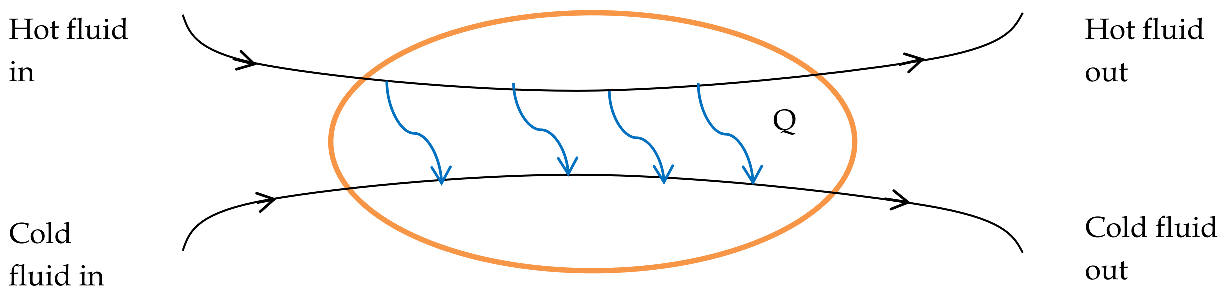
Heat Exchangers

Introduction

Heat Exchanger is an adiabatic steady flow device in which two flowing fluids exchange or transfer heat between themselves due to a temperature difference without losing or gaining any heat from the ambient atmosphere.



Heat Exchanger (Insulated device)

Some examples:

Heat Exchangers	Heat exchange occurs between
Steam condenser	Steam ⟶ Cooling water
Economiser	Flue gases ⟶ Feed water
Superheater	Flue gases ⟶ Saturated vapor
Cooling tower	Hot water ⟶ Atmospheric air
Air preheater	Flue gases ⟶ Combustion air

Classification of heat Exchangers

1. Direct transfer type heat exchanger

2. Direct contact type heat exchanger

3. Regenerative type heat exchanger

1. **Direct transfer type heat exchanger :-** In direct type heat exchanger both the fluids could not come into contact with each other but the transfer of heat occurs through the pipe wall of separation.

Examples:-

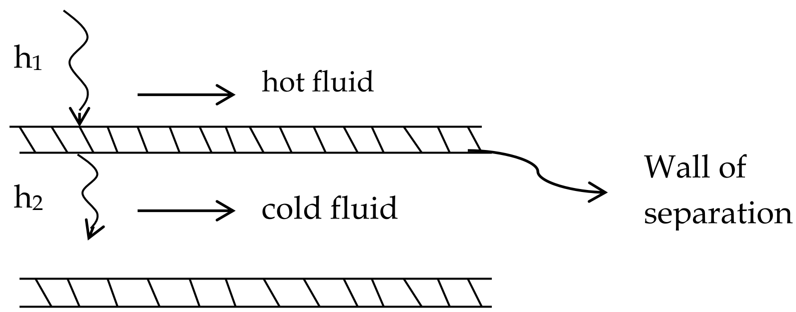
1. Concentric type heat exchanger

2. Economiser

3. Super heater

4. Double pipe heat exchanger

5. Pipe in pipe heat exchanger



h_1 = heat transfer rate on hot side in $W/m^2 - K$

h_2 = heat transfer rate on cold side in $W/m^2 - K$

2. **Direct Contact type heat exchanger :-** In direct contact type heat exchanger, the working fluids come in direct contact in order to exchange heat between each other. These type of exchangers are utilized when the mixing of two fluids is either harmless or desirable.

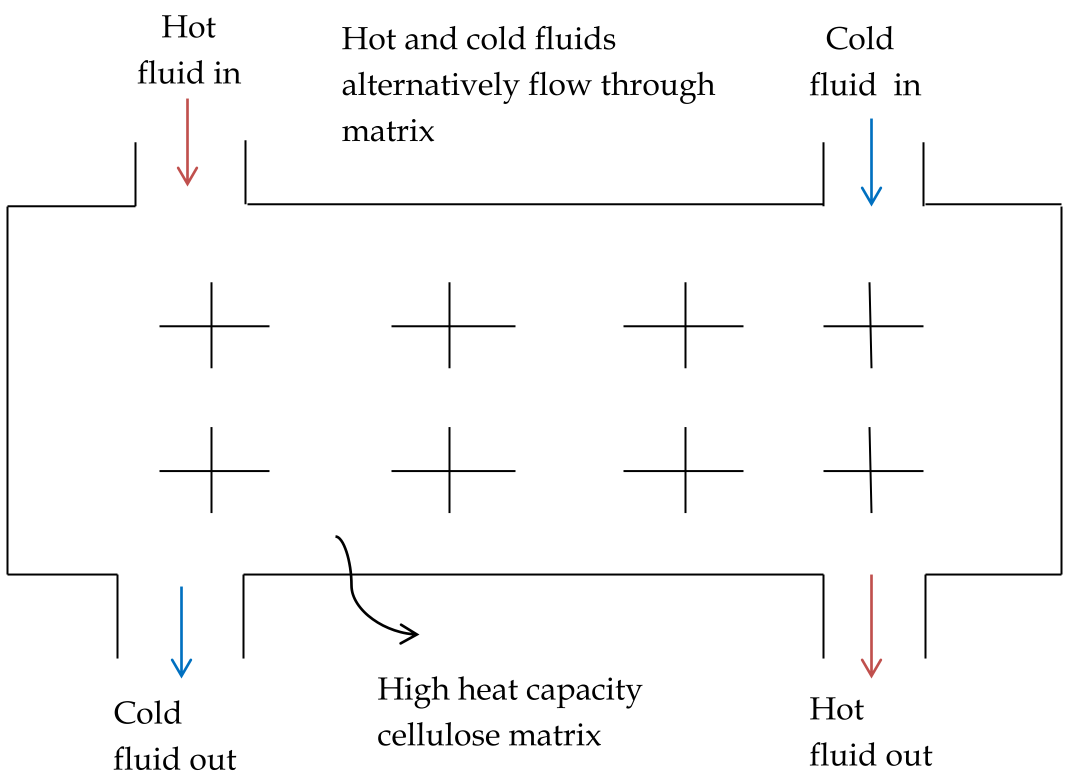
Examples :-

1. Cooling tower

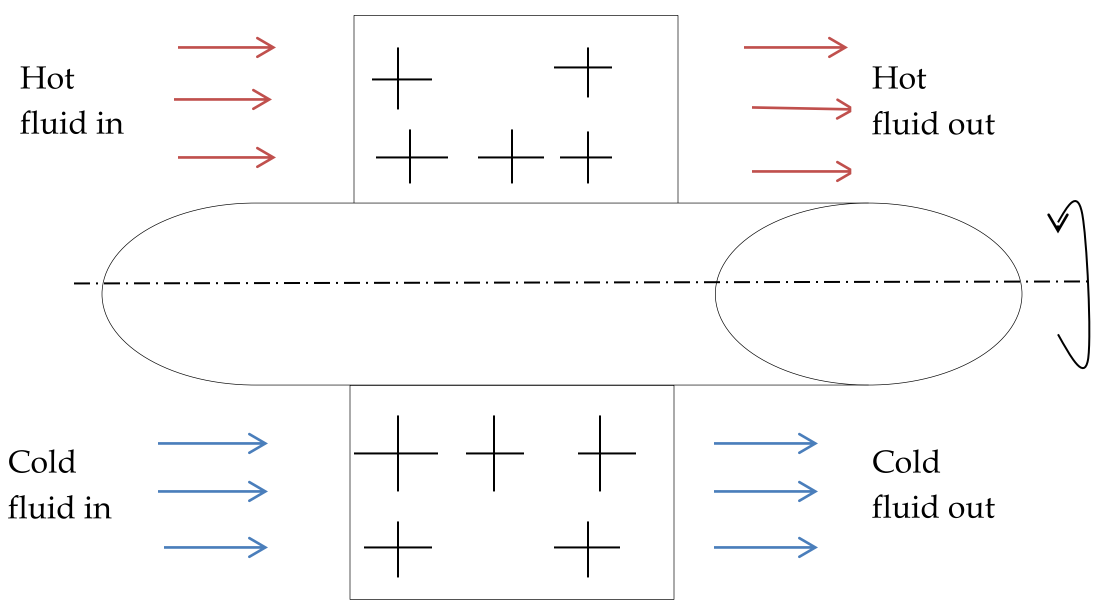
2. Jet Condenser

3. **Regenerative type heat exchanger :-** In this type of heat exchanger , hot and cold fluids alternatively pass through the high heat capacity material , one giving the heat to the material and the other picking up heat from it.

Example :- Ljungstorm air preheater use in gas turbine power plants



There can be a **rotating matrix type regenerative** heat exchanger which is shown below:



Classification of direct transfer type heat exchanger

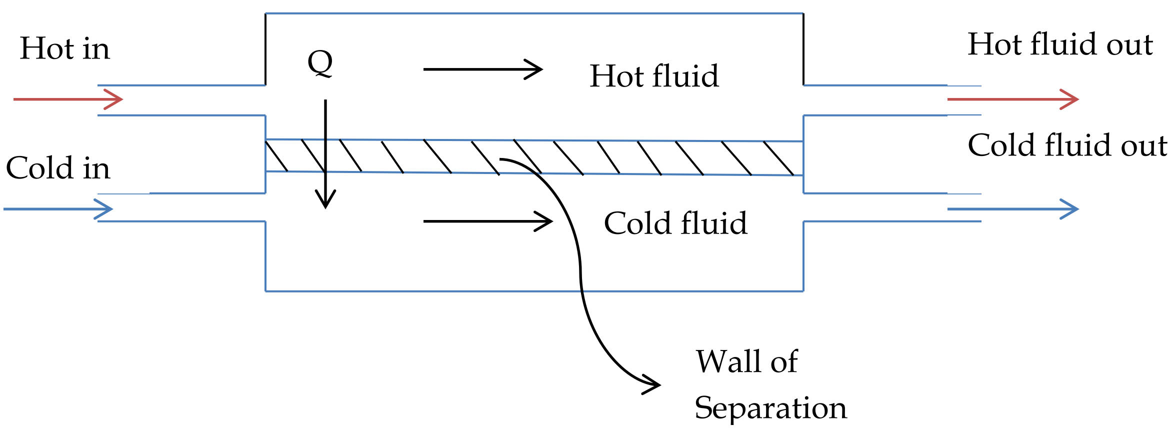
1. Parallel flow heat exchanger

2. Counter flow heat exchanger

3. Cross flow heat exchanger

1. Parallel flow heat exchanger

In this type of heat exchanger, both the fluids flow in same direction.

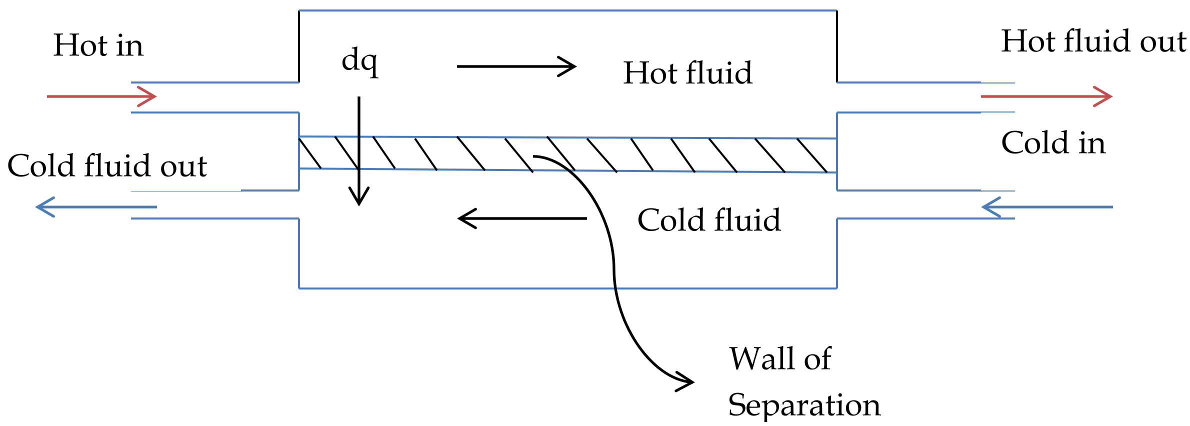


Q = heat transfer rate between hot and cold fluid (vector quantity)

(top to bottom heat transfer in above case)

2. Counter flow heat exchanger

In this type of heat exchanger, both the fluids flow in opposite direction.

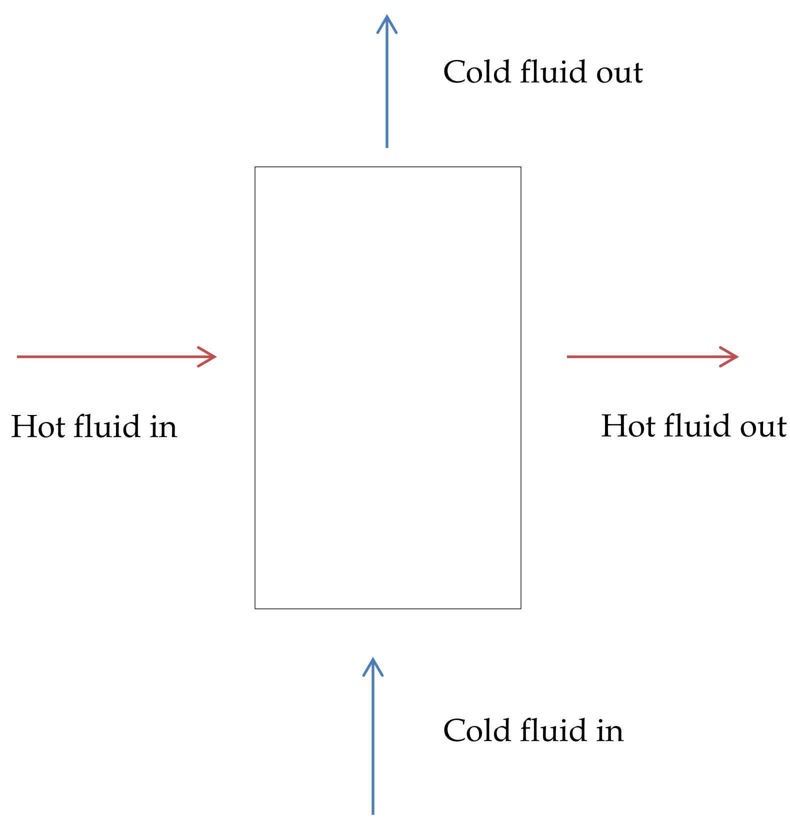


Note: Wall of separation should possess high thermal conductivity (K) values

3. Cross flow heat exchanger

In this type of heat exchanger, both the fluids flow in perpendicular direction with respect to each other.

Example:- Automobile radiator



First law of thermodynamics applied to heat exchanger

Heat exchanger is a steady flow adiabatic device

According to Steady flow energy equation

$Q' - \cancel{W} = \Delta H + \cancel{\Delta KE} + \cancel{\Delta PE}$

$(\Delta H)_{HE} = 0$

Where

Q' = heat transfer between heat exchanger and surroundings = 0

W = work done in heat exchanger

ΔH = net change in enthalpy (within heat exchanger)

ΔKE = Change in kinetic energy

ΔPE = Change in potential energy

$(\Delta H)_{HE} = 0$

$(\Delta H)_{hot\ fluid} + (\Delta H)_{cold\ fluid} = 0$

$\mathbf{-(\Delta H)_{hot\ fluid} = (\Delta H)_{cold\ fluid}}$

(negative sign shows that enthalpy of hot fluid is decreasing)

Rate of enthalpy decrease of hot fluid = Rate of enthalpy increase of cold fluid

Hence, energy balance equation or heat balance equation is given by

$$\mathbf{m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci}) \quad watts} \qquad \text{——— (1)}$$

where , m_h = mass flow rate of hot fluid in kg/sec

C_{ph} = specific heat capacity of hot fluid in J/kg-kelvin

m_c = mass flow rate of cold fluid in kg/sec

C_{pc}= specific heat capacity of cold fluid in J/kg-kelvin

T_{hi} = inlet temperature of hot fluid in kelvin

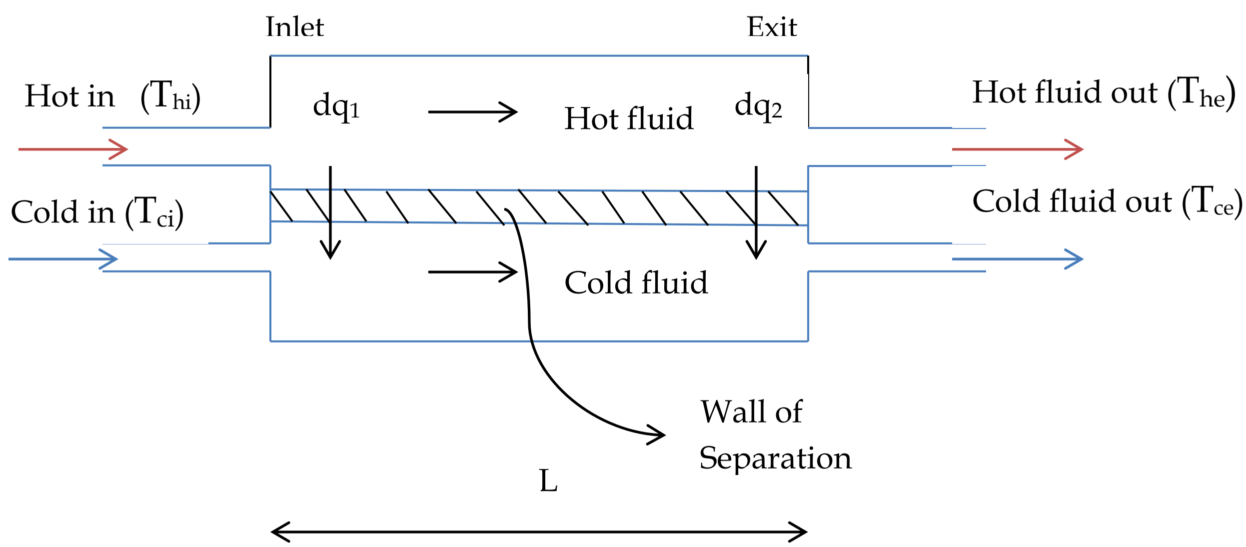
T_{he} = exit temperature of hot fluid in kelvin

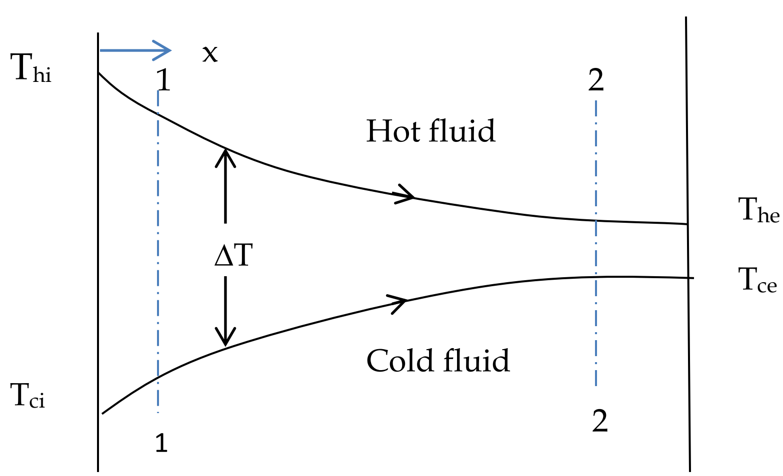
T_{ci} = inlet temperature of cold fluid in kelvin

T_{ce} = exit temperature of cold fluid in kelvin

Temperature profile of fluids in heat exchanger

For parallel flow





The differential heat transfer rate dq between hot and cold fluids varies with x i.e from inlet to exit because ΔT (the temperature difference between hot and cold fluids changes from one location to another location of the heat exchanger.

The differential heat transfer dq_1 at section 1-1 of the heat exchanger will be more as compared to differential heat transfer dq_2 at section 2-2 because ΔT at section 1-1 is more than that in section 2-2.

Differential heat transfer is given by

$$\mathbf{dq = U\,\Delta T\,dA} \qquad (2)$$

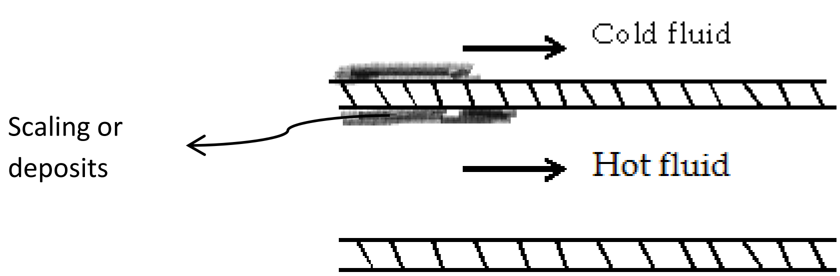
where U = overall heat transfer coefficient in Watt/m^2 –K

$$\frac{1}{U_{clean}} = \frac{1}{h_1} + \frac{1}{h_2} \qquad \text{(without scaling of the surface of pipe)}$$

dA = differential area of heat exchanger

$\Delta T = T_h - T_c$

Fouling factor: It is the factor which takes in to account the thermal resistance offered by any scaling or deposit that is formed on the surface of the pipe either on hot side or cold side.



Its unit is m^2 - K/watts

With fouling, U may be obtained from

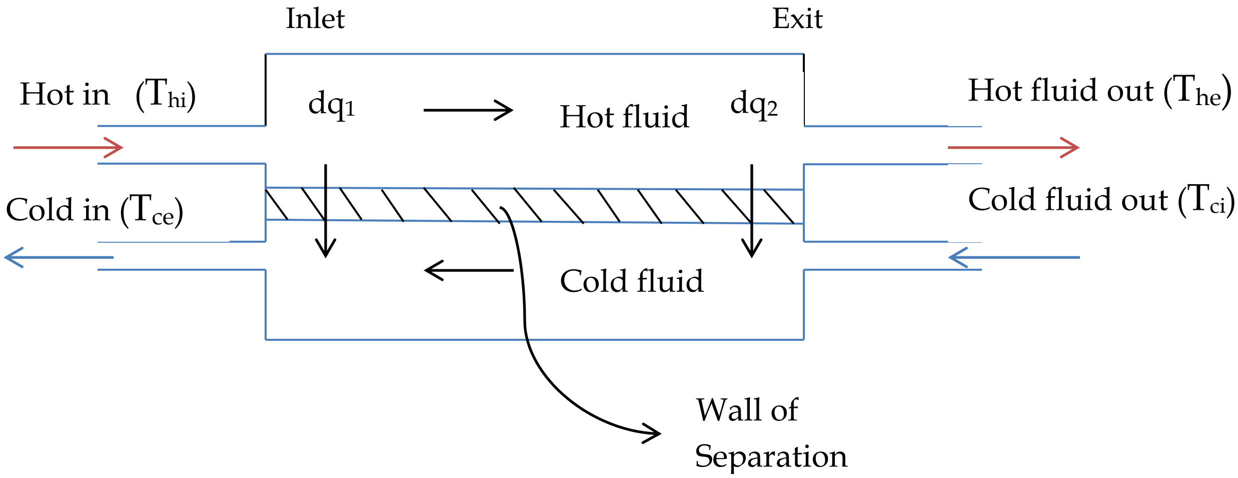
$$\frac{1}{U_{dirt}} = \frac{1}{h_1} + \frac{1}{h_2} + F_1 + F_2$$

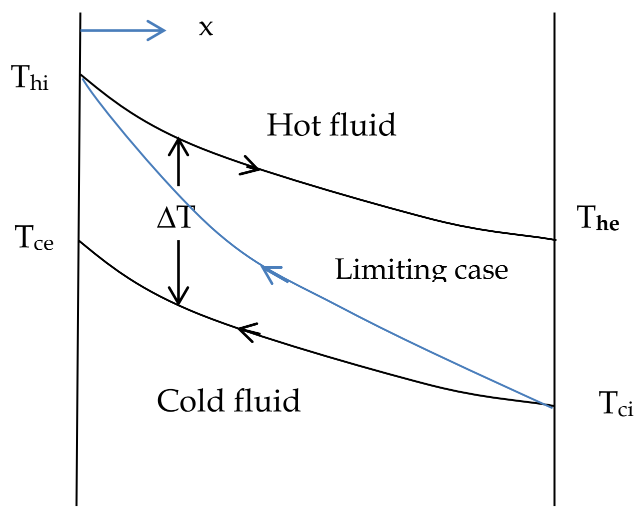
F_1: Fouling on hot side

F_2: Fouling on cold side

$$\frac{1}{U_{dirt}} = \frac{1}{U_{clean}} + F_1 + F_2$$

For Counter flow





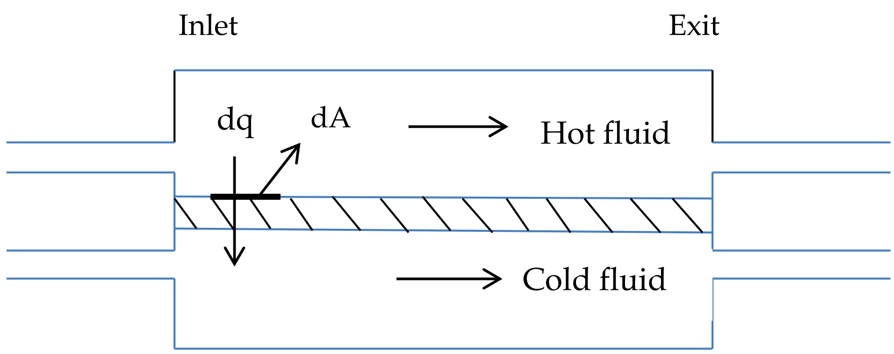
Limiting case:

T_{ce} can be greater than T_{he} in counter flow heat exchanger only (when infinite heat exchanger area is provided).

Note: The variation of ΔT with respect to x is less pronounced in counter flow heat exchanger as compared to that in parallel flow heat exchanger. Hence the heat transfer in counter flow heat exchanger is having lesser irreversibility associated with it as compare to that in parallel flow heat exchanger. Therefore, thermodynamically counter flow heat exchanger is more effective than parallel flow heat exchanger.

Hence for the same heat transfer area provided in both the heat exchangers, counter flow heat exchanger can have higher heat transfer rates than parallel flow heat exchanger.

Mean temperature difference



From equation (2)

$dq = U\,\Delta T\,dA$

Total heat transfer rate in heat exchanger,

$$Q = \int_{inlet}^{Exit} U\,\Delta T\,dA \qquad (3)$$

Mean temperature difference is the parameter which takes in to account the variation of ΔT with respect to x and hence averaging it from inlet to exit and defined from the equation

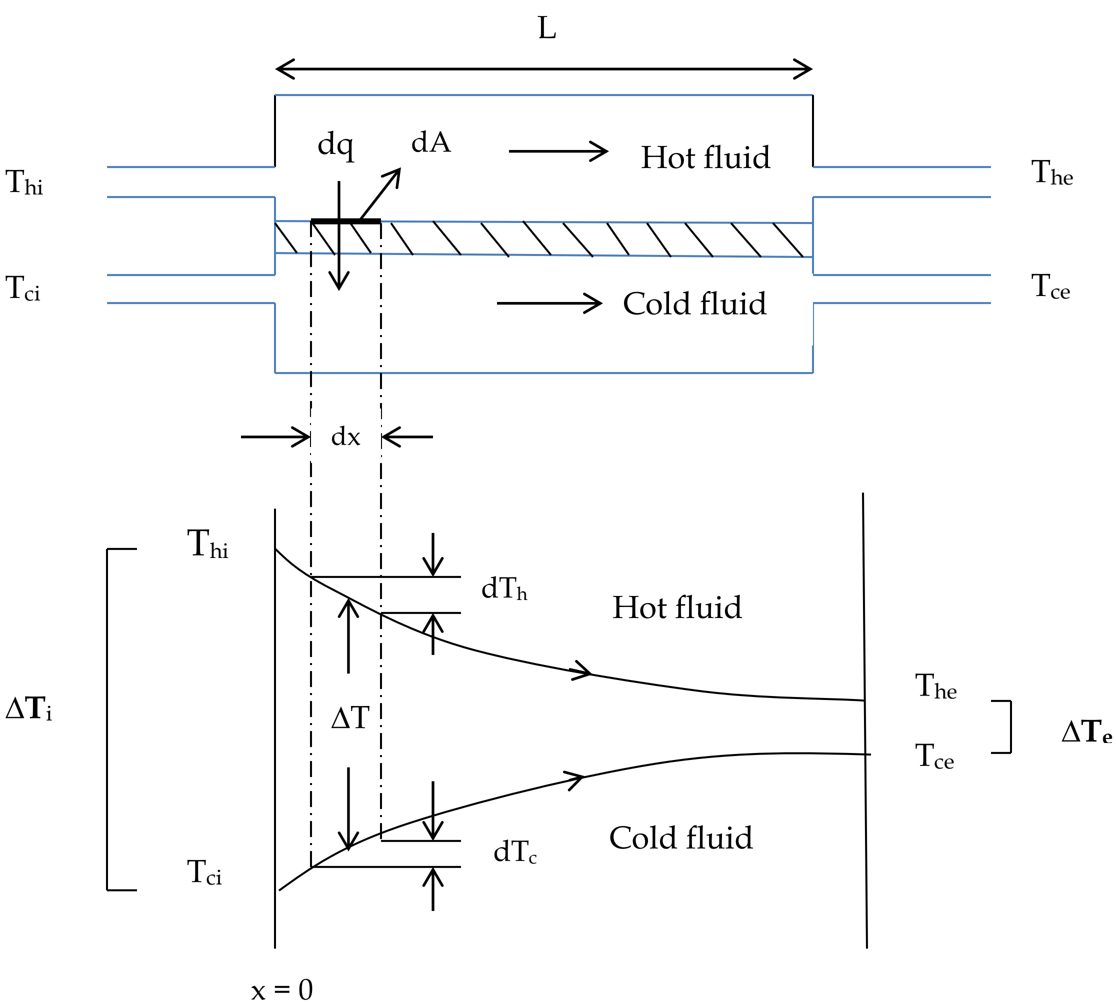
$$Q = U\,A\,\Delta T_m \qquad (4)$$

A = total area of heat transfer in the heat exchanger

Comparing eqation (3) and (4) and treating U as a constant, we get

$$\Delta T_m = \frac{1}{A}\int_{inlet}^{Exit} \Delta T\,dA$$

To derive for Mean temperature difference of a parallel flow heat exchanger



Let B = width of plate perpendicular to plane of figure

m_h = mass flow rate of hot fluid

C_{ph} = specific heat capacity of hot fluid

m_c = mass flow rate of cold fluid

C_{pc} = specific heat capacity of cold fluid

Consider a differential area of the heat exchanger of length dx through which the differential heat transfer rate between hot and cold fluids is dq.

Then $dA = Bdx$

$dq = U\Delta T\, B\, dx$

$dq = -\, m_h C_{ph}\, dT_h$

$= +\, m_c C_{pc}\, dT_c$

Let $\Delta T = f(x)$

At $x = 0$, $\Delta T = \Delta T_i = T_{hi} - Tc_i$

At $x = L$, $\Delta T = \Delta T_e = T_{he} - Tc_e$

$\Delta T = T_h - Tc$

$d(\Delta T) = d(T_h) - d(Tc)$

$$= \frac{-\,dq}{m_h C_{ph}} - \frac{dq}{m_c C_{pc}}$$

$$-d(\Delta T) = dq \left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right)$$

$$-d(\Delta T) = U\Delta T\, B\, dx \left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right)$$

$$\int_{\Delta T_i}^{\Delta T_e} \frac{-d\Delta T}{\Delta T} = \int_0^L U\, B \left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right)$$

$$\ln \frac{\Delta T_i}{\Delta T_e} = UBL \left(\frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right)$$

$Q = m_h C_{ph} (T_{hi} - T_{he})$

$= m_c C_{pc} (T_{ce} - T_{ci})$

$$\ln \frac{\Delta T_i}{\Delta T_e} = U\, A \left[\frac{T_{hi} - T_{he}}{Q} + \frac{T_{ce} - T_{ci}}{Q} \right]$$

$$= \frac{U\,A}{Q}\,[\Delta T_i - \Delta T_e]$$

$$Q = \frac{UA\,[\Delta T_i - \Delta T_e]}{\ln\frac{\Delta T_i}{\Delta T_e}} \qquad \text{———————} \qquad (5)$$

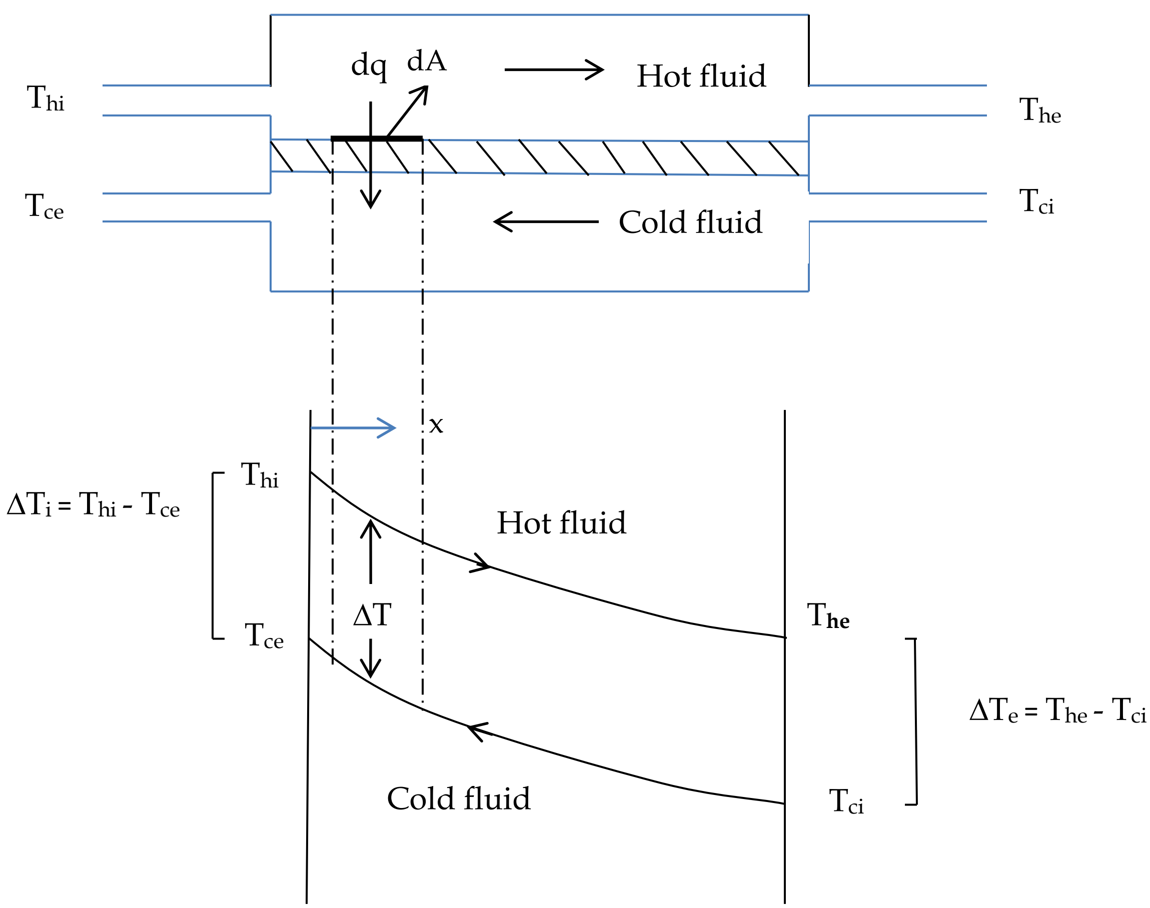
On comparing equation (4) and (5)

$Q = U\,A\,\Delta T_m$

$$\boldsymbol{\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\frac{\Delta T_i}{\Delta T_e}}} \qquad \text{———————} \qquad \textbf{(6)}$$

Equation (6) gives logarithmic mean temperature difference (LMTD) for parallel flow heat exchanger.

Mean temperature difference of a counter flow heat exchanger



Similarly, for counter flow heat exchanger we can derive

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\frac{\Delta T_i}{\Delta T_e}} \qquad (7)$$

Equation (7) gives logarithmic mean temperature difference (LMTD) for counter flow heat exchanger.

Note:

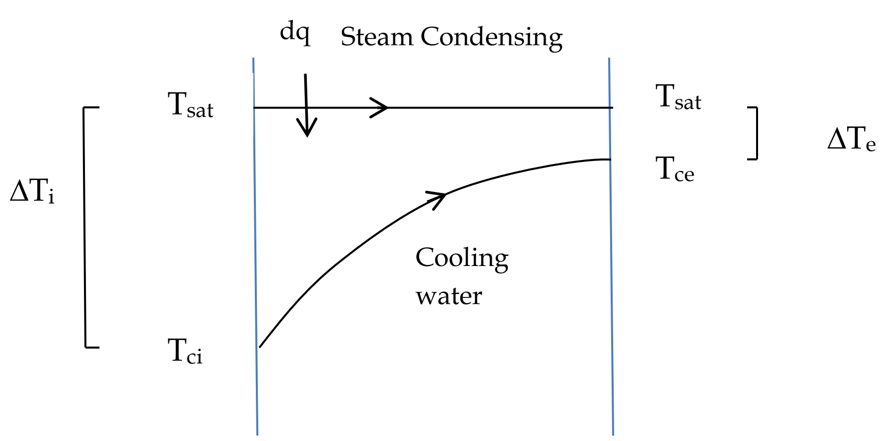
1. Even though the equations of LMTD is same in both parallel flow and counter flow heat exchangers the definitions of ΔT_i and ΔT_e are different for both of them.

2. ΔT_e may be more than ΔT_i in counter flow heat exchanger only.

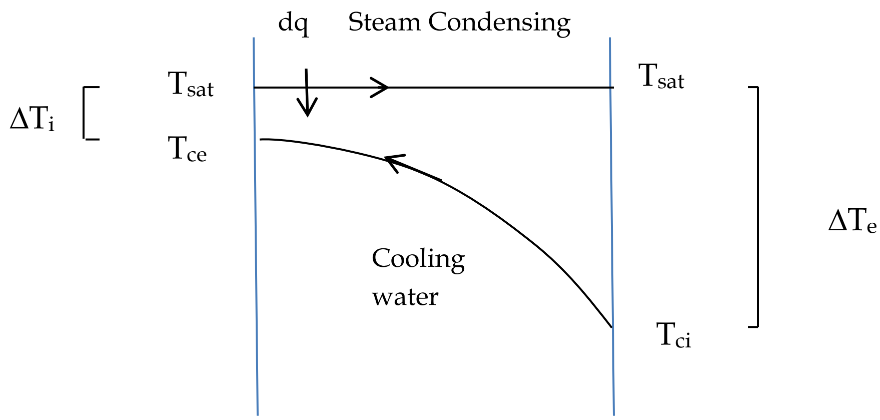
Special Cases of LMTD

1. If one of the fluids undergoing phase change like in steam condensers or evaporators or boiler.

(a) Steam Condenser



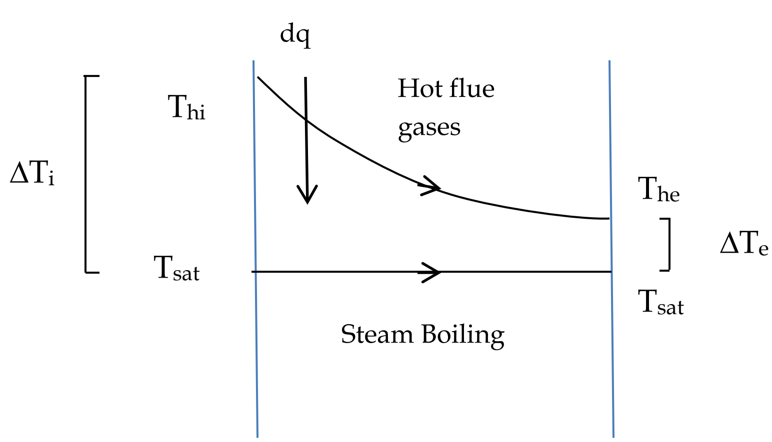
Parallel flow



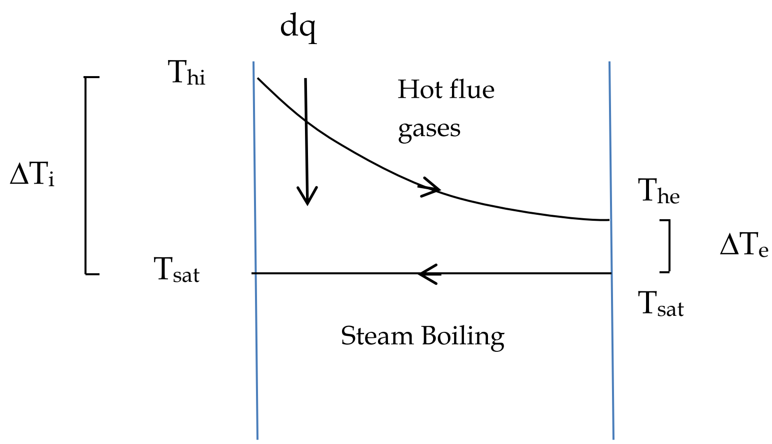
Counter flow

$$(\Delta T_m)_{\text{parallel flow}} = (\Delta T_m)_{\text{counter flow}}$$

(b) Boiler



Parallel flow



Counter flow

$$(\Delta T_m)_{\text{parallel flow}} = (\Delta T_m)_{\text{counter flow}}$$

Note:

(i) Whenever the change of phase occur the temperature of fluid does not change.

(ii) When one of the fluids is undergoing phase change, it does not matter what kind of heat exchanger is to be designed because LMTD value is same in both the cases.

2. When both the fluids have equal capacity rates (i.e. $m_h Cph = m_c C_{pc}$) in counter flow heat exchanger then from energy balance equation

$$m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci})$$

$$T_{hi} - T_{ce} = T_{he} - T_{ci}$$

$$\Delta T_i = \Delta T_e$$

Then $(LMTD)_{\text{counter flow}} = \dfrac{\Delta T_i - \Delta T_e}{\ln \dfrac{\Delta T_i}{\Delta T_e}}$

$$= \frac{0}{0}$$

From L hospital's rule

$(\Delta T_m)_{\text{counter flow}}$ = either ΔT_i or ΔT_e

$(\Delta T_m)_{\text{counter flow}} = T_{hi} - T_{ce}$ or $T_{he} - T_{ci}$

Design of Heat Exchangers

In any design of heat exchangers first it is required to find the area of the heat exchanger then we could find length of heat exchanger, diameter of each tube or the number of tubes required.

1. To find area of heat exchanger (A) (LMTD Method)

Given data :-

1. Both the mass flow rate of the hot and cold fluids (m_h and m_c).

2. Both the specific heat capacity of fluids (C_{ph} and C_{pc}).

3. Overall heat transfer coefficient (U).

4. Only three temperature among 4 temperature like T_{hi}, T_{ci}, T_{he}.

Solution :-

1. Find 4th unknown temperature from energy balance equation

$$m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci})$$

2. Draw the temperature profiles of fluids based on what type of heat exchanger is to be designed.

3. Obtain LMTD

4. Calculate heat transfer rate between hot and cold fluids

$$Q = m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci})$$

5. Obtain Area of the heat exchanger

$$A = \frac{Q}{U \Delta T_m}$$

Note:

For the same hot and cold fluids and for the same mass flow rate of both the fluids and for the same inlet and exit temperature of fluids, LMTD value for counter flow heat exchanger shall be more than that of parallel flow heat exchanger i.e. for the same heat transfer rate required the area of counter flow heat exchanger shall be lesser than parallel flow heat exchanger.

Effectiveness of heat exchanger

Effectiveness of a heat exchanger is defined as the ration between actual heat transfer rate between hot and cold fluids and the maximum possible heat transfer rate between them. It is denoted by ε.

$$\boldsymbol{\varepsilon = \frac{Q_{act}}{Q_{max}}}$$

Where,

$Q_{act} = m_h C_{ph} (T_{hi} - T_{he}) = m_c C_{pc} (T_{ce} - T_{ci})$

If fluid is condensing, then

$Q_{act} = m_{steam} (\Delta h)$

$h = h_f + x\, h_{fg}$

x is dryness fraction of steam

Q_{max} = Maximum possible heat transfer rate

$= (mC_p)_{small} \times (T_{hi} - T_{ci})$

$(mC_p)_{small}$ is the smaller capacity rate between $m_h C_{ph}$ and $m_c C_{pc}$.

Capacity rate ratio

It is defined as the ratio of smaller capacity rate to the bigger capacity rate .It is denoted by 'C'.

$$C = \frac{(m\,C_p)_{smaller}}{(m\,C_p)_{big}}$$

The value of C varies between 0 and 1.

Note: Capacity rate ratio will be zero (C= 0) if $(m\ C_p)_{big}$ is infinite or if one of the fluids is changing its phase. Example- Condenser, Evaporator and Boiler.

Number of Transfer Units (NTU)

NTU signifies overall size of the heat exchanger because it is directly proportional to area of the heat exchanger. It is given by

$$NTU = \frac{UA}{(mC_p)_{small}}$$

Effectiveness in terms of NTU

(1) For parallel flow heat exchanger,

$$(\varepsilon)_{\text{parallel flow}} = \frac{1-e^{-(1+c)NTU}}{1+C}$$

(2) For counter flow heat exchanger,

$$(\varepsilon)_{\text{counter flow}} = \frac{1-e^{-(1-c)NTU}}{1-Ce^{-(1-c)NTU}}$$

General Cases:

1. When one of the fluids is undergoing change of phase like in boiler, condenser and evaporator then C= 0 and

$$(\varepsilon)_{\text{parallel flow}} = 1 - e^{-NTU}$$

$$(\varepsilon)_{\text{counter flow}} = 1 - e^{-NTU}$$

2. If both the capacity rates are equal i.e. $m_h C_{ph} = m_c C_{pc}$ and C = 1

then

$$(\varepsilon)_{\text{parallel flow}} = \frac{1 - e^{-2NTU}}{2}$$

$$(\varepsilon)_{\text{counter flow}} = \frac{0}{0}$$

From L's hospital's rule

$$(\varepsilon)_{\text{counter flow}} = \frac{NTU}{1+NTU}$$

NOTE:

The effectiveness – NTU method is mainly useful whenever both the exit temperatures of hot and cold fluids (T_{he} and T_{ce}) are not known for a given heat exchanger area.

Additional Concept

1. To reduce the length of a heat exchanger passes are required in a heat exchanger. Also passes are required when the heat transfer area required is very large like in case of power plant condensers.

2.The diameter of each tube (D) of the heat exchanger can be determined from the area of heat exchanger using

Area of Heat Exchanger = $\pi D L \times n \times P$

Where

n = number of tubes

L = length of each tube per pass = length of heat exchanger

P= number of passes in a heat exchanger

References

1. Heat and Mass transfer by Cengel and Ghajar

2. NPTEL videos

HEAT EXCHANGER

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Contents

Convection, when a fluid is in communication with a solid with a difference of temperature between the two then heat transfer occurs due to bulk macroscopic motion of fluid with reference to solid surface (Advection) and intermolecular diffusion occurring randomly at microscopic level with in the fluid body (Diffusion). The heat transfer due to cumulative effect of the two is called convection.

Free Convection

- A Cup of Coffee
- Condenser of Domestic Refrigerator
- Babcock & Wilcox Boiler
- Emulsion Water Heater
- Ar Gas Within a Light Bulb

Forced Convection (Mixed Convection)

- Automobile Radiator (Fan type)
- Window Air conditioner (Blower type)
- Loeffler Boiler (Pump type)
- Metal Cutting
- Shell and Tube HX

The empirical governing law for convection heat transfer is Newtons law of cooling/heating.
Convection heat transfer coefficient (Film coefficient) is neither the property of solid nor the property of fluid. It is a complex parameter depending on a variety of factors like geometry of solid, flow is external or internal, whether convection is free or forced, whether the flow is laminar or turbulent and temperature under thermophysical properties of fluid (Cp, ν, k and μ).

h for liquid > h for gas
h for mixed > h for forced > h for free

Type of Convection	Air h (W/m² K)	Water h (W/m² K)
Free Convection	5	25
Forced Convection	100	500

Heat Exchanger, is broadly defined as a control volume where in exchange of heat takes place between two or more than two fluids at steady state steady flow conditions (SSSF).

Application

- Evaporators and condensers of domestic refrigerator.
- Automobile radiator.
- In thermal power plant intercooler, preheater, condenser, boiler, superheater, economiser.
- Oil cooler heat engines.
- Regenerators.
- Milk chilling of a pasteurising plants.

Classification of heat exchanger (TEMA)

1. On the basis of heat transfer process

Direct Contact Type (Mixing type)

- In this HX two fluids usually immiscible and typically a liquid and a gas or vapour that exchange heat when they come into direct physical contact with each other.
- Zero resistance to heat exchange means rate of heat transfer is very large because there is no separation between fluids.

Examples: Jet condensers, spray ponds, cooling towers, desuperheater and open feed water heater.

Indirect Contact Type (Surface HX Type)

- In this HX the two fluids are separated by an impervious solid surface during the heat exchange process.
- Here five resistance to heat transfer works are convective thermal resistance, fouling, internal thermal resistance, surface resistance and convective thermal resistance.

Examples: automobile radiator, water tube and fire tube boilers, Lancashire boiler, Cochran boiler, air preheater and economiser.

2. On the basis of compactness of the heat exchanger

Compactness is also called surface area density. It is defined as the surface area available for heat exchange per unit volume occupied by the heat exchanger. Its unit is (m^2/m^3). It offers large area per unit volume. It is denoted by symbol ‘C’ or ‘SAD’.

Categorization:

- Non compact HX ($70\ m^2/m^3 \leq C \leq 500\ m^2/m^3$).
- Medium compact HX ($500\ m^2/m^3 < C \leq 700\ m^2/m^3$)
- Compact HX ($C > 700\ m^2/m^3$)
- Automobile radiator for typical four wheeler $C \approx 1100\ m^2/m^3$.
- Gas turbine driven vehicle HX $C \approx 6600\ m^2/m^3$.
- Stirling heat engine regeneration HX $C \approx 15000\ m^2/m^3$.
- Human lungs (heat and mass exchanger) $C \approx 20000\ m^2/m^3$. Most compact HX till date in the world.

3. On the basis of hot and cold fluids in the HXs

- Recuperative (transfer type HX)
- Regenerative (storage type HX)
- **In recuperative HX** there is no time lag between the flow of hot and cold fluids in the HXs. This means the hot and cold fluids enter traverse to and leave the HX simultaneously.
- **In regenerative HX** there exist a time lag between hot and cold fluid with a matrix by some means coming into contact with hot and cold fluid alternatively. The matrix absorb or picks up the heat when it comes in contact of hot fluid and forgone to the cold fluid subsequently. The material chosen for matrix have a large thermal storage capacity.

Example: Matrix made of powders (material like aluminium oxide).

Regenerative HXs are further classified into two parts :

- **Stationary (Static HX):** In this HX the solid matrix would be stationary and using an appropriate flow switching device hot and cold fluids are made to pass through the matrix alternatively. Example: Stirling regenerative HX used in gas turbine plants.
- **Dynamic (Rotating HX):** In dynamic HX a rotating spindle possessing the matrix on its lateral surface comes alternatively in contact with hot and cold fluids. Example: Ljungstrom regenerative HX in air preheater application in steam power plant.

4. On the basis of relative direction of fluid flow

- **Co-current (Parallel flow) HX,** In parallel flow HX the hot and cold fluid flow in parallel direction.
- **Counter-current (Counter flow) HX,** In counter flow type HX the hot and cold fluid flow in opposite direction to each other. Enter and exit ends will be in opposite to each other.
- **Cross flow (Cross traverse flow) HX,** In cross flow type HX the hot and cold fluid cross each other or perpendicular to each other. The hot and cold fluid the entry sweeping and exit would be perpendicular.

Examples: Automobile radiator, condenser of a water chiller and evaporator and condenser of a room air conditioner.

5. On the basis of number of passes taken by hot and cold fluid

A HX fluid is set to have an executed pass if its sweeps or traverses to HX at once between its two ends.

- **Single pass HX,** In a single pass HX both the hot and cold fluid sweep the HX for only one time.
- **Multipass HX,** In a multipass HX either the hot or the cold or the both fluids passes through the HX for more than once.

6. On the basis of whether or not the heat exchanger fluids undergo phase change

- **Sensible HX,** In a sensible HX there are nearly a change in temperature of hot and cold fluid during heat exchange process. Example: Feed water heater, economiser, automobile radiator and air preheater.
- **Latent HX,** In a latent HX there is simultaneous change of phase in either hot or cold or both the fluid. Example: Boiler condenser and evaporator.

Backbone equation for HX:

Assumptions

- HX is assumed to be a control volume involved in SSSF process. Thus the continuity equation (law of conservation of mass) requires that there is no change with the mass flow rate of hot and cold fluid.
- The HX is assumed to be constant thermophysical properties.
- No heat crossing the control volume so the control volume is adiabatic.
- There is no work transfer is involved across the control volume.
- The hot and cold fluid undergo negligible changes in their potential energy and kinetic energy.

$$C_h (T_{hi} - T_{he}) = C_c (T_{ce} - T_{ci})$$

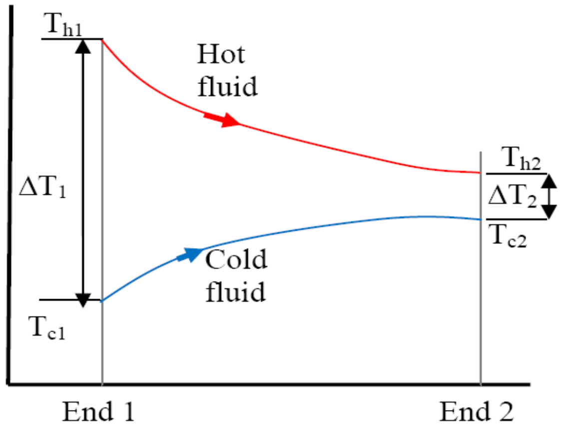
Heat exchanger analysis using LMTD approach

LMTD is given by Bowman et.al. It is noticed that the local temperatures of both the heat exchanger fluids undergo a continuous variations in a given typical HX. Thus the temperature difference also turns out to be local varying axially, a question arises at what value of ΔT has to be chosen in calculating HX rate. An acceptable answer is provided by Bowman et.al is known as LMTD.

1. Parallel flow heat exchanger:

$q = UA\ LMTD_{parallel\ HX}$;

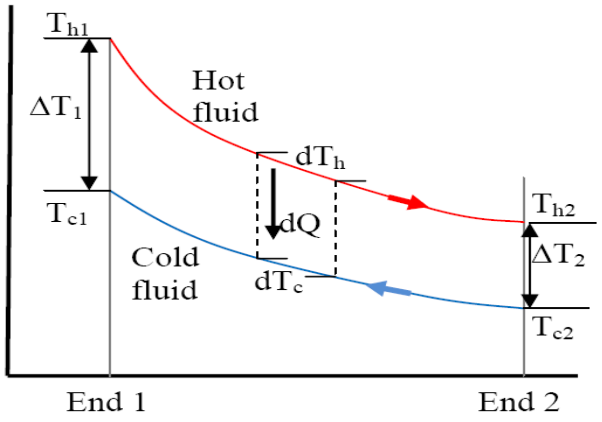
$$LMTD_{parallel HX} = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}}$$



2. Counter current heat exchanger:

$q = UA\ LMTD_{counter\ HX}$

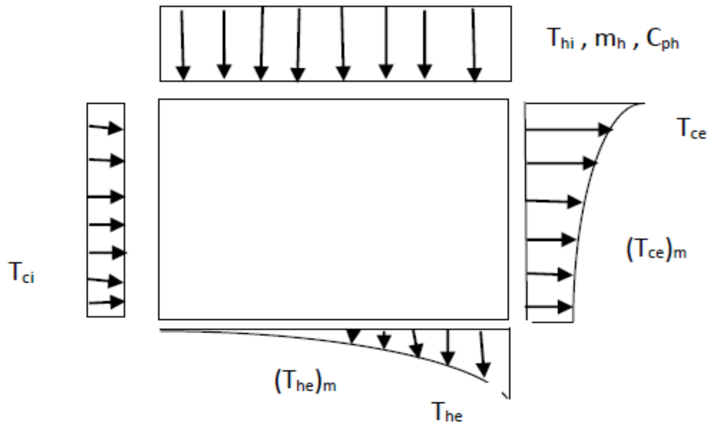
$$LMTD_{counter HX} = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}}$$



3. Wilhelm Nusselt's approach to LMTD of cross flow heat exchanger.

$q = UA\ LMTD_{cross\ HX}$

$$LMTD_{cross HX} = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}}$$



1. Counter flow heat exchanger with both fluids having identical heat capacity rates:

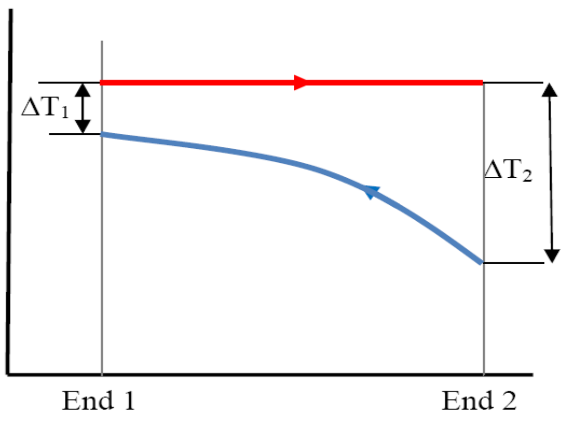
$$C_h = C_c$$

$$q = C_h\,(T_{hi} - T_{he}) = C_c\,(T_{ce} - T_{ci})$$

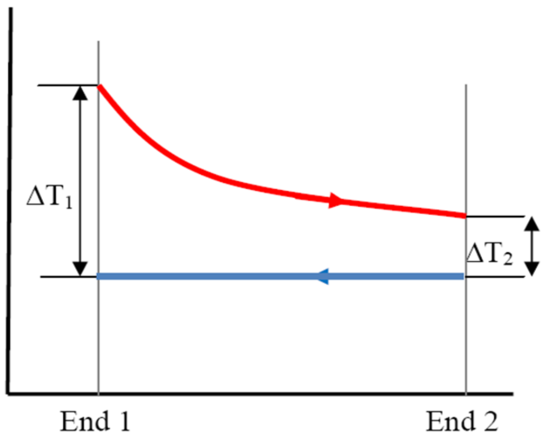
$$LMTD = AMTD = \frac{1}{2}(\Delta T_1 + \Delta T_2)$$

2. Latent (or) Phase-change HX (condenser or evaporator):

It can be clearly seen from the temperature profile that as long as the phase keeps changing the fluid enters and leaves the HX at saturated condition, the value of LMTD remains unaltered, not withstanding whether one uses co-current or counter current configuration.



Condenser



Evaporator

3. Bowman et.al chart solutions for multi pass HX

Let T_1 & T_2 indicate the inlet and exit temperatures of shell side fluid either hot or cold and t_1 & t_2 indicates inlet and exit temperatures of tube side fluid.

- Defining capacity ratio-

$$R = \frac{\left(\dot{m}\, c_p\right)_{tubefluid}}{\left(\dot{m}\, c_p\right)_{shellfluid}} = (T_1 - T_2)/(t_2 - t_1)$$

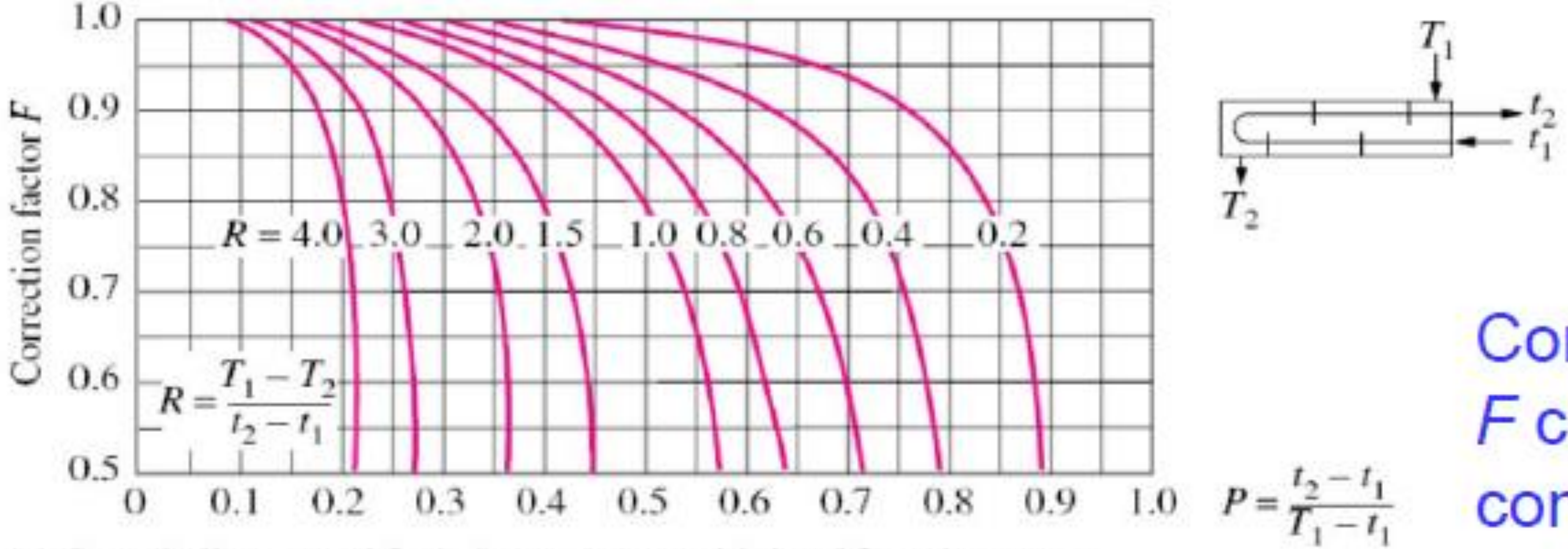
- Temperature ratio-

$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

- Correction factor (F), It is an indication of degree of departure of a given heat exchanger from the ideal counter current heat exchanger for given terminal temperature.

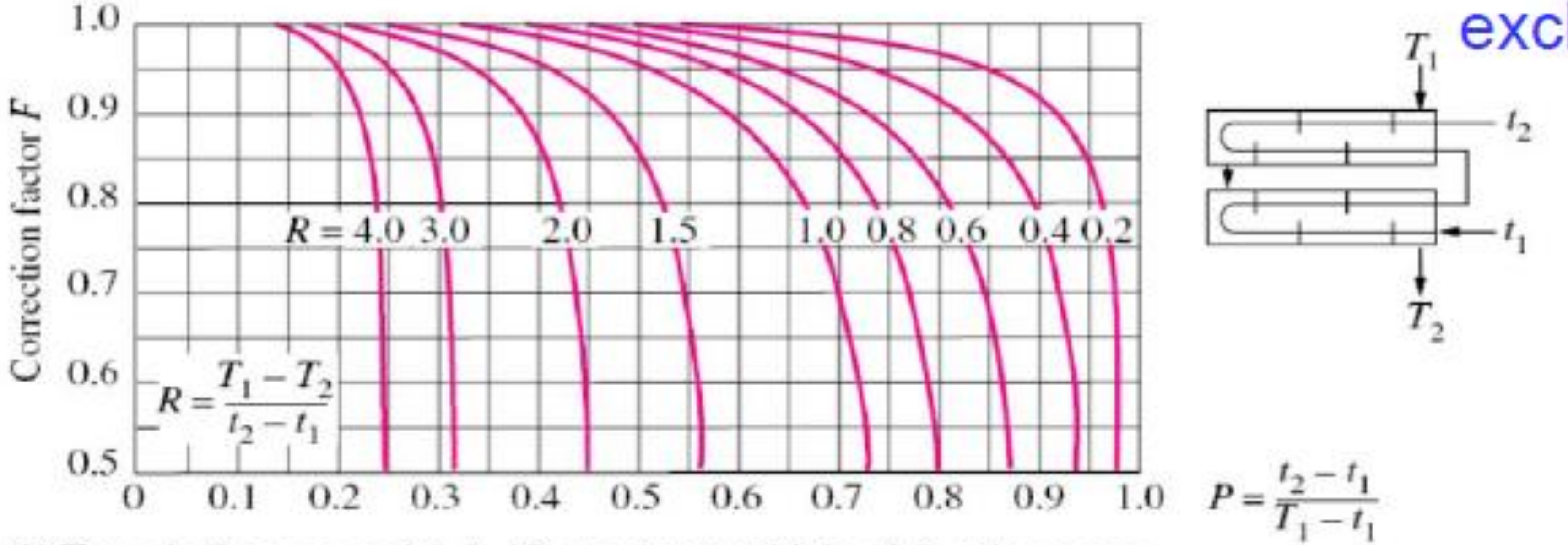
$$0 \le F \le 1$$

$$= f\,(R,P)$$



(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

Correction factor *F* charts for common shell-and-tube heat exchangers.



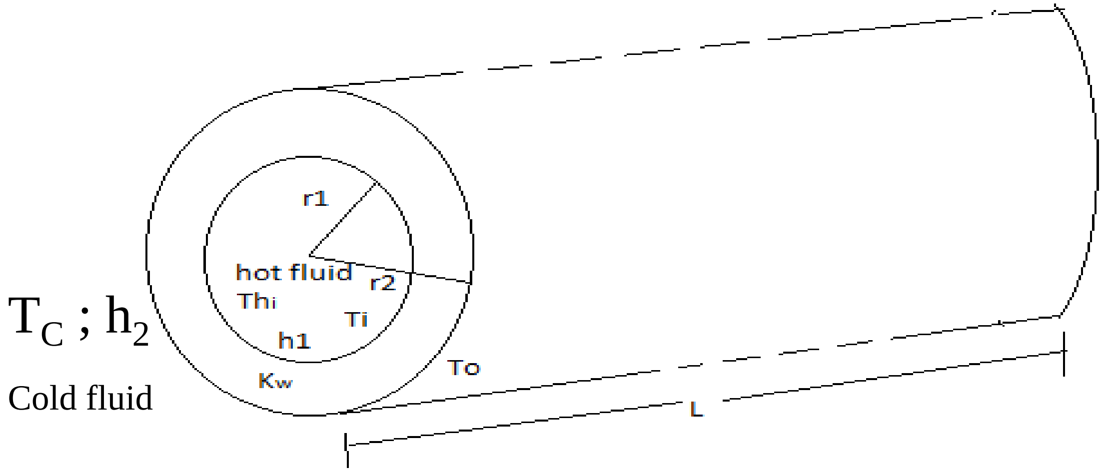
(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

Concept and expressions for overall heat transfer coefficient

There will be three resistance-

1. Surface convective resistance (hot fluid and inner surface of tube).
2. Internal thermal resistance.
3. Surface convective resistance (between outside wall of tube and cold fluid)

An equivalent coefficient of heat transfer is defined such that it has all the possible resistance integrate in it and on multiplying it with the appropriate heat transfer area and temperature difference it would give the net heat transfer rate in the heat exchanger. This equivalent heat transfer coefficient is called the overall heat transfer coefficient.



- A S.S. the rate of convective H.T. from the inner hot fluid to the inner hot surface of inner pipe is equal to the rate of H.T. by conduction through the tube material which is equal to the rate of H.T. from the outer surface of the pipe to cold surface.

$$q = h_1 * 2\pi * r_1 * L(T_h - T_i)$$

$$= \frac{2*\pi K_w * L(T_i - T_0)}{ln\frac{r_2}{r_1}}$$

$$= h_2 * 2\pi * r_2 * L(T_0 - T_c)$$

By componendo and dividend rule-

$$q = \frac{(T_h - T_c)}{\frac{1}{2\pi L} * (\frac{1}{h_1 * r_1} + \frac{ln\frac{r_2}{r_1}}{K_w} + \frac{1}{h_2 * r_2})}$$

$$= U_o * 2\pi r_2 L * (T_h - T_c)$$

$$\frac{1}{U_o} = [\frac{r_2}{r_1 * h_1} + \frac{r_2 * ln\frac{r_2}{r_1}}{K_w} + \frac{1}{h_2}]$$

- Let h_{si} and h_{so} stand for scale or fouling heat transfer coefficient on the inner surface of the tube, (watt/m^2 K) and h_{so} stand for the scale or fouling heat transfer coefficient based on outer surface of the tube (watt/m^2 K). then the fouled or inhibited HX would have a decreased overall H.T. coefficient "U_o" or increased unit "$1/U_o$" given by-

$$\frac{1}{U'_o} = \frac{r_2}{r_1 h_1} + \frac{r_2}{r_1 h_{si}} + \frac{r_2}{K_w}(ln\frac{r_2}{r_1}) + \frac{1}{h_{so}} + \frac{1}{h_2}$$

- Defining $\frac{1}{U'_o}$ and $\frac{1}{U_2}$ to be the following factor/ scaling factor/ fouling resistance-

$$\frac{1}{U'_o} - \frac{1}{U_o} = \text{fouling factor } (R_f)$$

$$R_f = \frac{U_o - U'_o}{U_o\, U'_o} (m^2\ K/Watts)$$

- There are no known theoretical methods for value of R_f for a given HX. It is generally taken based on empirical observation for standard fluid used in HX.

Heat Transfer Considerations (contd…):

- Fouling factor

  Material deposits on the surfaces of the heat exchanger tube may add further resistance. Such deposits are termed fouling and may significantly affect heat exchanger performance.

- Scaling, is the most common form of fouling and is associated with inverse solubility salts.

Examples: $CaCO_3$, $CaSO_4$, $Ca_3(PO_4)_2$, $CaSiO_3$, $Ca(OH)_2$, $Mg(OH)_2$, $MgSiO_3$, Na_2SO_4, $LiSO_4$, and Li_2CO_3.

- Corrosion fouling, is classified as a chemical reaction which involves the heat exchanger tubes. Many metals, copper and aluminum being specific examples, form adherent oxide coatings.

- Biological fouling, is common where untreated water is used as a coolant stream. Problems occurs like algae or other microbes to barnacles.

Contrast between AMTD and LMTD

- Defining the AMTD to be:

$$AMTD = \frac{1}{2}(\Delta T_1 + \Delta T_2) \qquad \text{……………..(1)}$$

Reconsidering the definition of LMTD

$$\text{LMTD} = \frac{\Delta T_2 - \Delta T_1}{\ln \dfrac{\Delta T_2}{\Delta T_1}} \qquad \text{………………(2)}$$

$$= \frac{(\Delta T_1 - \Delta T_2)}{\ln\left[\dfrac{1 + \dfrac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}}{1 - \dfrac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}}\right]} \qquad \text{………………(3)}$$

Defining a parameter, $\xi = \dfrac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}$ ……………….(4)

$$= \frac{\Delta T_1 - \Delta T_2}{\left[\xi + \frac{\xi^3}{3} + \frac{\xi^5}{5} + \dots\dots\right] - \left[-\xi - \frac{\xi^3}{3} - \frac{\xi^5}{5} - \dots\dots\right]}$$

$$= \frac{\Delta T_1 - \Delta T_2}{2\left[\xi + \frac{\xi^3}{3} + \frac{\xi^5}{5} + \dots\dots\dots\right]}$$

$$= \frac{\Delta T_1 - \Delta T_2}{2 \times \frac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}\left[1 + 2\left(\frac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}\right)^2 + \frac{1}{5}\left(\frac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}\right)^4 + \dots\dots\dots\right]} \quad \dots\dots\dots\dots\dots(5)$$

In general ξ < 1.0

* It follows that the contribution from higher order term could be neglected compare to unity in the denominator of equation (5)

$$= \frac{\Delta T_1 - \Delta T_2}{2 \times \frac{\Delta T_1 - \Delta T_2}{\Delta T_1 + \Delta T_2}\left[1 + somevalue\right]}$$

LMTD = AMTD/(1 + some value) ……………………(6)

in general, LMTD < AMTD

- q = UA LMTD
- q = UA AMTD
- $A_{design, LMTD} > A_{design, AMTD}$

Effect of varying overall heat transfer coefficient on LMTD of a heat exchanger:

Let us consider a single pass counter flow or counter current HX.

Consider a linear variation of overall heat transfer coefficient in the axial direction of HX

$$U = f(x)$$

$$U = f(\Delta T)$$

Assume:

$$U = a + b(\Delta T) \quad \text{………........(1)}$$

Let the two be known and let and indicate overall heat transfer coefficient (a) (1) and (2) (entry and exit) of the HX.

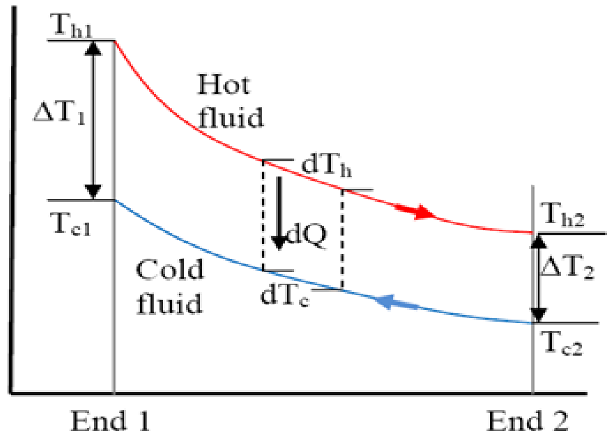
$$U_1 = a + b(\Delta T_1) \text{..........(2)}$$

$$U_2 = a + b(\Delta T_2) \text{..........(3)}$$

By equation (2) and (3) we get...

$$a = \frac{U_2 \Delta T_1 - U_1 \Delta T_2}{\Delta T_2 - \Delta T_1} \text{............(4)}$$

$$b = \frac{U_1 - U_2}{\Delta T_1 - \Delta T_2} \text{...............(5)}$$



Considering an element of length Δx at given x of HX as shown the local temperature difference which is a function of x

$$\Delta T = f(x) = T_h - T_c \text{......(6)}$$

Differentiating,

$$d(\Delta T) = dT_h - dT_c \text{.................(7)}$$

In the elemental length of there is elemental rate of heat exchange given by,

$$dq = -C_h dT_h \text{.........(8)}$$

$$dq = -C_c dT_c \text{...........(9)}$$

Put the value of dT_h and dT_c from equation 8 & 9 in equation (7)

$$d(\Delta T) = \frac{-dq}{C_h} + \frac{dq}{C_c}$$

$$d(\Delta T) = dq\left(\frac{1}{C_c} - \frac{1}{C_h}\right)\text{............}(10)$$

$$dq = UdA\Delta T = (a + b\Delta T)dA\Delta T\text{.....}(11)$$

Put equation (11) in (10)

$$d(\Delta T) = (a + b\Delta T)dA\Delta T\left(\frac{1}{C_c} - \frac{1}{C_h}\right)$$

$$\frac{d(\Delta T)}{\Delta T(a + b\Delta T)} = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)dA\text{.......}(12)$$

Considering the backbone equation for HX

$$q = C_h(T_{hi} - T_{h_e}) = C_c(T_{ce} - T_{ci})$$

$$C_h = \frac{q}{T_{hi} - T_{he}} ; C_c = \frac{q}{T_{ce} - T_{ci}}\text{..........}(13)$$

Substitute equation (13) in (12)

$$\frac{d(\Delta T)}{d(a+b\Delta T)} = \frac{dA}{q}(T_{ce} - T_{ci} - T_{hi} + T_{he})$$

$$\frac{d(\Delta T)}{\Delta T(a+b\Delta T)} = \frac{dA}{q}\left[(T_{he} - T_{ci}) - (T_{h2} - T_{ce})\right] \text{........(14)}$$

On integration-

$$\int_{T_1}^{T_2} \frac{d(\Delta T)}{\Delta T(a+b\Delta T)} = \frac{A}{q}(\Delta T_2 - \Delta T_1) \text{...............(15)}$$

Considering L.H.S. separately and performing the partial fraction-

$$\int_{\Delta T_2}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T(a+b\Delta T)} = \frac{1}{a}\left[\frac{1}{\Delta T} - \frac{b}{a+b(\Delta T)}\right] \text{.............(16)}$$

Now, put equation (16) in (15)

$$\frac{1}{a}\int_{\Delta T_1}^{\Delta T_2}\left[\frac{d\Delta T}{\Delta T}-\frac{bd(\Delta T)}{a+b(\Delta T)}\right]=\frac{A}{q}(\Delta T_2-\Delta T_1)$$

$$\frac{1}{a}\ln\left(\frac{U_1\Delta T_2}{U_2\Delta T_1}\right)=\frac{A}{q}(\Delta T_2-\Delta T_1)\text{.........}(17)$$

$$q=aA\frac{\Delta T_2-\Delta T_1}{\ln\left(\frac{U_1\Delta T_2}{U_2\Delta T_1}\right)}$$

Now, by substituting value of 'a' from equation (4) we obtain-

$$q_{HX}=A\left[\frac{U_1\Delta T_2-U_2\Delta T_1}{\ln\left(\frac{U_1\Delta T_2}{U_2\Delta T_1}\right)}\right]\text{ watts...........}(18)$$

Thank You