Control Systems

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Unit-II

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Lecture 1

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State space analysis

State space analysis is an excellent method for the design and analysis of control systems. The conventional and old method for the design and analysis of control systems is the transfer function method. The transfer function method for design and analysis had many drawbacks.

Advantages of state variable analysis.

- It can be applied to non linear system.
- It can be applied to tile invariant systems.
- It can be applied to multiple input multiple output systems.
- Its gives idea about the internal state of the system.

State Variable Analysis and Design

State: The state of a dynamic system is the smallest set of variables called state variables such that the knowledge of these variables at time $t=t_o$(Initial condition), together with the knowledge of input for $\geq t_0$, completely determines the behaviour of the system for any time $t \geq t_0$.

State vector: If n state variables are needed to completely describe the behaviour of a given system, then these n state variables can be considered the n components of a vector X. Such a vector is called a state vector.

State space: The n-dimensional space whose co-ordinate axes consists of the x_1 axis, x_2 axis,.... x_n axis, where x_1, x_2,..... x_n are state variables: is called a state space.

STATE VARIABLE MODELS

- We consider physical sytems described by nth-order ordinary differential equation. Utilizing a set of variables, known as state variables, we can obtain a set of first-order differential equations. We group these first-order equations using a compact matrix notation in a model known as the state variable model.

- The time-domain state variable model lends itself readily to computer solution and analysis. The Laplace transform is utilized to transform the differential equations representing the system to an algebraic equation expressed in terms of the complex variable ***s***. Utilizing this algebraic equation, we are able to obtain a transfer function representation of the input-output relationship.

- With the ready availability of digital computers, it is convenient to consider the time-domain formulation of the equations representing control system. The time domain techniques can be utilized for nonlinear, time varying, and multivariable systems.

A time-varying control system is a system for which one or more of the parameters of the system may vary as a function of time.

For example, the mass of a missile varies as a function of time as the fuel is expended during flight. A multivariable system is a system with several input and output.

The State Variables of a Dynamic System:

The time-domain analysis and design of control systems utilizes the concept of the state of a system.

The state of a system is a set of variables such that the knowledge of these variables and the input functions will, with the equations describing the dynamics, provide the future state and output of the system.

State Model

Lets consider a multi input & multi output system is having

r inputs $u_1 t, u_2 t, \ldots \ldots u_r(t)$

m no of outputs $y_1 t, y_2 t, \ldots \ldots . y_m(t)$

n no of state variables $x_1 t, x_2 t, \ldots \ldots . x_n(t)$

Then the state model is given by state & output equation

$$\dot{X}(t) = AX(t) + BU(t) \ldots \ldots \ldots \ldots \text{state equation}$$

$$Y(t) = CX(t) + DU(t) \ldots \ldots \ldots \text{output equation}$$

A is state matrix of size (n×n)

B is the input matrix of size (n×r)

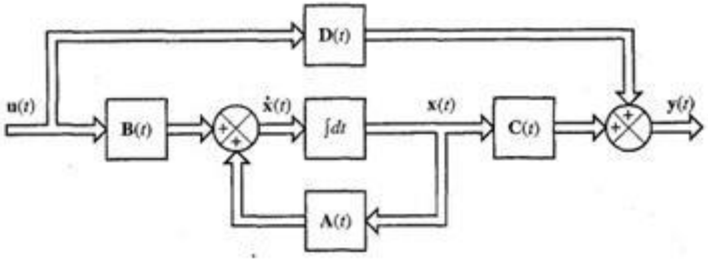
C is the output matrix of size (m×n)

D is the direct transmission matrix of size (m×r)

X(t) is the state vector of size (n×1)

Y(t) is the output vector of size (m×1)

U(t) is the input vector of size (r×1)



(Block diagram of the linear, continuous time control system represented in state space)

$$\dot{\mathbf{X}}(\mathbf{t}) = \mathbf{AX}(\mathbf{t}) + \mathbf{Bu}(\mathbf{t})$$

$$\mathbf{Y}(\mathbf{t}) = \mathbf{CX}(\mathbf{t}) + \mathbf{Du}(\mathbf{t})$$

For a dynamic system, the state of a system is described in terms of a set of state variables

$$[x_1(t) \quad x_2(t) \quad \ldots \quad x_n(t)]$$

The state variables are those variables that determine the future behavior of a system when the present state of the system and the excitation signals are known. Consider the system shown in Figure 1, where $y_1(t)$ and $y_2(t)$ are the output signals and $u_1(t)$ and $u_2(t)$ are the input signals. A set of state variables $[x_1 \ x_2 \ \ldots \ x_n]$ for the system shown in the figure is a set such that knowledge of the initial values of the state variables $[x_1(t_0) \ x_2(t_0) \ \ldots \ x_n(t_0)]$ at the initial time t_0, and of the input signals $u_1(t)$ and $u_2(t)$ for $t>=t_0$, suffices to determine the future values of the outputs and state variables.

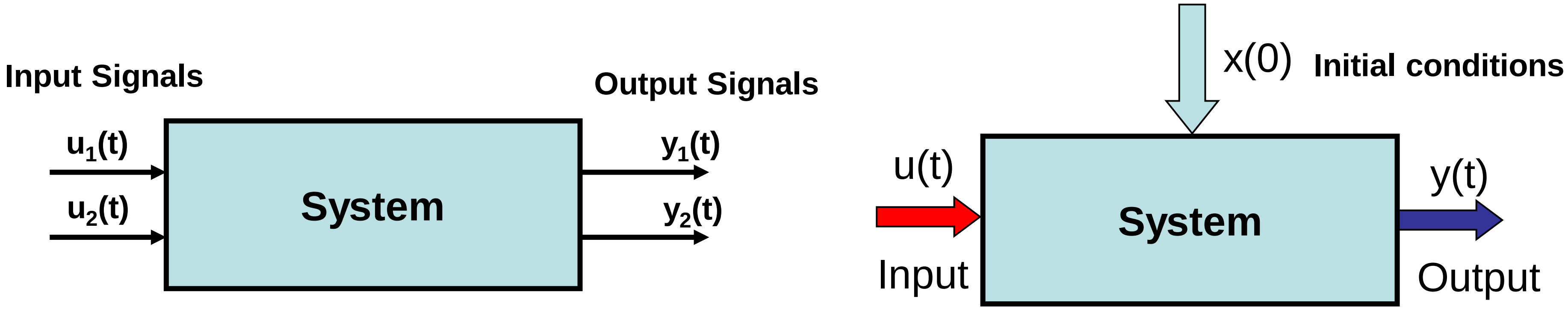


Figure 1. Dynamic system.

The state variables describe the future response of a system, given the present state, the excitation inputs, and the equations describing the dynamics.

A simple example of a state variable is the state of an on-off light switch. The switch can be in either the on or the off position, and thus the state of the switch can assume one of two possible values. Thus, if we know the present state (position) of the switch at t_0 and if an input is applied, we are able to determine the future value of the state of the element.

The concept of a set of state variables that represent a dynamic system can be illustrated in terms of the spring-mass-damper system shown in Figure 2. The number of state variables chosen to represent this system should be as small as possible in order to avoid redundant state variables. A set of state variables sufficient to describe this system includes the position and the velocity of the mass.

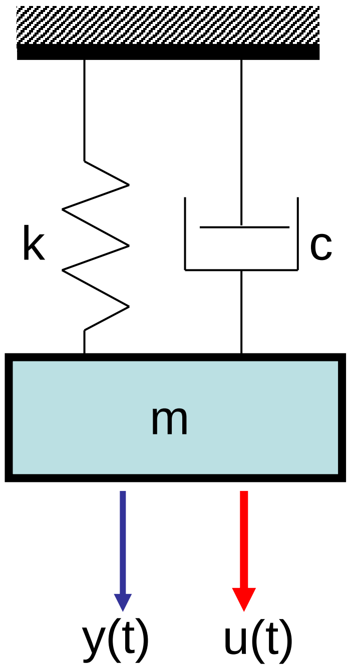


Figure 2. 1-dof system.