DYNAMICS OF MACHINES (BME-28)
B.Tech (Fifth Sem.)

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Syllabus
Dynamics of Machines (BME 28)

UNIT-I

STATIC & DYNAMIC FORCE ANALYSIS Static equilibrium of two/three force members, Static equilibrium of member with two forces and torque, Static force analysis of linkages, D'Alembert's principle, Equivalent offset inertia force, Dynamic force analysis of four link mechanism and slider crank mechanism, Dynamically equivalent system

TURNING MOMENT & FLYWHEEL Engine force analysis-Piston and crank effort, Turning moment on crankshaft, Turning moment diagrams-single cylinder double acting steam engine, four stroke IC engine and multi-cylinder steam engine, Fluctuation of energy, Flywheel and its design

UNIT-II

Governors Terminology, Centrifugal governors-Watt governor, Dead weight governors-Porter & Proell governor, Spring controlled governor-Hartnell governor, Sensitivity, Stability, Hunting, Isochronism, Effort and Power of governor

Gyroscopic Motion Principles, Gyroscopic torque, Effect of gyroscopic couple on the stability of aero planes, ships& automobiles

UNIT-III

BALANCING OF MACHINES Static and dynamic balancing, Balancing of several masses rotating in the same plane and different planes, Balancing of primary and secondary forces in reciprocating engine, Partial balancing of two-cylinder locomotives, Variation of tractive force, swaying couple, hammer blow, Balancing of two cylinder in-line engines

MECHANICAL VIBRATIONS Introduction, Single degree free & damped vibrations of spring-mass system, Logarithmic decrement, Torsional vibration, Forced vibration of single degree system under harmonic excitation, Critical speeds of shaft

UNIT-IV

Friction Introduction Friction in journal bearing-friction circle, Pivots and collar friction-Flat and conical pivot bearing Flat collar bearing, Belt drives-types, material, power transmitted, ratio of driving tensions for flat belt, centrifugal tension, initial tension, rope drive-types Laws of friction, Efficiency on inclined plane, Screw friction, Screw jack, Efficiency, Friction in journal bearing-friction circle, Pivots and collar friction-Flat and conical pivot bearing, Flat collar bearing

Clutches, Bakes & Dynamometers Single and multiple disc friction clutches, Cone clutch, Brakes-types, Single and double shoe brake, Simple and differential Band brake, Band and Block brake, Absorption and transmission dynamometers, Prony brake and rope brake dynamometers

Course Outcome

1. Ability to carry out static and dynamic force analysis of four bars mechanism and slider crank mechanism, and design of flywheels.
2. To understand types of centrifugal governors, the effects of characteristic parameters and controlling force diagrams and principles of gyroscopic effect and its engineering applications.
3. To Understand the balancing of rotating and reciprocating masses and ability to analyze single degree freedom systems subjected to free, damped and forced vibrations as well as calculation of critical speeds of shaft.
4. To Understand the applications of friction in pivot and collar bearings, belt drives, clutches, brakes and dynamometers.

Kinematics and Dynamics: Difference

- The objective of **kinematics** is to **develop various means of transforming motion to achieve a specific kind of applications**.

- The objective of **dynamics** is **analysis of the behavior of a given machine or mechanism when subjected to dynamic forces.**

- The role of **kinematics is to ensure the functionality of the mechanism**, while the **role of dynamics is to verify the acceptability of induced forces in parts**. The functionality and induced forces are subject to various constraints (specifications) imposed on the design.

Kinematics and Dynamics: Difference

- The term **machine** is usually applied to a complete product. A car is a machine. Similarly, a tractor, a combine, an earthmoving machine, etc are also machine. At the same time, each of these machines may have some devices performing specific functions, like a windshield wiper in a car, which are called **mechanisms.**

- The distinction between the **machine/mechanism and the structure** is more fundamental. The former must have moving parts, since it transforms motion, produces work, or transforms energy. The latter **does not have moving parts**; its function is purely structural, i.e., to **maintain its form and shape** under given external loads, like a bridge, a building, or an antenna mast.

Dynamics of Machine: Analyses the forces and couples on the members of the machine due to external forces (static force analysis), also analyses the forces and couples due to accelerations of machine members (Dynamic force analysis)

Rigid Body: Deflections of the machine members are neglected in general by treating machine members as rigid bodies (also called rigid body dynamics).

- The link must be properly designed to withstand the forces without undue deformation to facilitate proper functioning of the system.

- In order to design the parts of a machine or mechanism for strength, it is necessary to determine the forces and torques acting on individual links. Each component however small, should be carefully analysed for its role in transmitting force.

- The forces associated with the principal function of the machine are usually known or assumed.

Forces acting on machine elements

- Joint forces (or Reaction forces): the action and reaction between the bodies involved will be through the contacting kinematic elements of the links that form a joint. The joint forces are along the direction for which the degree-of-freedom is restricted.
- Physical forces
- Friction or resisting force
- Inertial forces

Reactions for different types of support

Constraint : The restriction to the motion of a body in any direction is called a constraint.

Fig. 1

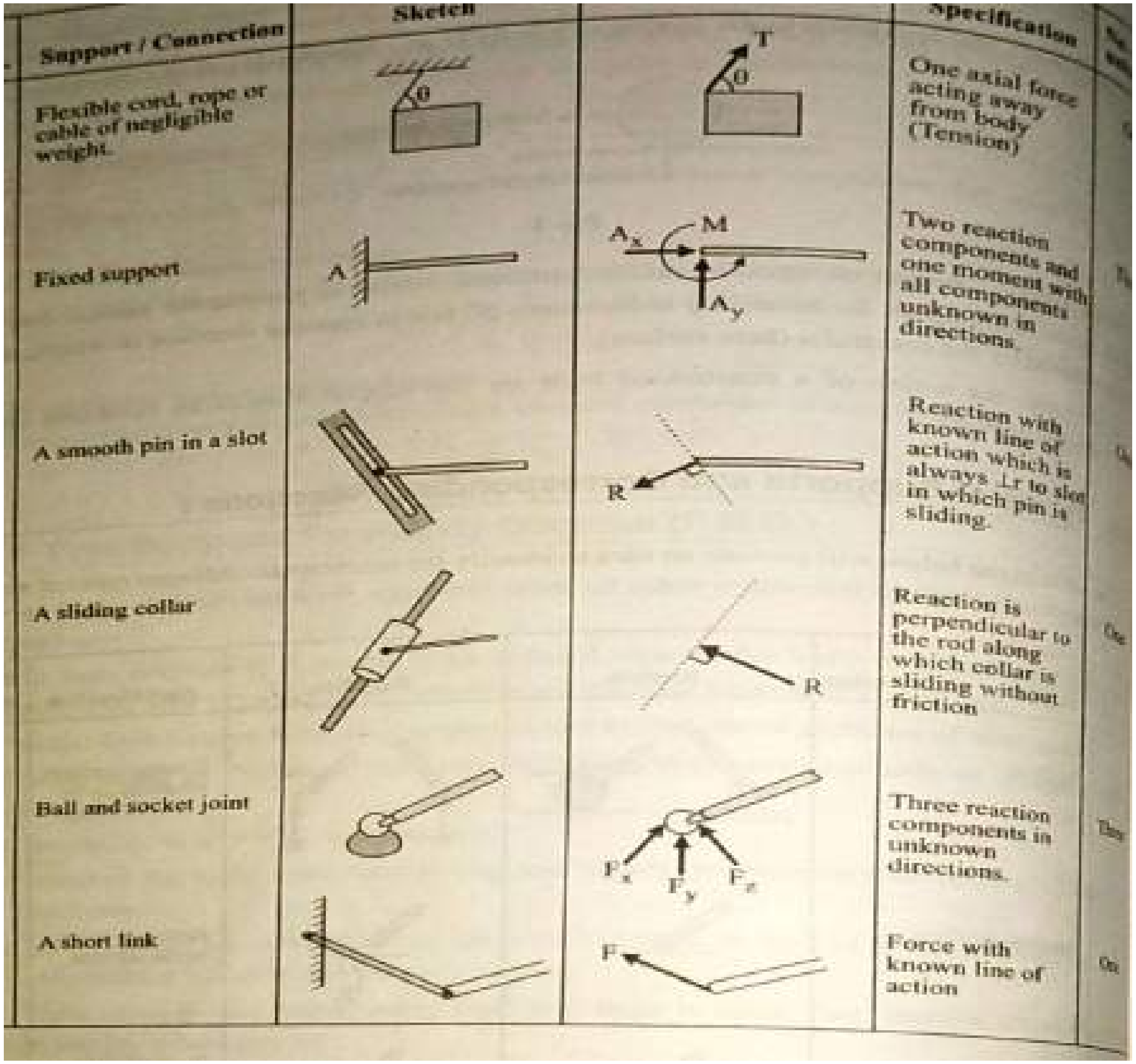
When a ball is resting on smooth surface horizontal motion is possible but vertical downward motion is restricted by plane. So according to Newton's 3rd law in opposite direction of weight, normal reaction is offered by the constraint (here surface).

In General, the action of a constrained body on any support induces an equal and opposite reaction from the support.

Eqm.5 Types of Supports and Corresponding Reactions :

The table given below will provide an idea to identify the reactions for different types of supports or connections.

Sr. No.	Support / Connection	Sketch	Reaction	Specification	No. of unknowns
1.	Rollers			Known reaction which is ⊥r to plane of roller	One
2.	Smooth surface			Reaction is ⊥r to the surface	One
3.	Rough surface			Two reaction components with unknown directions	Two
4.	Smooth pin or Hinge			Two reaction components with unknown directions	Two

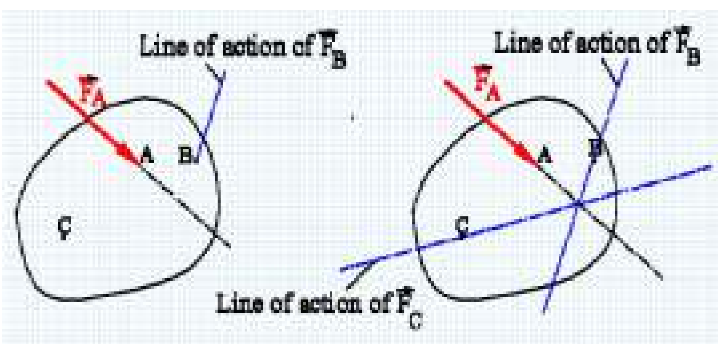


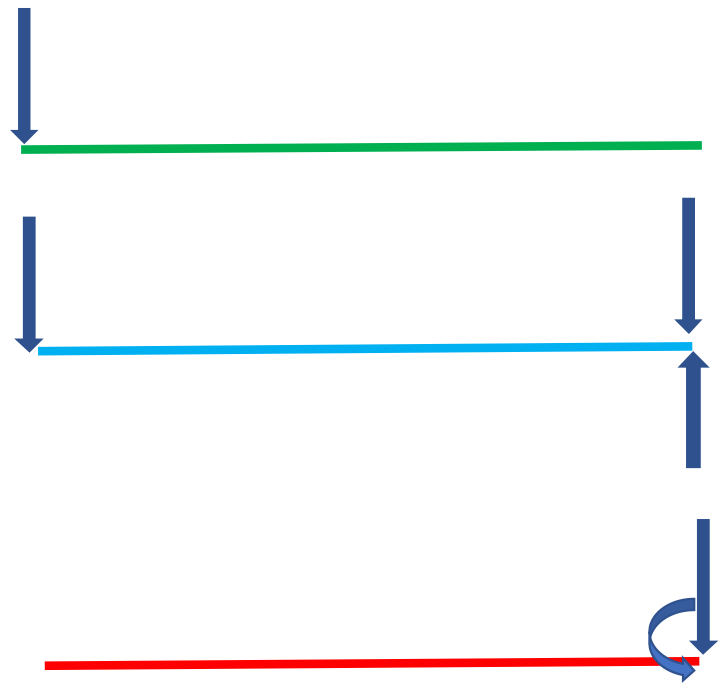
Support / Connection	Sketch		Specification
Flexible cord, rope or cable of negligible weight.	θ	T, θ	One axial force acting away from body (Tension)
Fixed support	A	A_x, M, A_y	Two reaction components and one moment with all components unknown in directions.
A smooth pin in a slot		R	Reaction with known line of action which is always ⊥r to slot in which pin is sliding.
A sliding collar		R	Reaction is perpendicular to the rod along which collar is sliding without friction
Ball and socket joint		F_x, F_y, F_z	Three reaction components in unknown directions.
A short link		F	Force with known line of action

Reactions for different types of support

Principle of Transmissibility: The point of application of a force can be transmitted anywhere along its line of action but within the body

Principle of Superposition: The effect of a force on a body remains unaltered if we add or subtract any system which is in equilibrium. It is very useful in application of parallel transfer of force.





Equivalent systems of forces: Two system are said to be equivalent if they can be reduced to the same force-couple System at a given point.

Two force system act on the same rigid body are equivalent if the sums of the forces (resultant) and sums of the moments about a point are equal.

IMPORTANCE OF FORCE ANALYSIS

Apart from static forces, mechanism also experiences inertia forces when subjected to acceleration, called dynamic forces.

Static forces **are predominant at lower speeds** and

Dynamic forces **are predominant at higher speeds**.

- Force analysis helps to determine the forces transmitted from one point to another, essentially from input to output.

- It is the starting point for strength design of a component/ system, basically to decide the dimensions of the components

- Force analysis is essential to avoid either **overestimation** or **under estimation** of forces on machine member.

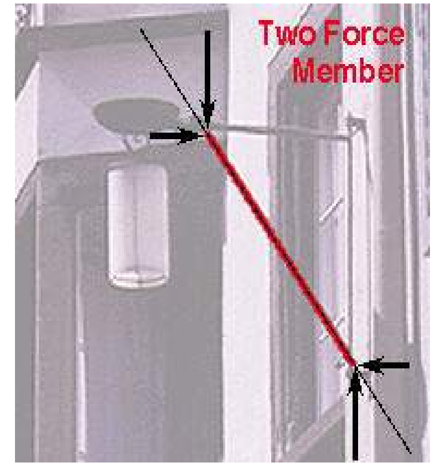
Overestimation: machine component would have more strength than required. Over design leads to heavier machines, costlier and becomes not competitive

Underestimation: leads to design of insufficient strength and to early failure.

- A machine is a device that performs work by transferring energy by means of mechanical forces from a power source to a driven load. It is necessary in the design of a mechanism to know the manner in which forces are transmitted from the input to output so that the components of mechanism can be properly sized to withstand the stresses induced.

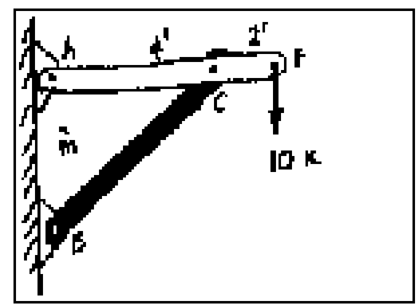
- **All links have mass, and if links are accelerating, there will be inertia forces associated with this motion. If the magnitudes of these inertia forces are small relative to the externally applied loads, then they can be neglected in the force analysis. Such an analysis is referred to as a STATIC FORCE ANALYSIS.**

Static Equilibrium of two force members



- There are many types of structural elements. The support condition has a significant influence on the behavior of the specific element. It is advantageous to identify certain types of structural elements which have distinct characteristics.

- If an element has pins or hinge supports at both ends and carries no load in-between, it is called a **two-force member**. These elements can only have two forces acting upon them at their hinges.



- If only two forces act on a body that is in equilibrium, then they must be **equal in magnitude, co-linear and opposite in sense**. This is known as the **two-force principle**.

- The two-force principle applies to ANY member or structure that has only two forces acting on it. This is easily determined by simply counting the number of places where forces act on that member. (REMEMBER: reactions are considered to be forces!) If they act in two places, it is a two-force member.

- One of the unique aspects of these members is the fact that the line of action of the resultants of the forces acting on the two ends of the member MUST pass along the center line of the structural element. If they did not, the element would not be in equilibrium.

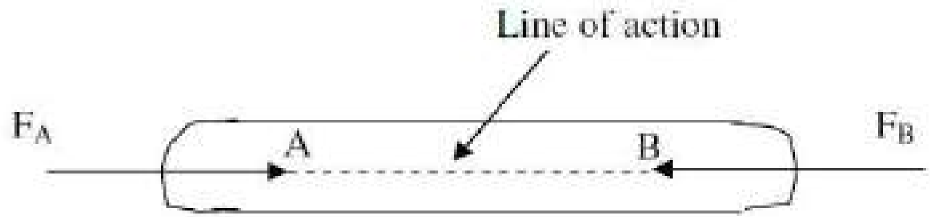
- Most, but not all, two-force members are straight. Straight elements are usually subjected to either tension or compression. Those members of other geometries will have bending across (or inside) their section in addition to tension or compression, but the two-force principle still applies.

- Some common examples of two-force members are columns, struts, hangers, braces, pinned truss elements, chains, and cable-stayed suspension systems.

TWO FORCE MEMBER

Very useful & important principles.

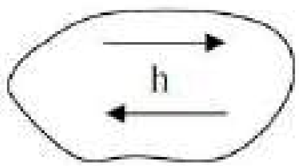
Equilibrium of a body under the action of two forces only (no torque)



For body to the in Equilibrium under the action of 2 forces (only), the two forces must the equal opposite and collinear. The forces must be acting along the line joining A&B.

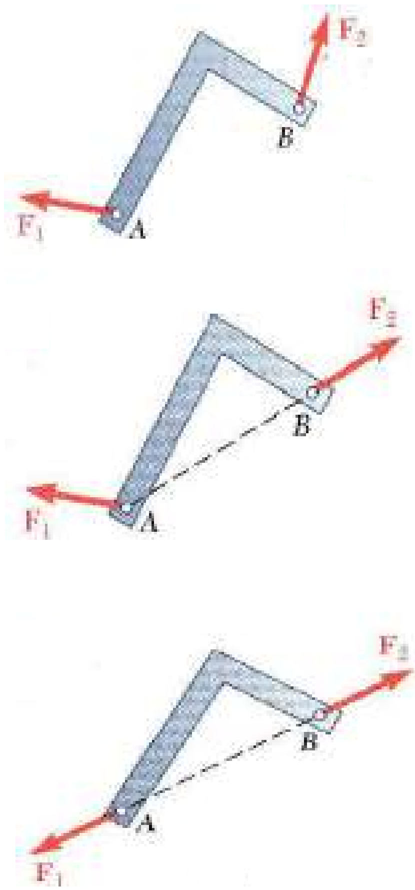
That is,

$F_A = - F_B$ (for equilibrium)

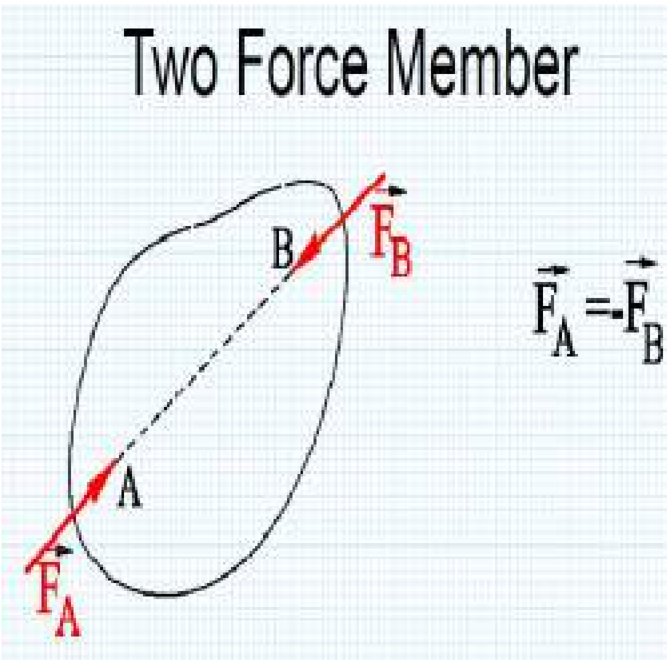


If this body is to be under equilibrium 'h' should tend to zero

Equilibrium of a Two-Force Body



- Consider a plate subjected to two forces F_1 and F_2
- For static equilibrium, the sum of moments about *A* must be zero. The moment of F_2 must be zero. It follows that the line of action of F_2 must pass through *A*.
- Similarly, the line of action of F_1 must pass through *B* for the sum of moments about *B* to be zero.
- Requiring that the sum of forces in any direction be zero leads to the conclusion that F_1 and F_2 must have equal magnitude but opposite sense.



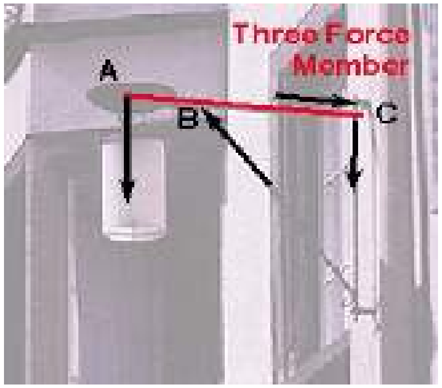
In a Two-Force member, the forces must be equal and opposite and must have the same line of action

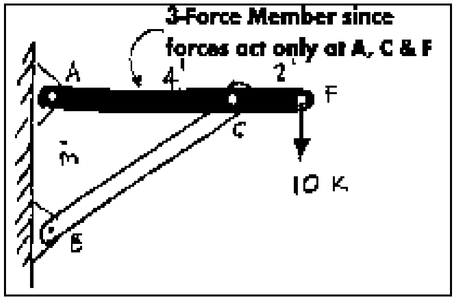
A member under the action of two forces will be in equilibrium if

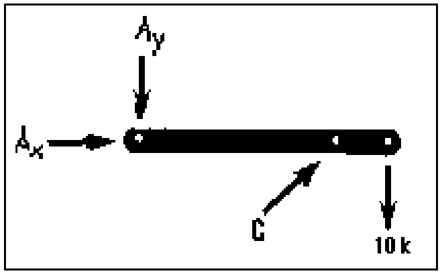
- – the forces are of the same magnitude,
- – the forces act along the same line, and
- – the forces are in opposite directions.

Static Equilibrium of three force members

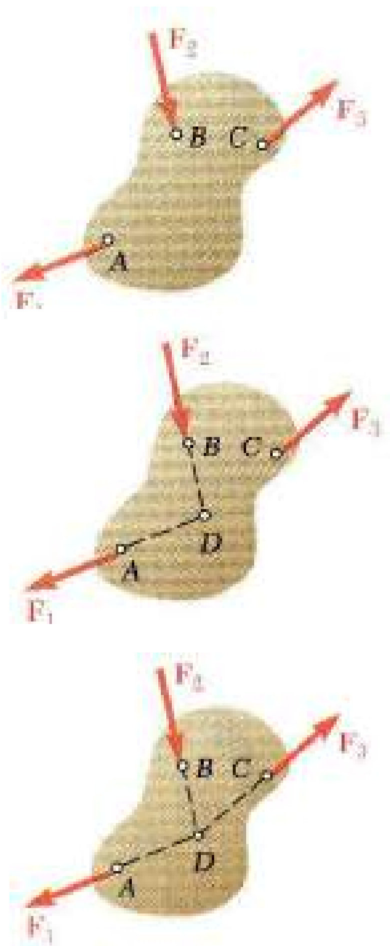
- If three non-parallel forces act on a body in equilibrium, it is known as a **three-force member**.
- The three forces interact with the structural element in a very specific manner in order to maintain equilibrium.
- If a **three-force member is in equilibrium** and the **forces are not parallel**, they must be concurrent. Therefore, the lines of action of all three forces acting on such a member must intersect at a common point; any single force is therefore the equilibrant of the other two forces.
- A three-force member is often an element which has a single load and two reactions. These members usually have forces which cause bending and sometimes additional tension and compression.
- The most common example of a three-force member is a simple beam.





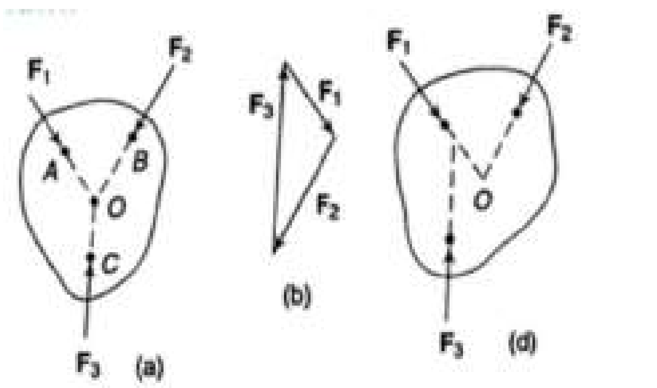


Equilibrium of a Three-Force Body



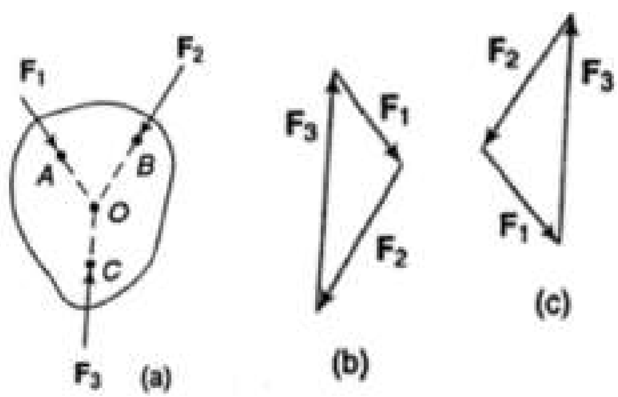
- Consider a rigid body subjected to forces acting at only 3 points.
- Assuming that their lines of action intersect, the moment of F_1 and F_2 about the point of intersection represented by *D* is zero.
- Since the rigid body is in equilibrium, the sum of the moments of F_1, F_2, and F_3 about any axis must be zero. It follows that the moment of F_3 about *D* must be zero as well and that the line of action of F_3 must pass through *D*.
- The lines of action of the three forces must be concurrent or parallel.

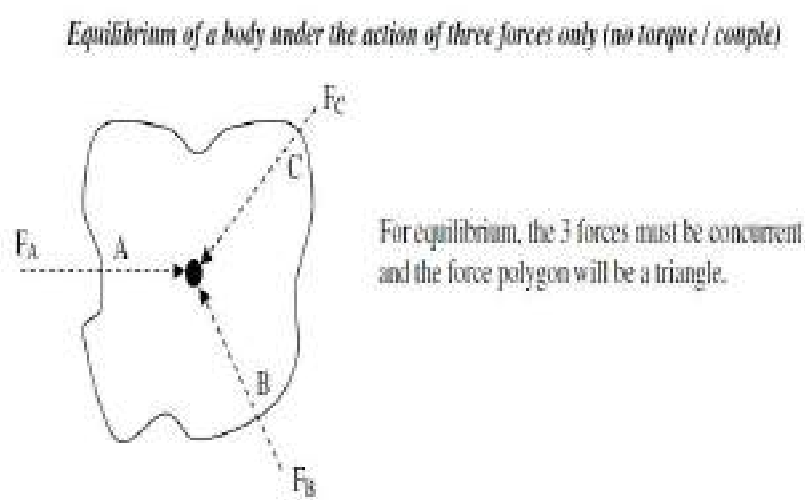
THREE FORCE MEMBER



A member under the action of three forces will be in equilibrium if –

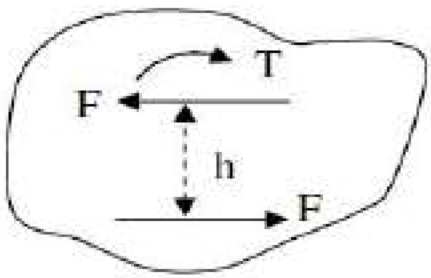
the resultant of the forces is zero, and –
the lines of action of the forces intersect at a point (known as point of concurrency).





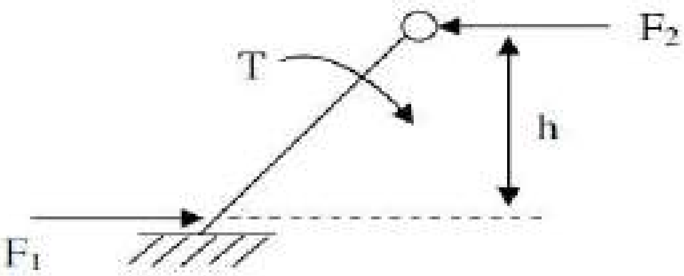
TWO FORCE and ONE MOMENT (TORQUE) MEMBER

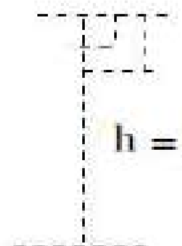
Equilibrium of a body acted upon by 2 forces and a torque.



For equilibrium, the two forces must form a counter couple. Therefore the forces must be equal, opposite and parallel and their senses must be so as to oppose the couple acting on the body

Example:

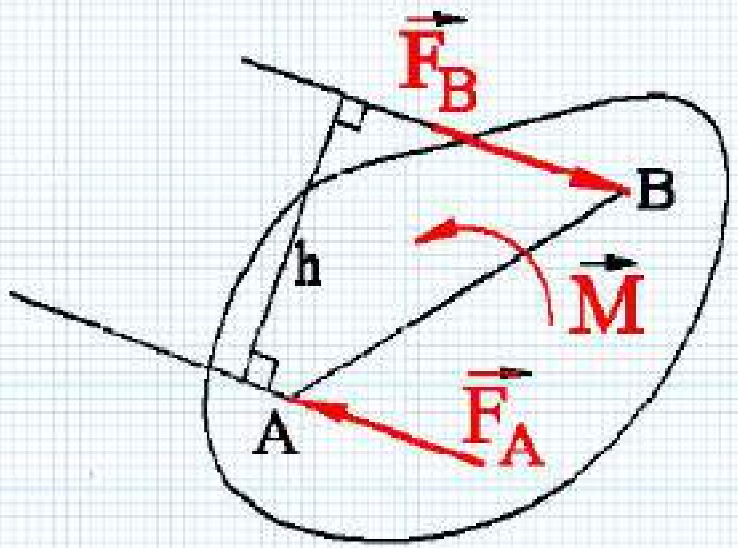




h = Perpendicular distance between F_1 & F_2

$F_1 = F_2 = F$ and $T = F \times h$

Two force and one moment member:



$$\vec{F}_A = \vec{F}_B$$

$$M = hF_A$$

Two forces form a couple whose moment is equal in magnitude but in opposite sense to the applied moment

Equilibrium

For a rigid body to be in Equilibrium

i) Sum of all the forces must be zero

ii) Sum of all the moments of all the forces about any axis must be zero

Static Equilibrium

$$\sum \vec{\mathbf{F}} = 0 \qquad \sum \vec{\mathbf{M}} = 0$$

In space, these two vector equations yield six scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

$$\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0$$

In case of coplanar force systems, there are three scalar equations:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0$$

D'Alembert's Principle

Inertia force:

- Inertia is a **property of matter** by virtue of which a body **resists any change in velocity**.
- It is an **imaginary force** which acts on a rigid body and brings it in equilibrium.
- It is mathematically **equal to the accelerating force** in magnitude but opposite in direction

Inertia force (F_i)= -(accelerating force)=- m x a   (a = linear acceleration of the CG of the body)

Inertia torque:

- Inertia torque **resists any change in the angular velocity of the body.**
- Inertia torque brings the body in equilibrium when applied on it.
- Inertia torque is equal to **accelerating couple** in magnitude but opposite in direction

Inertia Torque (T_i)= - (I x α) where (I =mass moment of inertia of the body about an axis passing through the CG of the body and perpendicular to the plane of the rotation of the body and **α** = angular acceleration

$I=mk^2$ (m= mass of body and k is radius of gyration)

D'Alembert's Principle states that the resultant force acting on a body together with the inertia force are in equilibrium. It is used to convert the dynamic problem into equivalent static problem.

$F_r + F_i = 0$ where F_r is the resultant external force act on the body and F_i is inertial force.

According to Newton's second law of motion

Resultant force = m x a

$F_r = ma$

$F_r - ma = 0$

But $F_r + F_i = 0$

Therefore $F_i = -(m \times a)$

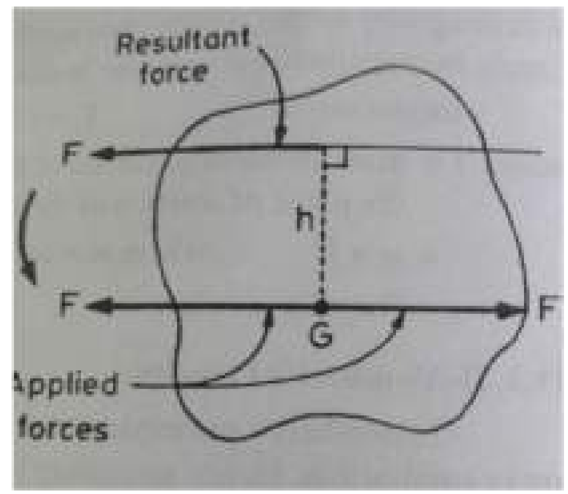
Equivalent offset inertia force:

- In plane motion involving acceleration the inertia force acts on a body through the its centre of mass.
- If the body acted upon by forces such that resultant do not pass through the centre of mass, a couple will also act on the body.
- It is necessary to replace the inertial force and inertia couple by an equivalent offset inertia force which can account for both.

Resultant force (F) due to large number of forces acting on the body does not pass through the CG and it is at the distance h from CG

Consider two equal opposite forces F at G so that the rigid body is now acted upon by

i) a couple of magnitude F x h in anti clockwise
ii) a force of magnitude F passing through G



The couple F x h causes angular acceleration of the rigid body
Force F causes linear acceleration of CG of the body.
Hence we will have two equations

Corresponding to couple:

Couple = $I \times \alpha$

$F \times h = m k^2 \times \alpha$

Corresponding to force:

Force = $m \times a$

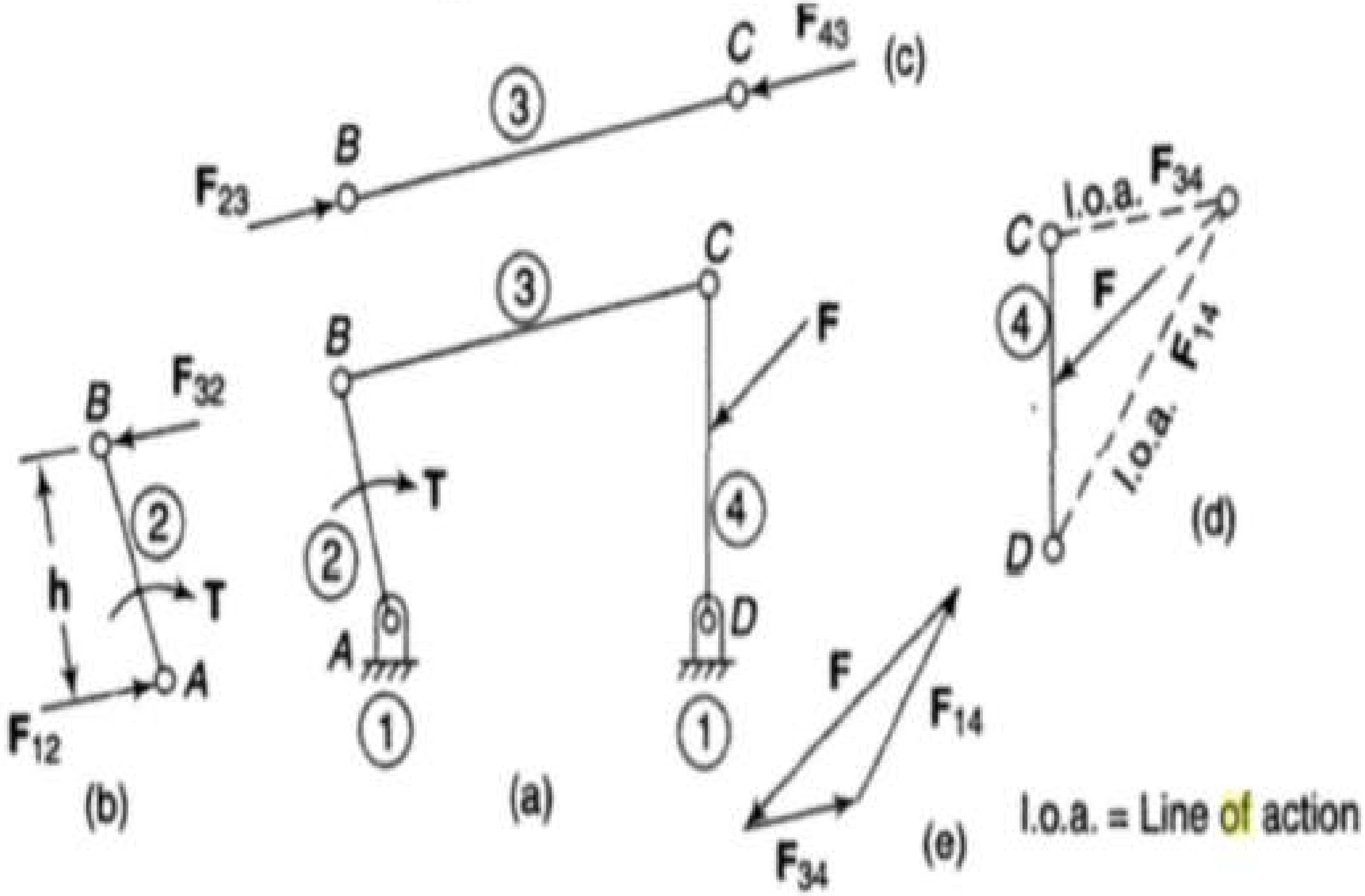
You can find the a and α for the above two equation. (if **F,h,m and k are known**)

But if only a and α are known then you have to find F and h

$F = m \times a$

$h = (I \times \alpha)/F = (mk^2 \times \alpha)/F$

h is the distance of the resultant force from CG



F43
C
(c)
3
B
F23
C
3
B
F
4
2
T
A
D
1
1
(a)
B
F32
2
h
T
A
F12
(b)
l.o.a. F34
C
F
4
F14
l.o.a.
D
(d)
F
F14
F34
(e)
l.o.a. = Line of action

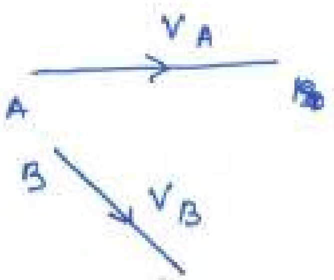
Relative Velocity Method

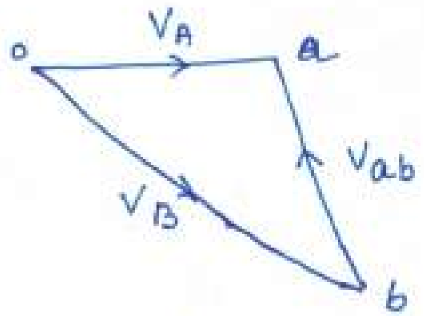
⇒ useful for determining linear & angular velocity in mechanism.

⇒ It can be used for acceleration analysis (adv. over instantaneous centre method)

V_{AB} = relative velocity of A w.r.t. B.

V_{BA} = " " " B w.r.t. A.

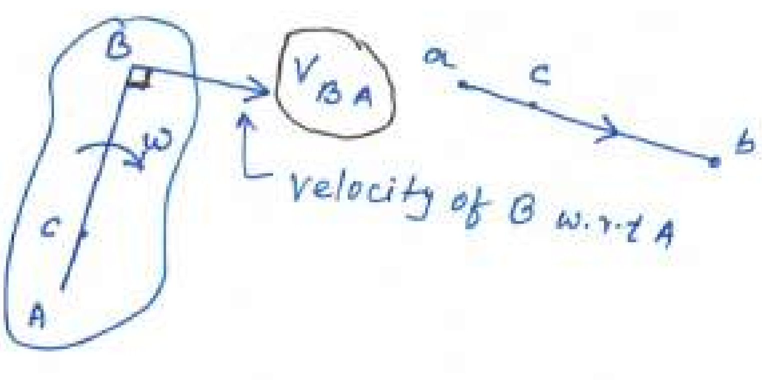




vector ba = V_{AB}

vector ab = V_{BA}.

Relative Velocity in Rigid Link.



Velocity at any pt on the link w.r.t another point on same link is always $\perp^r$ to the line joining these pts on space diag.

$V_{BA} = \vec{ab} = \omega \times AB$ [ω = angular vel. of link AB about A]

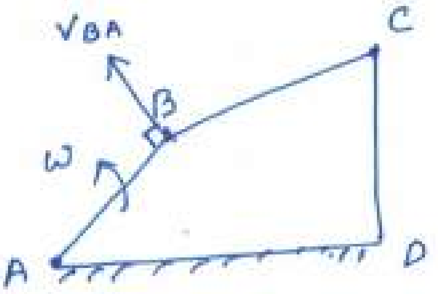
velocity of 'C' (any pt. on link AB) w.r.t. A (V_{CA}) will be $\perp^r$ to AC.

$$V_{CA} = \vec{ac} = \omega \times AC \quad \text{— ②}$$

From ① & ⑪

$$\frac{V_{CA}}{V_{BA}} = \frac{\vec{ac}}{\vec{ab}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB}$$

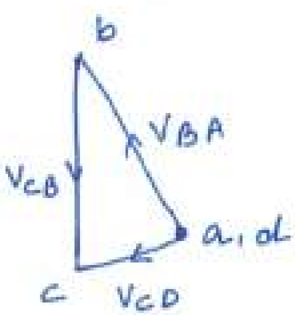
Velocity in 4-Bar Mechanism



Space dia.

$V_{BA} = \omega \times \text{Length } AB$
$= \omega \times AB$

$\because$ A is fixed, $\boxed{V_{BA} = V_B}$



Velocity dia.

Steps

(i) Take any pt. a

(ii) Draw $\vec{ab}$ $\perp^r$ to AB (V_{BA})

(iii) Fixed pts. (A, D) will coincide in velocity diag.

(iv) Draw perpend^r. to BC (V_{CB})

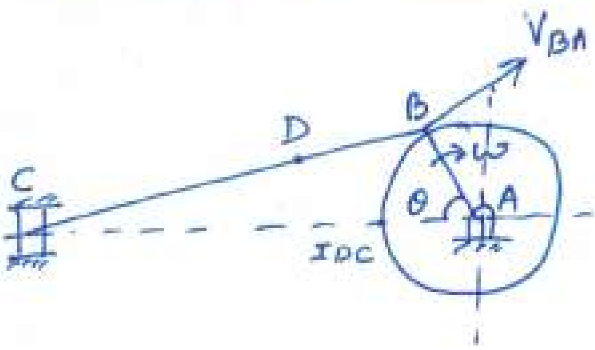
(v) Draw $\perp^r$ to CD, (V_{CB})

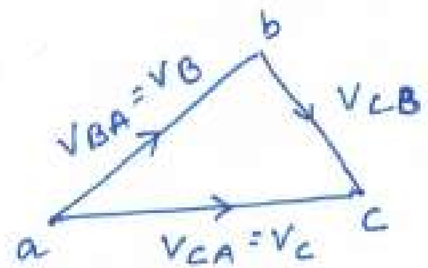
To find the angular vel. of BC & CD

$$\omega_{BC} = \frac{\text{vector } bc}{\text{length } BC}$$

$$\omega_{CD} = \frac{\text{vector } cd}{\text{Length } CD}$$

Velocities in Slider Crank Mechanism





N = speed of crank (BA) in rpm

ω = angular velocity $= \frac{2\pi N}{60}$

$$\boxed{V_{BA} (\text{or } V_B) = \omega \times AB}$$

$$\boxed{V_{BA} \perp^r AB}$$

V_{CA} remain horizontal

Absolute velocity of any pt 'D' on the connecting rod 'BC'

$$\frac{bd}{bc} = \frac{BD}{BC}$$

$$bd = \frac{BD}{BC} \times bc$$

Acceleration in Mechanism

Velocity ⇒ Accel^n ⇒ External force

⇓

Stress (at various pt. in mechanism)

are in addition to the stresses due to working load.

⇒ At very high speed, the forces & stresses due to ~~=~~ higher acc^n are more than stresses due to working load.

⇒ Acc^n $\propto$ sq. of speed

i.e, $\boxed{acc^n = v^2 \times r}$

∴

⇒ The acc^n diag. are fundamental to stress analysis of mechanism.

⇒ A body moving in circular path has two components of acc^n which are $\perp^r$ to each other. — ⓐ tangential acc^n & normal component of acc^n

Tangential Component $(f_t) = \boxed{r \times \alpha}$

$$f_t = \frac{dv}{dt} = \frac{d}{dt}(\omega \times r) = r \times \frac{d\omega}{dt} = r \times \alpha \quad \text{—(A)}$$

$\alpha \to$ ang. acc^n

Normal Comp. $(f_n) \Rightarrow$ normal to tangent.
It is directed towards centre of circular path. (also known as radial acc^n/ centripetal acc^n) $[f_n, f_c, f_r]$

$$f_n = \omega^2 r \text{ or } \frac{v^2}{r} \quad \text{—(B)}$$

$v = \omega \times r$
$\frac{d\theta}{dt} = \omega$

$$\text{Total acc}^n = f = \sqrt{f_t^2 + f_n^2} \quad \text{—(C)}$$

* Uniform Velocity. $\left(\frac{dv}{dt} = 0\right)$ $\therefore$ from A, $f_t = 0$
thus body has only normal/rad/ centripetal acc^n.

$$f_n = \frac{v^2}{r} = \omega^2 r$$

Total acc^n = radial/normal acc^n.

If body moves in straight path.
radius (r) will be infinity.

$$\therefore \frac{v^2}{r} = 0$$

$\therefore$ no normal acc^n, only tangential acc^n.

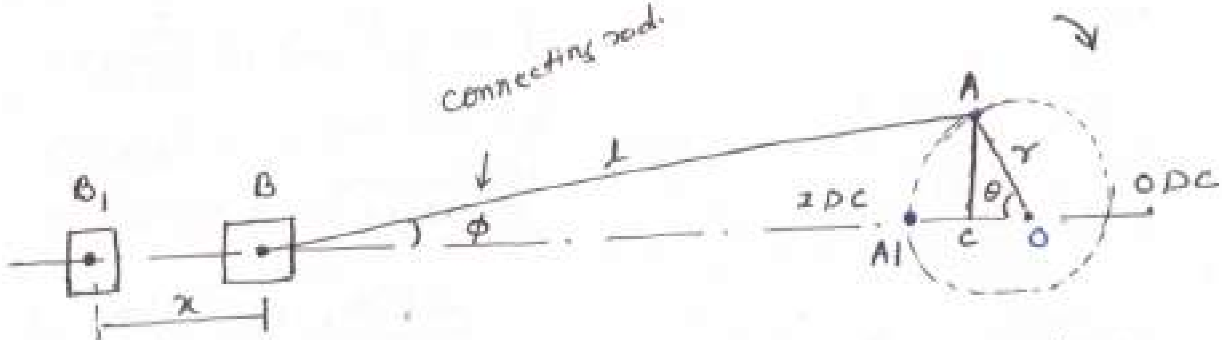
For body moving in straight path

$$\text{Total acc}^n = \alpha \times r$$

Body moving in circular path with uniform vel.

$$\boxed{\text{Total acc}^n = \omega^2 \times r}$$

Velocity and acceleration of reciprocating parts



crank is rotating with uniform angular velocity.

x = displacement of piston from inner dead centre
θ = angle turned by crank from " "
n = ratio of connecting rod length to crank radius $= l/r$

$\boxed{\text{At crank angle } \theta = 0}$ $A = A_1$, $B = B_1$
$A_1B_1 = l \rightarrow$ length of connecting rod
A_1O = length of crank.

~~When θ~~ = At θ crank angle.

$$x = B_1B = B_1O - BO$$
$$= (B_1A_1 + A_1O) - (BC + CO)$$
$$= (l + r) - (BC + CO)$$
$$= (l + r) - (AB\cos\phi + OA\cos\theta)$$
$$= (l + r) - (l\cos\phi + r\cos\theta)$$
$$= \frac{r}{r}\left[(l + r) - (l\cos\phi + r\cos\theta)\right]$$
$$= r\left[\left(\frac{l}{r} + 1\right) - \left(\frac{l}{r}\cos\phi + \cos\theta\right)\right]$$

$$= \boxed{r\left[(n+1) - (n\cos\phi + \cos\theta)\right]}$$

$\boxed{\text{convert } \phi \text{ into } \theta}$ in above equ^n.

$$\cos^2\phi + \sin^2\phi = 1$$

$$\Rightarrow \cos^2\phi = 1 - \sin^2\phi$$

$$\Rightarrow \cos\phi = \sqrt{1-\sin^2\phi}$$

In triangle ABC

$$AC = BA\sin\phi = l\sin\phi$$

In triangle ACO, $AC = r\sin\theta$

$$l\sin\phi = r\sin\theta$$

$$\Rightarrow \boxed{\sin\phi = \frac{r}{l}\sin\theta}$$

$$\therefore \cos\phi = \sqrt{1-\left(\frac{r}{l}\sin\theta\right)^2}$$

$$= \sqrt{1-\frac{r^2}{l^2}\sin^2\theta}$$

$$= \sqrt{1-\frac{\sin^2\theta}{n^2}}$$

$$= \sqrt{\frac{n^2-\sin^2\theta}{n^2}}$$

$$= \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\therefore x = r\left[(n+1) - \left(n \times \frac{1}{n}\sqrt{n^2-\sin^2\theta} + \cos\theta\right)\right]$$

$$= r\left[(n+1) - \left(\sqrt{n^2-\sin^2\theta} + \cos\theta\right]\right.$$

$$= r\left[(1-\cos\theta) + \left(n - \sqrt{n^2-\sin^2\theta}\right)\right]$$

If $n = \frac{l}{r} >> 1$

n^2 is very large, $\therefore \sqrt{n^2 - \sin^2\theta} \simeq \sqrt{n^2} = n$

$$\therefore x = r\left[(1-\cos\theta) + (n-n)\right]$$
$$= \boxed{r(1-\cos\theta)} \sim (2)$$

Eq (2) is an equ^n of simple harmonic motion. (only if connecting rod is very large)

<u>Velocity of Piston</u>

$$x = r\left[(1-\cos\theta) + (n + \sqrt{n^2 - \sin^2\theta})\right]$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} = \frac{dx}{d\theta}\cdot\omega$$ [ω = rate of change of angular displacement]

$$v = \frac{d}{d\theta}\left[r(1-\cos\theta) + (n - \sqrt{n^2-\sin^2\theta})\}\right]\cdot\omega$$

$$= r\left[(0+\sin\theta) + \left\{0 - \frac{1}{2}(n^2 - \sin^2\theta)^{-1/2}\cdot(0 - 2\sin\theta\cdot\cos\theta)\right\}\right]\cdot$$

$$= r\left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right]\cdot\omega$$

n^2 is very large compared to $\sqrt{n^2-\sin^2\theta}$ ~

then $\sqrt{n^2-\sin^2\theta} \approx n$

$$v = r\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]\cdot\omega$$

$$= r\omega\left[\sin\theta + \frac{\sin 2\theta}{2n}\right]$$

If n is very large $\frac{\sin 2\theta}{2n}$ becomes negligible

$$\boxed{v = r\omega\sin\theta}$$

Acceleration of piston

$$a = \frac{dv}{dt}$$

$$= \frac{dv}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left[r\omega\left(\sin\theta + \frac{\sin 2\theta}{2n} \right)\right] \cdot \frac{d\theta}{dt}$$

$$= r\omega\left(\cos\theta + \frac{2\cos 2\theta}{2n} \right) \cdot \omega.$$

$$= r\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

when $\theta = 0°$ ie crank is at IDC,

$$a = r\omega^2\left(1 + \frac{1}{n}\right)$$

when $\theta = 180° \longrightarrow$ ODC

$$a = r\omega^2\left(-1 + \frac{1}{n}\right)$$

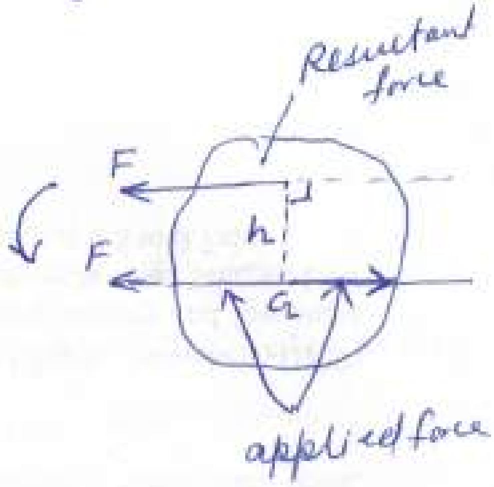
if 'n' is very large,

$$\boxed{a = r\omega^2 \cos\theta}$$

Equivalent offset Inertia forces

In graphical solⁿ, it is possible to replace inertia force & inertia couple by an equivalent offset inertia force which can account for both.

Effect of no. of forces acting on a rigid body.



Number of forces acting on a rigid body (not shown in Fig). The resultant of all these forces (F) is acting at a distance 'h' from the centre of Gravity.

Apply two equal & opposite forces at G.

Now the rigid body is acted upon by

ⓐ couple of magnitude $F \times h$ in CCW direction

b) Force 'F' ~~xx~~ passing through G

$\boxed{\text{Couple } F \times h}$ → causes angular acceleration of body about an axis passing through 'G' & perpendicular to plane of couple

$\boxed{\text{Force } F}$ → causes linear acceleration

$\boxed{T = C}$ Inertia couple

Couple $= I \times \alpha$

$F \times h = I \times \alpha$

$\Rightarrow F \times h = mk^2 \alpha$ ~~~ ⓐ

$F = ma$ ~~~ ⓑ

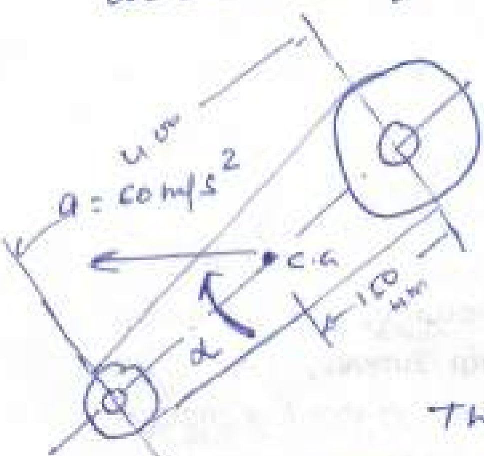
From ⓐ & ⓑ we can calculate linear accⁿ (a) & angular acceleration (α)

Magnitude of $F = ma$.

The line of action of 'F' is

$$h = \frac{I \times \alpha}{F} = \frac{mk^2\alpha}{F}$$

↓ is the distance of resultant force from centre of gravity of body. The direction of resultant force is obtained from the direction of linear acc^n & angular acc^n.



#1) Connecting rod of mass 12 kg and length 400 mm between the two centres. The C.G of connecting rod is at a distance of 150 mm from the centre of big end. The radius of gyration of connecting rod is 120 mm about an axis passing through C.G. Find the magnitude, direction & line of action of the resultant forces on connecting rod if the linear acceleration of the C.G of connected rod is 60 m/s² in the direction shown in dia. & angular acc^n of the rod is 100 rad/s² clockwise.

$m = 12$ kg.

$l = 400$ mm

Distance of C.G from big end = 150 mm = 0.15 m

$k = 120$ mm = 0.12 m

$a = 60$ m/s²

$\alpha = 100$ rad/s²

ⓐ Magnitude of resultant force

$F = m \times a = 12 \times 60 = 720$ N.

ⓑ Direction & line of action

The resultant force will act in the direction of linear acc^n.

To find the line of action of resultant force

$\frac{10144}{10000} \times 1$

$T = I \times \alpha$

$= mk^2 \times \alpha$

$= 12 \times 0.12^2 \times 100 \quad kg \times \frac{m^2}{} \times \frac{rad}{s^2}$

$= 12 \times 1.44 \; Nm$

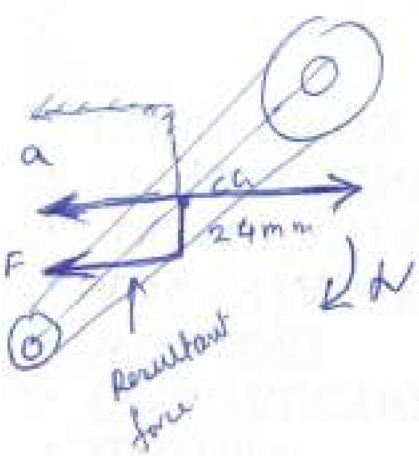
couple due to resultant force

$= F \times h = 720 \times h$

$F \times h = I \times \alpha$

$\boxed{h = 24mm}$

Resultant force must act in the downward direction at a distance of 24 mm from line of acceleration. Thus connecting rod will have angular acc^n clockwise.



Angular velocity and angular acceleration of connecting Rod

Angular velocity of crank = $\frac{d\theta}{dt}$ (ω)

" " connecting rod = $\frac{d\phi}{dt}$ (ω_c)

$$\boxed{\sin\phi = \frac{r}{l}\sin\theta} \sim (1)$$

differentiate (1) w.r.t. time.

$$\cos\phi \cdot \frac{d\phi}{dt} = \frac{r}{l}\cos\theta \cdot \frac{d\theta}{dt}$$

$$\cos\phi \cdot \omega_c = \frac{r}{l}\cos\theta \cdot \omega$$

$$\omega_c = \frac{r}{l}\frac{\cos\theta}{\cos\phi} \cdot \omega$$

$$\boxed{\cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}} \to$$ substitute this in above eqn.

$$\omega_c = \frac{r\omega}{l} \times \frac{\cos\theta}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}$$

$$= \frac{\omega}{n} \times \frac{\cos\theta}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}}$$

$$= \boxed{\frac{\omega\cos\theta}{\sqrt{n^2-\sin^2\theta}}}$$

α_c = Ang. acceleration of C.R.

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\omega_c}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \frac{d}{d\theta}\left(\frac{\omega\cos\theta}{\sqrt{n^2 - \sin^2\theta}}\right)\frac{d\theta}{dt}$$

$$= \omega\frac{d}{d\theta}\left[\cos\theta\cdot(n^2-\sin^2\theta)^{-1/2}\right]\cdot\omega \qquad \left[\frac{d\theta}{dt} = \omega\right]$$

$$= \omega^2\left[\cos\theta\cdot\left(-\tfrac{1}{2}\right)(n^2-\sin^2\theta)^{-3/2}(-2\sin\theta\cdot\cos\theta) + (n^2-\sin^2\theta)^{-1/2}(-\sin\theta)\right]$$

$$= \omega^2\left[\cos^2\theta\cdot\sin\theta(n^2-\sin^2\theta)^{-3/2} - \sin\theta(n^2-\sin^2\theta)^{-1/2}\right]$$

$$= \omega^2\sin\theta\left[\cos^2\theta(n^2-\sin^2\theta)^{-3/2} - (n^2-\sin^2\theta)^{-1/2}\right]$$

$$= \omega^2\sin\theta\left[\frac{\cos^2\theta}{(n^2-\sin^2\theta)^{3/2}} - \frac{1}{(n^2-\sin^2\theta)^{1/2}}\right]$$

$$= \omega^2\sin\theta\left[\frac{\cos^2\theta - (n^2-\sin^2\theta)}{(n^2-\sin^2\theta)^{3/2}}\right]$$

$$= \omega^2\sin\theta\left[\frac{\cos^2\theta + \sin^2\theta - n^2}{(n^2-\sin^2\theta)^{3/2}}\right]$$

$$= \omega^2\sin\theta\left[\frac{1-n^2}{(n^2-\sin^2\theta)^{3/2}}\right]$$

$$= -\omega^2\sin\theta\left[\frac{n^2-1}{(n^2-\sin^2\theta)^{3/2}}\right]$$

The -ve sign show that sense of acc^n of connecting rod is such that it reduce angle ϕ.

$$\alpha_c = \frac{d}{dt}\left(\frac{d\phi}{dt}\right) = \frac{d^2\phi}{dt^2}$$

If α_c is +ve → ϕ is increasing.
α_c = -ve → ϕ is reducing

The fig. shows that if ϕ increases, connecting rod is rotating anticlockwise & if ϕ is decreasing, CR rotates clockwise.

* In compare to (n^2) the value of $\sin^2\theta$ is very small. [max. value of $\sin^2\theta = 1$]

$$\therefore \; \omega_c = \frac{\omega\cos\theta}{n}$$

$$\alpha_c = \frac{-\omega^2\sin\theta(n^2-1)}{(n^2)^{3/2}} = \frac{-\omega^2\sin\theta(n^2-1)}{n^3}$$

$$\alpha_c = \frac{-\omega^2\sin\theta \times n^2}{n^3}$$ [as n^2 is very large compare to 1]

$$\boxed{\alpha_c = \frac{-\omega^2\sin\theta}{n}}$$

Acceleration diag. of Mech^n

Analytical

- Can be applied when displacement in terms of "time" is known.

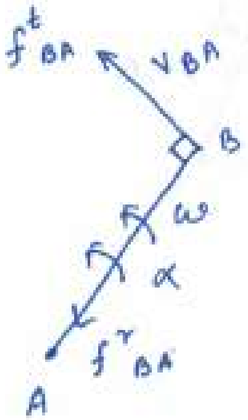
Graphical / acc^n diagram. suitable for

⇓

Practical cases

⇓ b'cos

expression for displacement cannot be determined easily.



$\Rightarrow f^r_{BA} = \frac{V^2_{BA}}{BA}$ or $\omega^2 \times BA$

↳ act along BA / parallel to BA

↳ due to angular vel.

$\Rightarrow f^t_{BA}$ = due to angular acc^n

↳ acts parallel to velocity.

↳ $\perp^r$ to AB

↳ $\alpha \times BA$.

$$f^t_{BA} = \alpha \times BA$$

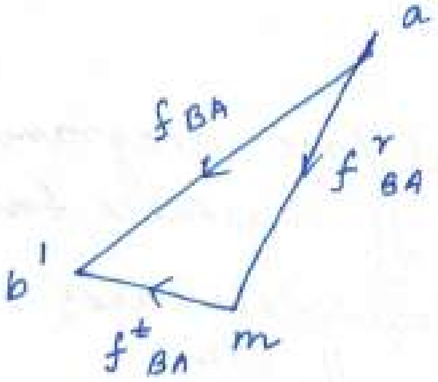
Total acc^n of B w.r.t. A is the vector sum of above two components.

$$f_{BA} = \text{vector sum of } \frac{V^2_{BA}}{BA} + \alpha . BA$$

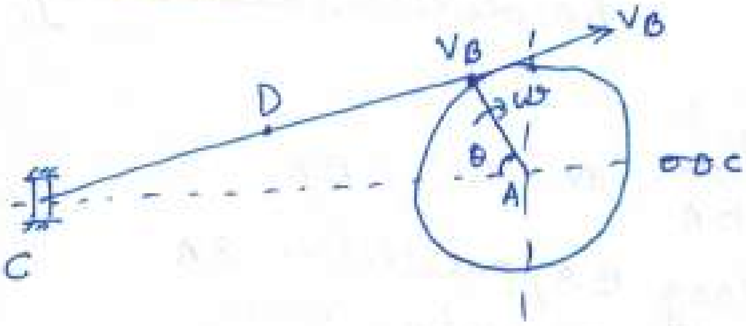
Acc^n Diag.

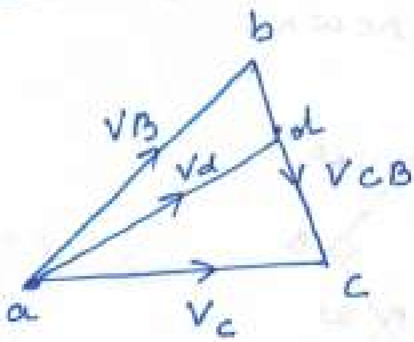
$$a'm = V^2_{BA} / BA$$

$$mb' =$$



Acc^n diag. of Slider Crank Mech





V.D

$$f^r_{BA} = \frac{V^2_{BA} / V_B^2}{AB}$$

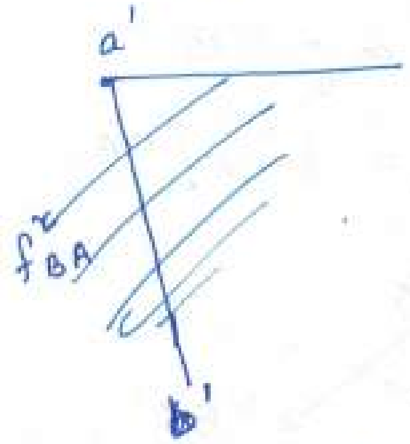
Point B is rotating with uniform angular velocity w.r.t. A, ∴ angular acc^n of B is zero.

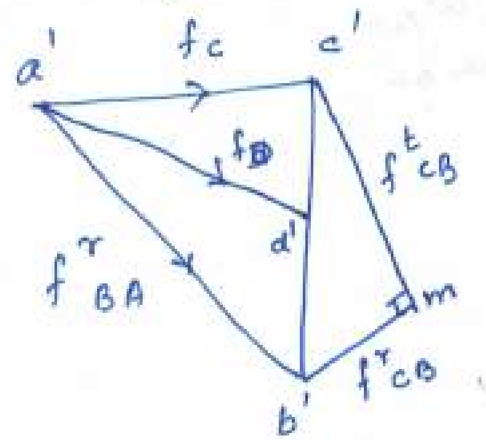
Tangential acc^n of B ($\alpha \times BA$) w.r.t. to A is zero.

$$f_{BA} \text{(total acc}^n) = f^r_{BA} = \frac{V^2_{BA}}{AB}$$

This acc^n act along BA. Total acc^n of B w.r.t. A is known.

Acc^n Diag.





a'b' ∥ ~~AB~~ BA

Pt. C has both radial & tangential comp. of acc^n w.r.t. B

Radial comp acts along CB, tangential is $\perp^r$ to BC

Draw b'm ∥ BC

$$b'm = f'_{CB} = \frac{V^2_{CB}}{BC}$$

Draw mc' $\perp^r$ to BC for tangential acc of C w.r.t. B (f^t_{CB})

Mag. of f^t_{CB} is not known.

Point C moves in St. line along CA.

$\therefore \quad f^r_{CA} = 0$

f^t_{CA} will be in direction of velocity C w.r.t. A

From a' draw a'c'.

vector mc' & a'c' intersect at c'

a'b'c' ⇒ acc^n dia. of S.C.M.

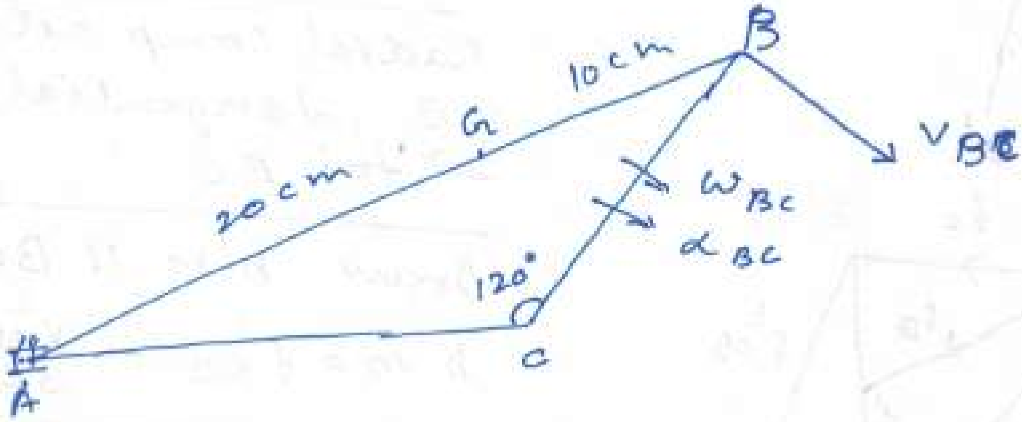
$$\frac{b'd'}{b'c'} = \frac{BD}{BC}$$

$$b'd' = \frac{BD}{BC} \times b'c'$$

Ang. acc^n of C.R CB

$$f^t_{CB} = \alpha_{CB} \times CB$$

$$\alpha_{CB} = f^t_{CB} / CB.$$



$\omega_{BC} = 75$ rad/sec

$\alpha_{BC} = 1200$ rad/sec^2

$CB = 10$ cm

Find (a) vel. & acc^n of G

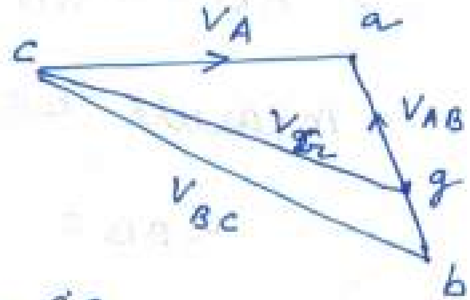
b) ang. vel. & ang. acc^n of AB

$$v_{BC} = \omega_{BC} \times BC = 75 \times 10 = 750 \text{ cm/s}$$
$$= 7.5 \text{ m/s.}$$

$\boxed{v_{BC} \perp^r BC}$

$$f^t_{BC} = \alpha_{BC} \times BC = 1200 \times 10 = 12000 \text{ cm/s}^2$$
$$= 120 \text{ m/s}^2$$

Vel. Diag.



Draw vector $cb \perp^r BC$

$cb = 7.5$ m/s.

$$\frac{bg}{ab} = \frac{BG}{AB}$$

$$bg = \frac{10}{30} \times ab = \frac{ab}{3}$$

Measure cg.

$cg = 6.8$ m/s.

$V_g = 6.8$ m/s

Angular Vel. of C.R.

$$\omega_{AB} = \frac{V_{AB}}{AB}$$

Measure $ba = 4$ m/s.

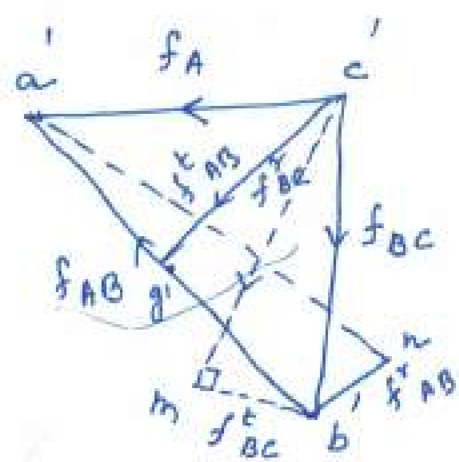
$$\therefore \omega_{AB} = \frac{4}{0.3} = 13.3 \text{ rad/s}$$

Acc^n

$$f^r_{BC} = \frac{V^2_{BC}}{BC} = \frac{7.5^2}{0.1} = 562.5 \text{ m/s}^2$$

$$f^r_{AB} = \frac{V^2_{AB}}{AB} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

$$f^t_{BC} = 120 \text{ m/s}^2$$



$$\frac{b'g'}{a'b'} = \frac{BG}{AB}$$

$$b'g' = \frac{BG}{AB} \times a'b' = \frac{10}{30} \times a'b' = \frac{a'b'}{3}$$

Measure $c'g' = 415$ m/s²

$f_G = 415$ m/s²

$f^t_{AB} = \alpha_{AB} \times AB$

measure $ra' = 550\ m/s^2 = f^t_{AB}$

$\alpha_{AB} = \frac{550}{0.3} = 1833.34\ rad/s^2$

Dynamically Equivalent System → is used to consider the weight of the connecting rod and inertia of the connecting rod. As the motion of connecting rod is not linear therefore it is difficult to find the inertia of the connecting rod.

To find the inertia of connecting rod (or any rigid body) it is used to replace the rigid body by two masses assumed to be concentrated at points and connected rigidly together. The two mass will be dynamically equal if.

$$m_1 + m_2 = m \quad \text{—(I)}$$

$$m_1 L_1 = m_2 L_2 \quad \text{—(II)}$$

$$m_1 L_1^2 + m_2 L_2^2 = mk^2 \quad \text{—(III)}$$

~ C.G of two masses coincides with rigid body.

$$m_1 = m - m_2 \quad \text{—(A)}$$

$$m_1 = \frac{m_2 L_2}{L_1} \quad \text{—(B)}$$

A = B

$$\Rightarrow m - m_2 = \frac{m_2 L_2}{L_1}$$

$$m = \frac{m_2 L_2}{L_1} + m_2$$

$$\Rightarrow m = \left[m_2\left(1 + \frac{L_2}{L_1}\right)\right]$$

$$= m_2\left(\frac{L_1 + L_2}{L_1}\right)$$

$$m_2 = \frac{mL_1}{L_1 + L_2} \quad \text{—(C)}$$

$$\therefore m_1 = (\text{from B}) = \frac{L_2}{L_1} \times m_2 = \frac{L_2}{L_1} \times \frac{mL_1}{L_1 + L_2}$$

$$= \frac{mL_2}{L_1 + L_2} \quad \text{—(D)}$$

$$m_1 L_1^2 + m_2 L_2^2 = mk^2$$

$$\Rightarrow \left[\frac{mL_2}{(L_1+L_2)}\right] \times L_1^2 + \left[\frac{mL_1}{(L_1+L_2)}\right] \times L_2^2 = mk^2$$

$$\Rightarrow \frac{mL_2L_1^2 + mL_1L_2^2}{L_1+L_2} = mk^2$$

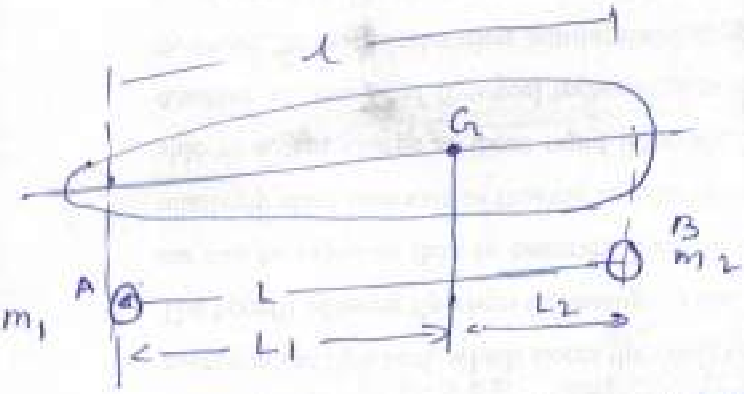
$$\Rightarrow \frac{mL_1L_2(L_1+L_2)}{L_1+L_2} = mk^2$$

$$\Rightarrow k^2 \; \cancel{m} L_1 L_2 \cancel{(L_1+L_2)} = \cancel{m}k^2 \cancel{(L_1+L_2)}$$

$$\Rightarrow K = \sqrt{L_1 L_2}$$

Equivalent length.

$$\boxed{L = L_1 + L_2}$$



of the two end masses one kept at the centre of two end bearing!

$$\boxed{BG = L_2}$$

Equivalent length of simple pendulum is

$L = L_1 + L_2$ is the equivalent length of rod is a suspended from the ~~end~~ point Ⓐ ~~a~~ ~~B~~ B is centre of oscillation

$$m_1 + m_2' = m$$

$$m_2' = \frac{mL_1}{L_1+L_2} = \frac{mL_1}{L}$$

$$m_1 = \frac{mL_2}{L_1+L_2} = \frac{mL_2}{L}$$

Intertia torque correction

~~$\Delta I = \alpha$~~ $T = I\alpha_c$

$$\Delta T = \alpha_c [mL_1L_2' - mL_1L_2)$$

$$= mL_1\alpha_c(L_2' - L_2)$$

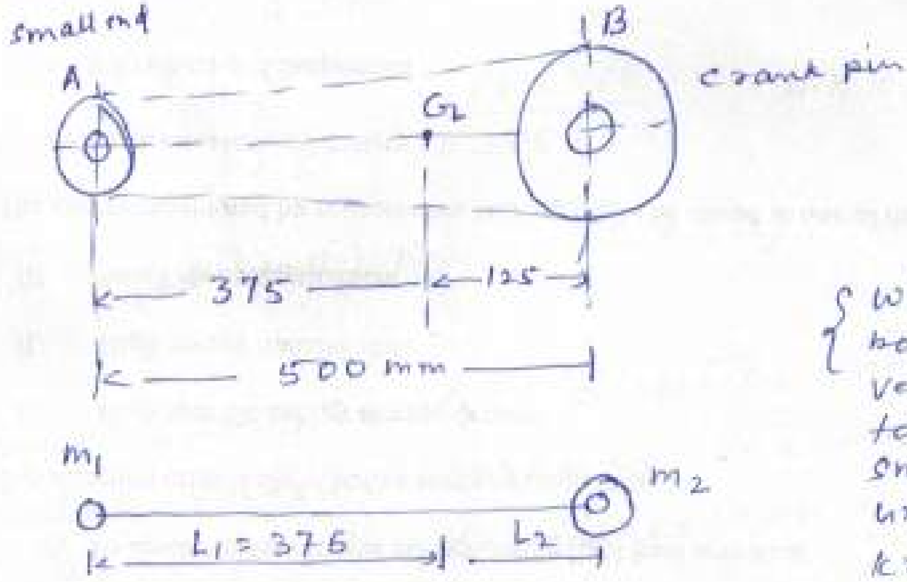
$$= mL_1\alpha_c\{(L_1'+L_1) - (L_1+L_2)\}$$

$$I' = mL_1L_2'$$

$$\therefore L_2' > L_2 \quad | \quad I' > I$$

dynamically Equivalent System

The length of connecting rod of an engine is 500 mm measured between the centres & its mass is 18 kg. The centre of gravity is 125 mm from the crank-pin centre & crank radius is 100 mm. Determine the dynamically equivalent system keeping one mass at small end. The frequency of oscillation of rod when suspended from the centre of small end is 43 vibrations per minute.



{ when a rigid body is suspended vertically is made to oscillate with small amplitude under gravity it is known as comp pendulum.

$r = 100$ mm (crank radius)

$L_1 = 375$ mm

No. of vibration of rod when suspended from end A $= 43/\text{min} = \frac{43}{60}$ vib/sec $= 0.716$ vib/sec.

$= 0.716$ cycles/sec

$k = ?$

$$n \text{ (freq. of oscillation)} = \frac{1}{2\pi}\sqrt{\frac{g \times h}{k^2 + h^2}}$$

h = distance of C.G & point of suspension

$$n = \frac{1}{2\pi}\sqrt{\frac{g \times L_1}{k^2 + L_1^2}}$$

$$\Rightarrow 0.716 = \frac{1}{2\pi}\sqrt{\frac{9.81 \times 0.375}{k^2 + (0.375)^2}}$$

$$\Rightarrow \boxed{k = 0.204 m}$$

As $L_1 \times L_2 = k^2$.

$$\Rightarrow L_2 = \frac{k^2}{0.375} = 0.111 m$$

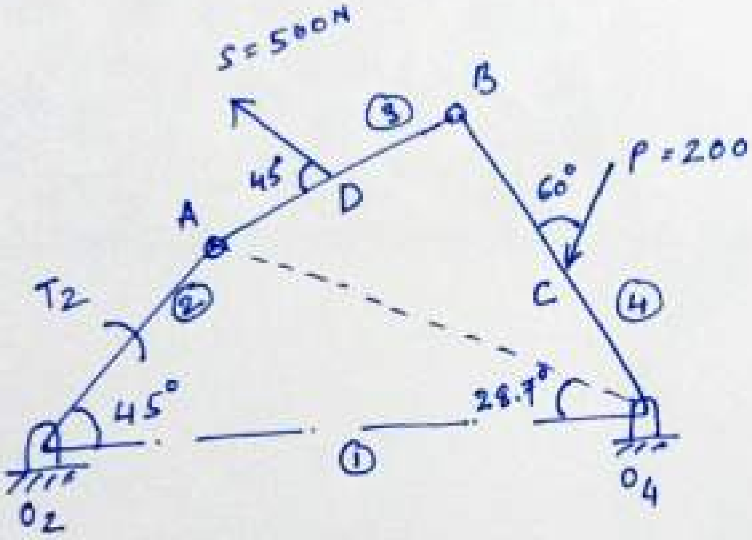
$$m_1 = \frac{m L_2}{L_1 + L_2} = \frac{18 \times 0.111}{0.375 + 0.111} = 4.11 kg.$$

$$m_2 = (18 - 4.11) kg = 13.89 kg.$$

* Correction couple (ΔT) direction is opp^n to applied inertia torque. Applied inertia torque is alway opposite to direction of angular acceleration.

Direction of correction torque & angular acceleration is same

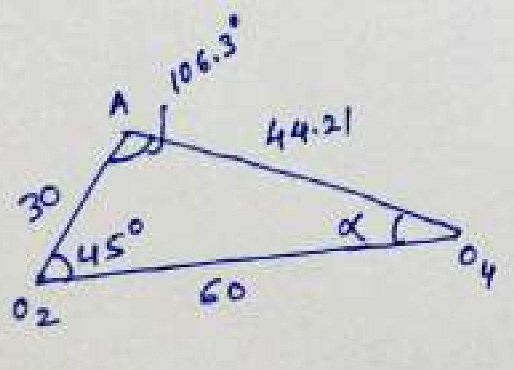
Determine torque T_2 required to keep mechanism in equilibrium



$O_2A = 30\,mm$
$AB = 30\,mm$
$O_4B = 30\,mm$
$O_2O_4 = 60\,mm$
$AD = 15\,mm$
$O_4C = 10\,mm$

$$AO_4 = \sqrt{O_2A^2 + O_2O_4^2 - 2\,O_2A \cdot O_2O_4 \cos 45^\circ}$$

$$= 44.21\,mm$$



Apply Sine law

$$\frac{O_2A}{\sin\alpha} = \frac{44.21}{\sin 45^\circ}$$

$$\boxed{\alpha = 28.7^\circ}$$

$$\angle O_2AO_4 = 106.3^\circ.$$

(2)

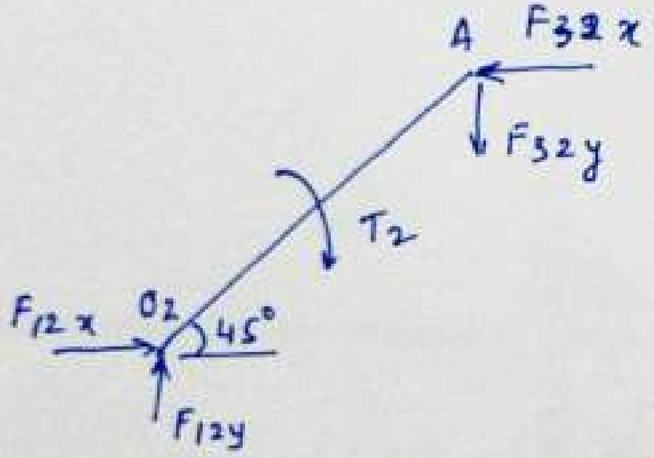
$\because$ (BAO_4) is an isosceles triangle

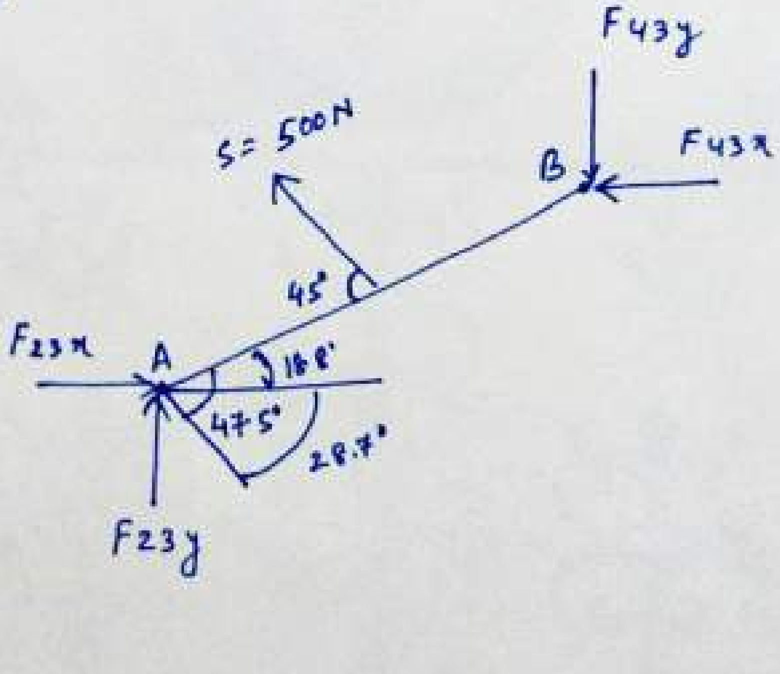
$AB = O_4B = 30\,mm$

$\therefore$ use cosine formula to find angle in ΔBAO_4 ($a^2 = b^2 + c^2 - 2bc\cos A$)

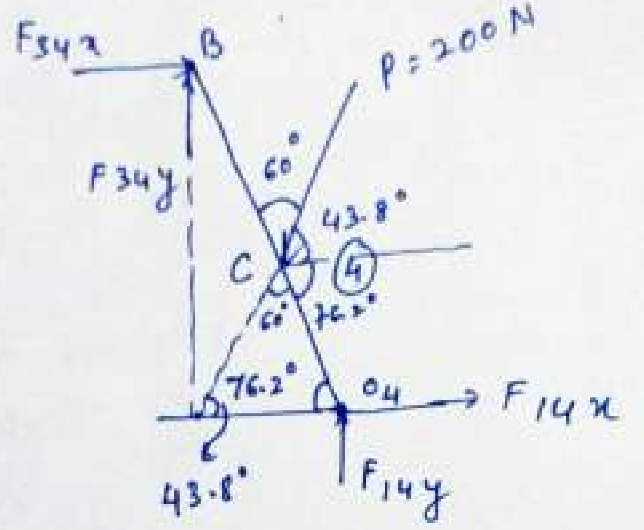
$\angle BAO_4 = 47.5^\circ$
$\angle BO_4A = 47.5^\circ$
$\angle ABO_4 = 85^\circ$

Now Draw Free Body diagram





Member-4



Force resolution in member (4)

$$\left.\begin{aligned} P_x &= 200 \cos 43.8° = 144.35\,N \\ P_y &= 200 \sin 43.8° = 138.43\,N \end{aligned}\right]$$

$\Sigma F_x = 0$ $(\rightarrow ve)$

$F_{14x} + F_{34x} - P_x = 0$

$$\boxed{F_{14x} + F_{34x} - 144.35 = 0}$$

$\Sigma F_y = 0$ $(\uparrow +ve)$

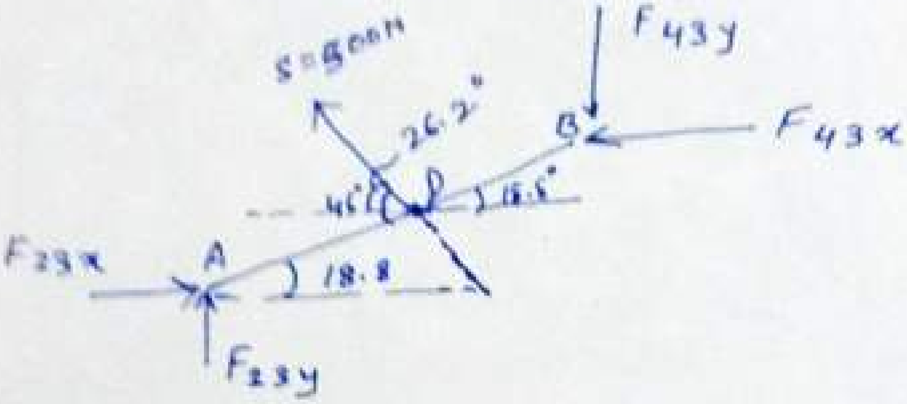
$F_{14y} + F_{34y} - 138.43 = 0$

$$\Rightarrow \boxed{F_{14y} + F_{34y} = 138.43}$$

Take moment about B. $\Sigma M_B = 0$ $(\circlearrowleft +ve)$

$$-P_y\, BC \cdot \cos 76.2° - P_x\, BC \sin 76.2° + F_{14y}\, 30 \cos 76.2°$$
$$+ F_{14x} \cdot 30 \sin 76.2° = 0 \quad \text{———} .$$

Member 3.



$$S_x = 500\cos 26.2° = 448.63\,N$$
$$S_y = 500\sin 26.2° = 220.75\,N$$

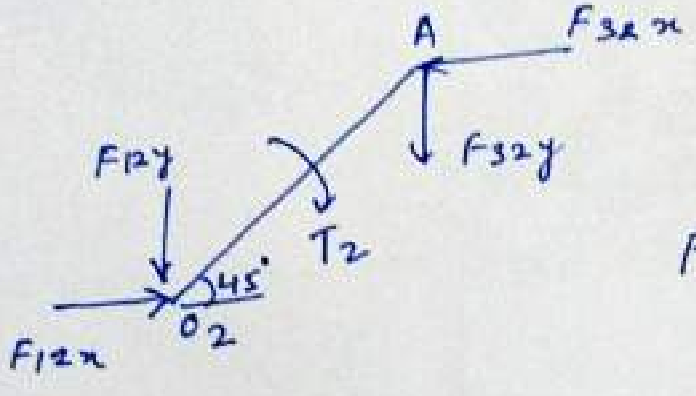
$$\Sigma F_x = F_{23x} - S_x - F_{43x} = 0$$
$$= \boxed{F_{23x} - F_{43x} = 448.63\,N}$$

$$\Sigma F_y = F_{23y} + S_y - F_{43y} = 0$$
$$\Rightarrow \boxed{F_{23y} - F_{43y} = -220.75}$$

$$\Sigma M_B = 0$$
$$F_{23x}\,AB\sin 18.8° - F_{23y}\,AB\cos 18.8° - S_x 15\sin 18.8°$$
$$- S_y \times 15\cos 18.8° = 0$$

$$F_{23x}\,30\sin 18.8° - F_{23}\,30\cos 18.8° - S_x 15\sin 18.8°$$
$$- S_y 15\cos 18.8° = 0$$

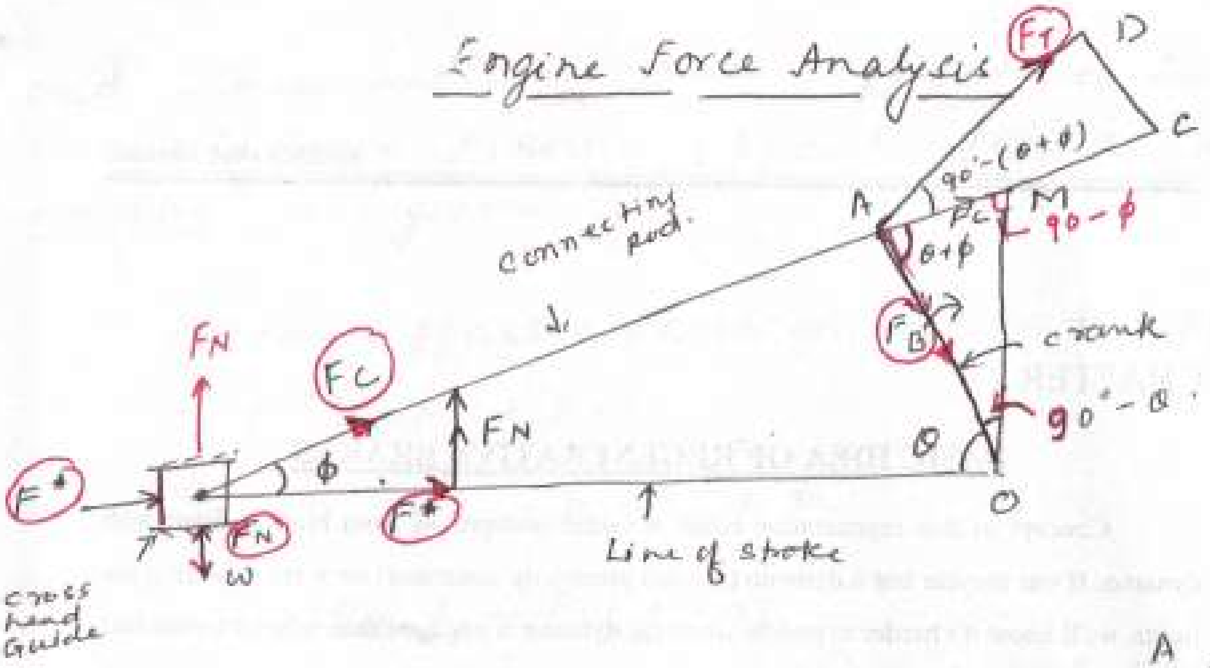
Member - 2

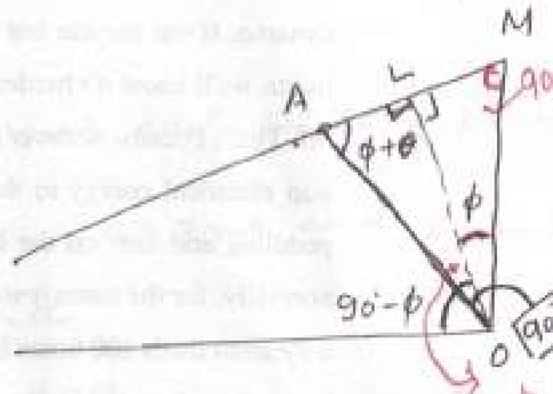


Moment about O_2

$F_{32x}\, O_2A \sin 45° - F_{32}\, O_2A$
$\cos 45° - T_2 = 0$

Engine Force Analysis





The forces are

(i) piston effort (F^*)

(ii) Force acting along connecting rod (F_C)

(iii) Thrust on the sides of cylinder wall or normal force (F_N)

(iv) Crank effort (F_T)

(v) Thrust on crank shaft bearing (F_B)

<u>Piston Effort</u> – It is also known as effective driving force. The piston is ~~ka~~ reciprocating with simple harmonic motion. During first half of the stroke, the reciprocating masses are accelerating. The inertia force due to acceleration of masses, opposes the force on the piston (due to steam or gas) & net forces decreases but during later half of the stroke, the reciprocting masses are retarding & inertia force opposes the retardation of inertia force act in the direction of applied gas pressure)

and thus increases the effective force on piston. Piston effort for a horizontal engine is given by

Piston Effort = Force on piston + Inertia force

$$F^* = F_P + F_i$$

$$= P_1 A_1 - P_2 A_2 + F_i$$

$$= P_1 A_1 - P_2 (A_1 - a)$$

P_1 = pressure on cover end (back end side of piston)

P_2 ——— piston end — (crank end side)

A_1 ——— area of cover end

A_2 ——— area of piston end.

a = cross sectional area of piston rod.

Force on piston $= F_P = P \times \frac{\pi}{4} \times D^2$

P = net pressure

D = diameter of cylinder

F_i = Inertia force on reciprocating parts

$= - m \times a$ [m = mass of piston, a = accn of reciprocating parts]

~~Force~~ $\left[\left(\cos\theta + \frac{\cos 2\theta}{n}\right) \omega^2 r\right]$

ω = uniform angular velocity

θ = angle of crank from IDC

$n = l/r$

of frictional resistance is considered (F^*)

$$\boxed{F^* = F_P + F_i - F_R}$$

For vertical Engine

$$\boxed{F^* = F_P + F_i \pm W - F_R}$$

W = weight of reciprocating parts (.mg)

In case of
F_P = -ve if piston is accelerated
F_i = +ve, if the piston is retarded.

W = [weight of reciprocating parts assist the piston effort during down stroke

Force on connecting Rod.

$$F_c \cos\phi = F^*$$

$$F_c = \frac{F^*}{\cos\phi}$$

$$\because \cos\phi = \frac{1}{n}\sqrt{n^2 - \sin^2\theta} = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$F_c = \frac{F^*}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

of frictional resistance is considered (F^*)

$$\boxed{F^* = F_P + F_i - F_R}$$

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$$F_C = \frac{F^*}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

Torque on crank-shaft / Turning moment on crank-shaft

T = crank effort × crank radius.

$$= F_T \times r$$

$$= F^* \frac{\sin(\theta+\phi)}{\cos\phi} \times r$$

$$= F^* \frac{(\sin\theta.\cos\phi + \cos\theta.\sin\phi)}{\cos\phi} \times r$$

$$= F^* \left(\sin\theta + \cos\theta.\tan\phi\right) \times r \quad \text{——— (A)}$$

$$\because \sin\phi = \frac{r}{l}\sin\theta = \frac{\sin\theta}{\left(\frac{l}{r}\right)} = \frac{\sin\theta}{n}$$

$$\cos\phi = \frac{1}{n}\sqrt{n^2-\sin^2\theta}$$

$$\tan\phi = \frac{\sin\theta/n}{\frac{1}{n}\sqrt{n^2-\sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2-\sin^2\theta}}$$

From (A)

$$T = F^* \left(\sin\theta + \cos\theta.\frac{\sin\theta}{\sqrt{n^2-\sin^2\theta}}\right) \times r$$

$$= F^* \left(\sin\theta + \frac{2\sin\theta.\cos\theta}{2\sqrt{n^2-\sin^2\theta}}\right) \times r$$

$$= F^* r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2-\sin^2\theta}}\right)$$

$\because \sin^2\theta$ is very small compare to n^2

$$T = F^* r \left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

From Fig. AOL

$$OL = OA \sin(\theta + \phi)$$
$$= r \sin(\theta + \phi) \quad \text{—— (B)}$$

In ΔOLM

$$OL = OM \cos\phi \quad \text{—— (C)}$$

From (B) & (C)

$$\boxed{r \sin(\theta + \phi) = OM \cos\phi}$$

$\because$ ~~$F_T =$~~

$$T = F_T \times r$$
$$= F^* \frac{\sin(\theta+\phi)}{\cos\phi} \times r$$
$$= F^* \frac{OM \cos\phi}{\cos\phi}$$
$$= F^* \times OM$$

#1) The length of the connecting rod of a vertical double acting steam engine is 1.5 m. The diameter of cylinder is 400 mm and stroke of engine is 600 mm. The crank is rotating at 200 rpm in clockwise. The crank has turned through 40° from top dead centre & piston is moving downward. The steam pressure above piston is 0.6 N/mm² and below the piston is 0.05 N/mm². The mass of reciprocating parts is 200 kg. The diameter of piston rod is given as 50 mm. Find the thrust on guide bars & crank-shaft bearing & also turning moment on crank-shaft.

$l = 1.5\,m$

$D = 400\,mm$

$r = \frac{\text{Stroke}}{2} = \frac{600}{2} = 300\,mm = 0.3\,m$

crank shaft radius

$N = 200\,rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 21\,rad/s.$

$\theta = 40^\circ$

$P_1 = 0.6\,N/mm^2 = 0.6 \times 10^6\,N/m^2$

$P_2 = 0.05\,N/mm^2 = 0.05 \times 10^6\,N/m^2$

$m = 200\,kg$

$d = 50\,mm = 0.05\,m.$

11-1

① Determine: F^*

$$F^* = F_p + F_i \pm W - F_R$$

$$= F_p + F_i \pm W$$

$$= F_p + \boxed{F_i} + W$$

$$=$$

$$F_p = P_1 A_1 - P_2 A_2$$

$$A_1 = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.4)^2 = 0.12566 \, m^2$$

$$\therefore A_2 = A_1 - a$$

$$= \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2$$

$$= 0.1237 \, m^2$$

$$F_p = P_1 A_1 - P_2 (A_1 - a)$$

$$= 0.6 \times 10^6 \times 0.12566 - 0.05 \times 10^6 \times 0.1237$$

$$\boxed{F_p = 69211 \, N}$$

$$F_i = -m \times a =$$

$$a = r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$n = \frac{l}{r} = \frac{1.5}{0.3} = 5$$

$$\boxed{= 105 \, m/s^2}$$

$$F_i = -200 \times 105 = -21000 \, N \quad | \quad W = m \times g = 200 \times 10 = 2000 \, N.$$

~~$F^* = 69211 + 105$~~ $F^* = 69211 - 21000 + 2000$

$$= 50{,}211 \, N$$

$F_N = F^* \tan\phi$

$\sin\phi = \frac{r}{l}\sin\theta$

$= \frac{0.3}{1.5}\sin 40^\circ = \underline{0.12855}$

$\boxed{\phi = 7.4^\circ}$

$\therefore\ F_N = 50{,}211 \times \tan 7.4^\circ$

$\approx \boxed{6{,}521\ N}$

(ii) $F_B = F^* \frac{\cos(\theta+\phi)}{\cos\phi} \simeq 34214.4\ N$

(iii) $T = F_T \times r$

$F_T = F^* \frac{\sin(\theta+\phi)}{\cos\phi}$

$= 37192.7\ N$

$\therefore\ T = 37192.7 \times 0.3$

$= \underline{11157.8\ Nm}$

Dynamically Eq. System

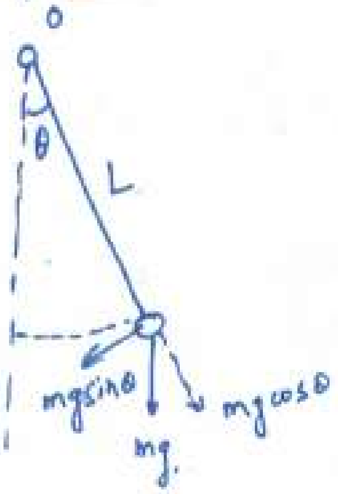
→ centre of mass of the equivalent link has same linear acc^n & link has same angular acc^n

$$F = m.a$$

$$F.d = I.\alpha$$

$$a = \frac{F}{m} \quad \Big| \quad \alpha = \frac{F.d}{I}$$

Simple & Compound Pendulum



Ang. acc^n of cord, $\alpha = \frac{T}{I} = \frac{mg\sin\theta L}{mL^2}$

$$\simeq \frac{mgL\theta}{L^2}$$

$$= \frac{mgL\theta}{mL^2}$$

$$= \frac{g}{L}\theta.$$

$$\frac{\alpha}{\theta} = \frac{g}{L} \simeq \text{const.}$$

Motion is SHM .

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

Compound Pend.

mass moment of inertia @ axis of suspension,

$$I = m(k^2+a^2)$$

Restoring couple

$$T = Wa\sin\theta$$
$$= mg\sin\theta . a$$

$$\alpha = \frac{T}{I} = \frac{ga}{(k^2+a^2)} \cdot \theta$$

$$\frac{\alpha}{\theta} = \frac{ga}{k^2+a^2} = \text{const.}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{ga}{k^2+a^2}}$$

Equivalent length of a simple pend. with same freq.

$$L = \frac{k^2+a^2}{a} = \frac{k^2}{a} + a.$$

#3) A single cylinder vertical engine has a bore of 300 mm and a stroke of 400mm. The connecting rod is 1m long. The mass of the reciprocating parts is 140 kg. On the expansion stroke with the crank at 30° from TDC the gas pressure is 0.7 MPa. If the engine runs at 250 rpm determine.

ⓐ net force acting on piston
b) resultant load on the gudgeon pin
c) thrust on the cylinder walls
d) the speed above which other thing remain same, the gudgeon pin load would be reversed in dirⁿ.

$D = 300mm = 0.3m$

Stroke: $400 mm \ (0.4m)$

crank radius $= \frac{0.4}{2} = 0.2mm$

$l = 1m$

$m_R = 140kg.$

$\theta = 30°$

$P = 0.7 MPa$
$= 0.7 \times 10^6 Pa.$

$N = 250 rpm$

$\omega = \frac{2\pi N}{60}$
$= 26.18 \ rad/s.$

$F^* = F_P + F_I + W$

$F_P = P \times A$
$= 0.7 \times \frac{\pi}{4} D^2 \times 10^6$
$= \boxed{49480 N}$

$F_I = -m \times a$
$= -140 \times r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$
$= \boxed{-18537.4 N}$

$W = m_R \times g = 140 \times 9.81 = \boxed{1373.4 N}$

$\therefore F^* = 32316. N$

(ii) Resultant load on gudgeon pin:

Two forces act on gudgeon pin

(a) F^* (net force)

(b) Normal reaction (F_N)

Resultant of F^* & F_N is the resultant load on gudgeon pin.

Resultant of F^* & F_N is the force acting along connecting rod (ie. F_c)

$$F_c = \frac{F^*}{\cos\phi}$$

$$l \sin\phi = r \sin\theta$$

$$\sin\phi = \frac{r}{l} \sin\theta$$

$$\sin\phi = \frac{\sin\theta}{n}$$

$$\sin\phi = 0.1$$

$$\phi = 5.739^\circ$$

$$F_c = \frac{32316}{\cos(5.739)}$$

$$= 32478.8 \text{ N.}$$

$\therefore$ Resultant load on gudgeon pin 32478.8

(iii) $F_N = F^* \tan\phi$

$= 3247.78N$

(iv) The gudgeon pin load is the force in connecting rod (F_c)

The gudgeon pin load would be reversed in dir^n if F_c is negative

$F_c = F^*/\cos\phi$

F^* will be negative if $F_i > F_p + W$

Let (N) is speed

$F_i = m \times a$

$= m_R \times r \times \omega^2 \times \cos\left(\theta + \frac{\cos 2\theta}{n}\right)$

$= 140 \times 0.2 \times \left(\frac{2\pi N_x}{60}\right)^2 \cos\left(30 + \frac{\cos 60}{5}\right)$

$= 0.2966\, N_x^{\,2}$

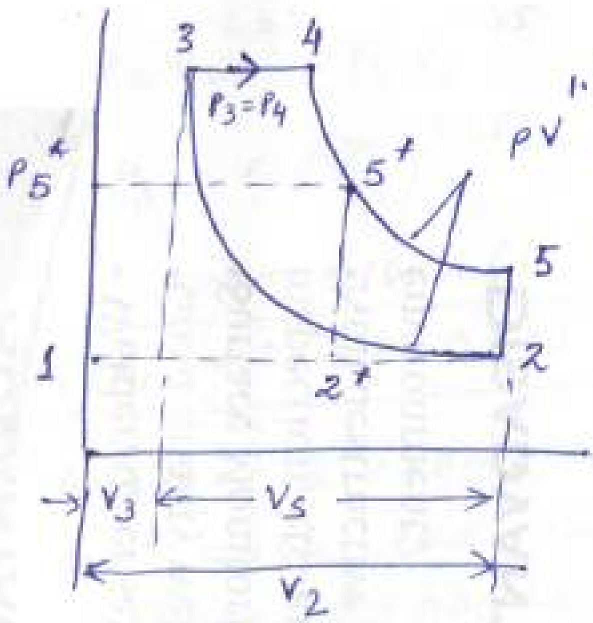
$0.2966\, N_x^{\prime 2} > 49480 + 1373.4$

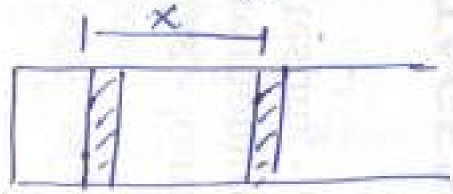
$N_x \geq \sqrt{\frac{50853.4}{0.2966}}$

$N' > 414.07$ rpm.

#4) The diameter of cylinder of a vertical single cylinder single acting diesel engine is 300 mm. The length of the crank and connecting rod are 250 mm & 1.125 m respectively. Reciprocating parts are having a mass of 140 kg & engine is running at 270 rpm. The ratio of compression is 14 & pressure remain constant during injection of oil for 1/10 th of the stroke. $pv^{1.35} = C$ for expansion & compression.

Find the torque on the crank-shaft when the crank makes an angle of 45° with the IDC during expansion stroke. Suction may be assumed at a pressure of 100 kN/m².





$D = 300\,mm = 0.3\,m$

$r = 250\,mm = 0.25$

$L = 2r = 2 \times 0.25$
$= 0.5\,m$

$l = 1.125\,m$

$m_R = 140\,kg.$

$N = 270\,rpm$

$\omega = \frac{2\pi N}{60} = 28.27\,rad$

Comp. Ratio $= \frac{V_2}{V_3} = 14$

$\theta = 45^\circ$

$P_1 = P_2 =$ (suction pressure) $= 100 \times 10^3\,N/m^2$

$V_S =$ Swept Vol $= \frac{\pi}{4} D^2 \times L = \frac{\pi}{4}(0.3)^2 \times 0.5$
$= 0.0353\,m^3$

$\frac{V_2}{V_3} =$ Comp Ratio $= \frac{V_3 + V_S}{V_3} = 1 + \frac{V_S}{V_3}$

$14 = 1 + \frac{V_S}{V_3}$

$\frac{V_S}{V_3} = 13.$

$V_3 = \frac{0.0353}{13} = 0.0027\,m^3$

Volume at the end of injection of oil (V_4)

$V_4 = V_3 +$ Vol. bet^n 3–4
$= 0.0027 + \frac{1}{10} \times V_S$
$= 0.00623\,m^3$

$$P_2 V_2^{1.35} = P_3 V_3^{1.35}$$

$$\Rightarrow P_3 = P_2\left(\frac{V_2}{V_3}\right)^{1.35}$$

$$= 100\times10^3 \times (14)^{1.35}$$

$$\boxed{P_3 = P_4 = 3.5259\times10^6\ N/m^2}$$

When the crank makes an angle of 45° with the inner dead centre during expansion stroke [process 4-5 expansion] let V_5^* correspond to expansion

$\theta = 45^\circ$, the displacement of piston (x)

$$x = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right]$$

$$= 0.25\left[(1-\cos 45^\circ) + \frac{\sin^2 45^\circ}{2\times4.5}\right] \quad \left\{ n = \frac{l}{r} = \frac{1.125}{0.2\ldots} \right.$$

$$= 0.25\left[(1-0.707) + \frac{0.5}{9}\right] = \boxed{0.08714\,m}$$

volume correspond to $[x]$

$$\frac{\pi}{4} D^2 x = \frac{\pi}{4}\times(0.3)^2\times 0.08714$$

$$= 0.00616\ m^3.$$

$V_5^* = V_3 + \text{Vol for displacement } x$

$$= 0.0027 + 0.00616 = 0.00886\ m^3$$

Expansion between 4 & 5*

$$P_4 V_4^{1.35} = P_5^* V_5^{*1.35}$$

$$P_5^* = 3.5259 \times 10^6 \times \left[\frac{0.00623}{0.00886}\right]^{1.35}$$

$$= 2.192 \times 10^6 \ N/m^2$$

When the crank is at $\theta = 45°$ during expansion stroke the pressure on one side of piston is P_5^* & other side is P

$$\therefore P_2^* = P_2 = 100 \times 10^3 \ N/m^2 = 0.1 \times 10^6 \ N$$

~~Force~~ Net pressure on piston $= p = P_5^* - P_2$

$$= (2.192 - 0.1) \times 10^6$$

$$= 2.092 \times 10^6 \ N/m^2$$

Force due to gas pressure

$$F_p = p \times \frac{\pi}{4} D^2$$

$$= (2.092 \times 10^6) \times \frac{\pi}{4} \times (0.3)^2$$

$$= 147874 \ N$$

$$F^* = F_p + F_I + W \qquad r\omega^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$F_i = -m \times a$$

$$= -140 \times (0.25) \times (28.27)^2 \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= -140 \times (0.25) \times (28.27)^2 \left[\cos 45° + \frac{\cos 90}{4.5}\right]$$

$$= -19776 \ N.$$

$$F^* = F_P + F_I + W$$

$$= \cancel{148}\ 147874 - 19776 + 1373.4$$

$$= 129471.4\ N$$

Torque on crankshaft

$$T = F_T \times r$$

$$F_T = F^* \frac{\sin(\theta + \phi)}{\cos\phi}$$

$$\boxed{\sin\phi = \frac{\sin\theta}{n}} \Rightarrow \phi = 9.04^\circ$$

$$F_T = 129471.4 \times \frac{\sin(45^\circ + 9.04^\circ)}{\cos 9.04^\circ}$$

$$= 106112.2\ N$$

$$\therefore \text{Torque} = F_T \times r = 106112.2 \times 0.25$$

$$= 26528.05\ Nm$$

Turning moment diagrams

Turning moment dia (crank effort dia) is the graphical representation of the turning moment or crank-effort for various positions of the crank. Generally the effect of ~~vate~~ inertia of connecting rod is neglected while drawing T-M diagram.

$$T = F_t \times r$$
$$= F^* r \times \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right]$$

F_T = tangential force (or force normal to crank).

T can be calculated if net force on piston (F^*) & θ are known.

T-M diagram for single-cylinder double acting steam Engine.

$$T = F_t^* r$$
$$= F_p \times r \left[\sin\theta + \frac{\sin 2\theta}{\sqrt{n^2 - \sin^2\theta}} \right]$$
$$= F_p \times r \left[\sin\theta + \frac{\sin 2\theta}{n} \right] \quad \boxed{n >> 1}$$

$\theta = 0, \quad M = 0$

$\theta = 90^\circ, \; M$ is max^m

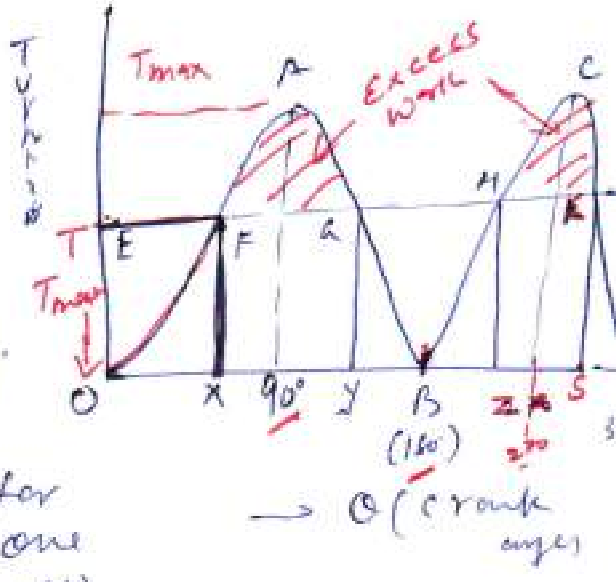
$\theta = 180^\circ \quad M = 0$

$\therefore$ work done $= \underline{T \times \theta}$

$\therefore$ work done / rev $= \underline{T \times 2\pi}$

But $\underline{T \times \theta}$ = area of T·M dig.

$T \times 2\pi$ = " " " " for one rev



work done / rev = Area of turning Moment diagram for one rev^n

$= \boxed{OABCD}$

Normally the engine is assumed to work against the uniform mean resisting torque.

$$\underline{T_{mean}} = \frac{\text{area } OABCD}{2\pi}$$

Area of rectangle $\underline{OEMD} = \underline{OE \times OD}$

$= T_{mean} \times 2\pi$

$=$ Area of T M diagram $OABCD$

$=$ work done/rev

Area of rectangle OEMD represents the work done against the mean resisting torque.

Let [T] is torque at any instant on the crank shaft.

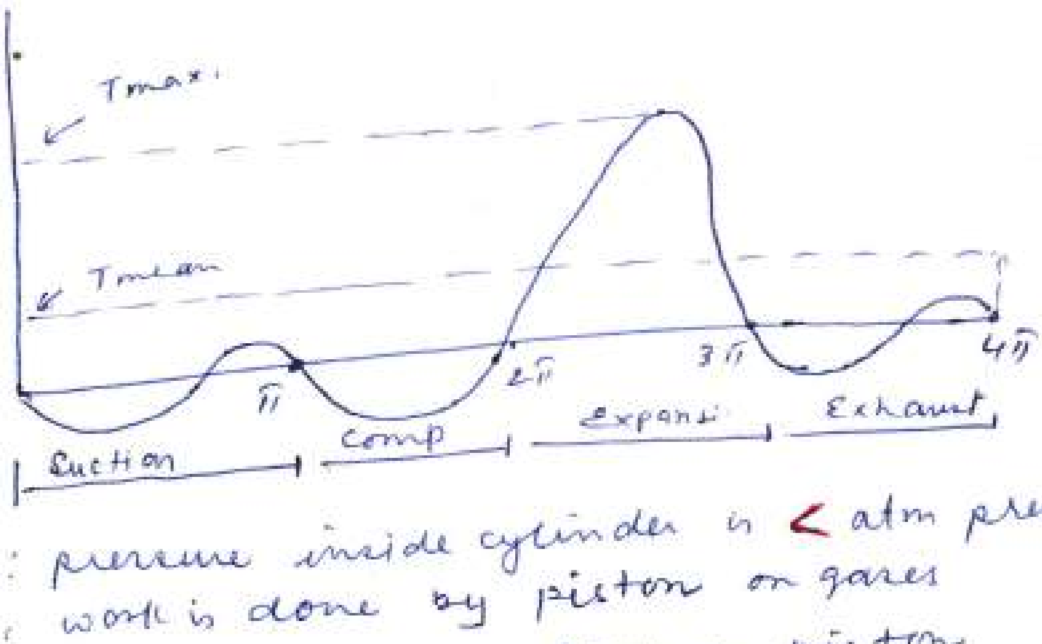
Tmean = mean resisting torque

T − Tmean = accelerating torque on the rotating parts of the engine (flywheel)

If T > Tmean, the flywheel accelera

<u>Turning</u> <u>moment</u> <u>diagram</u> <u>four stroke</u> <u>I.C. Eng.</u>

Turning moment diag. repeat itself after every ~~one~~ two revolution



Suction: pressure inside cylinder is < atm press

Comp : work is done by piston on gases

Expansion: " " " " gases on piston

Exhaust: " " " " piston on gases

Fluctuation of Energy and Fluctuation of speed of crankshaft

The torque which resist the rotation of the crank shaft is app. uniform. But the torque exerted on the crank shaft by steam/gas ~~is~~ fluctuate considerably. Hence there is unbalance torque which tends to either increase/decrease the speed of rotation of crankshaft.

The area of turning moment diag [OFx] But the area due to resisting torque or mean torque is OEFx.

Area of T.M. diagram = Work done by Eng

Area of Resisting Torque/mean torque = work required for external resistance.

i.e. OEFx > OFx [work done by engine is less than work required for extunal ~~work~~] resistance

The loss of work is made up by the flywheel which gives up some of its energy & speed of flywheel decreases.

Area XFAGY > XFGY (work done by Eng is > work reqd for external resistance

Excess work FAG is stored in flywheel & speed increase

Similarly for Y-Z

Area YGBHZ < YAHZ [Work Eng < Work reqd for ext. work resistance]

Speed of flywheel decreases

Z-S

Excess work HCK is stored in flywheel.

S-D = Loss of work equal to KDM is made by flywheel & speed decreases

The excess work or loss of work represented by diff areas are fluctuation of energy. The difference betn maxm & minm energies are maximum fluctuation of energy.

Crank Moves from X-Y, the flywheel starts absorbing energy when crank is at Y' the maxm energy (FAG) has been absorbed in flywheel. Speed ↑

Crank moves from Y-Z, flywheel starts giving energy. When crank is at Z' the maxm energy (GBH) has been given out by flywheel.

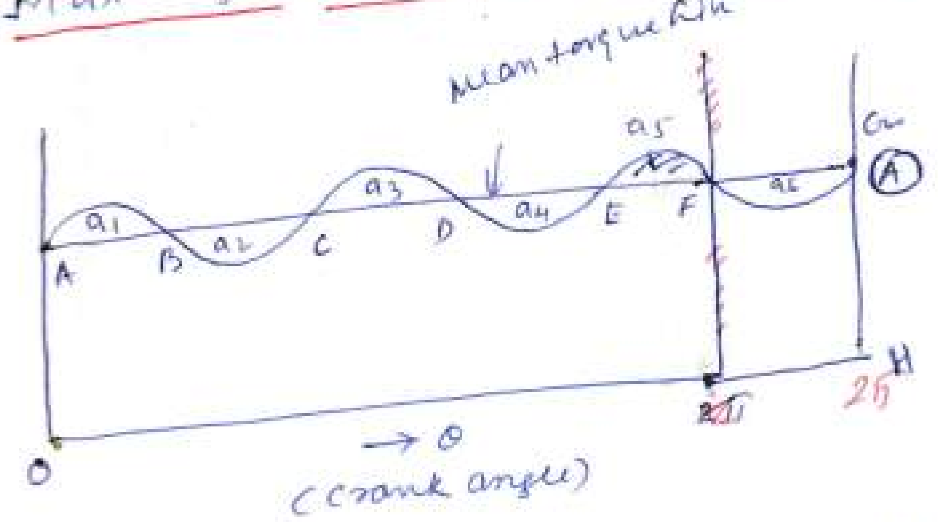
Speed of flywheel is maxm at S & minimum at X. There are two maximum & two minimum speed of crank shaft in one revolution.

The fluctuation of speed is the difference between the greatest & the lowest speed of crank-shaft for one ~~ve~~ revolution.

Co-efficient of Fluctuation of Energy.

$$K_e = \frac{\text{Max}^m \text{ fluctuation of energy}}{\text{work done per cycle.}}$$

Max^m fluctuation of energy (ΔE)



$a_1, a_3, a_5 \rightarrow$ energy added to flywheel

$a_2, a_4, a_6 \rightarrow$ —— taken from "

Let E = Energy in flywheel correspond to A

Energy at $B = E + a_1 = \underline{B}$

$C = \underline{E + a_1} - a_2 = B - a_2$

$D = \underline{E + a_1 - a_2} + a_3 = C + a_3$

$E = E + a_1 - a_2 + a_3 - a_4 = D - a_4$

$F = E + a_1 - a_2 + a_3 - a_4 + a_5 = E + a_5$

$G =$ ———————— $- a_6 = F - a$

NF17

#9) The equation of turning moment curve of a three ~~crank angle~~ crank engine is 2500+750 sin3θ Nm, θ is crank angle in radians. The mean speed of the engine is 300 rpm. The flywheel & other rotating ~~mass~~ parts attached to the engine have a mass of 500 kg at a radius of gyration 1m.

calculate ⓐ power of engine
ⓑ total fluctuation of the speed of flywheel in percentage when
⇒ ① resisting torque is constant
⇒ ⓘⓘ the resisting torque is 2500+ 300 sin θ.

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 300}{60} = 10\pi \text{ rad/s.}$$

$m = 500\,kg.$

$k = 1m$

$T = 2500 + 750 \sin 3\theta.$

Since the expression for torque is function of 3θ, the cycle is repeated after every 120° $\left(\frac{360}{3} = 120^\circ\right)$ or $\frac{2\pi}{3}$ radians of crank rotation.

Work done per cycle = Area of TM diag. for one cycle

$$= \int_0^{\frac{2\pi}{3}} T \times d\theta$$

$$= \int_0^{2\pi/3} (2500 + 750 \sin 3\theta)\, d\theta.$$

$$= \left[2500 \times \theta + 750 \times \left(\frac{-\cos 3\theta}{3}\right)\right]_0^{\frac{2\pi}{3}} = \left[2500 \times \theta - 250 \cos 3\theta\right]_0^{\frac{2\pi}{3}}$$

$$= 2500\left(\frac{2\pi}{3} - 0\right) - 250\left[\cos 3\left(\frac{2\pi}{3}\right) - \cos(3 \times 0)\right]$$

$$= \frac{5000\pi}{3} - 250\left[\cos 2\pi - \cos 0\right]$$

$$= \frac{5000\pi}{3} - 250\left[1 - 1\right]$$

$$= \frac{5000\pi}{3} \text{ Nm.}$$

Let Tmean = Mean Torque

Area of T-M diagram corresponding to mean torque for one cycle.

$= T_{mean} \times$ Length of one cycle in radn

$$= T_{mean} \times \frac{2\pi}{3}$$

$$T_{mean} \times \frac{2\pi}{3} = \frac{5000 \times \pi}{3}$$

$$T_{mean} = 2500 \text{ Nm.}$$

(i) Power $= T_{mean} \times \omega$

$= 2500 \times 10\pi \ \frac{Nm}{s}$

$= 78539.8$ W

(ii) Constant Resisting Torq

K_s = fluctuation of speed in %

∵ Resisting torque is constant

∴ Torque exerted on shaft = mean resisting torque on flywheel

$$T = T_{mean}$$

NF15

$T = T_{mean}$

$\Rightarrow 2500 + 750 \sin 3\theta = 2500$

$\Rightarrow \sin 3\theta = 0$

$3\theta = 0 \text{ or } 180°$

$\boxed{\theta = 0 \text{ or } 60°}$

Max^m fluctuation of energy.

$$\Delta E = \int_0^{60} (T - T_{mean})\, d\theta$$

$$= \int_0^{60} [(2500 + 750 \sin 3\theta) - 2500]\, d\theta$$

$$= \left[750 \left(\frac{-\cos 3\theta}{3}\right)\right]_0^{60}$$

$$= \left[-250 \cos 3\theta\right]_0^{60}$$

$$= -250 \left[\cos 180° - \cos 0°\right]$$

$$= 500 \text{ Nm.}$$

$\Delta E = I \omega^2 K_s$

$\Rightarrow 500 = m k^2 \omega^2 K_s$

$\Rightarrow K_s = 0.00101$

$\boxed{K_s = 0.101\%}$

When resisting Torque is $(2500 + 300 \sin\theta)$

Find the points at which the curve representing the torque exerted on the shaft intersect the resisting torque line. At intersection two torque are equal.

Torque on shaft = Resisting torque

$2500 + 750 \sin 3\theta = 2500 + 300 \sin\theta$

$\cancel{2.5(3\sin\theta - 4\sin^3\theta) = \sin\theta}$

$2.5 \sin 3\theta = \sin\theta$

$\Rightarrow 2.5(3\sin\theta - 4\sin^3\theta) = \sin\theta$

$\Rightarrow \dfrac{3\sin\theta - 4\sin^3\theta}{\sin\theta} = \dfrac{1}{2.5} = 0.4$

$\dfrac{\sin\theta(3 - 4\sin^2\theta)}{\sin\theta} = 0.4$

$3 - 4\sin^2\theta = 0.4$

$\boxed{\sin\theta = \pm 0.8062}$

$\theta = \cancel{126.28^\circ} / \cancel{306.28^\circ}$

Ans If $\sin\theta = 0.8062$
$\theta = 53.72^\circ$
$\theta = 126.28^\circ$

When $\sin\theta = -0.8062$
$\theta = 233.72^\circ$
$\theta = 306.28^\circ$

Since the cycle is repeated after 120°.

$\therefore \theta = 53.72°$ & $126.28°$

$$\Delta E = \int_{53.72°}^{126.28°} (\text{Torque on shaft} - \text{resisting torque})\, d\theta$$

$$= \int_{53.72}^{126.28} [(2500 + 750 \sin 3\theta) - (2500 + 300 \sin\theta)]\, d\theta$$

$$= \int_{53.72}^{126.28} (750 \sin 3\theta - 300 \sin\theta)\, d\theta$$

$$= \left[750\left(\frac{-\cos 3\theta}{3}\right) + 300 \cos\theta \right]_{53.72}^{126.28}$$

$= -828.24$ Nm

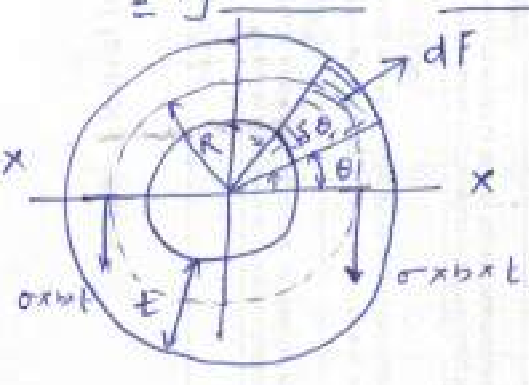
$\Delta E = I \omega^2 k_s$

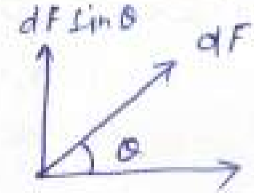
$= m k^2 \omega^2 k_s$

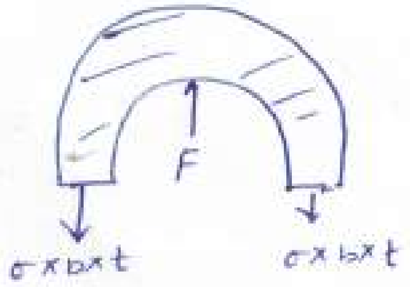
$-828.24 = m k^2 \omega^2 k_s$

$k_s = -0.00168 \approx 0.168\%$

Flywheel's Rim Dimensions







R = mean radius of rim

σ = tensile stress/hoop stress due to centrifugal force

$dF = \frac{mv^2}{R}$ [dF is centrifugal force]

$= \frac{m\omega^2 R^2}{R} = m\omega^2 \times R$

mass of element (δm) = $\rho \times [\overbrace{(R \times \delta\theta) \times t}^{\text{area of element}} \times b]$ ↳ vol of element

$dF = [\rho \times R \times \delta\theta \times t \times b] \times \omega^2 \times R.$

vertical component of this force is $dF \sin\theta$.

Total vertical upward force across x-x

$F = \int_0^{\pi} dF \sin\theta = \int_0^{\pi} [\rho R\, d\theta \times t \times b \times \omega^2 R] \sin\theta.$

$= \rho R^2 t b \omega^2 \int_0^{\pi} \sin\theta \cdot d\theta.$

$= \rho R^2 t b \omega^2 [-\cos\theta]_0^{\pi} = 2\rho v^2 bt$

This total vertical force tends to burst the rim across dia X–X. If σ is tensile stress/ hoop stress due to centrifugal force, then the resisting force is

$$\boxed{2\sigma \times b \times t}$$

$$2\sigma \times b \times t = 2\rho v^2 b \times t$$

$$\Rightarrow \boxed{v = \sqrt{\frac{\sigma}{\rho}}}$$

#1) A steam engine runs at 150 rpm. Its T.M.D gives following area moments taken in order above & below mean torque line.

500, −250, 270, −390, 190, −340, 270, −250 (all in sq. mm)

<u>Scale</u>: Turning moment: 1mm = 500Nm
Crank displacement : 1mm = 5°

If the total fluctuation of speed is 1.5% of the mean speed, determine the cross-section of the rim of the flywheel assumed rectangular with axial dimension equal to 1½ times the radial dimension. The hoop stress is limited to $3 N/mm^2$, $\rho = 7500 kg/m^3$

$N = 150$ rpm

$\omega = \frac{2\pi N}{60} = 5\pi$ rad/s.

1 mm = 500 Nm

1 mm = $5° = 5 \times \frac{\pi}{180} = \frac{\pi}{36}$ rad.

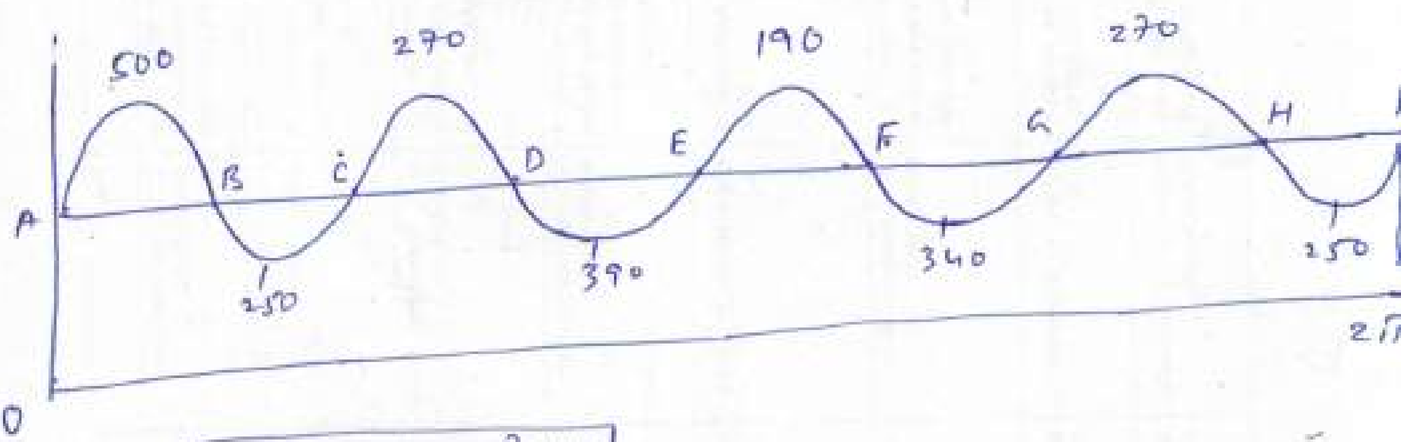
1 mm² of T.M. diagram = 1 mm (TMD) × 1 mm (crank angle)

$= 500 \times \frac{\pi}{36} = 43.63$ Nm.

$K_s = \frac{\text{Total fluctuation of speed}}{\text{Mean speed}} = 0.015$

$\boxed{b = 1.5t}$ — Given

hoop stress = 3 N/mm² = 3×10^6 N/m²



Let $\boxed{x = 43.63 \text{ Nm}}$

Let energy at A = E Nm.

Energy at B = 500x + E

= E + 500x − 250x = E + 250x

" " C = E + 250x + 270x = E + 520x

" " D = E + 520x − 390x = E + 130x

" " E = E + 130x + 190x = E + 320x

" " F = E + 320x − 340x = E − 20x

G = E − 20x + 270x = E + 250x

H = ...

$\Delta E =$ Max^m Energy − Min^m Energy.

$= (E + 520x) - (E - 20x) = 500x.$

$= 500 \times 43.63$ ~ (i)

$\Delta E = mv^2 k_s$ ~ (ii)

(i) = (2)

$500 \times 43.63 = mv^2 k_s$

$v = \sqrt{\frac{\sigma}{\rho}} = \sqrt{\frac{3 \times 10^6}{7500}} = 20 \text{ m/s}$

$\therefore m = \frac{500 \times 43.63}{20^2 \times 0.015} = 3635.8 \text{ kg}$

$m = \rho \times V$

$\Rightarrow 3635.8 = \rho \times [\pi D \times \text{Area of cross section}]$

$\Rightarrow 3635.8 = \rho \times [\pi D \times b \times t]$

$\Rightarrow 3635.8 = \rho \times [\pi D \times 1.5 t^2]$ — (iii)

$v = \frac{\pi D N}{60}$

$20 = \frac{\pi \times D \times 150}{60}$

$\underline{D = 2.546 \text{m}}$

$3635.8 = 7500 \times [\pi \times 2.546 \times 1.5 t^2]$

$\Rightarrow t = 0.201 \text{m}$

$= \underline{201 \text{mm}}$

$b = 1.5 \times t = 301.5$ m.

Co-efficient of fluctuation of speed

$$k_s = \frac{\text{Max}^m \text{ fluctuation of speed}}{\text{Mean speed}}$$

$$= \frac{\text{diff bet}^n \text{ max}^m \text{ \& min}^n \text{ speed / cycle}}{\text{Mean speed}}$$

Max^m speed corresponds to max^m K.E. enrgy in flywheel.

Min^n speed correspond to min K.E.

N_1 = max^m speed in rpm during cycle
N_2 = min^m " " "

$$N = \frac{N_1 + N_2}{2}$$

$\omega_1, \omega_2 \,\&\, \omega$ = angular speed.

$$k_s = \frac{\omega_1 - \omega_2}{\omega} = \frac{N_1 - N_2}{N}$$

$$= \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \left[\omega_1 = \frac{2\pi N_1}{60} \right.$$

$\boxed{k_s \text{ decreases as } \omega_1 - \omega_2 \text{ decreases}}$

But there is no point in decreasing / increasing the value to max^m/min^n.
The value of $[k_s]$ depends on the purpose for which the engine is to be used.

Purpose	K_s
Engine driving agricultural machinery	0.05
" " workshop shafting	0.03
" " weaving & spinning m/c	0.02 - 0.01
" " direct current generator	0.006

Co-efficient of fluctuation of energy (K_e)

$$\boxed{K_e = \frac{\text{Excess energy betn max}^m \text{ \& min}^n \text{ speed}}{\text{Indicated work done per cycle}}}$$

Flywheel

(a) Limits the fluctuation of speed during each cycle.

(b) Absorbs energy when Turning moment is greater than resisting moment & gives out the energy when resisting moment is greater than turning moment.

(c) It regulate the speed over short intervals of times. It tends to keep the speed within the required limits from revolution to revolution.

Let. K = K.E of flywheel at mean speed.

$$\boxed{E = \frac{1}{2} I \omega^2 = \frac{1}{2} m k^2 \omega^2}$$

ΔE = Max^m fluctuation of energy

$$\boxed{\Delta E = K.E_{max} - K.E_{min}}$$

$$= \frac{1}{2} I \omega_1^2 - \frac{1}{2} I \omega_2^2$$

$$= \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2) \quad \left[\frac{\omega_1 + \omega_2}{2} = \omega \right]$$

$$= I \times \omega (\omega_1 - \omega_2)$$

$$= I \times \omega^2 \times \frac{\omega_1 - \omega_2}{\omega}$$

$$= I \times \omega^2 \times K_s$$

$$= \boxed{\frac{1}{2} I \omega^2} \times 2 K_s = E \times 2 K_s = \boxed{2 E K_s}$$

Case II

$$\boxed{\begin{aligned}\Delta E &= I \times \omega^2 \times K_s \\ &= mK^2 \times \omega^2 \times K_s\end{aligned}}$$

If the thickness of the rim of the flywheel is very small compared to the diameter of the flywheel, then K = mean radius of flywheel

$$\Delta E = mK^2 \times \omega^2 \times K_s$$
$$= m r^2 \omega^2 K_s$$
$$= m V^2 K_s \quad [r\omega = V]$$

* Flywheel reduces the speed fluctuation during a cycle for a constant load i.e. it controls inside cycle fluctuations only due to ~~vibration~~ variation in turning moment.

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* If the material & mass of different flywheels are same, then the flywheel which have maximum radius of gyration will have the max^m mass moment of inertia about the axis of rotation passing through C.G. For max^m ~~moment of inertia~~ radius of gyration more material should be present at the periphery & less material should be at the centre.

NFI

Numericals

#1) The max^m & min^m speed of a flywheel are 242 rpm & 238 rpm respectively. The mass of flywheel is 2600 kg & radius of gyration is 1.8 m. Find
(a) mean speed of flywheel
(b) max^m fluctuation of energy.
(c) co-eff. of fluctuation of speed.

(i) $N = \frac{N_1 + N_2}{2} = 240$ rpm

(ii) $\Delta E = \frac{1}{2} I \times \omega_1^2 - \frac{1}{2} I \times \omega_2^2$

$= \frac{1}{2} \times 8424 \times (25.34)^2 - \frac{1}{2} \times 8424 \times (24.92)^2$

$= \boxed{88911 \text{ Nm}}$

$I = mk^2 = 2600 \times 1.8^2 = 8424 \text{ kg m}^2$

$\omega_1 = \frac{2\pi N_1}{60} = \frac{2 \times \pi \times 242}{60} = 25.34$ rad/s

$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 238}{60} = 24.92$ rad/s.

(iii) $K_s = ?$

$\Delta E = 2 E K_s$

$K_s = \frac{\Delta E}{2E}$

$E = \frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 8424 \times (25.13)^2$ $\left[\omega = \frac{\omega_1 + \omega_2}{2} = 25.13 \text{ rad/s} \right]$

$= 2659949$ Nm

$K_s = \frac{88911}{2 \times 2659949}$

$= 0.0167$

NF2

$$k_s = \frac{\omega_1 - \omega_2}{\omega} = \frac{25.34 - 24.92}{25.13} = \underline{0.0167}$$

#2) Find the max^m & min^m speed of flywheel of mass 5200 kg and radius of gyration 1.8 m when the fluctuation of energy is 100800 Nm. The mean speed of engine is 180 rpm.

$m = 5200$ kg.

$k = 1.8$ m

$\Delta E = 100800$ Nm.

$N = 180$ rpm.

$$\omega = \frac{2\pi N}{60} = 18.85 \text{ rad/s.}$$

$$\omega = \frac{\omega_1 + \omega_2}{2} = 18.85$$

N_1 = max speed

N_2 = min speed.

$$\Delta E = \frac{1}{2} I (\omega_1^2 - \omega_2^2)$$

$$= I \times \omega (\omega_1 - \omega_2)$$

$$= I \times 18.85 (\omega_1 - \omega_2)$$

$$= mk^2 \times 18.85 (\omega_1 - \omega_2)$$

$$N_1 = \frac{60\omega_1}{2\pi} = 181.5 \text{ rpm}$$

$$N_2 = 178.5 \text{ rpm.}$$

$$100800 = 5200 \times (1.8)^2 \times 18.85 \times (\omega_1 - \omega_2)$$

$$\left.\begin{aligned} \omega_1 - \omega_2 &= 0.3174 \\ \omega_1 + \omega_2 &= 37.70 \end{aligned}\right] \quad \begin{aligned} \omega_1 &= 19.0087 \text{ rad/s} \\ \omega_2 &= 18.6913 \text{ rad/s.} \end{aligned}$$

NF3

#3) A gas engine working on otto cycle is provided with two flywheels each weighing 580 kg & radius of gyration 52 cm. The diameter of the cylinder is 24 cm, stroke 27 cm & mean speed 250 rpm. The mean pressure during the cycle are

Suction : atm.
Comp : $1.06\ kg/cm^2$
Firing : $6.2\ kg/cm^2$
Exhaust : $0.3\ kg/cm^2$

If resistance is constant find percentage var^n of speed of engine.

Work done during firing stroke

$$\frac{\pi}{4} d^2 \times l \times P_1$$

$$\Rightarrow \frac{\pi}{4}(24)^2 \times (0.27) \times \cancel{1.06}\ 6.2.$$

$$= 757\ kgf\ m.$$

Work expanded during other stroke

$$= (P_2 + P_3)\ a\ l$$

$$= (1.06 + 0.3) \times \frac{\pi}{4}(24)^2 \times 0.27$$

$$= 166\ kgf\ m.$$

Net work done $= 757 - 166\ kgf\ m$

$= 591\ kgf\ m.$

Since the number of stroke in a cycle is 4, work done ~~is~~ per stroke

$= \frac{1}{4}(591)$

$E_f = \Delta E$ = fluctuation of energy

= (work done during power stroke) − ($\frac{1}{4}$ work done in a cycle)

$= 757 - \frac{591}{4} = 609$ kgf m.

But $\boxed{\Delta E = I K_s \omega^2}$

$\frac{W}{g} k^2 K_s \omega^2 = 609$

$\Rightarrow \frac{2 \times 580}{g} \times (0.52)^2 \times K_s \times \left(\frac{\pi \times 250}{30}\right)^2 = 609$

$\boxed{K_s = 2.8\%}$

1.4% above & below the mean speed

NF5

#5) The speed of an engine varies from 210 rad/s to 190 rad/s. during cycle the change in K.E is 400 Nm Find the inertia of flywheel in ~~N-m~~ kg-m²

$\omega_{max} = 210$ rad/s

$\omega_{min} = 190$ rad/s

$\Delta E = 400$ N-m

$K_s = \dfrac{\omega_{max} - \omega_{min}}{\omega}$

$= \dfrac{210-190}{\frac{210+190}{2}} = \dfrac{210-190}{200} = \underline{0.1}$

$\Delta E = I\omega^2 K_s$

$400 = I \times (200)^2 \times 0.1$

$\boxed{I = 0.10 \text{ kg-m}^2}$

NFG

#6) The radius of gyration of flywheel is 1 metre & the fluctuation of speed is not to exceed 1% of the mean speed of the flywheel. If the mass of the flywheel is 3340 kg & steam engine develops 150 kW at 135 rpm, then find

a) Max^m fluctuation of energy.

b) Co-efficient of fluctuation of energy.

$K = 1m$ — (radius of gyration)

fluctuation of speed = 1% of mean speed

$$\boxed{\frac{\omega_1 - \omega_2}{\omega} = 0.01}$$

$\underline{K_S = 0.01}$;

$$\Delta E = mK^2 \omega^2 K_S \qquad \left[\omega = \frac{2\pi \times 135}{60} = 14.137 \right]$$

$$= 3340 \times 1^2 \times (14.137)^2 \times 0.01$$

$$= \underline{6675.13 \text{ Nm}}$$

$$K_E = \frac{\text{Max}^m \text{ fluctuation of energy}}{\text{work done per cycle}}$$

$$= \frac{6675.13}{T_{mean} \times \theta} = \frac{6675.13}{T_{mean} \times 2\pi} = \frac{6675.13}{10610.45 \times 2\pi}$$

$$= \underline{\frac{6675.13}{66667.42} = 0.1}$$

$P = T_{mean} \times \omega$

$$T_{mean} = \frac{P}{\omega} = \frac{150000}{14.137} = \underline{10610.45 \text{ Nm}}$$