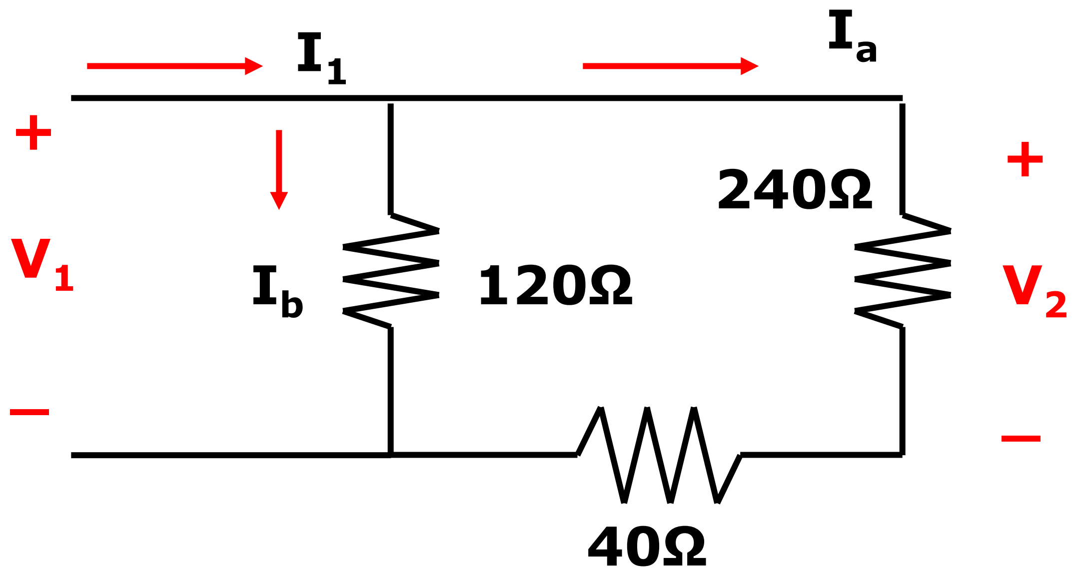
Solution

i) $I_2 = 0$(open circuit port 2). Redraw the circuit.



$$V_1 = 120 I_b \ldots\ldots(1)$$

$$I_b = \frac{280}{400} I_1 \ldots\ldots(2)$$

$$\text{sub } (1) \rightarrow (2)$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = 84\Omega$$

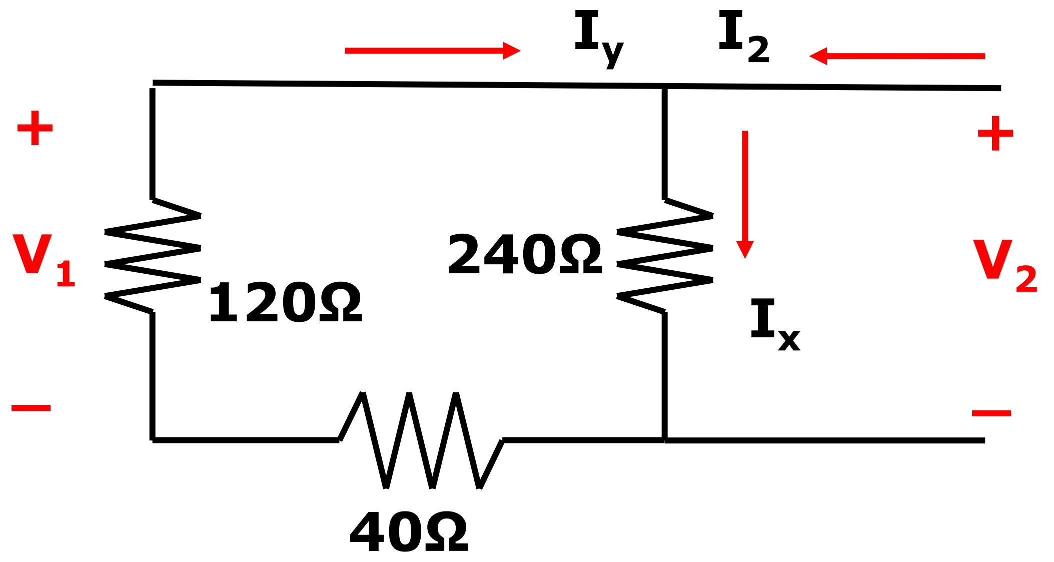
$$V_2 = 240 I_a \ldots\ldots(3)$$

$$I_a = \frac{120}{400} I_1 \ldots\ldots(4)$$

$$\text{sub } (4) \rightarrow (3)$$

$$\therefore Z_{21} = \frac{V_2}{I_1} = 72\Omega$$

ii) I_1 = 0 (open circuit port 1). Redraw the circuit.



$$V_2 = 240 I_x \ldots\ldots.(1)$$

$$I_x = \frac{160}{400} I_2 \ldots\ldots.(2)$$

sub $(1) \rightarrow (2)$

$$\therefore \mathrm{Z}_{22} = \frac{V_2}{I_2} = 96\Omega$$

$$V_1 = 120 I_y \ldots\ldots.(3)$$

$$I_y = \frac{240}{400} I_2 \ldots\ldots.(4)$$

sub $(4) \rightarrow (3)$

$$\therefore Z_{12} = \frac{V_1}{I_2} = 72\Omega$$

In matrix form:

$$[Z] = \begin{bmatrix} 84 & 72 \\ 72 & 96 \end{bmatrix}$$

EXAMPLE 13.1 Find the **Z**-parameters of the two-port circuit in Fig. 13-2.

Apply KVL around the two loops in Fig. 13-2 with loop currents $\mathbf{I}_1$ and $\mathbf{I}_2$ to obtain

$$\begin{aligned} \mathbf{V}_1 &= 2\mathbf{I}_1 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = (2 + \mathbf{s})\mathbf{I}_1 + \mathbf{s}\mathbf{I}_2 \\ \mathbf{V}_2 &= 3\mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{s}\mathbf{I}_1 + (3 + \mathbf{s})\mathbf{I}_2 \end{aligned} \tag{3}$$

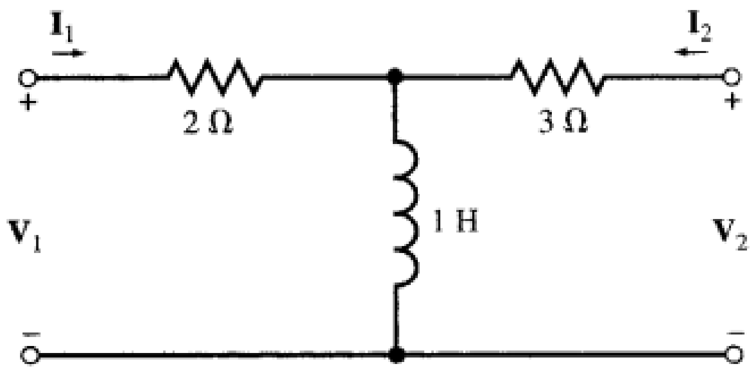


Fig. 13-2

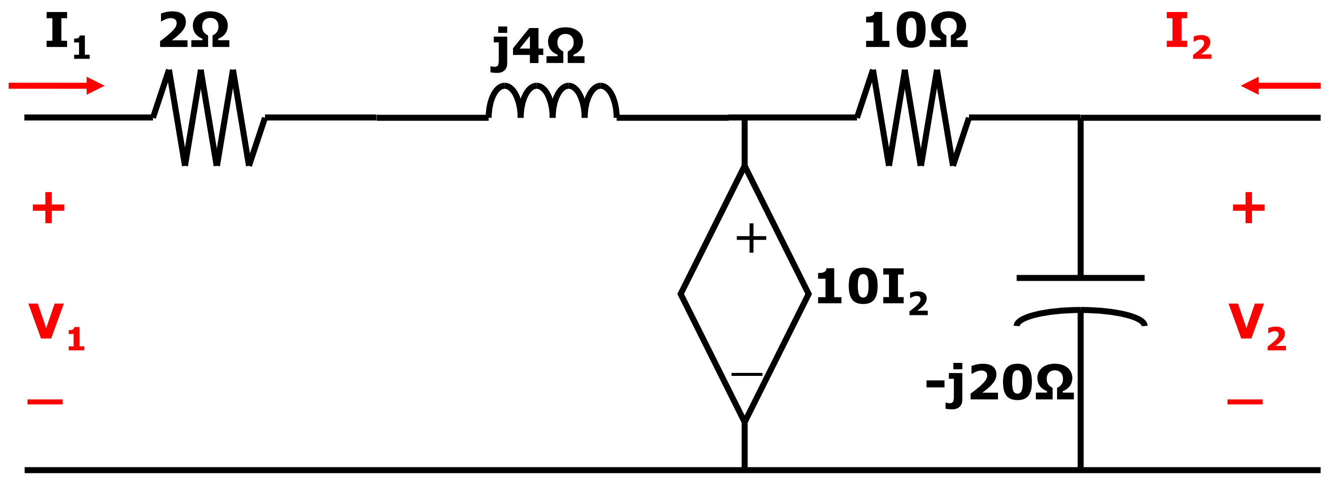
By comparing (*1*) and (*3*), the **Z**-parameters of the circuit are found to be

$$\begin{aligned} \mathbf{Z}_{11} &= \mathbf{s} + 2 \\ \mathbf{Z}_{12} &= \mathbf{Z}_{21} = \mathbf{s} \\ \mathbf{Z}_{22} &= \mathbf{s} + 3 \end{aligned} \tag{4}$$

Note that in this example $\mathbf{Z}_{12} = \mathbf{Z}_{21}$.

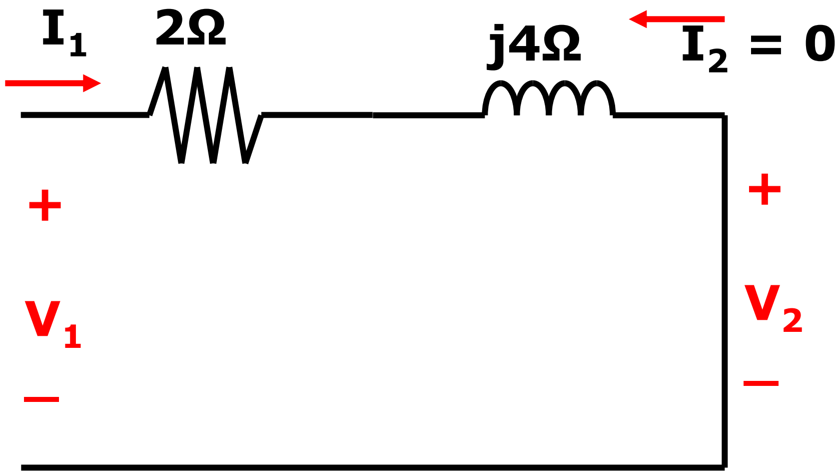
Example 2

Find the Z – parameter of the circuit below



Solution

i) $I_2 = 0$ (open circuit port 2). Redraw the circuit.



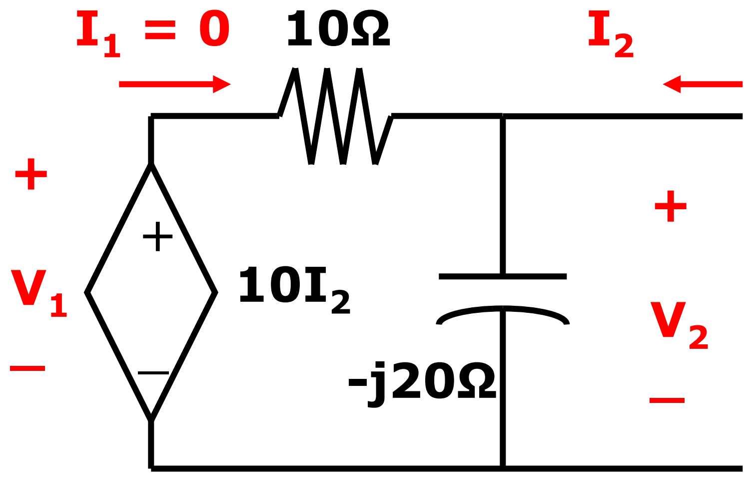
$$V_1 = I_1 (2 + j4)$$

$$\therefore Z_{11} = \frac{V_1}{I_1} = (2 + j4)\Omega$$

$$V_2 = 0 \text{ (short circuit)}$$

$$\therefore Z_{21} = 0\Omega$$

ii) $I_1 = 0$ (open circuit port 1). Redraw the circuit.



$$V_1 = 10I_2$$

$$\therefore Z_{12} = \frac{V_1}{I_2} = 10\Omega$$

$$I_2 = \frac{V_2}{-j20} + \frac{V_2 - 10I_2}{10}$$

$$2I_2 = V_2\left(\frac{j}{20} + \frac{1}{10}\right)$$

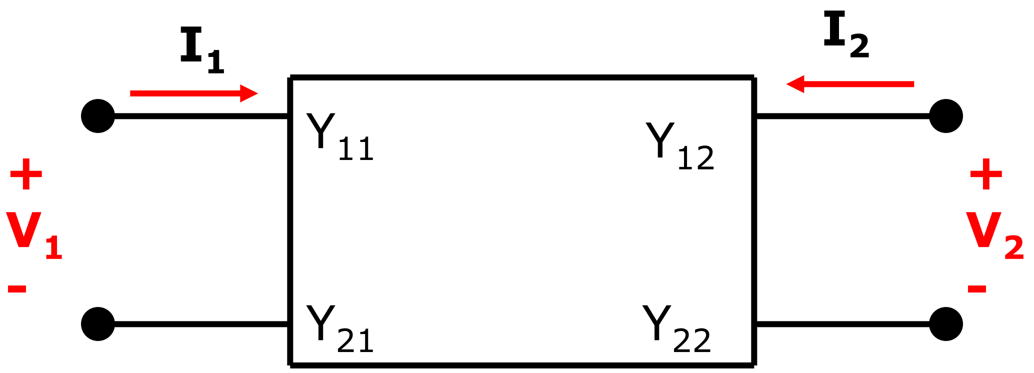
$$\therefore Z_{22} = \frac{V_2}{I_2} = (16 - j8)\ \Omega$$

In matrix form;

$$[Z] = \begin{bmatrix} (2 + j4) & 0 \\ 10 & (16 - j8) \end{bmatrix}$$

Y - PARAMETER

- Y – parameter also called admittance parameter and the units is siemens (S).
- The “black box” that we want to replace with the Y-parameter is shown below.



- The terminal current can be expressed in term of terminal voltage as:

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad \text{——— } \mathbf{(1)}$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad \text{——— } \mathbf{(2)}$$

- In matrix form:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$