Example

A C-frame subjected to a force of 15 kN is shown in the figure. It is made of grey cast iron FG 300. Taking a factor of safety of 2.5, determine the dimensions of the section of frame.



Solution

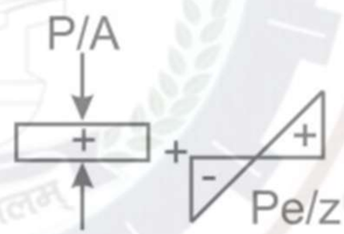
DATA and FBD: Material FG 300, (fs) = 2.5

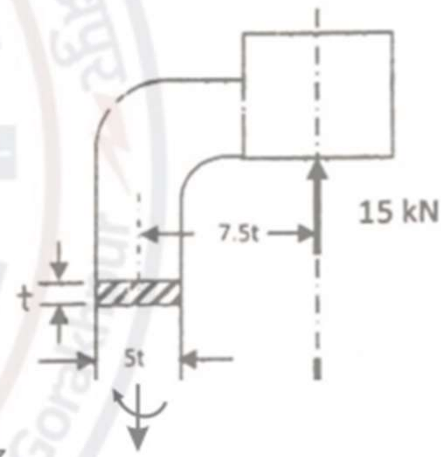
P = 15 kN = 15000 N, Thickness t

Analysis:

- The section is subjected to direct load P
- Moment Pe.

$$Allow. Stress\ \sigma_a = \frac{Su}{(fs)} = \frac{300}{2.5} = 120\ \ N/mm^2$$





Free Body Diagram

$$Direct\ Stress = \frac{P}{A} = \frac{15000}{(5t^2)} = \frac{3000}{t^2}\ N/mm^2$$

Max stress due to moment = Pe/z

$$= \frac{(15000)(7.5t)}{(t(5t)^2)/6} = \frac{27000}{t^2}\ N/mm^2$$

$$Thus\ 120 = \frac{3000}{t^2} + \frac{27000}{t^2}\ \ or\ t^2 = 250 = +15.8\ mm$$

Comment: Use t = 16 mm

Principal Stresses and Mohr's Circle

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{(\frac{\sigma_x-\sigma_y}{2})^2 + (\tau_{xy})^2} \quad \text{and} \quad \tau = 0$$

- Stresses σ_1 and σ_2 are called the principal stresses.
- The planes on which the principal stresses act are called principal planes.
- it may be observed that shear stress is zero on these planes.

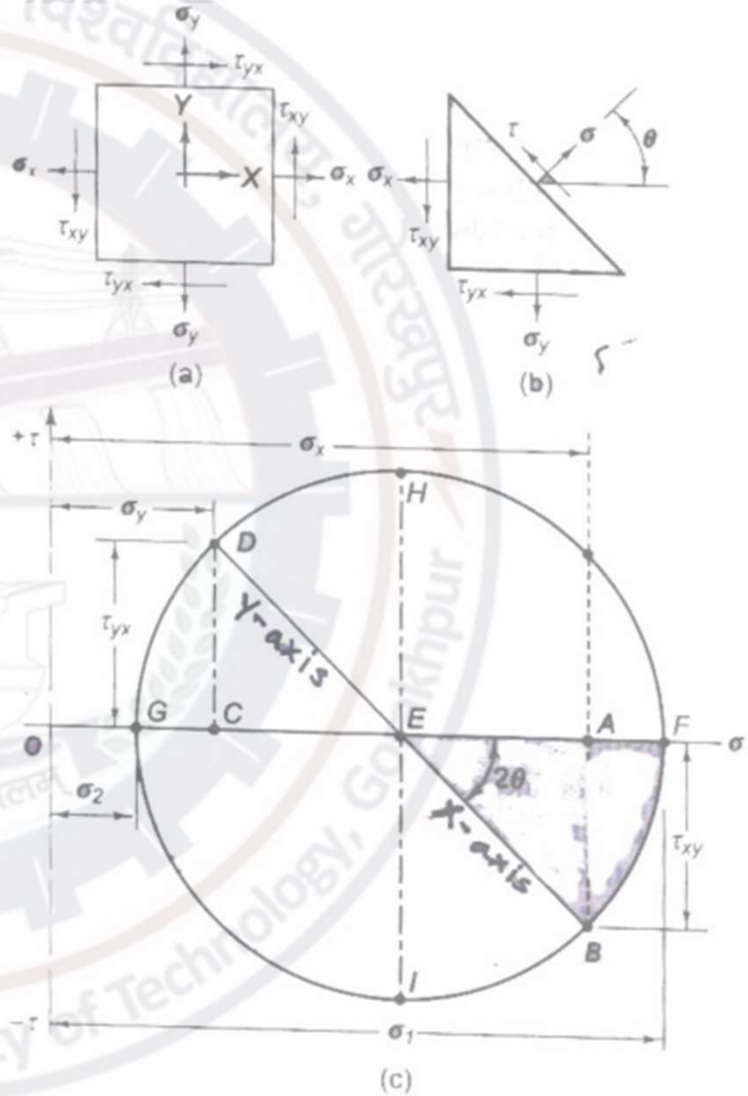
Principal Stresses and Mohr's Circle

Principal Stresses and Mohr's Circle

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Max. In-plane shear stress

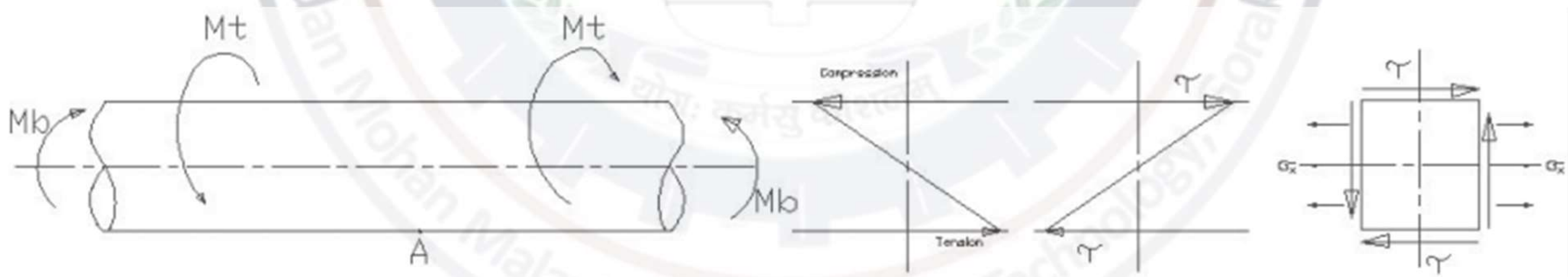
$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\,\tau_{xy}}\right)$$



a) Two dimensional state of stress
b) Stresses on oblique plane
c) Mohr's circle diagram

Combined Bending & Torsion

- Consider a cylindrical element subjected to a bending moment as well as a torsional moment leading to a two dimensional stress system.
- It may be noticed that both the bending stress and torsional stress have linear distribution, the maximum being at the outermost fiber from the neutral axis.
- In such cases we calculate principal stresses first.

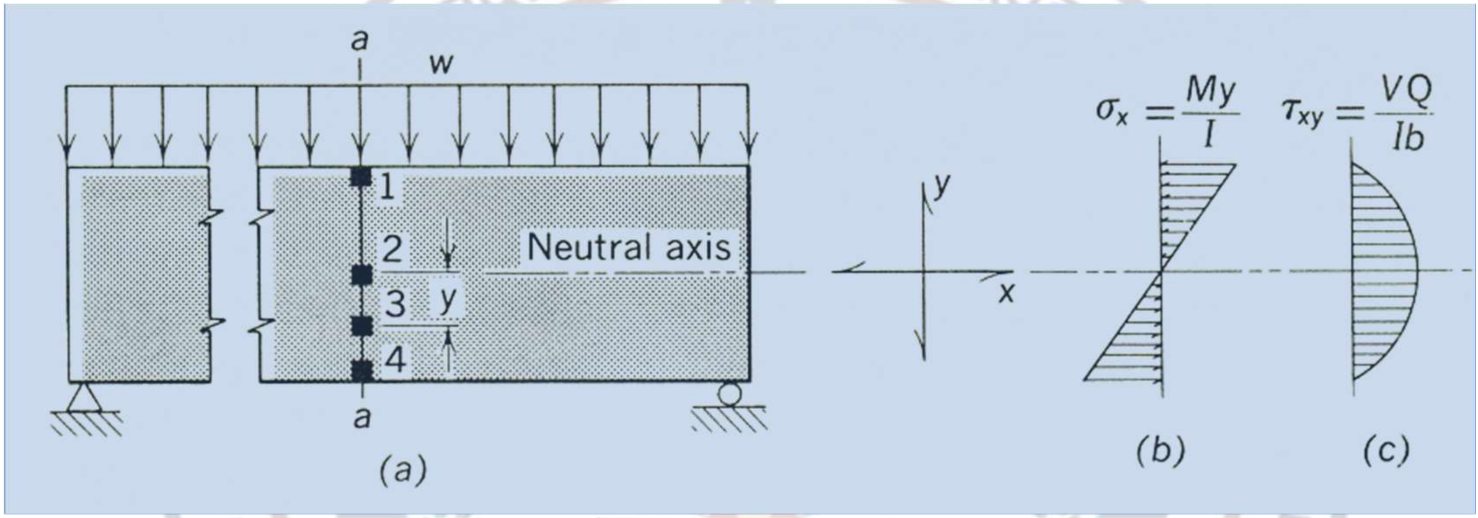


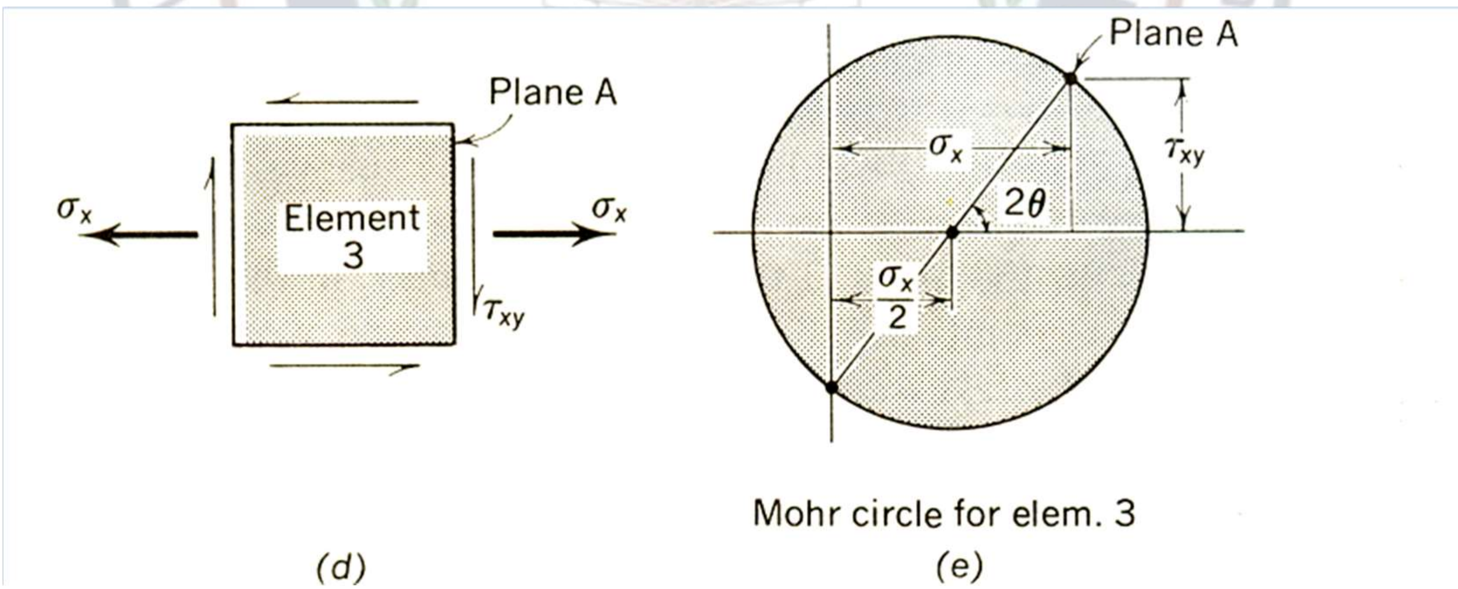
A

Tension bending stress

Combined Stress at A

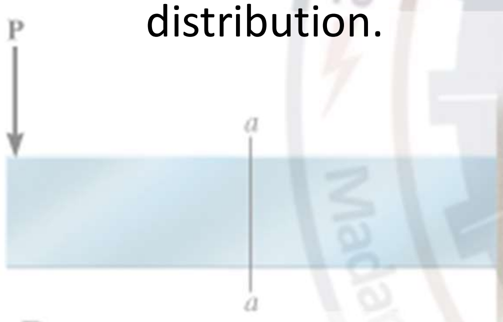
Stress variation throughout a prismatic beam



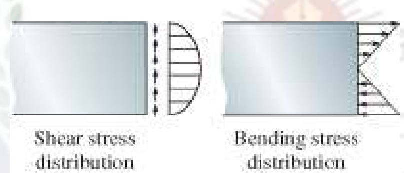


Stress variation throughout a prismatic beam

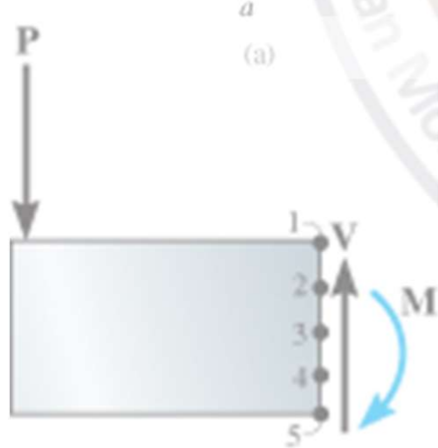
- A cantilevered beam that has a rectangular x-section and supports a load **P** at its end.
- At arbitrary section *a-a* along beam's axis, internal shear **V** and moment **M** are developed from a parabolic shear-stress distribution, and a linear normal-stress distribution.



(a)

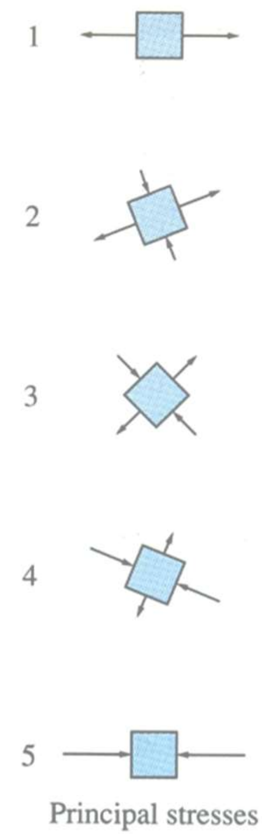


(c)



(b)

(e)

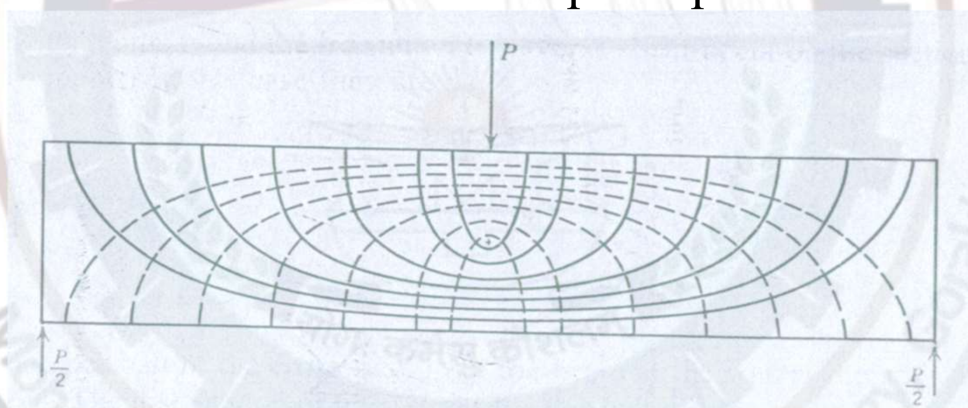


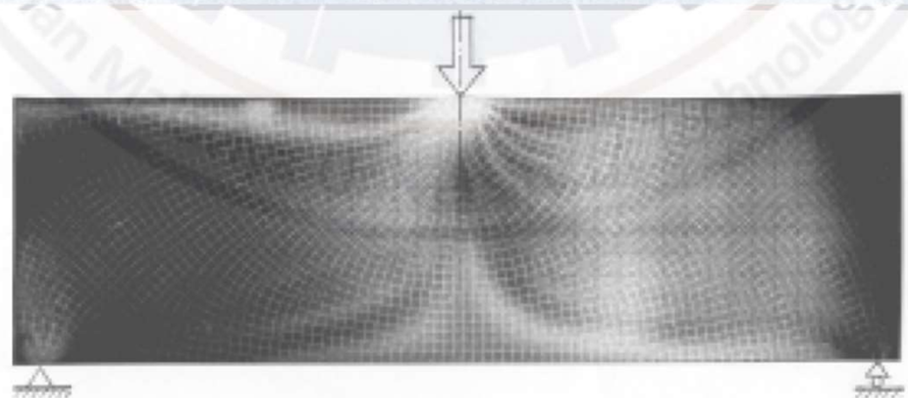
(f)

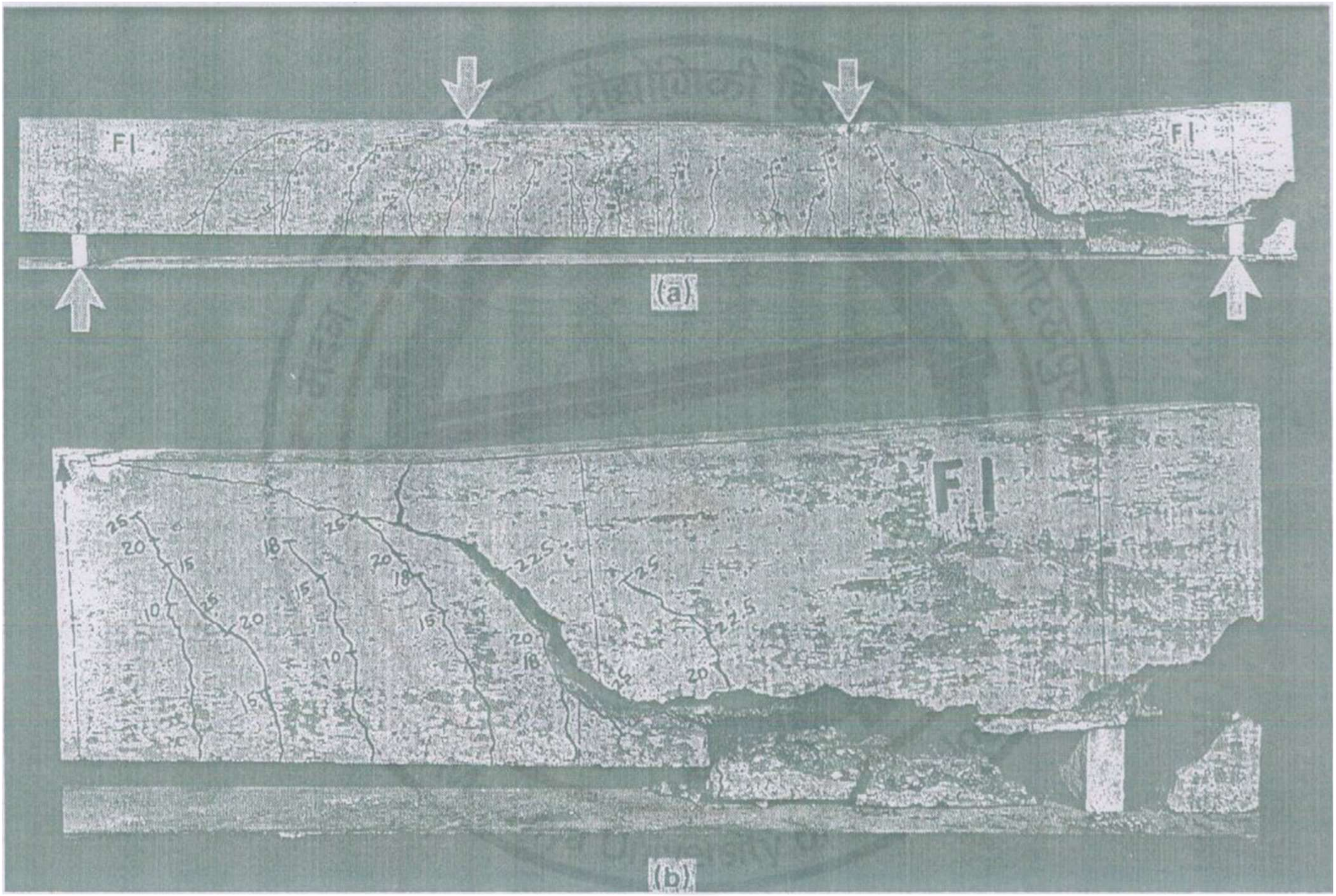
In each case, the state of stress can be transformed into principal stresses, using either stress-transformation equations or Mohr's circle.

Stress Trajectories

- In each case the state of stress can be transformed into *principal stresses,* using either the stress-transformation equations or Mohr's circle.
- If this analysis is extended to many vertical sections along the beam other an *a-a,* a profile of the results can be represented by curves called *stress trajectories.*
- Each curve indicate the direction of a principal stress having a constant magnitude.






F1
F1
(a)
F1
(b)

Theories of Failure

- Most machine members are subjected to several types of forces simultaneously .
- In order to design such components, it will be ideal to load a few test pieces to similar forces simultaneously, and determine failure forces and design accordingly .
- Practically, such an approach will be very expensive and time consuming.
- Therefore, several theories of failure have been proposed to relate the failure under two-or-three-dimensional stresses with the mechanical properties under tensile test.
- Thus the objective of theories of failure is to relate failure under two- or three-dimensional stress system with results of failure under simple tensile or other simple tests.

Theories of Failure

1. Maximum principal / normal stress theory (Rankine's theory)
2. Maximum shear stress theory (Coulomb, Tresca & Guest Theory)
3. Maximum strain energy theory (Beltrami-Haigh's theory)
4. Maximum principal strain theory (St. Venant theory)
5. Distortion energy theory (Von-Mises & Hencky)

Maximum Principal stress theory

- According to the maximum principal stress theory, failure occurs whenever the largest principal stress in a component exceeds the strength of the material in tensile or compressive test.
- Let the three principal stresses be σ_1, σ_2 and σ_3 and $\sigma_1 > \sigma_2 > \sigma_3$

<u>If yielding is the criteria of failure, then failure will occur whenever</u>

$\sigma_1 = S_{yt}$ or $\sigma_3 = S_{yc}$

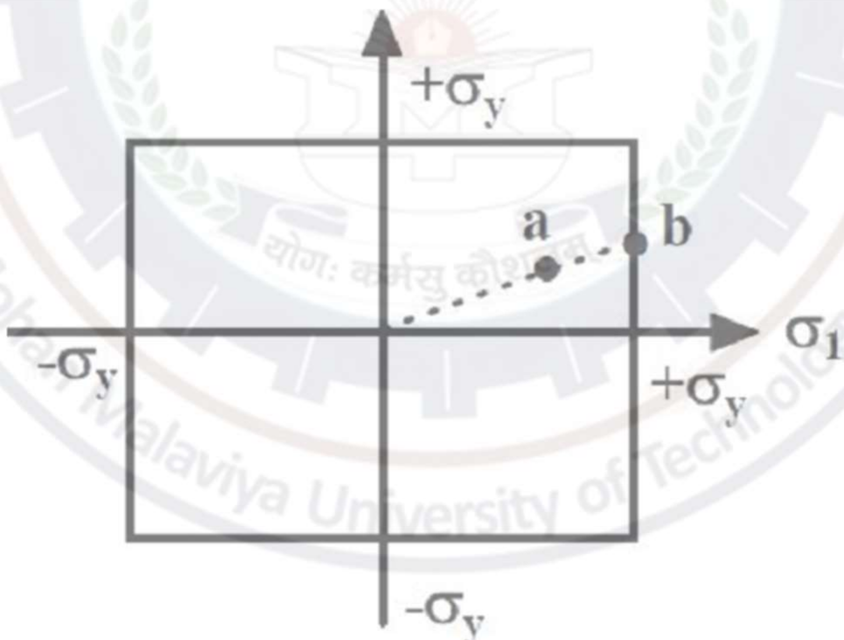
- For ductile materials: $S_{yt} = S_{yc} = S_y$

<u>For brittle materials, failure will occur whenever</u>

- $\sigma_1 = S_{ut}$ or $\sigma_3 = S_{uc}$
- For parts subjected to pure torsion, $\sigma_1 = \tau = -\sigma_3$ and $\sigma_2 = 0$. Thus according to this theory, such a part in pure torsion will fail when $\tau = S_y$, but in practice such parts fail when τ is only about 58% of S_y. Hence, this theory is hardly used.

Yield surface corresponding to maximum principal stress

- Yielding occurs when the state of stress is at the boundary of the rectangle. Consider, for example, the state of stress of a thin walled pressure vessel. Here $\sigma_1 = 2\sigma_2$, σ_1 being the circumferential or hoop stress and σ_2 the axial stress. As the pressure in the vessel increases the stress follows the dotted line.



Maximum Shear Stress Theory

- According to maximum-shear-stress theory, failure occurs whenever the maximum shear stress in a component exceeds the shear strength of the material at the point of yielding.
- The theory is based on yield stress and hence it is applied to ductile materials only.
- The shear strength of the material is half the yield strength in a tensile test, i.e., $S_{sy} = 0.5\, S_{yt}$

Let the three principal stresses be σ_1, σ_2 , and σ_3 and $\sigma_1 > \sigma_2 > \sigma_3$. Three maximum shear stresses acting on three mutually perpendicular planes are

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}, \quad \tau_{23} = \frac{\sigma_2 - \sigma_3}{2} \text{ and } \tau_{31} = \frac{\sigma_3 - \sigma_1}{2}$$

Equating the largest of these shear stresses to τ_{max}, for failure to occur

$$\tau_{max} = S_{sy} = \frac{S_yt}{2} = \frac{\sigma_1 - \sigma_2}{2} \text{ etc.}$$

Maximum shear stress theory

Let us say that the failure will occur whenever either of the following three conditions is met

$$\sigma_1 - \sigma_2 = S_{yt}, \quad \text{or} \quad \sigma_2 - \sigma_3 = S_{yt}, \quad \text{or} \quad \sigma_3 - \sigma_1 = S_{yt},$$

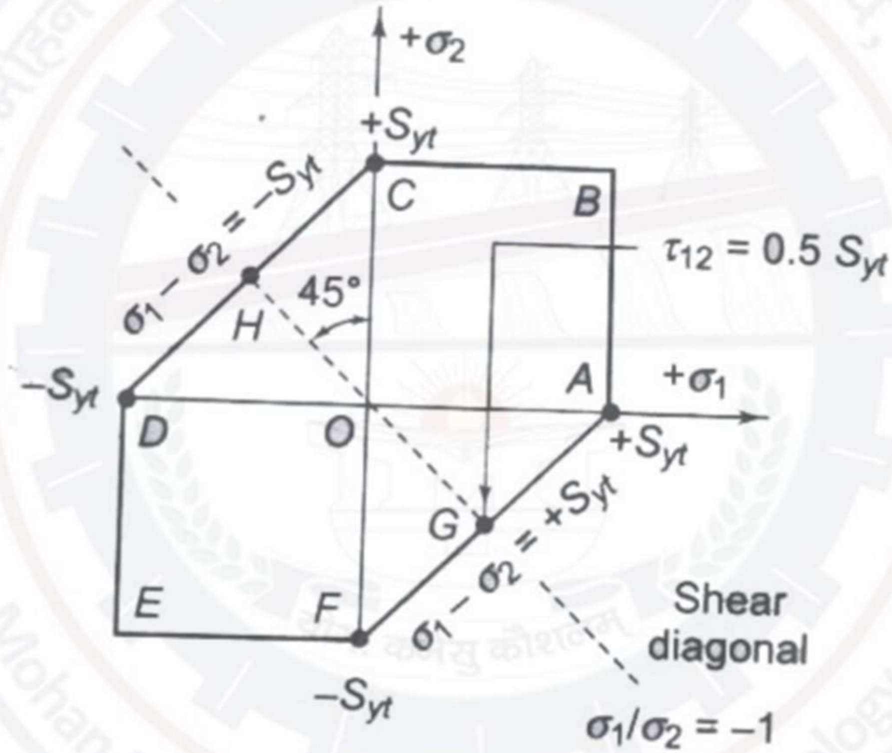
Since S_{yc} is assumed to be equal to S_{yt}, the above equations may be written as

$$\sigma_1 - \sigma_2 = \pm S_{yt}, \quad \text{or} \quad \sigma_2 - \sigma_3 = \pm S_{yt}, \quad \text{or} \quad \sigma_3 - \sigma_1 = \pm S_{yt},$$

For a biaxial stress system, $\sigma_3 = 0$ and the above equations become

$$\sigma_1 - \sigma_2 = \pm S_{yt}, \quad \text{or} \quad \sigma_2 = \pm S_{yt}, \quad \text{or} \quad \sigma_1 = \pm S_{yt},$$

Yield surface corresponding to Maximum shear stress



Boundary for Maximum Shear Stress Theory under bi-axial stress

Example

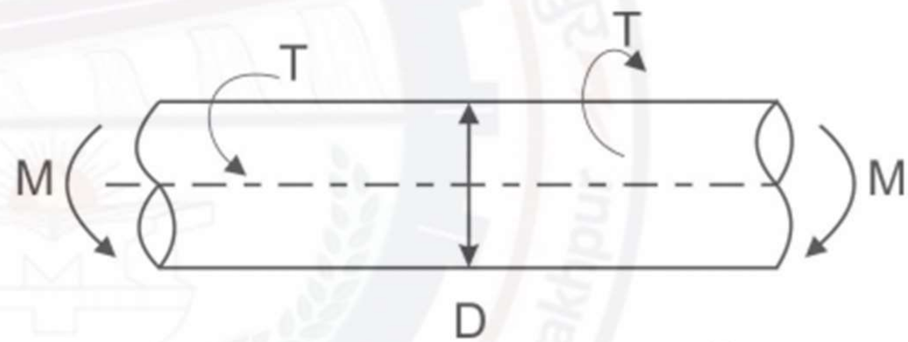
A mild steel shaft of 4 cm diameter is subjected to a bending moment of 1,50,000 N cms and a Torque T. If the yield point of the steel in tension is 20000 N/cm^2 find the maximum value of this torque T without causing yielding of the shaft according to

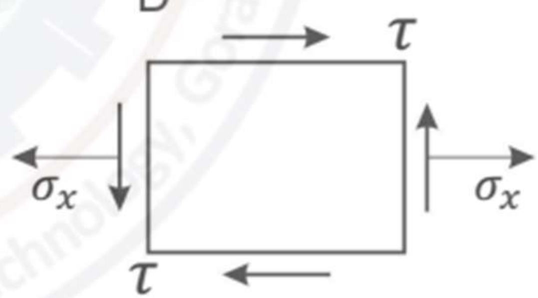
(i) maximum principle stress theory

(ii) the maximum shear stress theory

Solution

1. Data and FBD





D = 4 cm = 40 mm

M = 150000 N.cm = 1.5 x 10^6 N.mm

Sy = 20000 N/cm^2 = 200 MPa

2. To calculate max value of T without causing yielding

(a) Using maximum principal stress theory

(b) Using maximum shear stress theory

Example

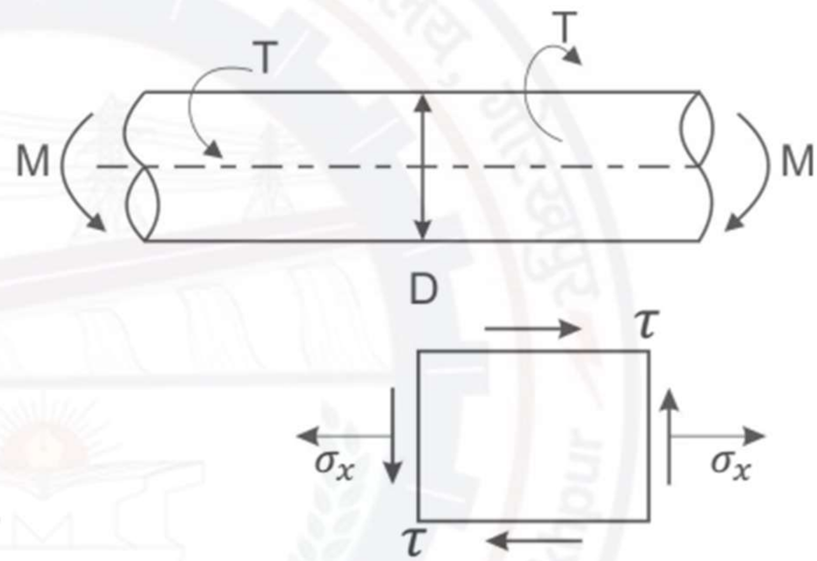
3. Analysis

$$Z = \frac{\pi 40^3}{32} = 6283\ mm^3$$

$$Z_p = 2Z = 12566\ mm^3$$

$$\sigma_x = \frac{M}{Z} = \frac{1.5\times10^6}{6283} = 238.7\,\frac{N}{mm^2}$$

$$\tau = \frac{T}{12566} = T \times (79.58) \times 10^{-6}$$



(a) Max Principal Stress theory

$\sigma_x = 238.7\ N/mm^2 > (Sy = 200\ N/mm^2)$

Hence the shaft will fail with out any torque.

(b) Max SS theory =

(S_{sy} = 100 N/mm²) < 119.35 N/mm², the shear stress due to M alone

Hence, the shaft will fail due to M. No T can be supported.

4. **<u>Comment</u> :** Either decrease the moment on the shaft or increase the diameter.

For M only and T = 0, σ_x = 200;

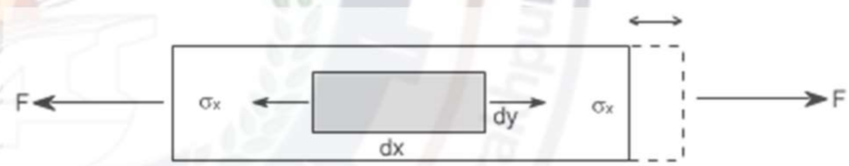
i.e. $Z = \frac{M}{Sy} = \frac{1.5\times10^6}{200} = 7500\ mm^3$

$d^3 = (32 \times 7500)/\pi$ d = 42.4 mm say 43 mm

Maximum Strain Energy Theory

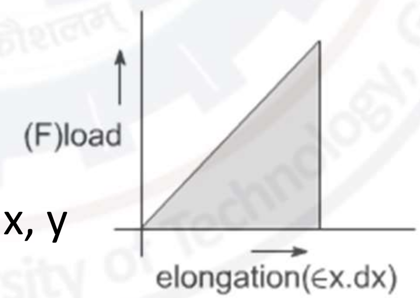
- According to this theory, failure of a mechanical component subjected to bi-axial or tri-axial stresses occurs when the strain energy per unit volume exceeds the strain energy per unit volume of specimen under tensile test.
- Let σ_1, σ_2, and σ_3 be the principal stresses, the final forces generated on each face of an element of dimensions d_x, d_y and d_z are:

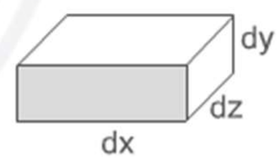
$$\left.\begin{aligned} F_x &= \sigma_1 \, d_y \, d_z \\ F_y &= \sigma_2 \, d_z \, d_x \\ F_z &= \sigma_3 \, d_x \, d_y \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(a)$$



The total strain energy will be

$$U = \frac{1}{2}\left[F_x\delta_x + F_y\delta_y + F_z\delta_z\right] \quad \ldots\ldots\ldots\ldots\ldots(b)$$





Where δ_x, δ_y, and δ_z are the elongations in x, y and z directions, respectively

Maximum Strain Energy Theory

$$\left.\begin{aligned} \delta_x = \in dx = \frac{I}{E}(\sigma_1 - \mu\,\sigma_2 - \mu\,\sigma_3)dx \\ \delta_y = \in dy = \frac{I}{E}(\sigma_2 - \mu\,\sigma_3 - \mu\,\sigma_1)dy \\ \delta_z = \in dz = \frac{I}{E}(\sigma_3 - \mu\,\sigma_1 - \mu\,\sigma_2)dz \end{aligned}\right\} \quad \text{...............(c)}$$

where μ is the Poisson's ratio.
Substituting Eq. (a) and Eq. (c) in Eq. (b) and arranging the terms gives:

$$u = \frac{dx\,dy\,dz}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Therefore, the strain energy per unit volume is:

$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

Maximum Strain Energy Theory

- The strain energy per unit volume in a tensile test may be obtained from the above equation by substituting

  $\sigma 2 = \sigma 3 = 0$, i.e.

$$u = \frac{I}{2E} S_y^2$$

Thus the failure equation for the material under triaxial stress system is

$$[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] = S_y^2$$

For (fs) as factor of safety, the design equation is:

$$[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \leq (\frac{S_y}{fs})^2$$

Maximum Principal strain theory

- This Theory assumes that failure occurs when the maximum strain for a complex state of stress system becomes equals to the strain at yield point in the tensile test for the three dimensional complex state of stress system.
- For a 3 - dimensional state of stress system the total strain energy U_t per unit volume is equal to the total work done by the system and given by the equation

$$U_t = 1/2\sigma_1 \epsilon_1 + 1/2\sigma_2 \epsilon_2 + 1/2\sigma_3 \epsilon_3$$

substituting the values of ϵ_1, ϵ_2 and ϵ_3

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \gamma(\sigma_2 + \sigma_3)]$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \gamma(\sigma_1 + \sigma_3)]$$

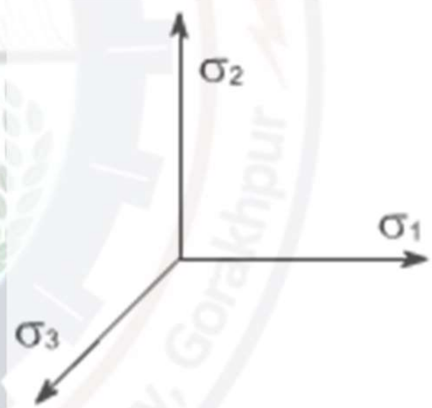
$$\epsilon_3 = \frac{1}{E}[\sigma_3 - \gamma(\sigma_1 + \sigma_2)]$$

Thus, the failure criterion becomes

$$\left(\frac{\sigma_1}{E} - \gamma\frac{\sigma_2}{E} - \gamma\frac{\sigma_3}{E}\right) = \frac{\sigma_{yp}}{E}$$

or

$$\boxed{\sigma_1 - \gamma\sigma_2 - \gamma\sigma_3 = \sigma_{yp}}$$



Distortion energy theory

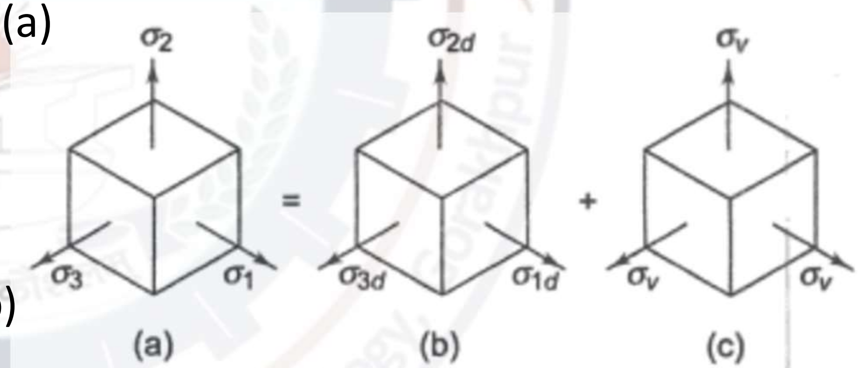
- The theory state that the failure of mechanical component subjected to bi-axial or tri-axial stresses occurs when the strain energy of distortion per unit volume at any point in the component, becomes to the strain energy of distortion per unit volume in the standard specimen of tension test, when yielding starts.
- A unit cube subjected to the three principal stresses σ_1, σ_2, and σ_3. the total strain energy U of the cube is given by,

$$U = \frac{1}{2}\sigma_1\varepsilon_1 + \frac{1}{2}\sigma_2\varepsilon_2 + \frac{1}{2}\sigma_3\varepsilon_3 \quad \text{...............(a)}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1 + \sigma_3)] \quad \text{......(b)}$$

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)]$$



(a) Element with Tri-axial stresses
(b) Stress components due to distortion of element
(c) Stress components due to change of volume

Distortion energy theory

Substituting Eq. (b) in Eq. (a), we get

$$U = \frac{1}{2E}[(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)] \quad ....(c)$$

The total strain energy U is resolved into two components:

- U_v - corresponding to change of volume with no distortion of the element
- U_d - corresponding to distortion without change of volume.

Therefore, $U = U_v + U_d$ ....(d)

Corresponding stresses are resolved into 2 components as shown in figure:

$$\sigma_1 = \sigma_{1d} + \sigma_v, \quad \sigma_2 = \sigma_{2d} + \sigma_v, \quad \sigma_3 = \sigma_{3d} + \sigma_v \quad ....(e)$$

Since $\sigma_{1d}, \sigma_{2d}, \sigma_{3d}$ do not change the volume of the cube,

$\epsilon_{1d} + \epsilon_{2d} + \epsilon_{3d} = 0$, i.e. ....(f)

$$\left.\begin{aligned} \varepsilon_{1d} &= \frac{1}{E}[\sigma_{1d} - \mu(\sigma_{2d} + \sigma_{3d})] \\ \varepsilon_{2d} &= \frac{1}{E}[\sigma_{2d} - \mu(\sigma_{1d} + \sigma_{3d})] \\ \varepsilon_{3d} &= \frac{1}{E}[\sigma_{3d} - \mu(\sigma_{1d} + \sigma_{2d})] \end{aligned}\right\} ......(g)$$

Distortion energy theory

$$\frac{I}{E}[\sigma_{1d}\text{-}\mu(\sigma_{2d}+\sigma_{3d})]+\frac{I}{E}[\sigma_{2d}\text{-}\mu(\sigma_{3d}+\sigma_{1d})]+\frac{I}{E}[\sigma_{3d}\text{-}\ \mu(\sigma_{1d}+\sigma_{2d})]=0$$

or $(1-2\mu)\ (\sigma_{1d}+\sigma_{2d}+\sigma_{3d}) = 0$

Since $(1-2\mu)$is not zero, $(\sigma_{1d}+\sigma_{2d}+\sigma_{3d}) = 0$ ……………(h)

From Eq. (h) and (e)

$$\sigma_v = \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3) \quad \text{……………(j)}$$

The strain energy U_v corresponding to change of volume for the cube is given by

$$U_v = 3\left[\frac{\sigma_v \varepsilon_v}{2}\right] \quad \text{……………(k)}$$

Also $\varepsilon_v = \frac{1}{E}[\sigma_v - \mu(\sigma_v + \sigma_v)]$

or $\varepsilon_v = \frac{(1-2\mu)\sigma_v}{E}$ ……………(l)

From expressions (k) and (l),

$$U_v = \frac{3(1-2\mu)\sigma_v^2}{2E} \quad \text{……………(m)}$$

Substituting expression (j) in the Eq. (m),

$$U_v = \frac{(1-2\mu)\ (\sigma_1+\sigma_2+\sigma_3)^2}{6E} \quad \text{……………(n)}$$

Distortion energy theory

From expressions (c) and (n),

$$U_d = U - U_v$$

or $$U_d = \left(\frac{1+\mu}{6E}\right)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

In simple tension test, when the specimen starts yielding,

$$\sigma_1 = S_{yt} \quad \text{and} \quad \sigma_2 = \sigma_3 = 0$$

Therefore, $$U_d = \left(\frac{1+\mu}{3E}\right)S_{yt}^2$$

the criterion of failure for the distortion energy theory is expressed as

$$2S_{yt}^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

or $$S_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

Distortion energy theory

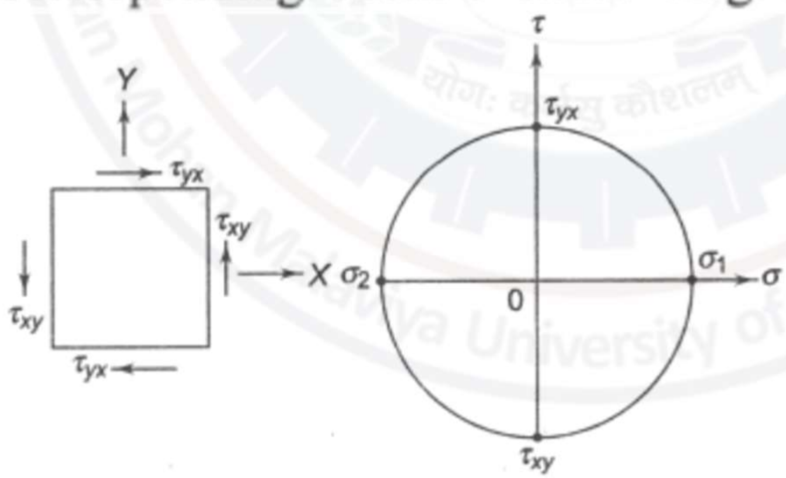
Considering the factor of safety,

$$\frac{S_{yt}}{(fs)} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]}$$

For bi-axial stresses ($\sigma_3 = 0$),

$$\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}$$

A component subjected to pure shear stresses and the corresponding Mohr's circle diagram is



(a) (b)

Distortion energy theory

From the figure,

$$\sigma_1 = -\sigma_2 = \tau_{xy} \quad \text{and} \quad \sigma_3 = 0$$

Substituting these values

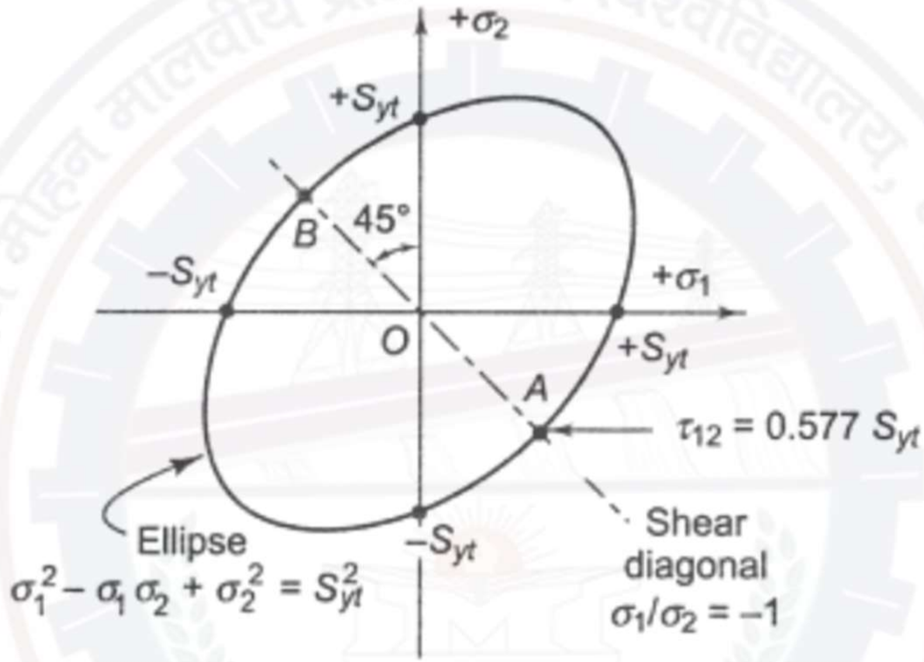
$$S_{yt} = \sqrt{3}\,\tau_{xy}$$

Replacing (τ_{xy}) by S_{sy},

$$S_{sy} = \frac{S_{yt}}{\sqrt{3}} = 0.577\,S_{yt}$$

Therefore, according to the distortion-energy theory, the yield strength in shear is 0.577 times the yield strength in tension.

Region of Safety



For bi-axial stresses,

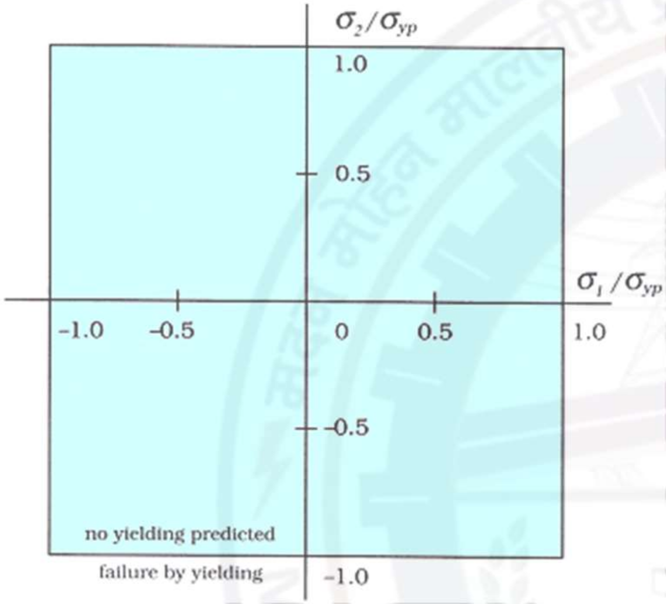
$$\sigma_3 = 0$$

Substituting this value in Eq. (4.42),

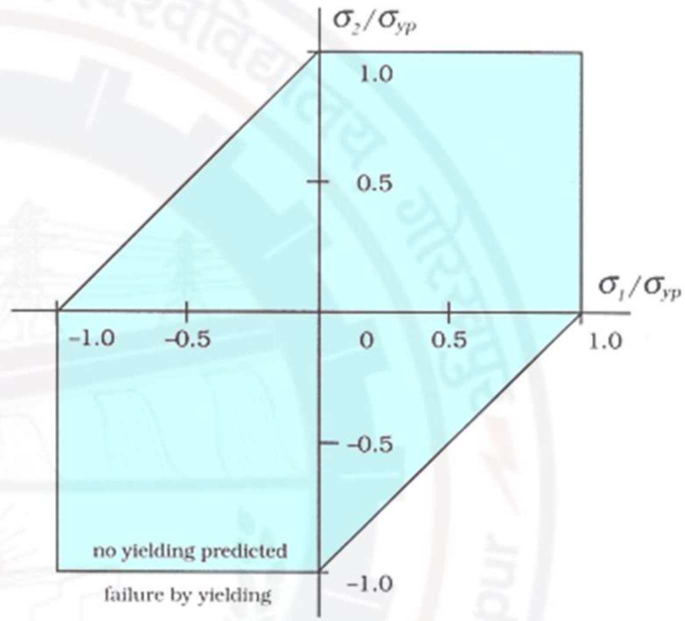
$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = S_{yt}^2$$

The above equation indicates an ellipse whose semi-major axis is $(\sqrt{2}\ S_{yt})$ and semi-minor axis is $\left[\sqrt{\frac{2}{3}}S_{yt}\right]$.

Region of Safety

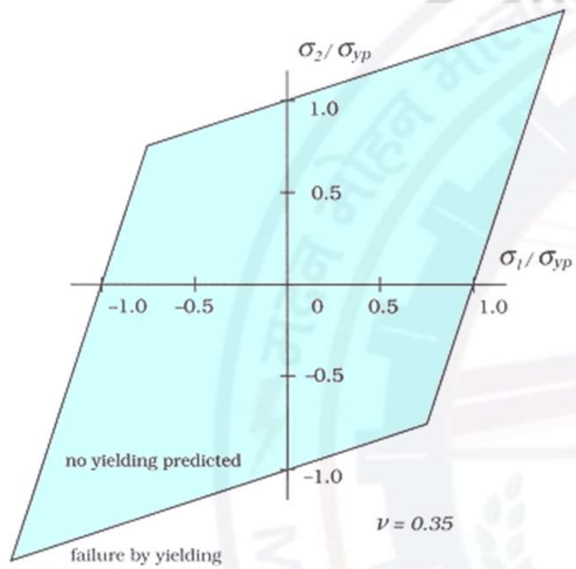


Maximum principal / normal stress theory (Rankine's theory)



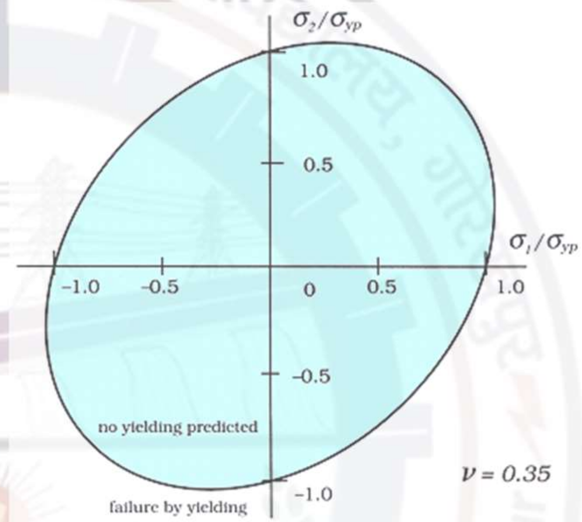
Maximum shear stress theory (Coulomb, Tresca & Guest Theory)

Region of Safety



Maximum strain energy theory

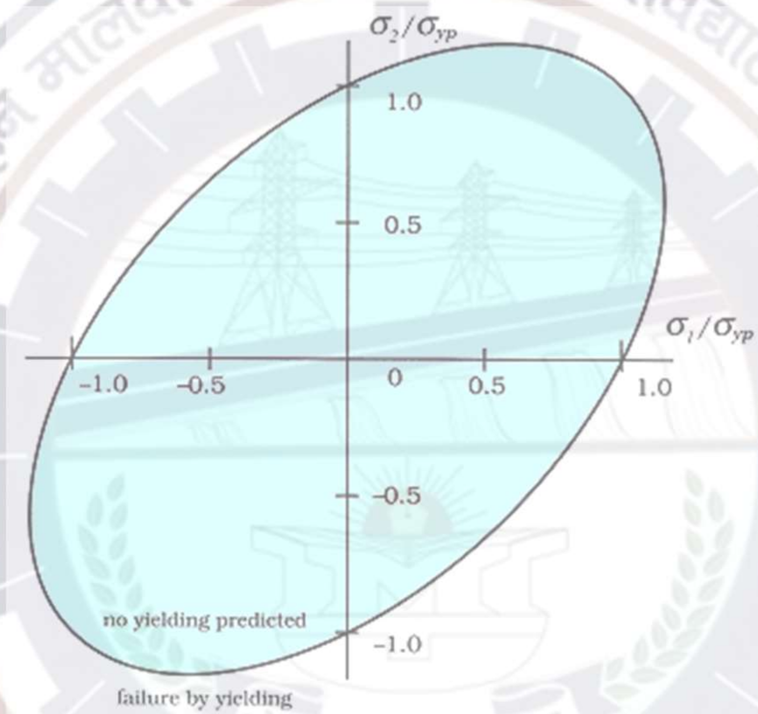
(Beltrami-Haigh's theory)



Maximum principal strain theory

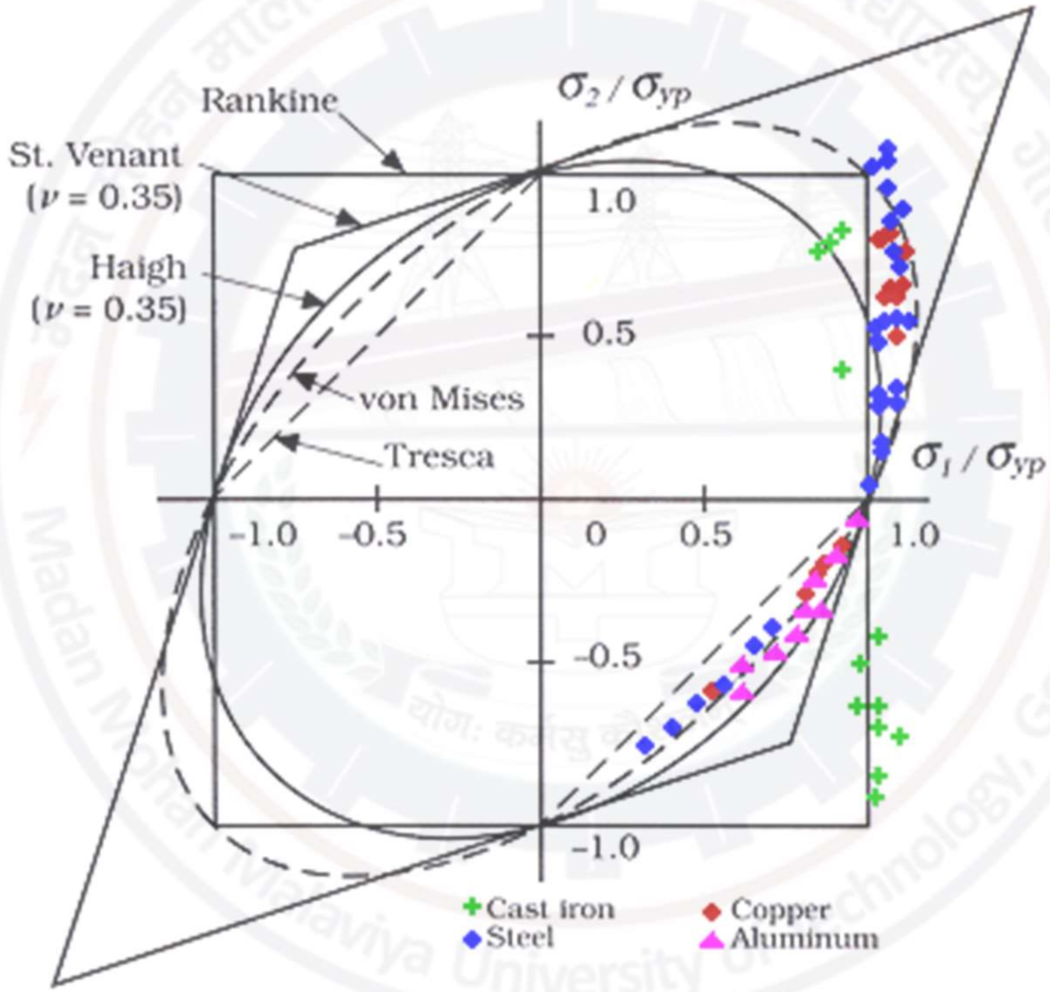
(St. Venant theory)

Region of Safety



Distortion energy theory
(Von-Mises & Hencky)

Selection and use of Failure theories



Example :1

The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5kN. Find the diameter of bolt required according to

1. Maximum principal stress theory
2. Maximum shear stress theory
3. Maximum principal strain theory
4. Maximum strain energy theory
5. Maximum distortion energy theory

Permissible tensile stress at elastic limit =100MPa and Poisson’s ratio =0.3

Solution 1

- Cross – sectional area of the bolt,

$$A = \frac{\pi}{4} d^2 = 0.7854 d^2$$

- Axial stress,

$$\sigma_1 = \frac{P}{A} = \frac{10}{0.7854 d^2} = \frac{12.73}{d^2} kN / mm^2$$

- And transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854 d^2} = 6.365 kN / mm^2$$

According to maximum principal stress theory

- Maximum principal stress,

$$\sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \left(\frac{12.73}{2d^2}\right) + \sqrt{\left[\left(\frac{12.73}{2d^2}\right)^2 + \left(\frac{6.365}{d^2}\right)^2\right]}$$

$$\sigma_1 = \frac{15365}{d^2} N/mm^2$$

- According to maximum principal stress theory, $S_{yt} = \sigma_1$

$$100 = \frac{15365}{d^2} \Rightarrow d = 12.4mm$$

According to maximum shear stress theory

- Maximum shear stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left[\left(\frac{12.73}{d^2}\right)^2 + \left(\frac{6.365}{d^2}\right)^2\right]} = \frac{9}{d^2} kN/mm^2 = \frac{9000}{d^2} N/mm^2$$

- According to maximum shear stress,

$$\tau_{max} = \frac{S_{yt}}{2} \Rightarrow \frac{9000}{d^2} = \frac{100}{2}$$

$$d = 13.42 mm$$

According to maximum principal strain theory

- The maximum principal stress,

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{15365}{d^2}$$

- And minimum principal stress,

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \left(\frac{\sigma_x}{2}\right) - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{12.73}{d^2} - \sqrt{\left(\frac{12.73}{2d^2}\right)^2 + \left(\frac{6.365}{d^2}\right)^2}$$

$$\sigma_2 = \frac{-2635}{d^2} N/mm^2$$

- And according to maximum principal strain theory,

$$\frac{\sigma_1}{E} - \frac{\sigma_2}{mE} = \frac{S_{yt}}{E}$$

$$\sigma_1 - \frac{\sigma_2}{m} = S_{yt} \Rightarrow \frac{15365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100$$

$$d = 12.7mm$$

- According to maximum strain energy theory

$$\sigma_1^2 + \sigma_2^2 - \frac{2\sigma_1 \sigma_2}{m} = S_{yt}^2$$

$$\left[\frac{15365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2\times\frac{15365}{d^2}\times\frac{-2635}{d^2}\times 0.3 = 100^2$$

$$d = 12.78mm$$

- According to maximum distortion theory

$$S_{yt} = \sqrt{\left(\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2\right)}$$

$$100 = \sqrt{\left(\left[\frac{15365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - \frac{15365}{d^2}\times\frac{-2635}{d^2}\right)}$$

$$d = 13.4mm$$

Any question?

UNIT-II
Part-2

Design against fluctuating load

R. B. Prasad

Assistant Professor

Learning objectives

Learning objectives are:

- Understand the importance of stress concentration and the factors responsible for it.
- Determination of stress concentration factor.
- Different techniques to reduce stress concentration.
- Notch sensitivity factor and Fatigue strength reduction factors.
- Different types of fluctuating loads.
- Fatigue test.
- Difference between Endurance Strength and Endurance Limit.
- Design of machine element subjected to fluctuating loads.
- Soderberg , Goodman and Gerber Criteria.
- Modified Goodman Diagram.

Stress Concentration

- In design of machine element, the following three fundamental equations are used:

$$\sigma_t = \frac{P}{A}$$

$$\sigma_b = \frac{M_b y}{I}$$

$$\tau = \frac{M_t r}{J}$$

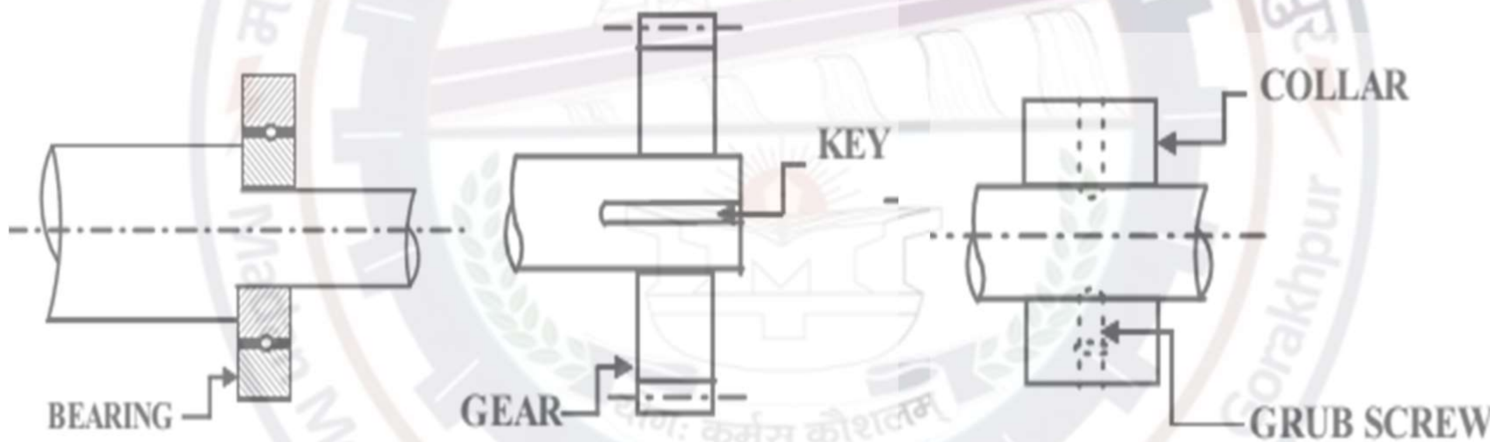
<u>Assumption</u>: there are no discontinuities in the cross-section of the component.

- If there are discontinuities or abrupt change in the cross-sectional area

  - ➢ Above Fundamental equations are not valid.
  - ➢ Sudden rise in the magnitude of stresses in the vicinity of the hole.
  - ➢ Localised stresses are far greater.

Discontinuities

- In practice, discontinuities and abrupt changes in cross-section are unavoidable. Due to oil holes and grooves, keyways and splines, screw threads and shoulders.



Any such discontinuity in a member affects the stress distribution in the neighbourhood and the discontinuity acts as a stress raiser.

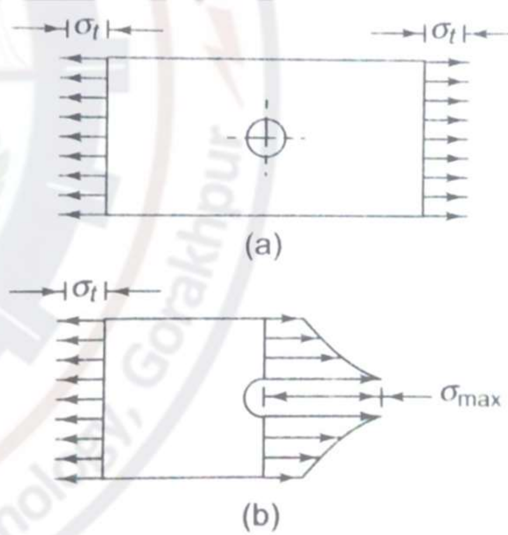
Stress Concentration Factor

- Stress concentration factor is defined as the localization of high stresses due to the irregularities present in the component and abrupt changes of the cross-section.

$$K_t = \frac{\textbf{highest value of } \textit{actual stress} \textbf{ near discontinuity}}{\textit{nominal stress} \textbf{ obtained by elementary equations}}$$

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

$$K_t = \frac{\sigma_{max.}}{\sigma_0} = \frac{\tau_{max.}}{\tau_0}$$



K_t = Theoretical stress concentration factor

= The value of K_t depends upon the material and geometry of the part.

Stress Concentration Factor

Stress concentration factors determined by 2 Methods:

Theoretical Methods

1. Theory of Elasticity
2. Finite element analysis

Experimental Method

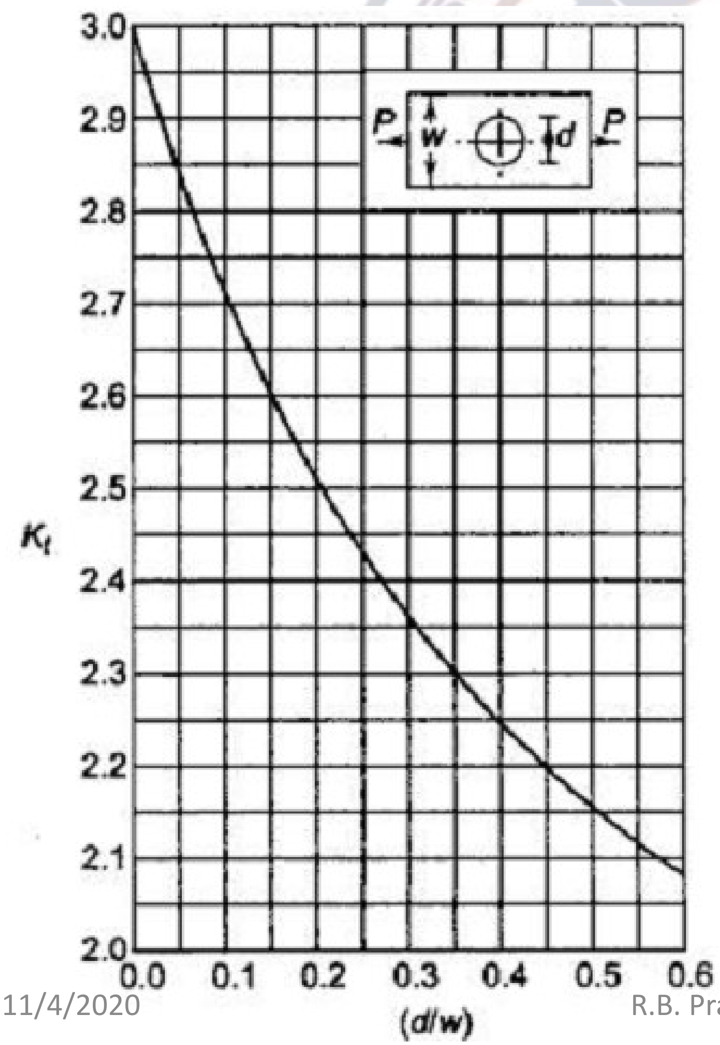
1. Strain gage method
2. Photo elasticity method
3. Brittle coating technique
4. Grid method

Causes of Stress Concentration

- Variation in properties of materials
- Load applications
- Abrupt change in cross sections
- Discontinuities in the component
- Machining scratches

Stress Concentration Factor

R. E Peterson (1953) prepared the charts for the stress concentration factors for different geometric shapes and condition of loading:



Stress Concentration Factor: Rectangular Plate with Transverse Hole in Tension or Compression

$$K_t = \frac{\sigma_{max.}}{\sigma_0}$$

$$\sigma_o = \frac{P}{(w-d)t}$$

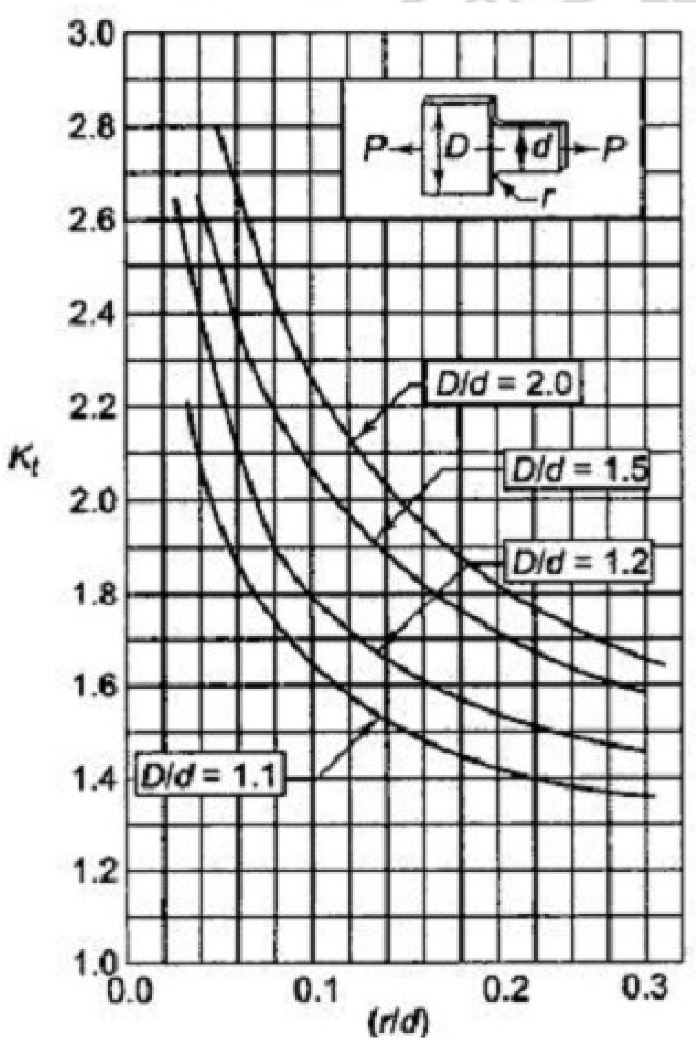
Given: d/w

K_t can be determined

Therefore,

$$\sigma_{max} = K_t \sigma_o$$

Stress Concentration Factor



Stress Concentration Factor: Flat Plate with Shoulder Fillet in Tension or Compression

$$K_t = \frac{\sigma_{max.}}{\sigma_0}$$

$$\sigma_o = \frac{P}{dt}$$

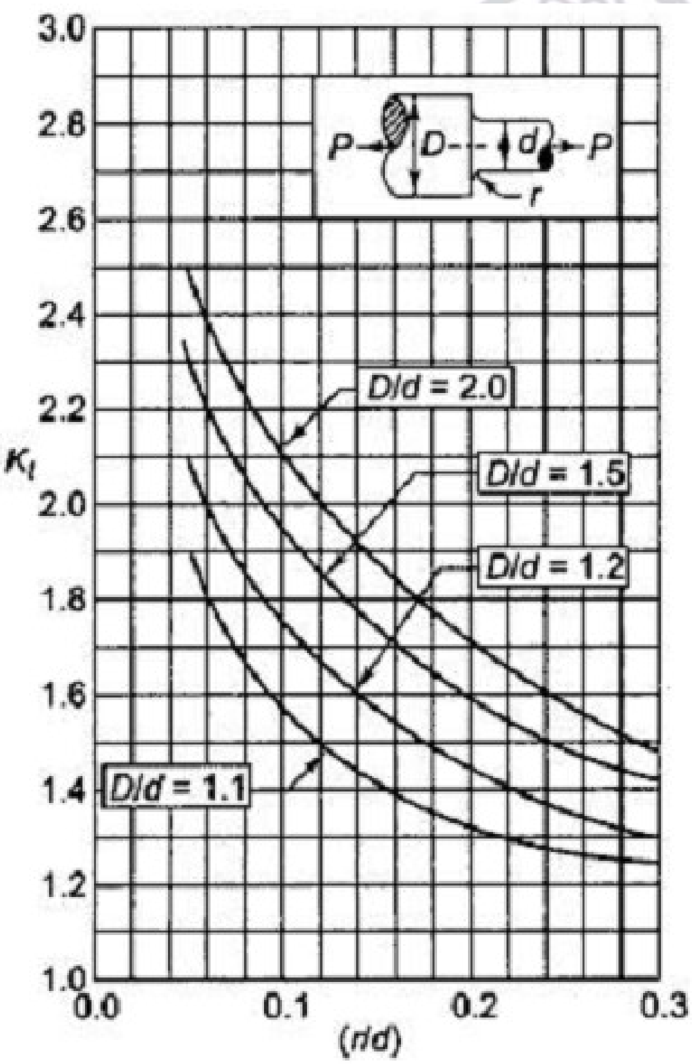
Given: r/d and D/d

K_t can be determined

Therefore,

$$\sigma_{max} = K_t \sigma_o$$

Stress Concentration Factor



Stress Concentration Factor: Round Shaft with Shoulder Fillet in Tension

$$K_t = \frac{\sigma_{max.}}{\sigma_0}$$

$$\sigma_o = \frac{P}{\left[\frac{\pi}{4} d^2\right]}$$

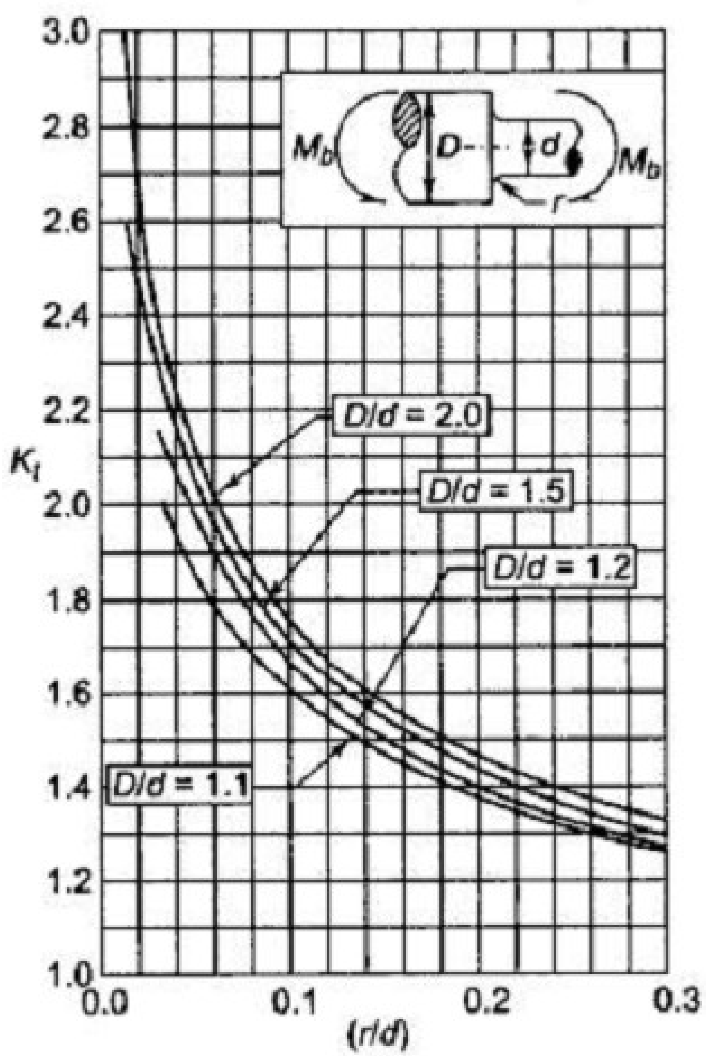
Given: r/d and D/d

K_t can be determined

Therefore,

$$\boldsymbol{\sigma_{max} = K_t \sigma_o}$$

Stress Concentration Factor



Stress Concentration Factor: Round shaft with Shoulder Fillet in Bending

$$K_t = \frac{\sigma_{max.}}{\sigma_0}$$

$$\sigma_o = \frac{M_b y}{I}$$

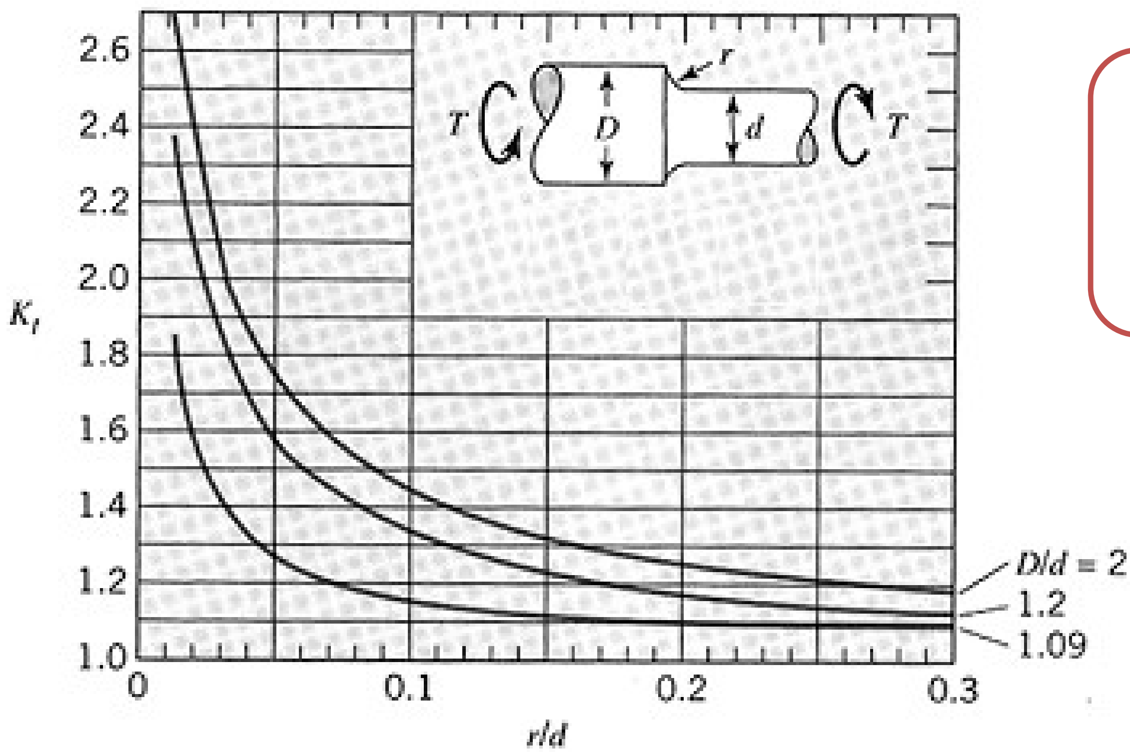
Given: r/d and D/d

K_t can be determined

Therefore,

$$\boldsymbol{\sigma_{max} = K_t \sigma_o}$$

Stress Concentration Factor



Stress Concentration Factor: Round shaft with Shoulder Fillet in Torsion

$$K_t = \frac{\sigma_{max.}}{\sigma_0} = \frac{\tau_{max.}}{\tau_0}$$

$$\tau_0 = \frac{M_t r}{J}$$

Given: r/d and D/d

K_t can be determined

Therefore,

$$\tau_{max} = K_t \tau_o$$

Stress Concentration Factor

- ✓ The above charts are based on photo-elastic studies of the models.
- ✓ Under static loads ductile materials are not affected by stress concentration.
- ✓ When the stress in the vicinity of the discontinuity reaches the yield point, there is plastic deformation resulting in a redistribution of stresses.
- ✓ Therefore, K_t is not used for Ductile material under static loading.
- ✓ The effect is more severe in case of brittle materials (due to inability to plastic deformation).
- ✓ Therefore stress concentration factors are used for components made of brittle materials subjected to static loads.
- ✓ However, when the load is fluctuating the endurance strength of the ductile material is greatly reduced due to stress concentration. This accounts for the use of stress concentration factors for ductile material as well.

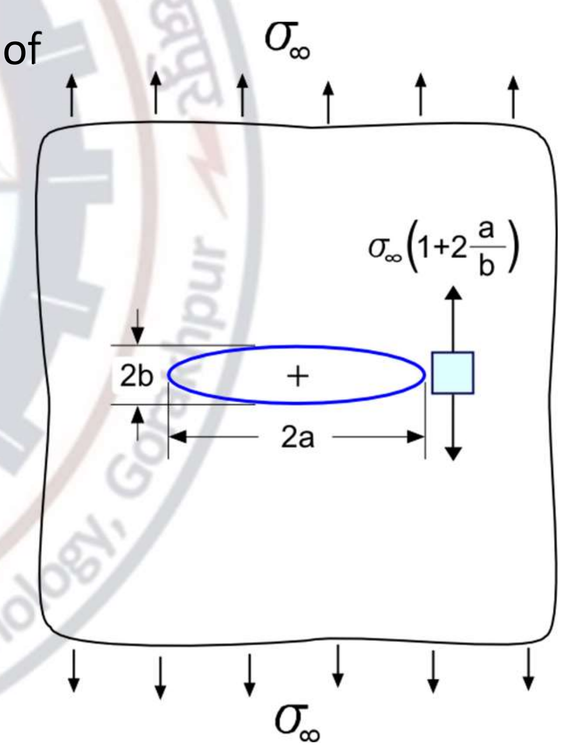
Theory of elasticity

- For the case of elliptical holes in an infinitely wide plate (e.g., plate width 2w → ∞, a/w → 0) subject to remote tensile loading, the theoretical stress concentration factor Kt at the edge of the hole can be obtained analytically, and is given by:

  $$Kt = 1 + 2\ (a/b) \ \ldots\ldots\ldots\ldots\ldots (a)$$

  where A= semi-major axis (Perpendicular to the direction of load),

  B= semi-minor axis (in the direction of load)



- If b= 0 , then Kt = ∞,

  As the width of elliptical hole in the direction of load approaches zero, the stress concentration factor becomes infinity
- If a= b, then Kt = 3

  Kt for smaller circular hole in a flat plate is 3.
- Equation (a) may also be expressed as

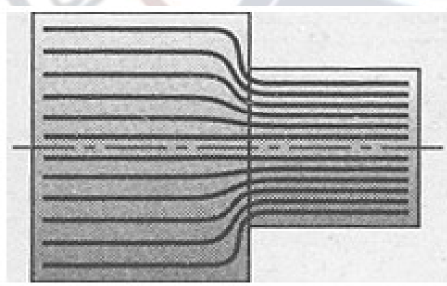
  $$Kt = 1 + 2\sqrt{(b/\rho)}\ ,$$

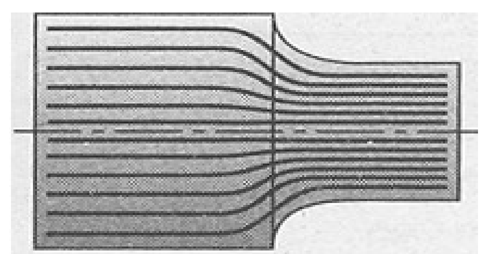
  where ρ is the local radius of curvature of the edge of the ellipse near the 2aaxis; for an ellipse,

  at this location, ρ = b2/a.

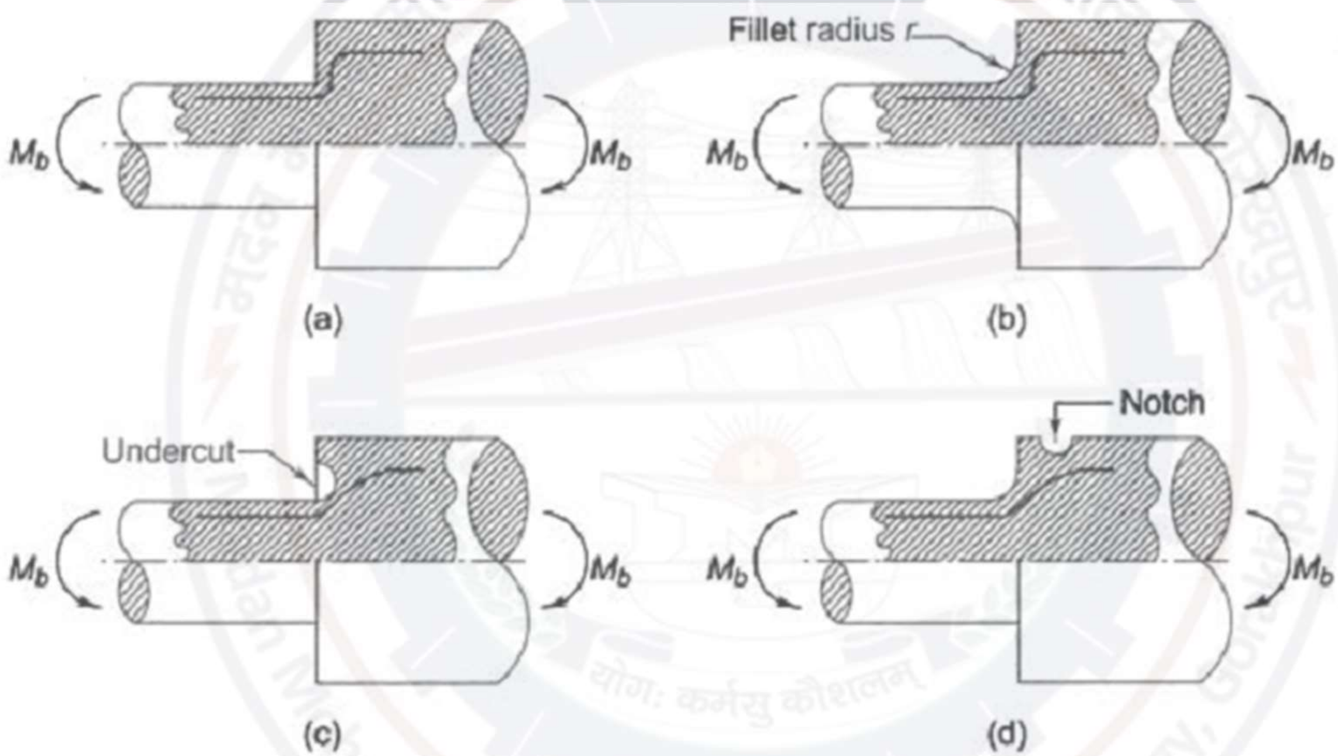
Methods to reduce stress concentration

- The presence of stress concentration can not be totally eliminated but it may be reduced to some extent.
- A technique that is useful in assisting a design engineer is to visualize the presence of stress concentration and stress flow lines.
- The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.



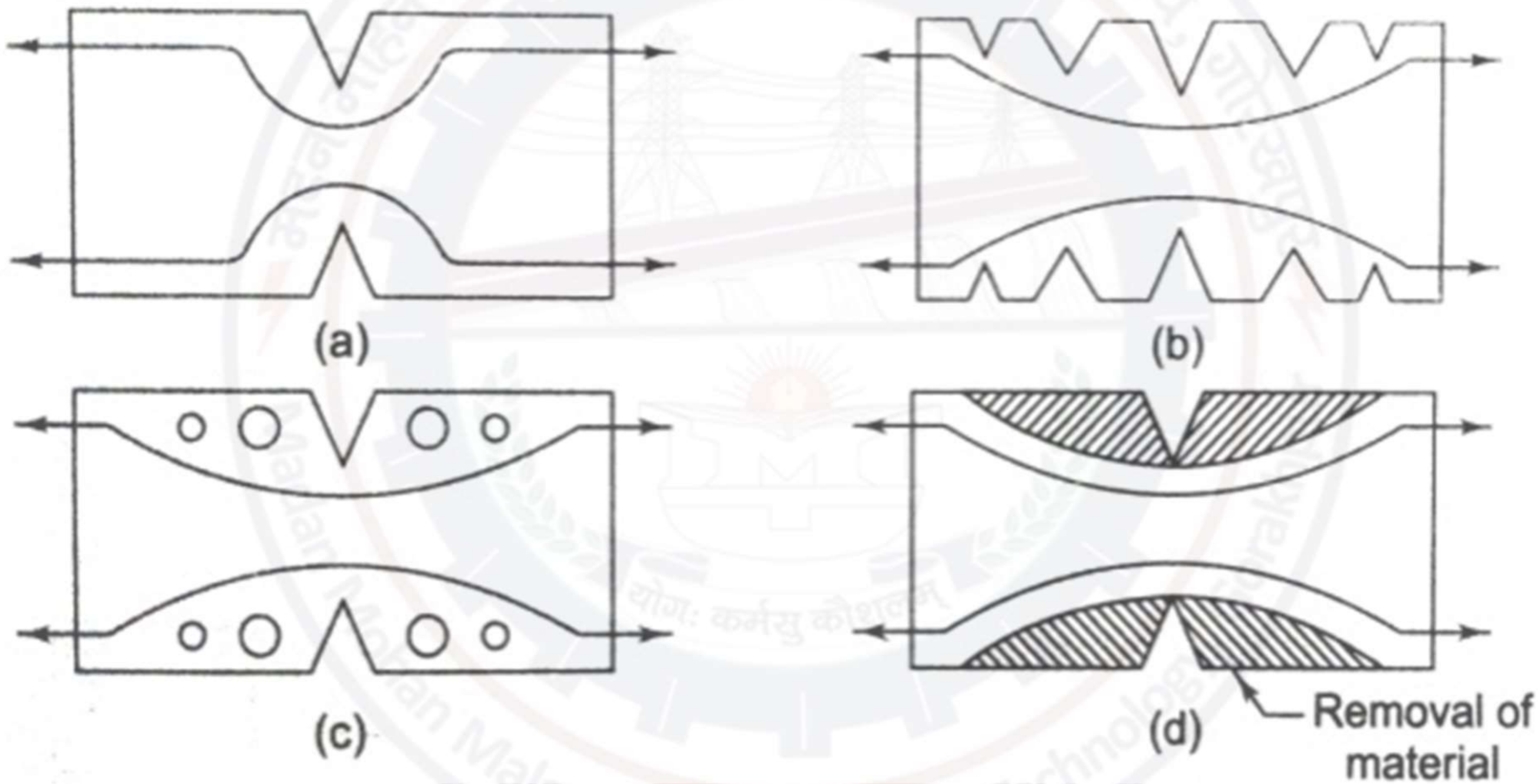


Methods to reduce stress concentration



Reduction of Stress Concentration due to Abrupt Change in Cross-section: (a) Original Component (b) Fillet Radius (c) Undercutting (d) Addition of Notch

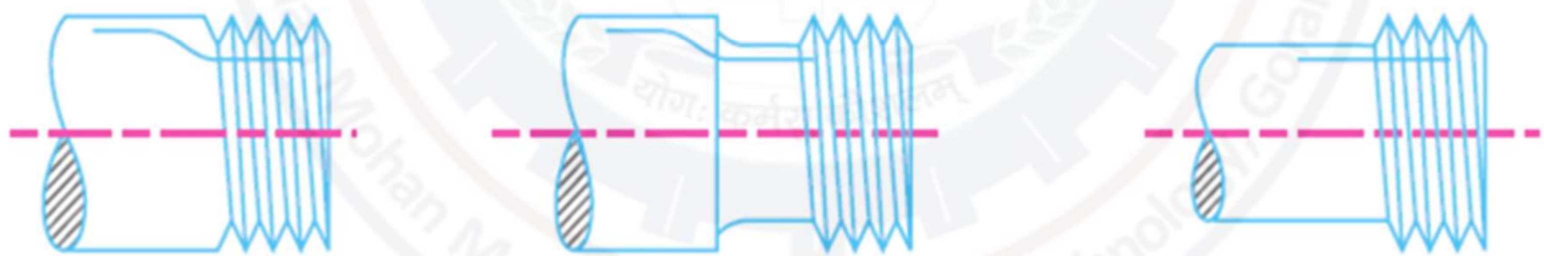
Methods to reduce stress concentration



Reduction of Stress Concentration due to V-notch: (a) Original Notch (b) Multiple Notches (c) Drilled Holes (d) Removal of Undesirable Material

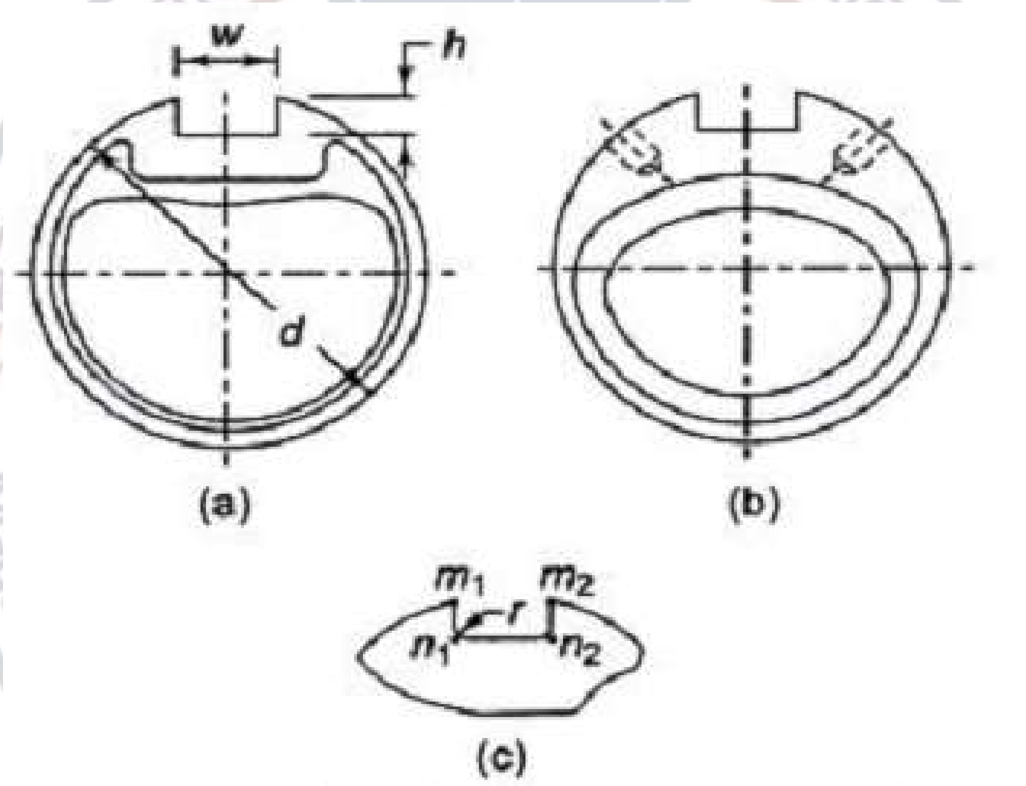
Methods to reduce stress concentration

(a) Poor (b) Preferred



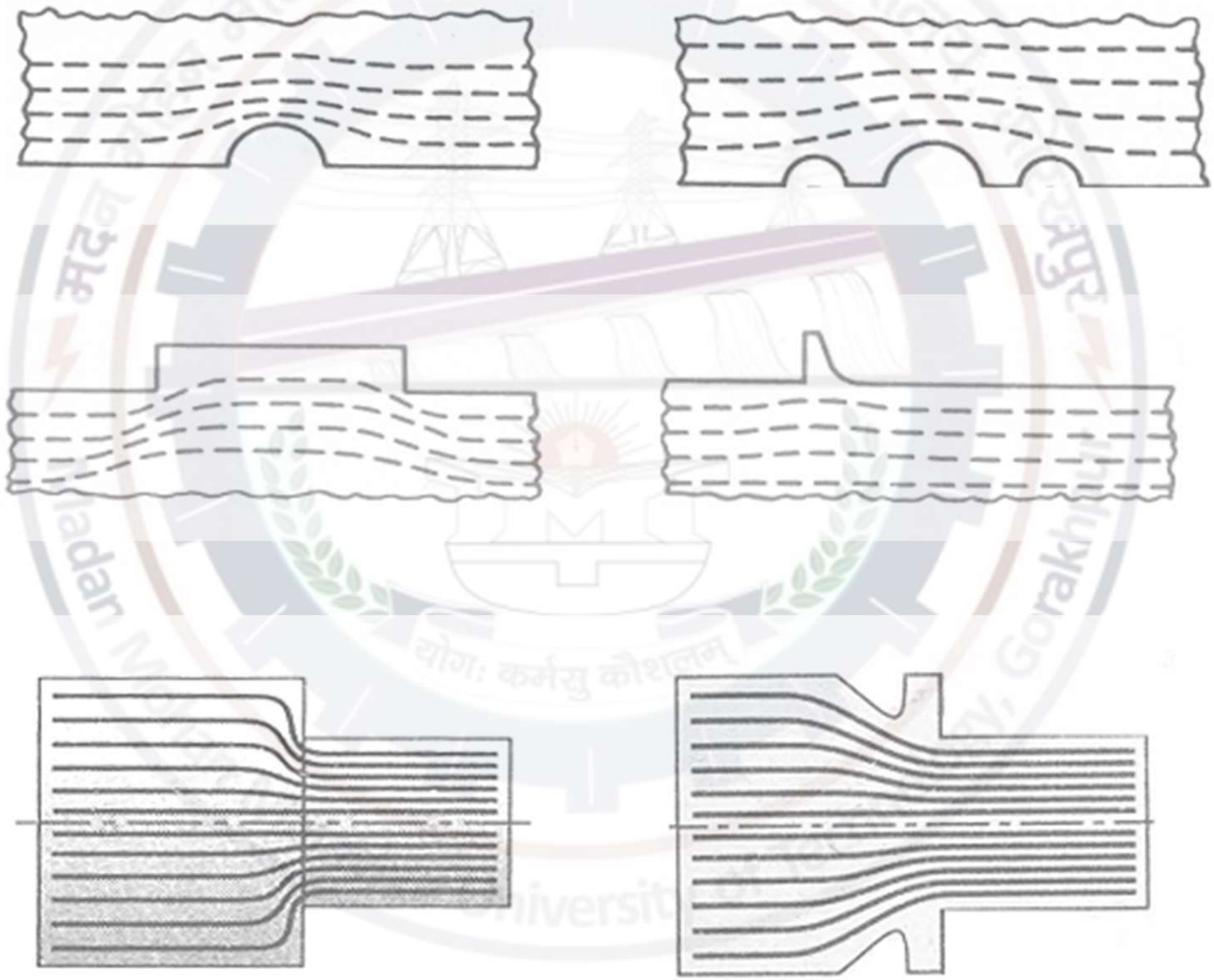
(a) Poor (b) Good (c) Preferred

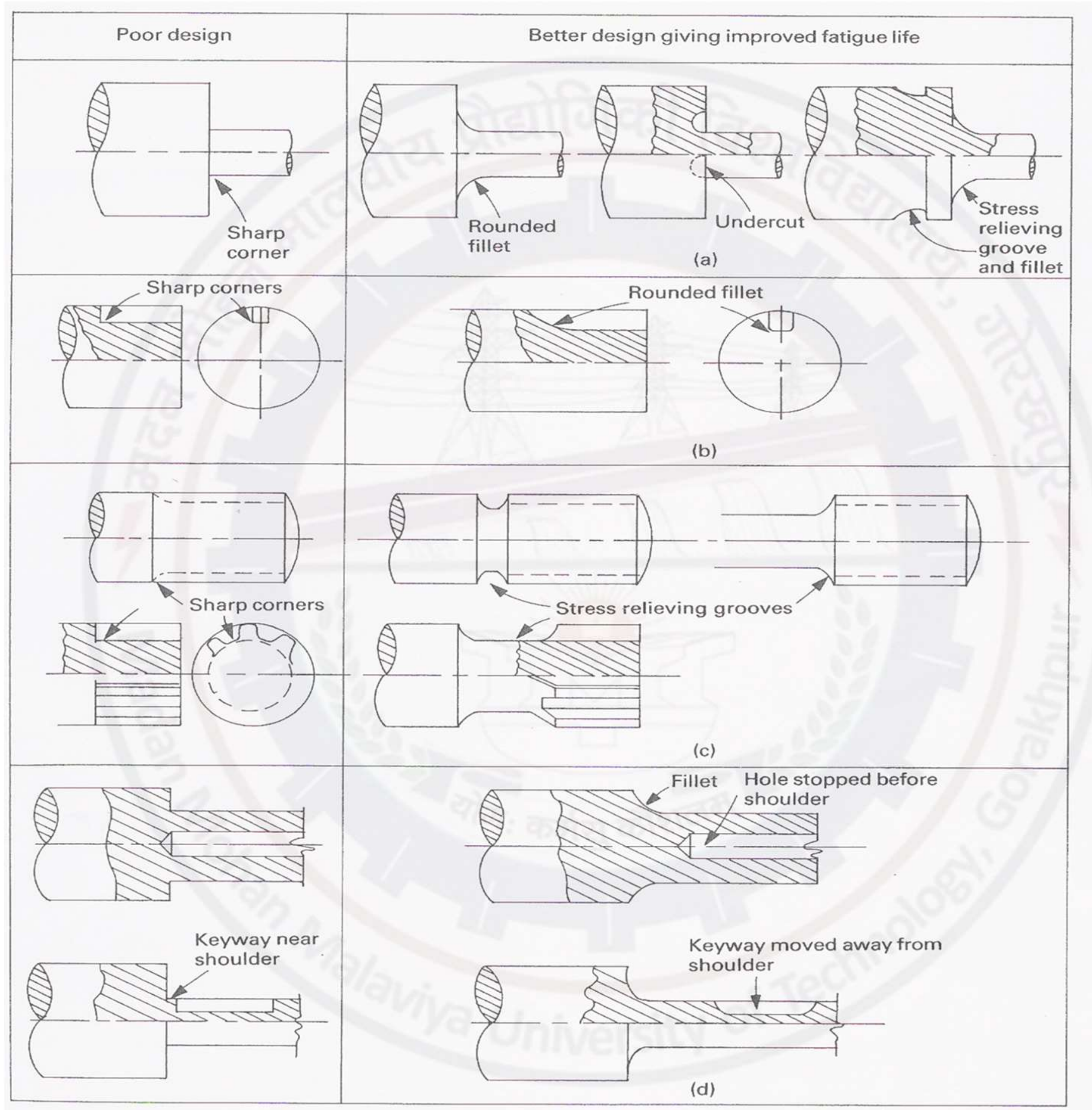
Methods to reduce stress concentration



Reduction of Stress Concentration in Shaft with Keyway: (a) Original Shaft (b) Drilled Holes (c) Fillet Radius

Methods to reduce stress concentration

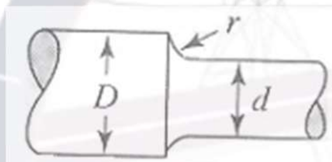




Design guideline of shafts subjected to fatigue loading

Example

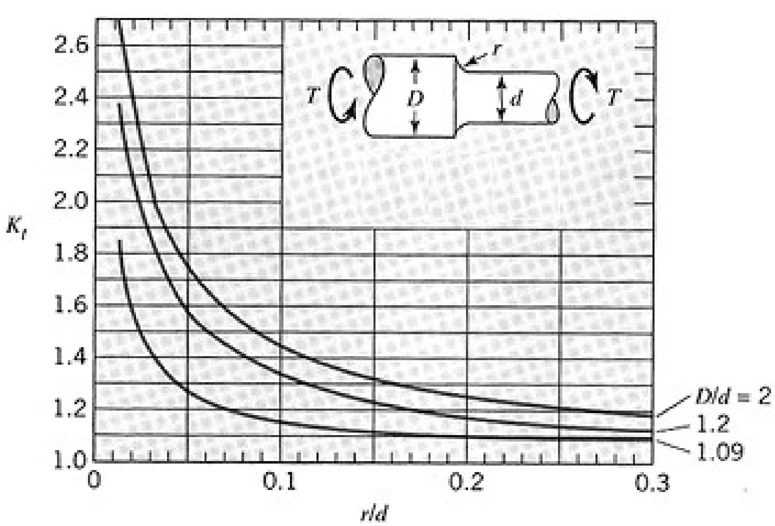
Find the maximum stress developed in a stepped shaft subjected to a twisting moment of 100 Nm as shown in figure. What would be the maximum stress developed if a bending moment of 150 Nm is applied.

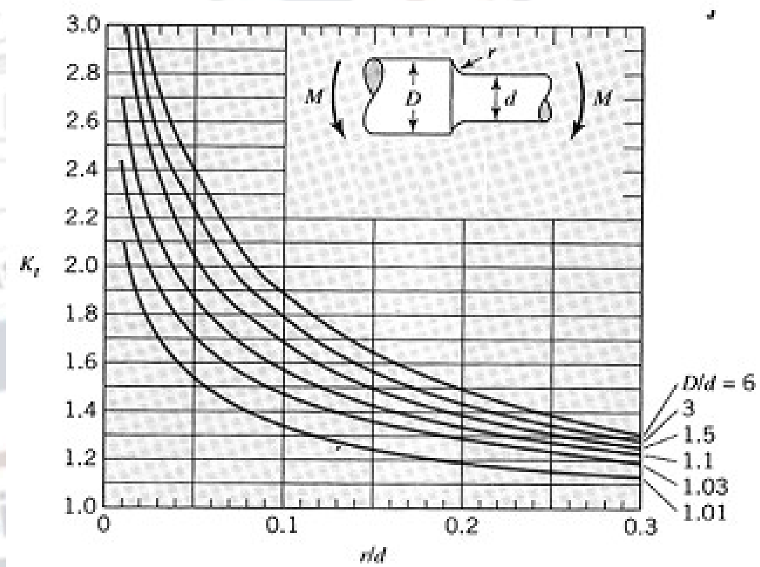


r = 6 mm

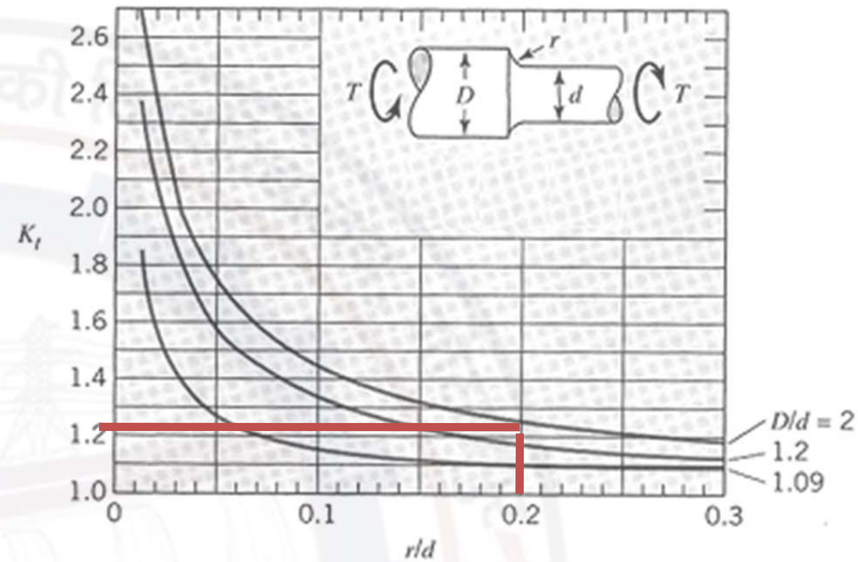
d = 30 mm

D = 40 mm.





Referring to the stress- concentration plots for stepped shafts subjected to torsion for r/d = 0.2 and D/d = 1.33, Kt ≈1.23.



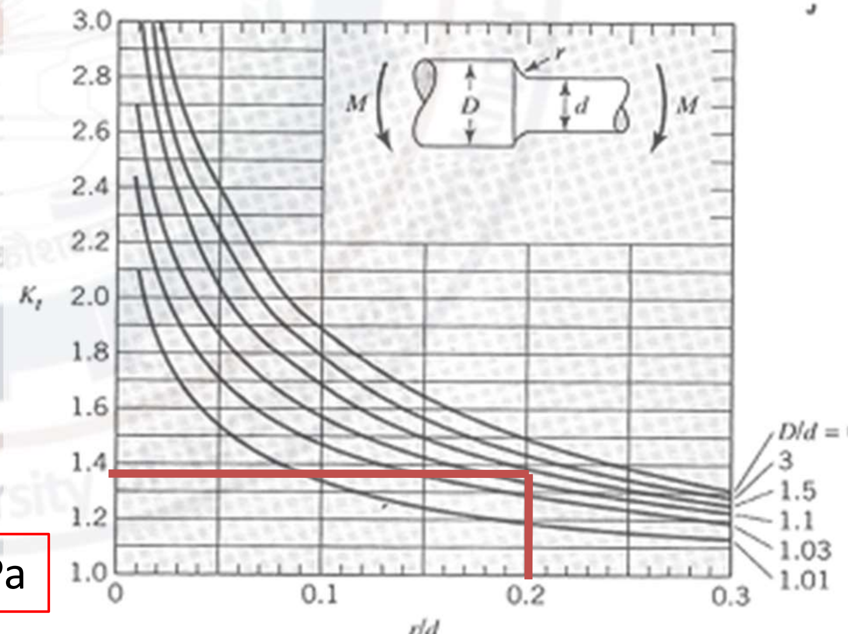
Torsional shear stress is given by τ= 16T/πd³. Considering the smaller diameter and the stress concentration effect at the step, we have the maximum shear stress as

$$\tau_{max} = K_t \frac{16x100}{\pi(0.03)^3}$$

This gives τ_{max} = 23.20 MPa

Similarly referring to stress-concentration plots for stepped shaft subjected to bending , for r/d = 0.2 and D/d = 1.33, Kt ≈1.38

Bending stress is given by $\sigma = \frac{32M}{\pi d^3}$



Considering the smaller diameter and the effect of stress concentration at the step, we have the maximum bending stress as

$$\sigma_{max} = K_t \frac{32x150}{\pi(0.03)^3}$$

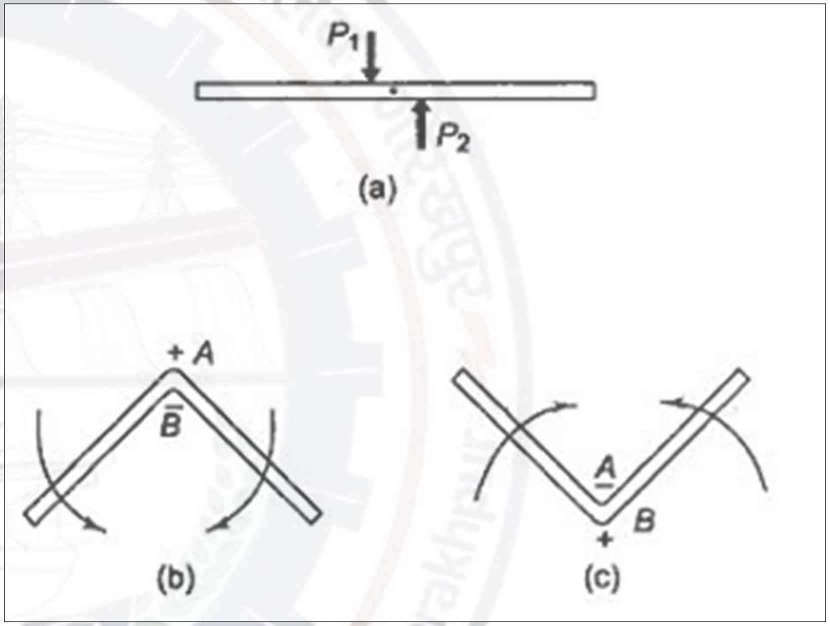
This gives σ_{max} = 78.13 MPa

Design for Fluctuating Loads

- A large number of mechanical components are subjected to loads that are not static but fluctuating or cyclic.
- Even a static load causes cyclic stresses in a rotating shaft.
- Fluctuating stresses are also called cyclic as they are repetitive when plotted against time as abscissa.
- The shape of repetitive stress-time curve is not important but the mean and amplitude of stresses is.
- Ultimate and yield strengths were used in the design under static loads. In the case of fluctuating loads, in addition to ultimate and yields strengths, we shall need the strength of materials under fluctuating stresses.
- It is called fatigue strength of a material which is much lower than ultimate or yield strength.

Fatigue Failure

- Materials fail under fluctuating stress at stress magnitude which is lower than ultimate tensile strength.
- Sometimes, even lower than the yield strength.
- Magnitude of the stress, causing fatigue failure decreases as the no. of stress cycles increase.
- This phenomenon of decreseased resistance of the materials to the fluctuating stresses is the main characteristics of fatigue failure.
- Fatigue failure is defined as time delayed fracture under cyclic loading.
- Ex- shafts, connecting rods, gears, springs, ball bearing etc.
- It depends upon no. of cycle, mean stress, stress amplitude, corrosion, creep etc



Shear and Fatigue failure of Wire:
(a) Shearing of wire
(b) Bending of wire
(c) Unbending of wire

Static load Failure and Fatigue Failure

<u>Failure under static load</u>

- The failure due to static load is illustrated by the simple tension test.
- Load is gradually applied and there is sufficient time for the elongation of fibres.
- In the ductile material, there is considerable plastic flow prior to fracture.

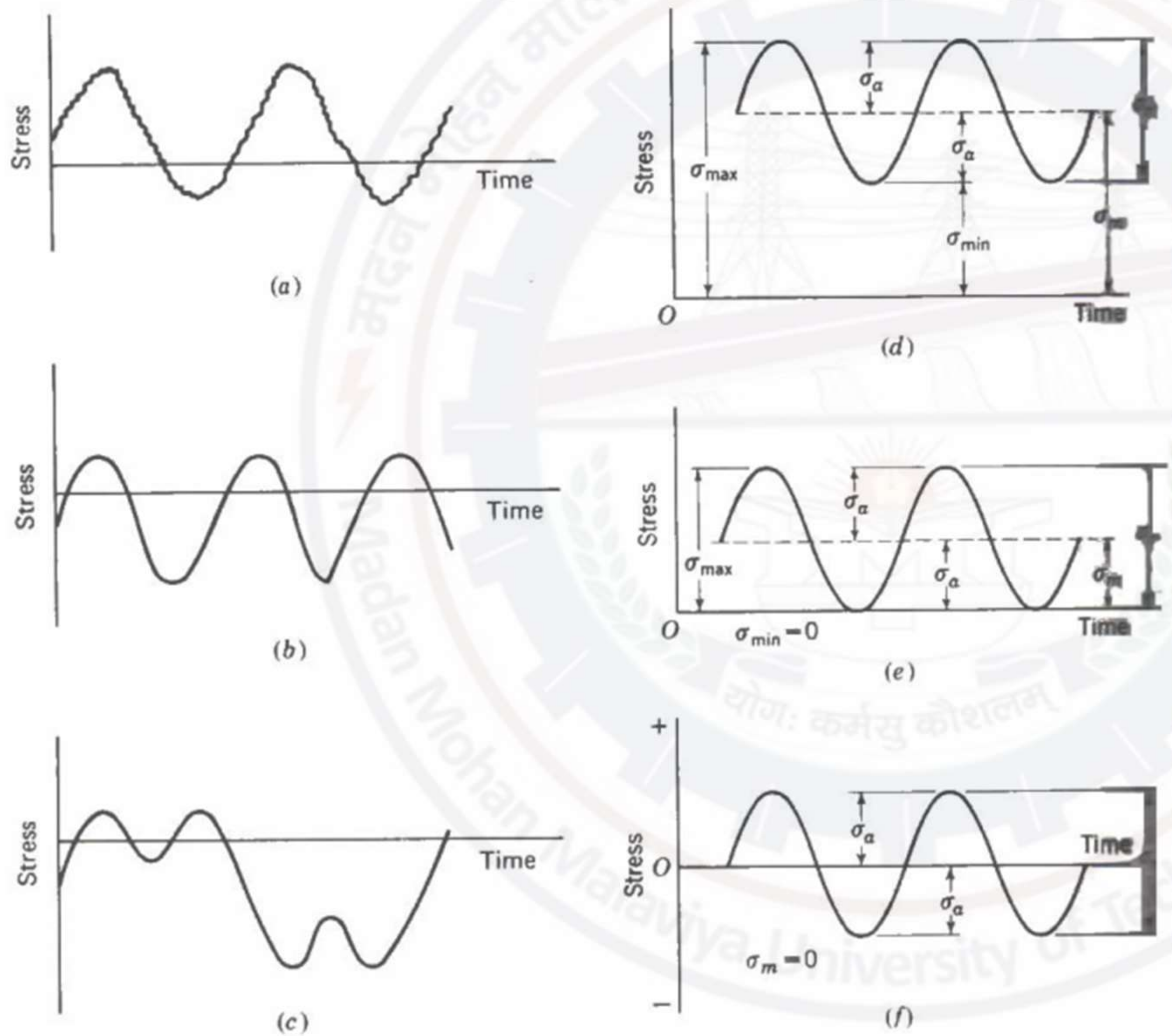
<u>Failure under fluctuating load</u>

- Fatigue failure begins with crack at some point in the material.
- The cracks are likely to occur in the region of discontinuity, irregularities, internal cracks due to defects in the materials.
- These regions are subjected to stress concentration due to crack.
- The crack spreads due to fluctuating stresses until the cross-section is so reduced that the remaining portion is subjected to sudden fracture.
- The fatigue failure is sudden and total.

Fluctuating stresses

- In many application, the components are subjected to forces, which vary in magnitude with respect to time.
- The stresses induced due to such forces are called fluctuating stresses.
- It is observed that about 80% of failure of mechanical components are due to ‘fatigue failure’ resulting from fluctuating stresses.

Stress-time relations



(a) Fluctuating stress with high frequency ripple.
(b) Non sinusoidal fluctuating stress.
(c) No-sinusoidal fluctuating stress.
(d) Sinusoidal fluctuating stress.
(e) Repeated stress.
(f) Completely reversed sinusoidal stress.

Classification of Cyclic Stresses

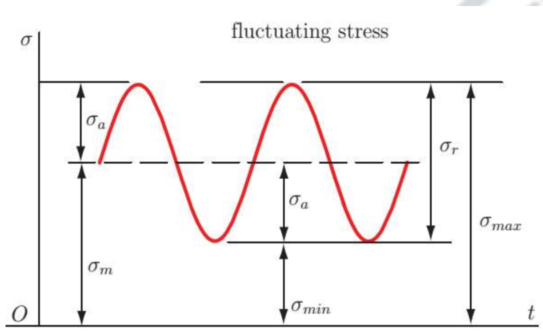
- Stress-time relations show that the stresses are not static but vary in magnitude with time.
- Cyclic stresses cause 'fatigue failure' and the objective here is to design machine elements that do not fail due to fatigue during the specified design life.
- Cyclic stresses are classified according to the relative magnitudes of mean stress σ_m and stress amplitude σ_a as follows:

Mean stress: $\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min})$

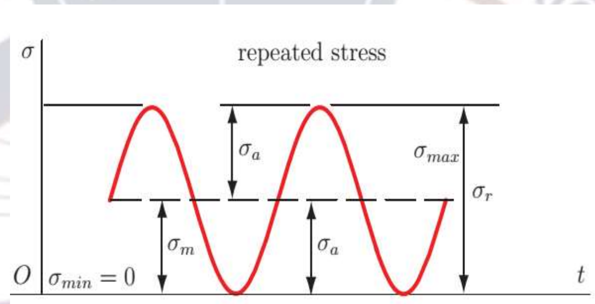
Stress amplitude: $\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min})$

Where σ_{max} and σ_{min} are the maximum and minimum stresses, respectively.

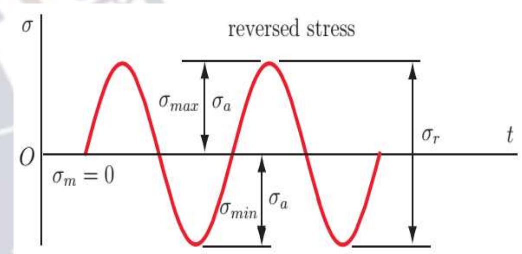
Cyclic Stresses



Fluctuating stresses :
σ_{max} , σ_{min} and σ_{mean} are all non-zero



Repeated or pulsating stresses: σ_{max} *or* σ_{min} = 0



Reversed stresses : σ_m = 0,

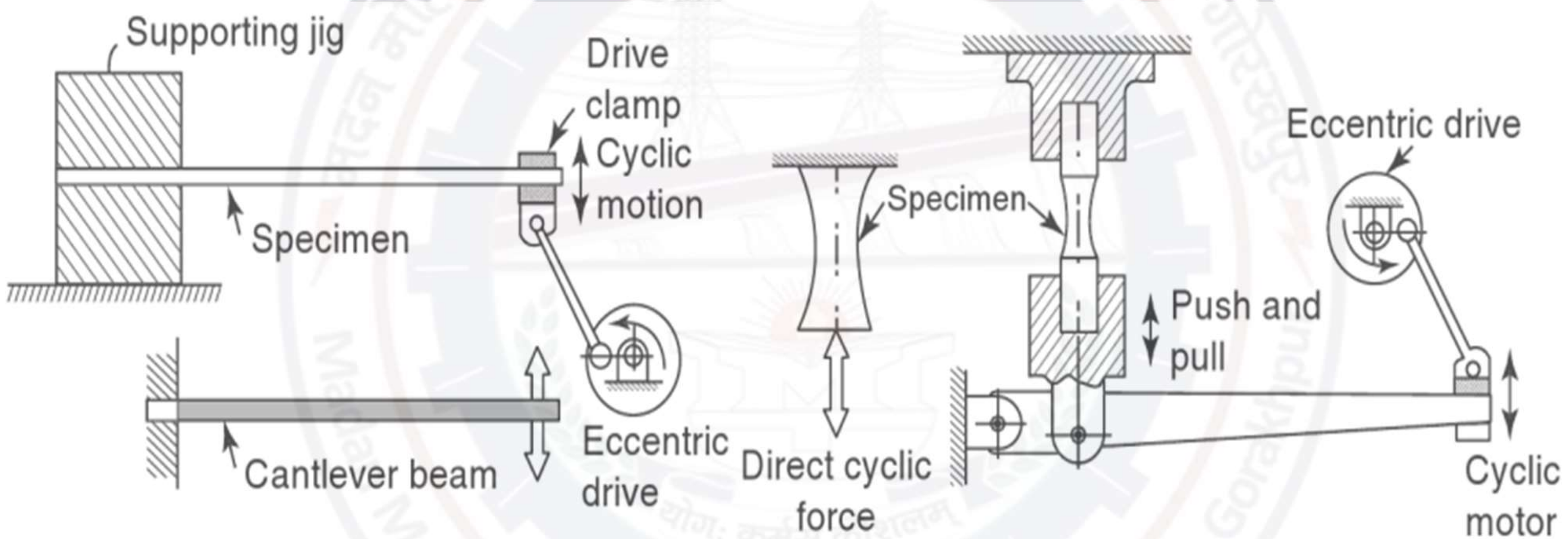
$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

σ_{max} = maximum stress σ_a = stress amplitude

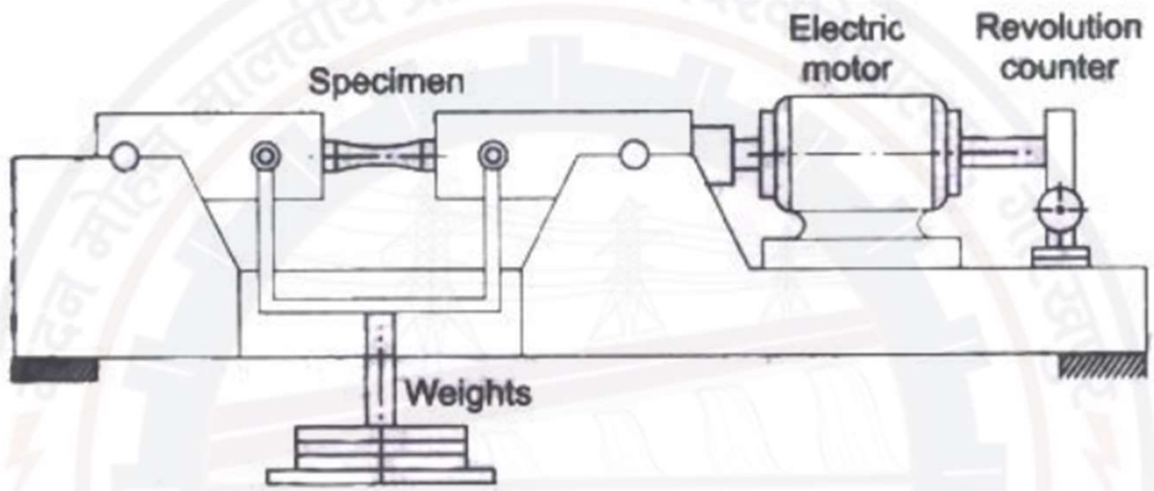
σ_{min} = minimum stress σ_m = mean stress

Fatigue Testing Machine

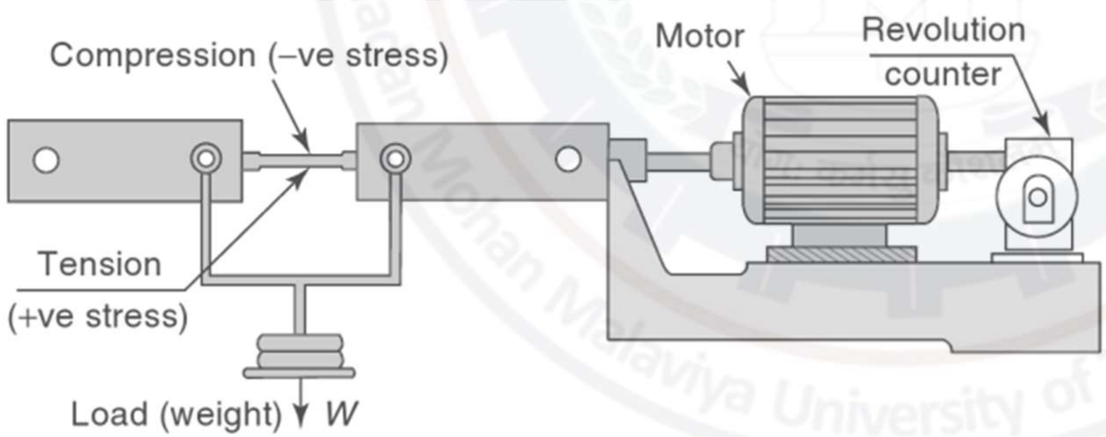


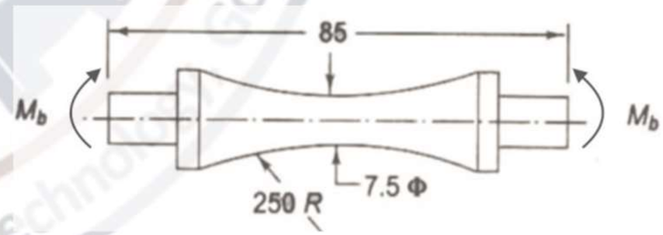
(b) Cyclic bending (hogging and sagging bending) (c) Cyclic axial force (tension and compression)

Rotating Beam Fatigue Testing Machine



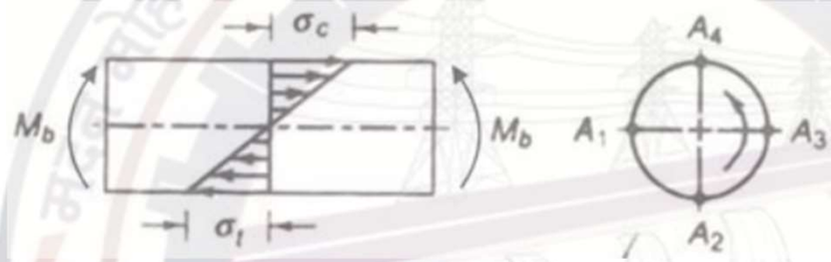
Rotating beam machine developed by R. R. Moore





(a) Rotating (cyclic)-beam fatigue testing machine

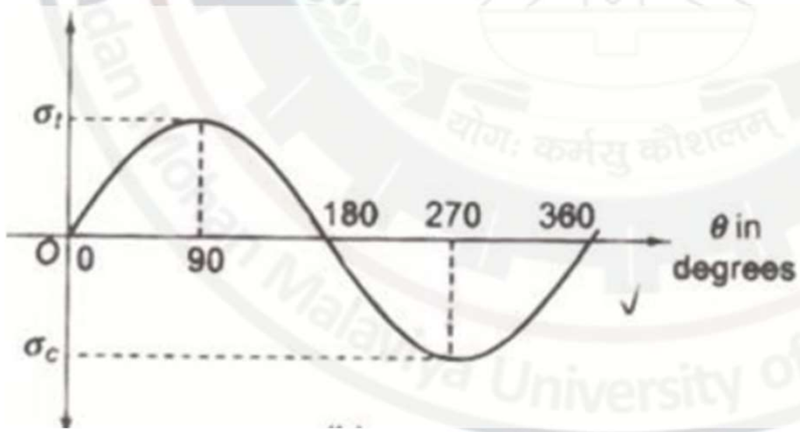
Variation of Stresses in rotating beam machine



Beam

Rotating Beam Subjected to Bending Moment

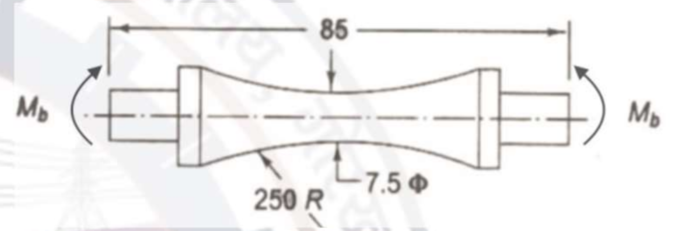
$$\sigma_t \text{ or } \sigma_c = \frac{M_b y}{I}$$



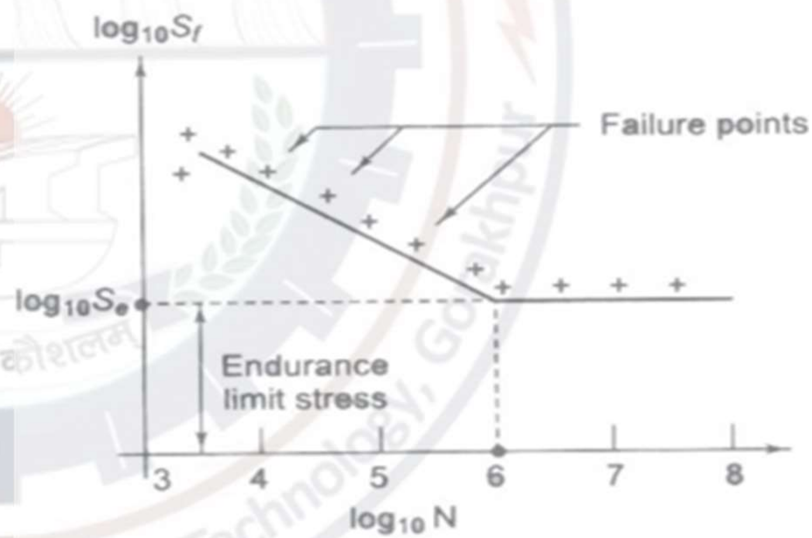
Stress cycle at point A

Endurance Limit, Fatigue Life and S-N Curve

- The fatigue or endurance limit of a material is defined as the maximum amplitude of completely reversed stress that the standard specimen can sustain for an *unlimited number of cycles* without fatigue failure.
- Since the fatigue test can not be conducted for unlimited or infinite no. of cycle, 10^6 cycles is considered as sufficient to define endurance limit.
- Fatigue life is defined as the number of stress cycles that the standard specimen can complete during the test before the appearance of the first fatigue crack.



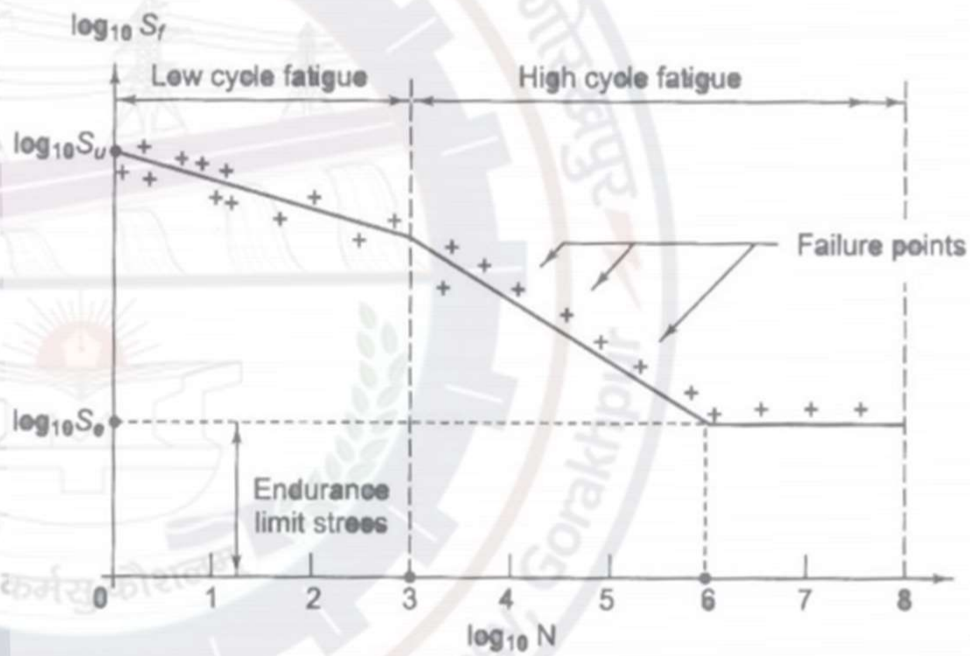
Dimension (mm) of standard fatigue test specimen



S-N Curve for steels

Low-cycle and High-cycle Fatigue

- Earlier S-N curve drawn was for small cycles.
- There are two main regions.
- Any Fatigue failure less than 1000 cycles is low cycle fatigue & more than 1000 is high cycle fatigue.
- Failures of studs on truck wheel, setscrews for locating gears on shafts, missiles are examples of low cycle fatigue
- Failure is springs, ball bearings, gears are high cycle.
- Low cycle fatigue involves plastic yielding at localized area of the components.
- Fatigue effect can be neglected when the no. of stress cycle is less than 1000.



Low and High Cycle fatigue

Fatigue Stress Concentration Factor

- When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor.
- Mathematically, fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

- **Notch Sensitivity:** It may be defined as the degree to which the theoretical effect of stress concentration is actually reached.
- **Notch Sensitivity Factor "q":** Notch sensitivity factor is defined as the ratio of increase in the actual stress to the increase in the nominal stress near the discontinuity in the specimen.

$$\text{actual stress} = K_f \sigma_o$$
$$\text{theoretical stress} = K_t \sigma_o$$

$$\text{increase of actual stress over nominal stress} = (K_f \sigma_o - \sigma_o)$$
$$\text{increase of theoretical stress over nominal stress} = (K_t \sigma_o - \sigma_o)$$

$$q = \frac{(K_f \sigma_o - \sigma_o)}{(K_t \sigma_o - \sigma_o)}$$

$$q = \frac{K_f - 1}{K_t - 1}$$

$$K_f = 1 + q(K_t - 1)$$

Where, K_f and K_t are the fatigue stress concentration factor and theoretical stress concentration factor.

- The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material.

Notch Sensitivity

$$K_f = 1 + q(K_t - 1)$$

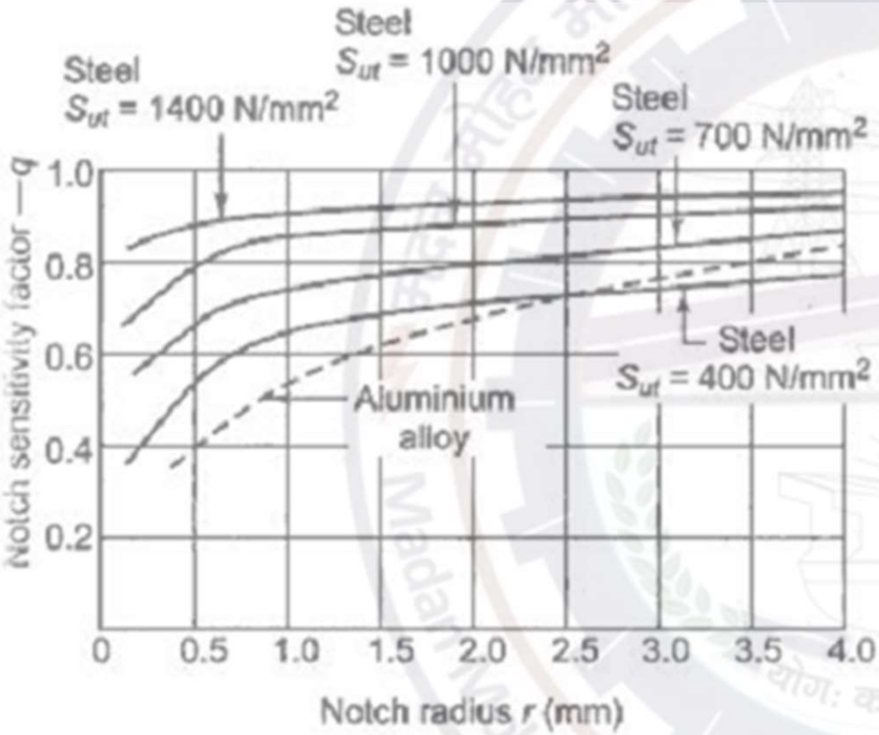
The following conclusions can be drawn with the help of above equation

1. When the material has no sensitivity to notches,
   $q = 0$, and $K_f = 1$

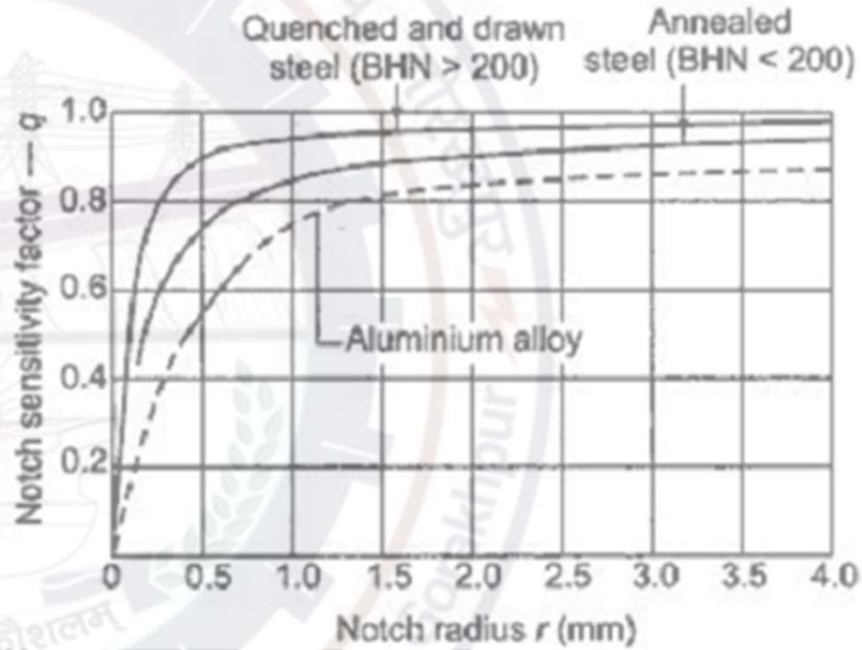
2. When the material is fully sensitive to notches,
   $q = 1$, and $K_f = K_t$

- In general, magnitude of notch sensitivity factor q varies between 0 to 1.
- In case of doubt, the designer should use ($q=1$) or ($K_f = K_t$) and design will be on the safe side.

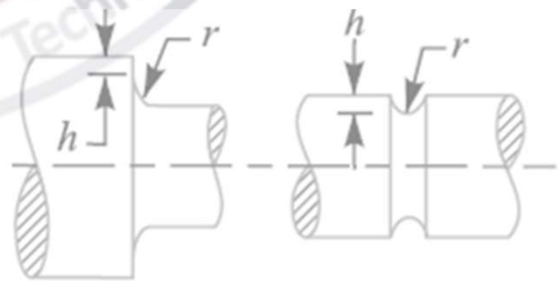
- Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in figure may be used for determining the values of q for two steels.



Notch Sensitivity Chart
(for Reversed Bending and Reversed Axial Stresses)



Notch Sensitivity Chart
(for Reversed Torsional Shear Stresses)



Approximate Estimation of Endurance Limit

- The laboratory method for determining the endurance limit of materials, although more precise, is laborious and time consuming.
- No of tests are required to prepare one S-N curve and each test takes considerable time.
- When the laboratory data regarding the endurance limit of the materials is not available, following procedure should be adopted

$$S_e = K_a K_b K_c K_d S'_e$$

S_e = Endurance limit stress of a ***particular mechanical component*** subjected to reversed bending stress.

S'_e = Endurance limit stress of a ***rotating beam specimen*** subjected to reversed bending stress.

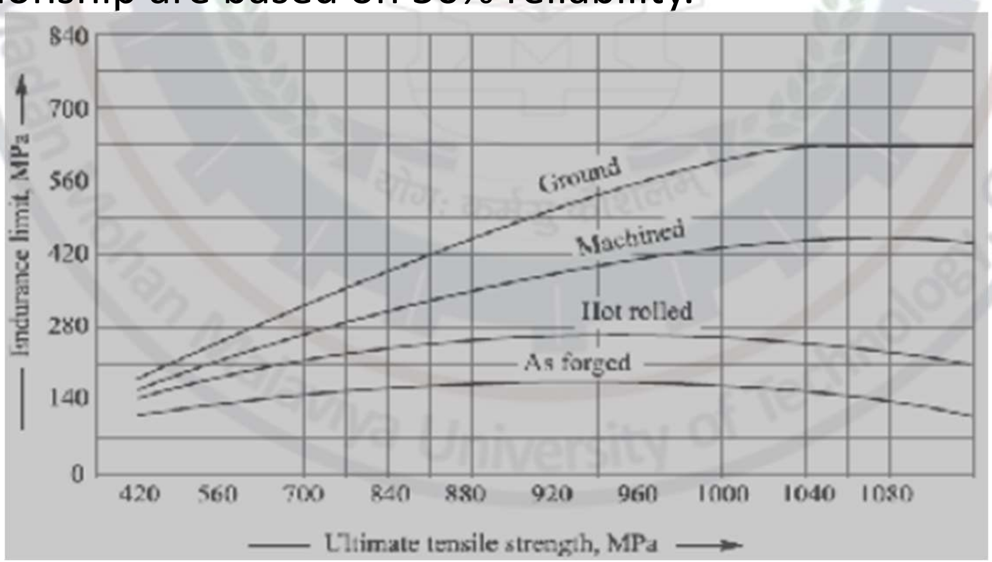
K_a = Surface finish factor
K_b = Size factor
K_c = Reliability factor
K_d = Modifying factor to account for stress concentration

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that **endurance limit (S'_e)** of a material subjected to fatigue loading is a function of **ultimate tensile strength (S_{ut}).**

<u>For Steel</u>, $S'_e = 0.5\ S_{ut}$
<u>For cast iron and cast steels</u>, $S'_e = 0.4\ S_{ut}$
<u>For wrought aluminium alloys</u>, $S'_e = 0.4\ S_{ut}$
<u>For cast aluminium alloys</u>, $S'_e = 0.3\ S_{ut}$

The above relationship are based on 50% reliability.



Surface finish factor (K_a)

The endurance limit of a ***component*** is different from the endurance limit of a ***rotating beam specimen*** due to no. of factors.

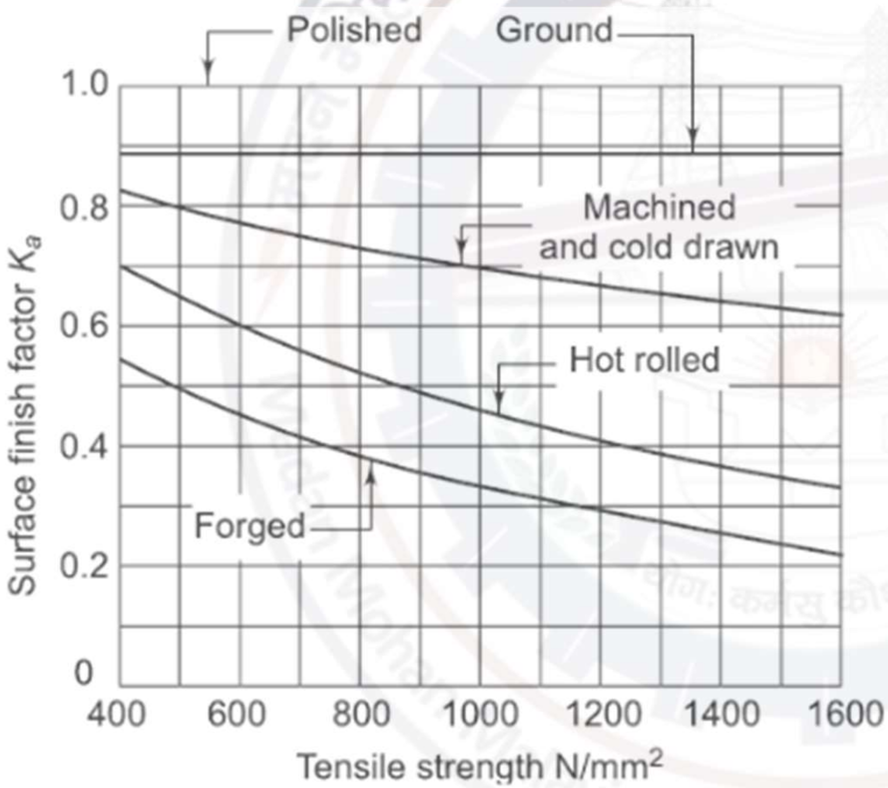


Fig. 5.24 *Surface Finish Factor*

Surface finish factor for steel components

- The surface of rotating beam specimen is polished to mirror finish.
- The final polishing is carried out in the axial direction to smooth out any circumferential scratches.
- This makes the specimen almost free from surface scratches and imperfection.
- Surface finish factor for cast iron parts is always taken as 1.

Size Factor (K_b)

•The ***rotating beam specimen*** is small with 7.5 mm diameter.
•The larger the machine part, the greater the probability that a flaw exists somewhere in the components.

Table 5.1 Values of Factor K_b for rotating Shafts (Load type: bending or torsional)

Diameter d (mm)	K_b
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75
For axial loading	1.00

Reliability Factor (K_c)

•The laboratory values of endurance limit are usually mean values.
•There is considerable dispersion of data when a number of tests are conducted even using the same material and same conditions.
•The reliability factors based on a standard deviation of 8% are given below:

Table 5.3 *Reliability factor*

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Modifying stress concentration factor (K_d)

•The endurance limit is reduced due to stress concentration.
•To apply the effect of stress concentration the endurance limit can be reduced by (K_d) or increase the stress amplitude by (K_f).

$$q = \frac{K_f - 1}{K_t - 1}$$

$K_f = 1 + q\,(K_t - 1)$ and modifying factor $K_d = 1 / K_f$

When all the values of S'_e , K_a, K_b, K_c, and K_d are known, endurance limit can be calculated by

$$S_e = K_a K_b K_c K_d S'_e$$

Endurance stress of fluctuating torsional stresses

Endurance limit (S_{se}) of a component subjected to **fluctuating torsional shear stresses** is obtained from the endurance limit in reversed bending (S_e) using Theories of Failures.

According to Maximum shear stress theory,

$$S_{se} = 0.5\ S_e$$

According to distortion energy theory,

$$S_{se} = 0.577\ S_e$$

When the component is subjected to ***axial fluctuating load*** the endurance limit can be obtained by,

$$(S_e)_a = 0.8\ S_e$$

Reversed stresses – Design for Finite and Infinite Life

- There are two types of problems in fatigue design
  - Components subjected to completely reversed stresses
    - Design for infinite life.
    - Design for finite life.
  - Components subjected to fluctuating stresses

Design for infinite life

- When the component is to be designed for infinite life, the endurance limit becomes the criterion of failure.
- The amplitude stress induced should be lower than the endurance limit in order to withstand the infinite number of cycles.

$$\sigma_a = \frac{S_e}{fs}$$

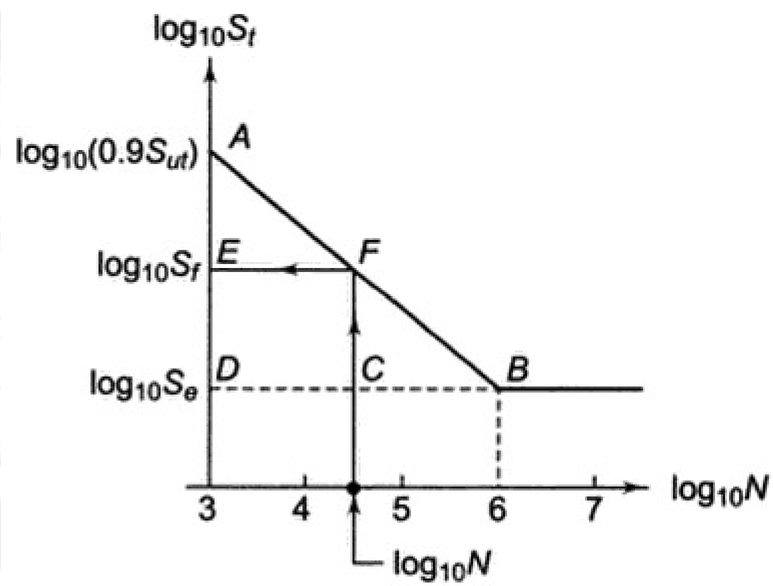
$$\tau_a = \frac{S_{se}}{fs}$$

(σ_a) and (τ_a) = Stress amplitudes in the components.
S_e and S'_e = Corrected endurance limits in reversed bending and torsion respectively.

Design for Finite life

- When the component is to be designed for finite life, the S-N Curve can be used.
- The curve is valid for steels
- It consists of a straight line AB drawn from ($0.9\ S_{ut}$) at 10^3 cycles to (S_e) at 10^6 cycles on a log-log paper.

(i) Locate the point A with coordinates
[3, $\log_{10}(0.9\ S_{ut})$] since $\log_{10}(10^3) = 3$
(ii) Locate the point B with coordinates
[6, $\log_{10}(S_e)$] since $\log_{10}(10^6) = 3$
(iii) Join AB, which is used as a criterion of failure for finite-life problems.
(iv) Depending upon the life N of the component, draw a vertical line passing through $\log_{10}(N)$ on the abscissa. The line intersects AB at point F.
(v) Draw a line FE parallel to the abscissa.
The ordinate at the point E, i.e. $\log_{10}(S_f)$, gives
The fatigue strength corresponding to N cycles.
(vi) The value of fatigue strength (S_f) is used for the design calculation.



Example

- A rotating bar made of steel 45C8 (S_{ut} = 630 N/mm2)is subjected to a completely reversed bending stress. The corrected endurance limit of the bar is 315 N/mm2. Calculate the fatigue strength of the bar for a life of 90,000 cycles.

Example

Solution

Given $S_{ut} = 630$ N/mm^2 $S_e = 315$ N/mm^2
$N = 90000$ cycles

Step I Construction of S–N diagram

$0.9S_{ut} = 0.9\,(630) = 567$ N/mm^2

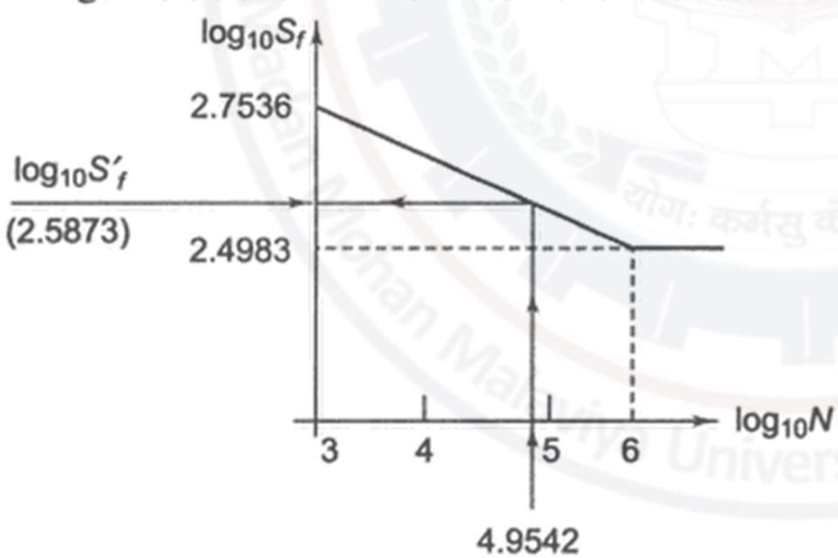
$\log_{10}(0.9S_{ut}) = \log_{10}(567) = 2.7536$

$\log_{10}(S_e) = \log_{10}(315) = 2.4983$

$\log_{10}(90\,000) = 4.9542$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10)^6 = 6$

Figure 5.30 shows the S–N curve for the bar.



Step II Fatigue strength for 90000 cycles

Referring to Fig. 5.30,

$$\log 10\,(S'_f) = 2.7536 - \frac{(2.7536 - 2.4983)}{(6-3)} \times (4.9542 - 3) = 2.5873$$

$$S'_f = 386.63 \text{ N/mm}^2$$

Example

- A forged steel bar, 50 mm in diameter is subjected to a reversed bending stress of 250 N/mm2. The bar is made of steel 40C8 (Sut = 600 N/mm2). Calculate the life of the bar for a reliability of 90%.

Example

Solution

Given $S_f = \sigma_b = 250$ N/mm^2
$S_{ut} = 600$ N/mm^2 $R = 90\%$

Step I Construction of S–N diagram

$S'_e = 0.5S_{ut} = 0.5(600) = 300$ N/mm^2

From Fig. 5.24, ($S_{ut} = 600$ N/mm^2 and forged bar),

$K_a = 0.44$

For 50 mm diameter, $K_b = 0.85$

For 90% reliability, $K_c = 0.897$

$S_e = K_a K_b K_c S'_e = 0.44(0.85)(0.897)(300)$
$= 100.64$ N/mm^2

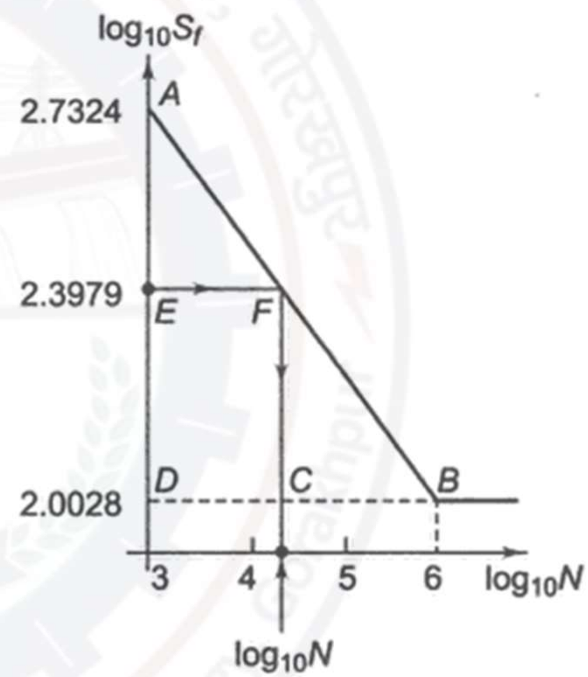
$0.9S_{ut} = 0.9(600) = 540$ N/mm^2

$\log_{10}(0.9S_{ut}) = \log_{10}(540) = 2.7324$

$\log_{10}(S_e) = \log_{10}(100.64) = 2.0028$

$\log10(S_f) = \log_{10}(250) = 2.3979$

Also, $\log_{10}(10^3) = 3$ and $\log_{10}(10^6) = 6$



Step II Fatigue life of bar

From Fig. 5.31,

$$\overline{EF} = \frac{\overline{DB} \times \overline{AE}}{\overline{AD}} = \frac{(6-3)(2.7324 - 2.3979)}{(2.7324 - 2.0028)}$$

$= 1.3754$

Therefore,

$\log_{10} N = 3 + \overline{EF} = 3 + 1.3754$

$\log_{10} N = 4.3754$

$N = 23\ 736.2$ cycles

Cumulative Damage in Fatigue

- In certain application, the mechanical component is subjected to different stress levels for different parts of the work cycle.
- Suppose that a component is subjected to completely reversed stresses σ_1 for n_1 cycles, σ_2 for n_2 and so on.
- Let N_1 be the number of stress cycles before fatigue failure if only the alternating stress σ_1 is acting.
- One stress cycle will consume $(1/N_1)$ of the fatigue life.
- Since there are n_1 at the stress level, the proportionate damage of fatigue life will be (n_1/N_1).
- Similarly for σ_2 will be (n_2/N_2).

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_x}{N_x} = 1$$

Known as Miner's Equation

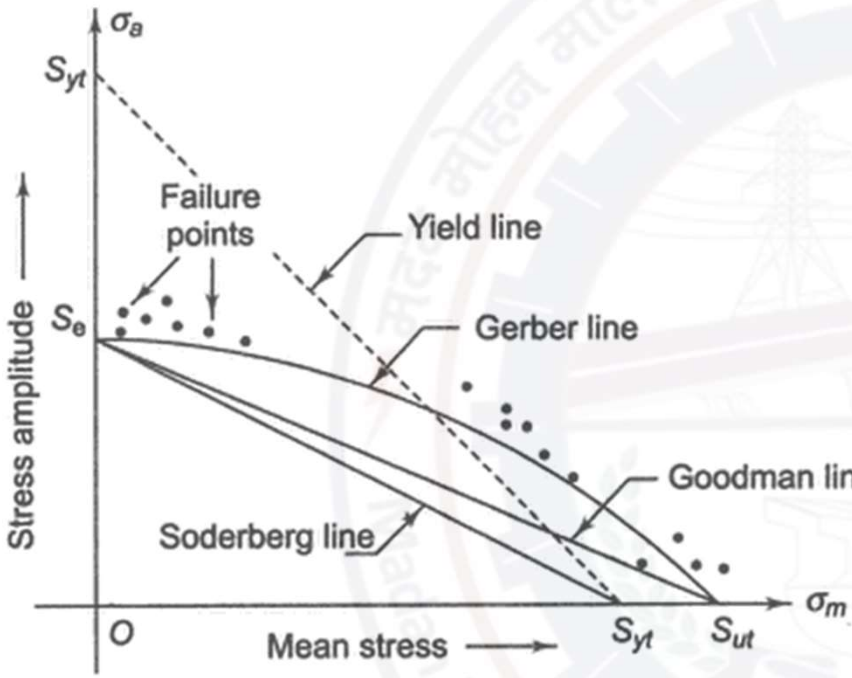
- Suppose proportion of total life is given as α_1 , α_2 .... α_x corresponding to stress level σ_1, σ_2 .... σ_x
- Let N be the total life of the components, then

$$n_1 = \alpha_1 N$$
$$n_2 = \alpha_2 N$$

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \ldots + \frac{\alpha_x}{N_x} = \frac{1}{N}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots\ldots + \alpha_x = 1$$

Soderberg, Goodman and Gerber lines



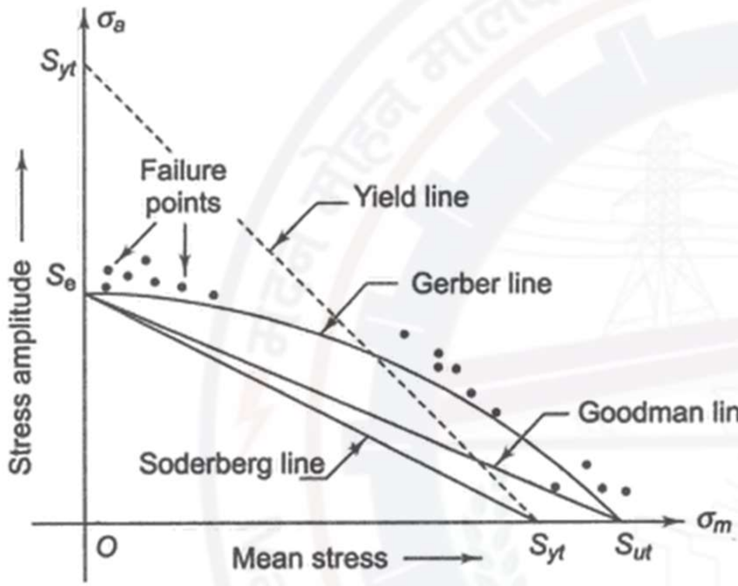
Gerber Line A parabolic curve joining S_e on the ordinate to S_{ut} on the abscissa is called the Gerber line.

Soderberg Line A straight line joining S_e on the ordinate to S_{yt} on the abscissa is called the Soderberg line.

Goodman Line A straight line joining S_e on the ordinate to S_{ut} on the abscissa is called the Goodman line.

- Gerber parabola fits the failure points of test data in the best possible way.
- Goodman line is more safe from design considerations.
- Soderberg line is a more conservative failure criterion.

Soderberg, Goodman and Gerber lines



The Goodman line is widely used as the criterion of fatigue failure when the component is subjected to mean stress as well as stress amplitude. It is because of the following reasons:

(i) The Goodman line is safe from design considerations because it is completely inside the failure points of test data.

(ii) The equation of a straight line is simple compared with the equation of a parabolic curve.

(iii) It is not necessary to construct a scale diagram and a rough sketch is enough to construct fatigue diagram.

Equation of Soderberg Line

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_e} = 1$$

Equation of Goodman Line

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{S_e} = 1$$

Equation of Gerber Line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$S_a = S_e\left[1 - \left(\frac{S_m}{S_{ut}}\right)^2\right]$$

Modified Goodman Diagram

The components, which are subjected to fluctuating stresses, are designed by constructing the modified Goodman diagram. For the purpose of design, the problems are classified into two groups:

(i) components subjected to fluctuating axial or bending stresses; and

(ii) components subjected to fluctuating torsional shear stresses.

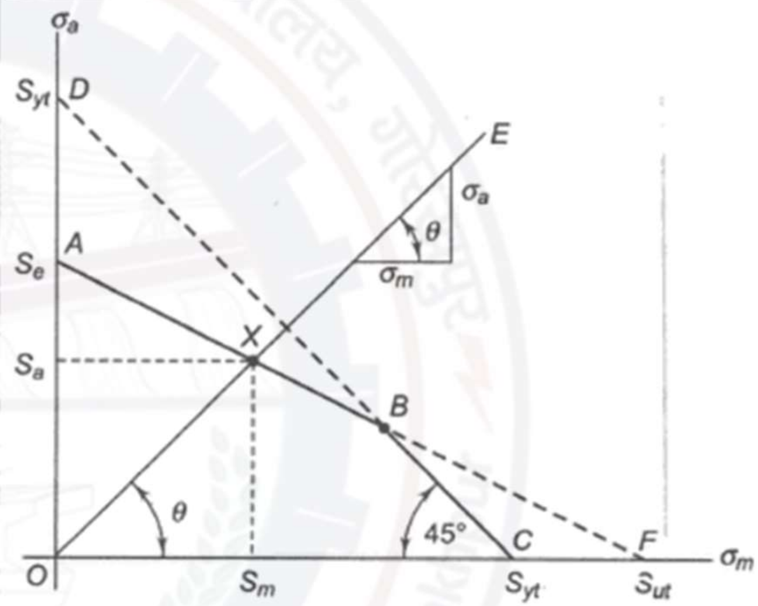


Fig. 5.40 *Modified Goodman Diagram for Axial and Bending Stresses*

A line OE is drawn, the slope of this line,

$$\tan\theta = \frac{\sigma_a}{\sigma_m}$$

$$\frac{\sigma_a}{\sigma_m} = \frac{(P_a / A)}{(P_m / A)} = \frac{P_a}{P_m}$$

$$\tan\theta = \frac{P_a}{P_m}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m}$$

$$\sigma_a = \frac{S_a}{(fs)} \quad \text{and} \quad \sigma_m = \frac{S_m}{(fs)}$$

Region of safety = OABC

Modified Goodman Diagram

A Fatigue failure is indicated if

$$\tau_a = S_{se}$$

The permissible stresses

$$\tau_a = \frac{S_{se}}{(fs)}$$

$$\tau_{max} = \frac{S_{sy}}{(fs)}$$

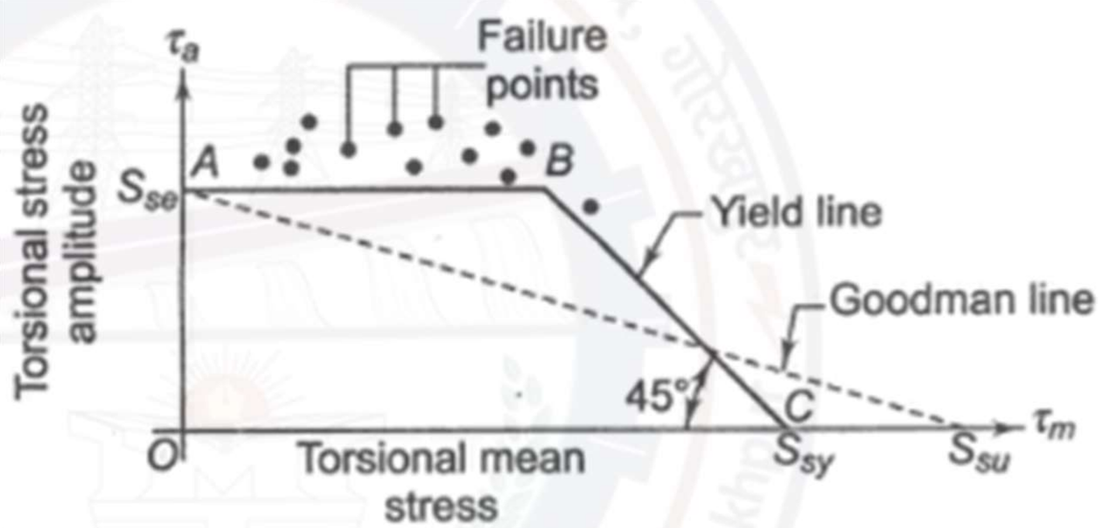
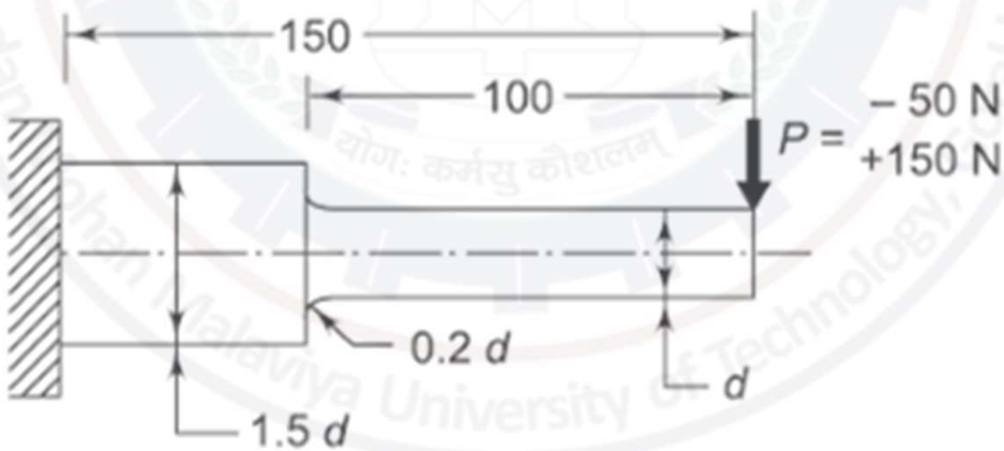


Fig. 5.41 *Modified Goodman Diagram for Torsional Shear Stresses*

Region of safety = OABC

Example

Example 5.12 *A cantilever beam made of cold drawn steel 4OC8 (S_{ut} = 600 N/mm^2 and S_{yt} = 380 N/mm^2) is shown in Fig. 5.42. The force P acting at the free end varies from –50 N to +150 N. The expected reliability is 90% and the factor of safety is 2. The notch sensitivity factor at the fillet is 0.9. Determine the diameter 'd' of the beam at the fillet cross-section.*



Example

Solution

Given $P = -50$ N to $+150$ N $S_{ut} = 600$ N/mm^2
$S_{yt} = 380$ N/mm^2 $R = 90\%$ $(fs) = 2$ $q = 0.9$

Step I Endurance limit stress for cantilever beam

$S'_e = 0.5 S_{ut} = 0.5\,(600) = 300$ N/mm^2

From Fig. 5.24 (cold drawn steel and $S_{ut} = 600$ N/mm^2),

$$K_a = 0.77$$

Assuming $7.5 < d < 50$ mm,

$$K_b = 0.85$$

For 90% reliability, $K_c = 0.897$

Since, $\frac{r}{d} = 0.2$ and $\frac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.44 - 1) = 1.396$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

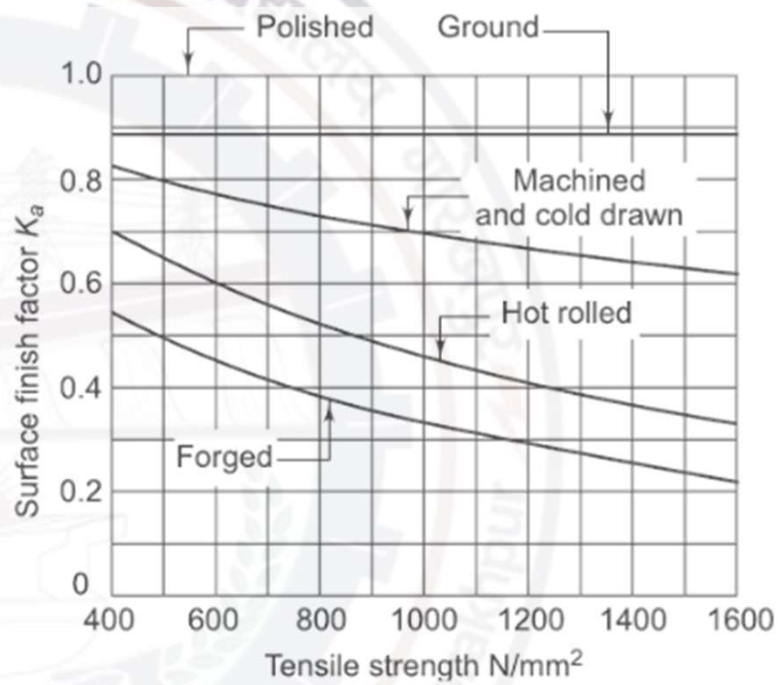


Fig. 5.24 *Surface Finish Factor*

Table 5.2 *Values of size factor*

Diameter (d) (mm)	K_b
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75

Example

Solution

Given $P = -50$ N to $+150$ N $S_{ut} = 600$ N/mm^2
$S_{yt} = 380$ N/mm^2 $R = 90\%$ $(fs) = 2$ $q = 0.9$

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$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

Table 5.3 *Reliability factor*

Reliability R (%)	K_c
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Example

Solution

Given $P = -50$ N to $+150$ N $S_{ut} = 600$ N/mm^2
$S_{yt} = 380$ N/mm^2 $R = 90\%$ $(fs) = 2$ $q = 0.9$

Step I *Endurance limit stress for cantilever beam*

$S'_e = 0.5 S_{ut} = 0.5\,(600) = 300$ N/mm^2

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$$K_a = 0.77$$

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$$K_b = 0.85$$

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Since, $\dfrac{r}{d} = 0.2$ and $\dfrac{D}{d} = 1.5$

From Fig. 5.5, $K_t = 1.44$

From Eq. (5.12),

$K_f = 1 + q\,(K_t - 1) = 1 + 0.9\,(1.44 - 1) = 1.396$

$$K_d = \frac{1}{K_f} = \frac{1}{1.396} = 0.716$$

$$S_e = K_a K_b K_c K_d S'_e$$
$$= 0.77\,(0.85)\,(0.897)\,(0.716)\,(300)$$
$$= 126.11 \text{ N/mm}^2$$

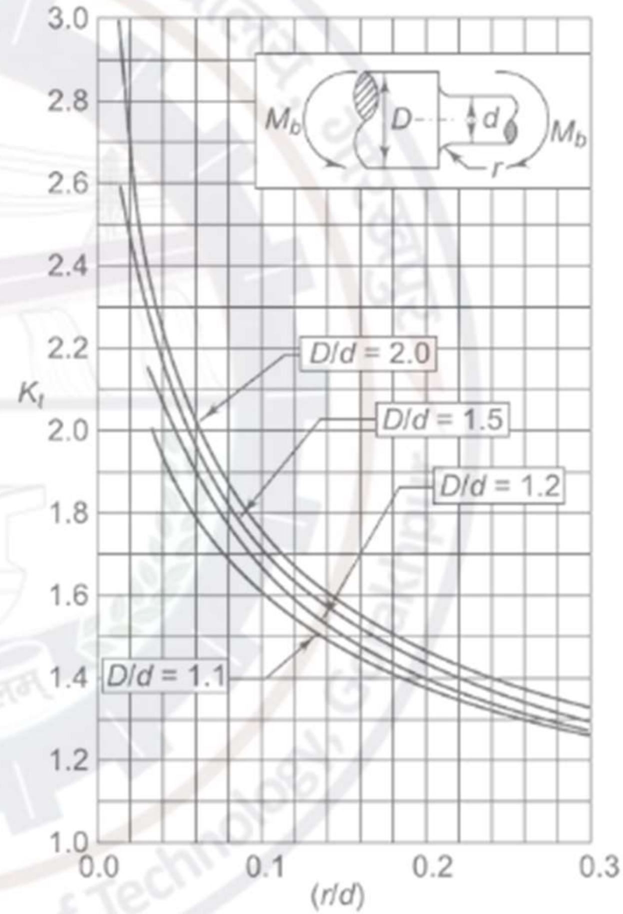


Fig. 5.5 *Stress Concentration Factor (Round Shaft with Shoulder Fillet in Bending)*

Example

Step II *Construction of modified Goodman diagram*

At the fillet cross-section,

$$(M_b)_{max.} = 150 \times 100 = 15000 \text{ N-mm}$$

$$(M_b)_{min.} = -50 \times 100 = -5000 \text{ N-mm}$$

$$(M_b)_m = \frac{1}{2}[(M_b)_{max.} + (M_b)_{min.}]$$

$$= \frac{1}{2}[15000 - 5000] = 5000 \text{ N-mm}$$

$$(M_b)_a = \frac{1}{2}[(M_b)_{max.} - (M_b)_{min.}]$$

$$= \frac{1}{2}[15000 + 5000] = 10\,000 \text{ N-mm}$$

$$\tan\theta = \frac{(M_b)_a}{(M_b)_m} = \frac{10000}{5000} = 2$$

$$\theta = 63.435°$$

The modified Goodman diagram for this example is shown in Fig. 5.43.

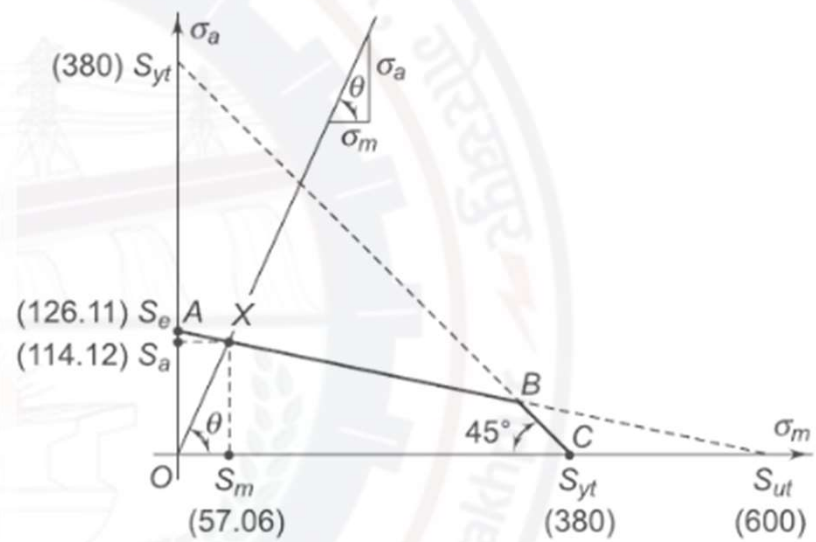


Fig. 5.43

Example

Step III *Permissible stress amplitude*

Refer to Fig. 5.43. The coordinates of the point X are determined by solving the following two equations simultaneously.

(i) Equation of line AB

$$\frac{S_a}{126.11} + \frac{S_m}{600} = 1 \qquad \text{(a)}$$

(ii) Equation of line OX

$$\frac{S_a}{S_m} = \tan\theta = 2 \qquad \text{(b)}$$

Solving the two equations,

$S_a = 114.12$ N/mm^2 and $S_m = 57.06$ N/mm^2

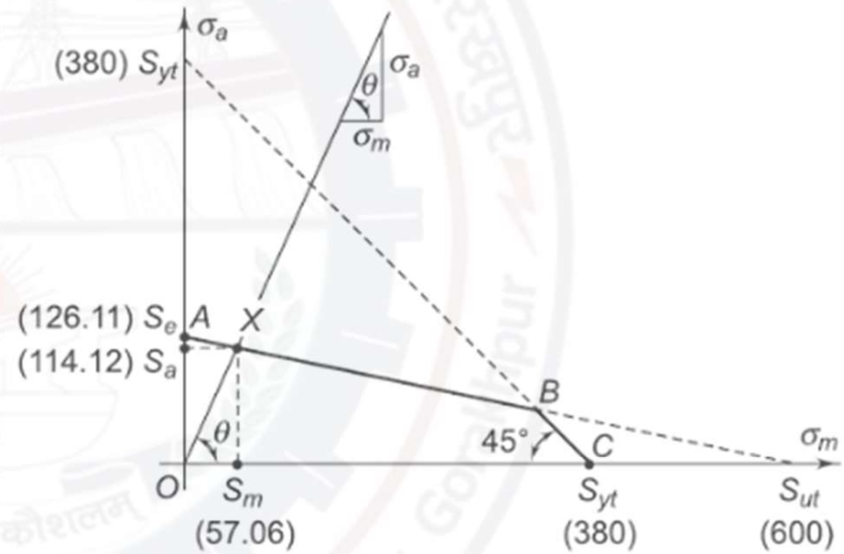


Fig. 5.43

Step IV *Diameter of beam*

Since $\sigma_a = \frac{S_a}{(fs)}$ $\quad\therefore\quad \frac{32(M_b)_a}{\pi d^3} = \frac{S_a}{(fs)}$

$$\frac{32(10\,000)}{\pi d^3} = \frac{114.12}{2}$$

$d = 12.13$ mm

Fatigue design under combined stresses

- The problems involving combination of stresses are solved by the distortion energy theory of failure.
- The general equation of distortion energy theory of failure

$$\sigma^2 = \frac{1}{2}[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]$$

In the case of 2-dimensional stresses, the components of stresses are σ_x and σ_y in X and Y directions

Substituting $\sigma_z = \tau_{xy} = \tau_{yz} = \tau_{zx} = 0$

$$\sigma = \sqrt{(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2)}$$

The mean and alternating stresses of σ_x are σ_{xm} and σ_{xa} and σ_y are σ_{ym} and σ_{ya}

The mean and alternating stresses are separately combined by

$$\sigma_m = \sqrt{(\sigma_{xm}^2 - \sigma_{xm}\sigma_{ym} + \sigma_{ym}^2)}$$

Similarly,

$$\sigma_a = \sqrt{(\sigma_{xa}^2 - \sigma_{xa}\sigma_{ya} + \sigma_{ya}^2)}$$

The two stresses σ_m and σ_a obtained by the above equations are used in the modified Goodman Diagram to design the component.

Fatigue design under combined stresses

In the case of combined bending and torsional moments, there is a normal σ_x and torsional shear stress τ_{xy}.

Substituting $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$

$$\sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

The mean and alternating components of σ_x are σ_{xm} and τ_{xy} and τ_{xy} are τ_{xym} and τ_{xya}. Combining these components separately

$$\sigma_m = \sqrt{\sigma_{xm}^2 + 3\tau_{xym}^2}$$

$$\sigma_a = \sqrt{\sigma_{xa}^2 + 3\tau_{xya}^2}$$

The two stresses σ_m and σ_a obtained by the above equations are used in the modified Goodman Diagram to design the component.

Impact stress

- Impact is defined as a collision of one component in motion with a second component, which may be either in motion or at rest.
- The stress induced in the machine component due to impact load is called impact stress.
- Example- driving a nail with a hammer, breaking a coconut.
- Hoisting ropes, hammers, spring, punches and shear, clutches and brakes

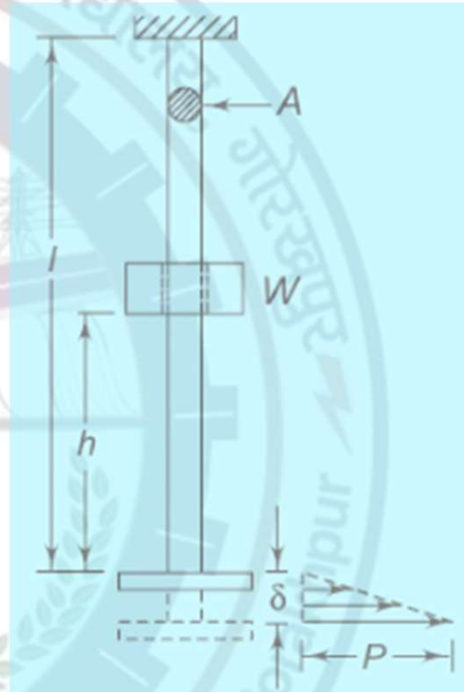
Impact stress

The weight W falls through the height h and strikes the collar of the bar. In this process, the potential energy released by the falling weight is absorbed by the bar and stored in the form of strain energy.

Energy released by falling weight = potential energy = $W(h + \delta)$

Energy absorbed by the system = strain energy = average load × deflection

$$= \left(\frac{1}{2}P\right)\delta$$



Equating the above two expressions,

$$\left(\frac{1}{2}P\right)\delta = W(h + \delta) \quad \text{..........(a)}$$

Also,

$$P = \sigma_i A \quad \text{..........(b)}$$

$$\frac{\delta}{l} = \varepsilon = \frac{\sigma_i}{E} \quad \text{or} \quad \delta = \frac{\sigma_i l}{E} \quad \text{..........(c)}$$

A = cross-sectional area of the bar (mm^2)
P = impact force which produces deflection δ (N)
E = modulus of elasticity of bar material (N/mm^2)
σ_i = impact stress in the bar (N/mm^2)

Impact stress

Substituting (b) and (c) in Eq. (a),

$$(\sigma_i)^2\left(\frac{Al}{2E}\right)-(\sigma_i)\left(\frac{Wl}{E}\right)-Wh=0$$

The above equation is a quadratic equation. Solving the equation and using positive sign for getting maximum value,

$$\sigma_i=\frac{W}{A}\left[1+\sqrt{1+\frac{2hAE}{Wl}}\right]$$

Substituting the above expression for impact stress in Eq. (b),

$$P=W\left[1+\sqrt{1+\frac{2hAE}{Wl}}\right]$$

or,

$$\frac{P}{W}=\left[1+\sqrt{1+\frac{2hAE}{Wl}}\right]$$

The quantity $\left(\frac{P}{W}\right)$ is called *shock factor*, which indicates the magnification of the load W into the impact force P during impact.

We will consider a special case when the weight is applied instantaneously without any initial velocity,

$$h=0$$

$$\sigma_i=2\left(\frac{W}{A}\right)$$

This means that the stress in the bar is double when the load is suddenly applied compared with a gradually applied load.

Summary

Types of Loading	**Load/stress**	**Solved by**
Static load (axial, bending, torsion)	Gradually applied load	Axial stress, bending stress, torsion stress
Static load (combined stresses)	Gradually applied load	Principal stresses and Theories of failure
Fluctuating load (reversed)	($\sigma_m = 0$)	S-N Curve (Infinite Life)
Fluctuating load (reversed)	($\sigma_m = 0$)	S-N Curve (Finite Life)
Fluctuating load (reversed)	Different stress levels for different cycles	Miner's Equation (Cumulative Damage)
Fluctuating load (axial, bending, torsion)	($\sigma_a \neq 0$, $\sigma_m \neq 0$)	Soderberg, Goodman, Gerber Modified Goodman Diagram
Fluctuating load (combined stresses)	($\sigma_a \neq 0$, $\sigma_m \neq 0$)	Distortion energy theory and Modified Goodman Diagram
Impact load	Suddenly applied load	Impact Stress

Learning objectives

- ✓ Understand the importance of stress concentration and the factors responsible for it.
- ✓ Determination of stress concentration factor.
- ✓ Different techniques to reduce stress concentration.
- ✓ Notch sensitivity factor and Fatigue strength reduction factors.
- ✓ Different types of fluctuating loads.
- ✓ Fatigue test.
- ✓ Difference between Endurance Strength and Endurance Limit.
- ✓ Design for finite and infinite life.
- ✓ Cumulative damage in fatigue
- ✓ Design of machine element subjected to fluctuating loads.
- ✓ Soderberg, Goodman and Gerber Criteria.
- ✓ Modified Goodman Diagram.
- ✓ Fatigue design under combined stresses.
- ✓ Impact stress

Any question?