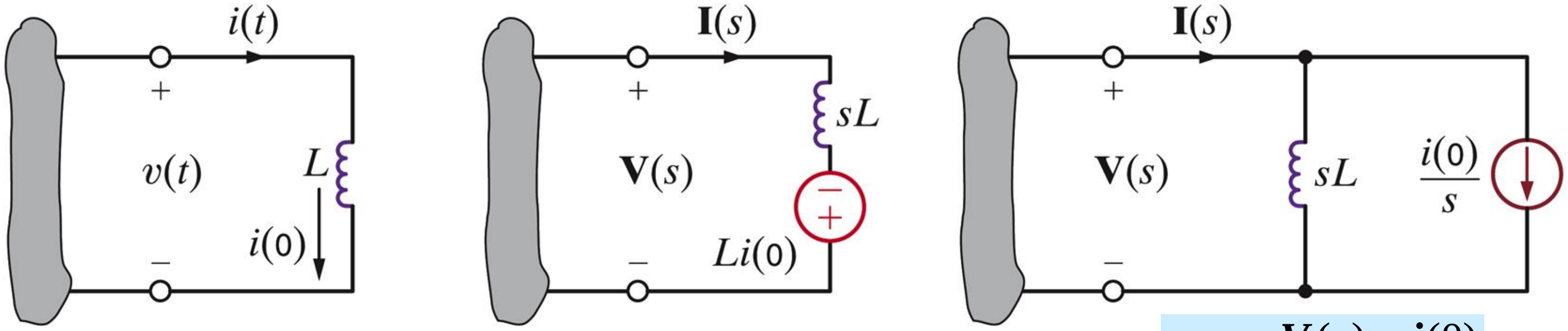
Inductor Models



$$v(t) = L\frac{di}{dt}(t) \Rightarrow V(s) = L(sI(s) - i(0))$$

$$I(s) = \frac{V(s)}{Ls} + \frac{i(0)}{s}$$

$$\mathcal{L}\left[\frac{di}{dt}\right] = sI(s) - i(0)$$

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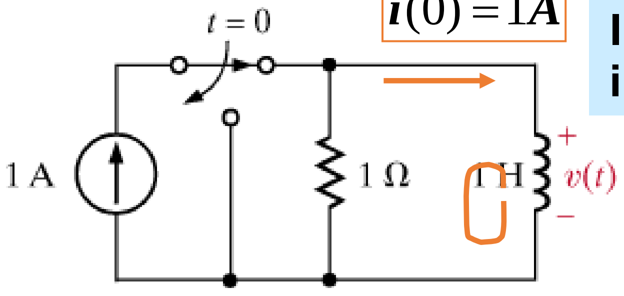
LEARNING BY DOING

Determine the model in the s-domain and the expression for the voltage across the inductor

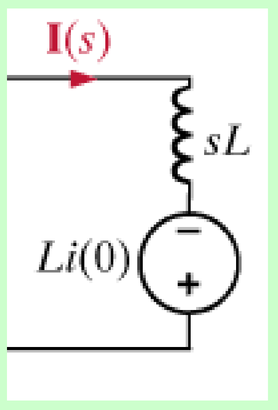
Steady state for t<0

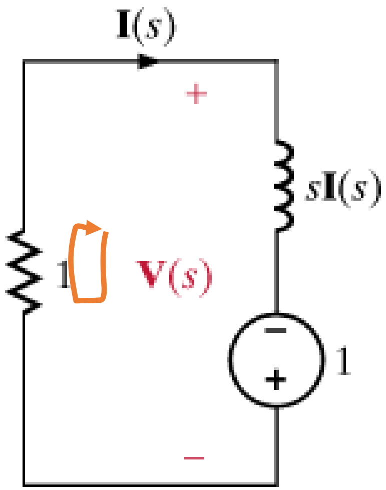
$i(0) = 1A$

Inductor with initial current



(a)





KVL: $1 = (1+s)I(s)$

Ohm's Law

$$V(s) = -1 \times I(s) \Rightarrow V(s) = -\frac{1}{s+1}$$

Equivalent circuit in s-domain

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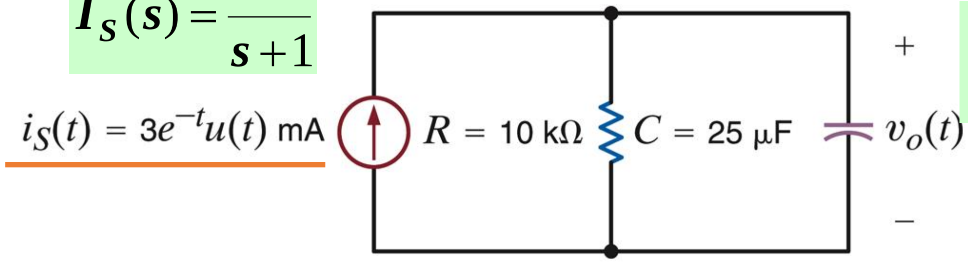
ANALYSIS TECHNIQUES

All the analysis techniques are applicable in the s-domain

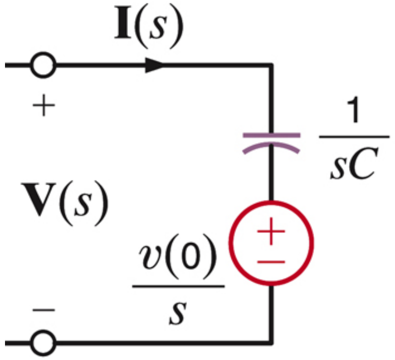
LEARNING EXAMPLE

Draw the s-domain equivalent and find the voltage in both s-domain and time domain

$$I_S(s) = \frac{3}{s+1}$$

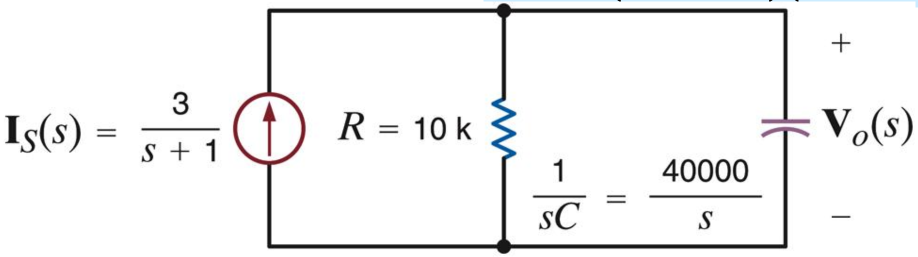


One needs to determine the initial voltage across the capacitor



$$i_S(t) = 0,\ t < 0 \Rightarrow v_o(0) = 0$$

$$RC = (10\times10^{3})(25\times10^{-6}) = 0.25$$



$$V_o(s) = \left(R \,\|\, \frac{1}{Cs}\right) I_S(s)$$

$$V_o(s) = \frac{\frac{R}{Cs}}{R + \frac{1}{Cs}} I_S(s) = \frac{1/C}{s + 1/RC} \times \frac{3\times10^{-3}}{s+1}$$

$$V_o(s) = \frac{120}{(s+4)(s+1)} = \frac{K_1}{s+4} + \frac{K_2}{s+1}$$

$$K_1 = (s+4)V_o(s)\,|_{s=-4} = -40$$

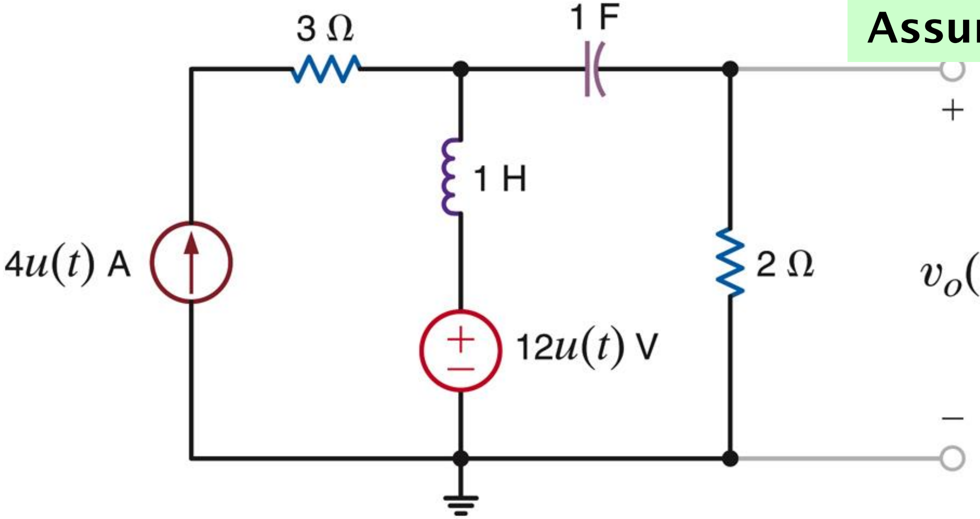
$$K_2 = (s+1)V_o(s)\,|_{s=-1} = 40$$

$$v_o(t) = 40\left[e^{-t} - e^{-4t}\right]u(t)$$

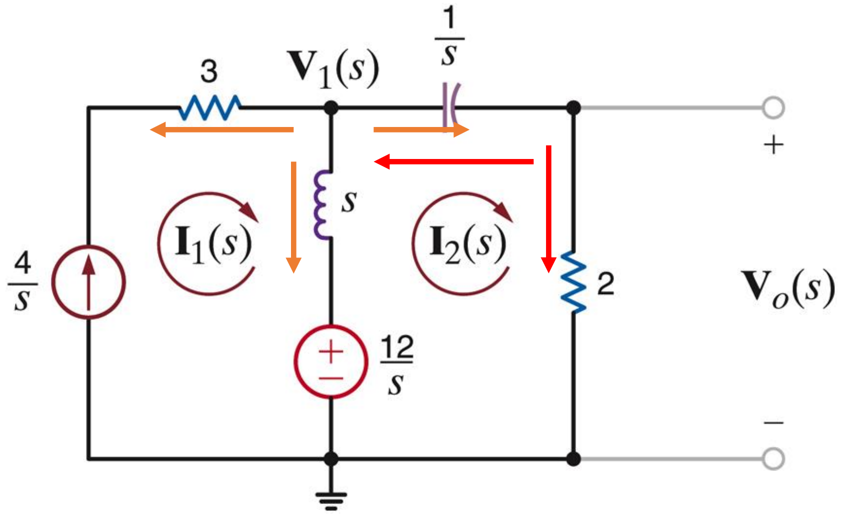
LEARNING EXAMPLE

Find $v_o(t)$ using node analysis, loop analysis, superposition, source transformation, Thevenin's and Norton's theorem.

Assume all initial conditions are zero



Node Analysis



KCL @ V_1

$$-\frac{4}{s} + \frac{V_1(s) - \frac{12}{s}}{s} + \frac{V_1(s) - V_o(s)}{\frac{1}{s}} = 0 \quad \times s$$

$$\frac{V_1(s)}{2 + \frac{1}{s}}$$

KCL@V_o

$$\frac{V_o(s)}{2} + \frac{V_o(s) - V_1(s)}{\frac{1}{s}} = 0 \times 2$$

Could have used voltage divider here

$$(1+s^2)V_1(s) - s^2V_o(s) = \frac{4s+12}{s} \quad \times 2s$$

$$-2sV_1(s) + (1+2s)V_o(s) = 0 \quad \times (1+s^2)$$

$$V_o(s) = \frac{8(s+3)}{(1+s)^2}$$

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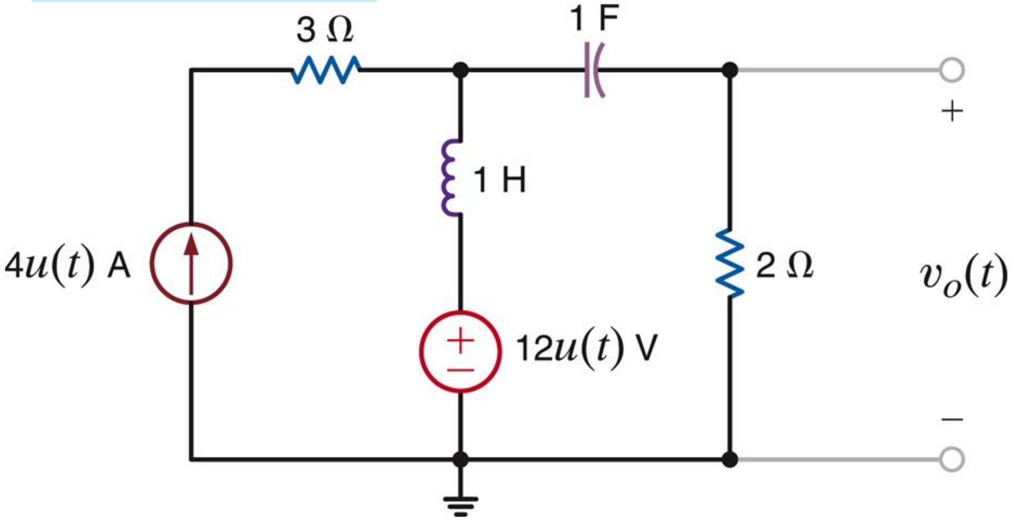
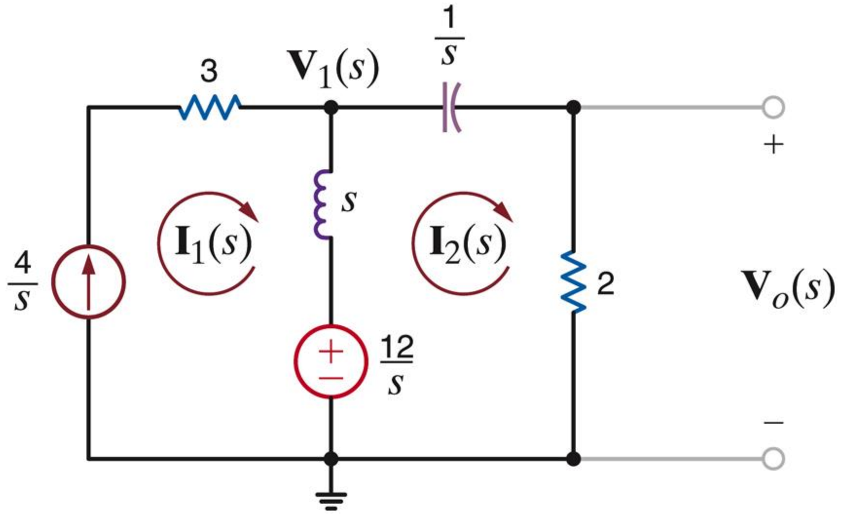
Loop Analysis





Loop 1

$$I_1(s) = \frac{4}{s}$$

Loop 2

$$s(I_2(s) - I_1(s)) + \frac{1}{s} I_2(s) + 2I_2(s) = \frac{12}{s}$$

$$I_2(s) = \frac{4(s+3)}{(s+1)^2}$$

$$V_o(s) = 2I_2(s) = \frac{8(s+3)}{(s+1)^2}$$

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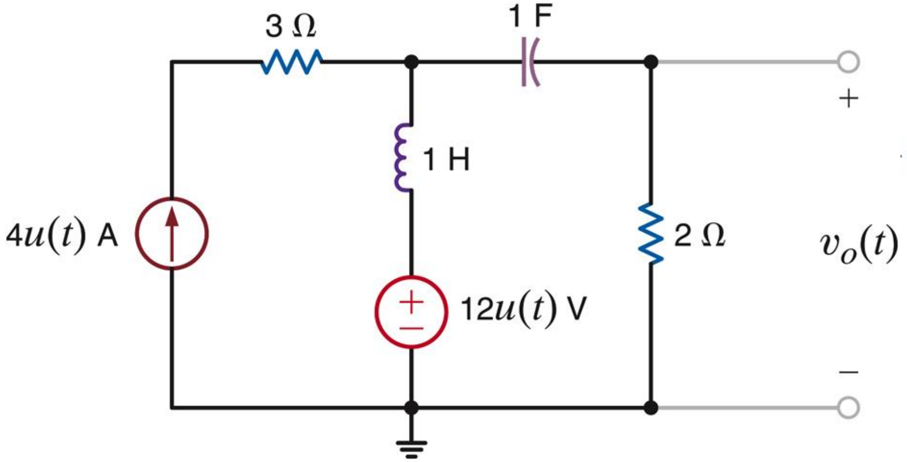
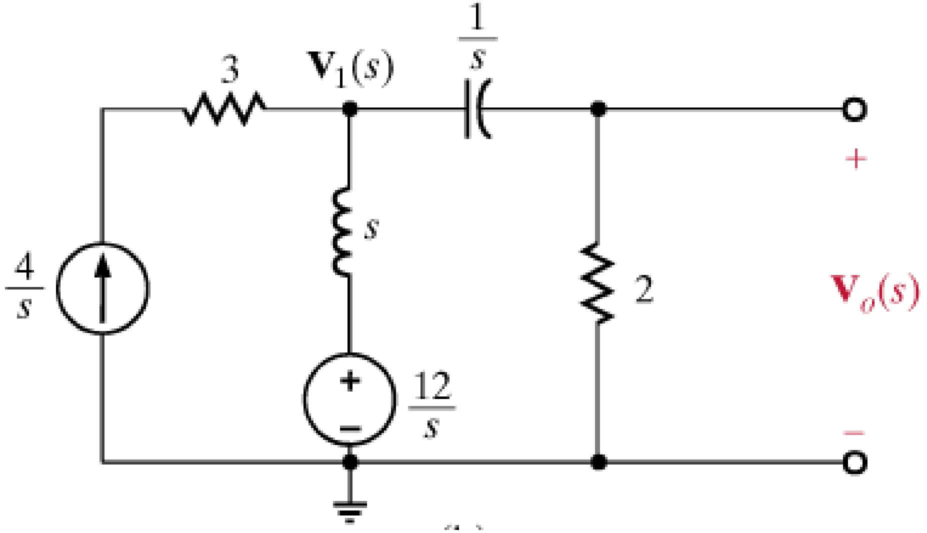
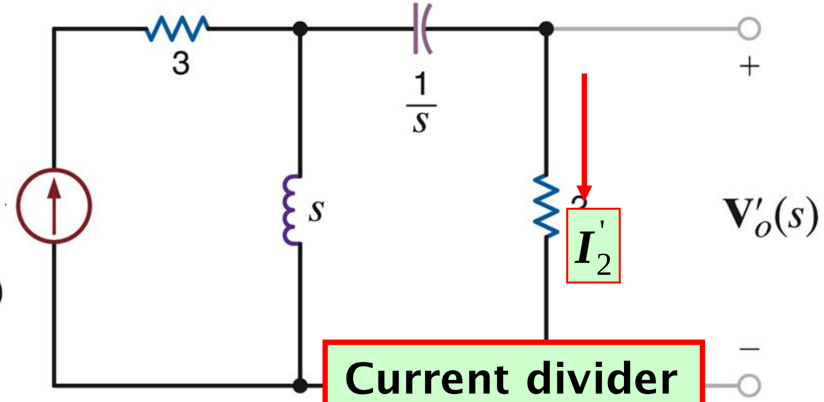
Source Superposition





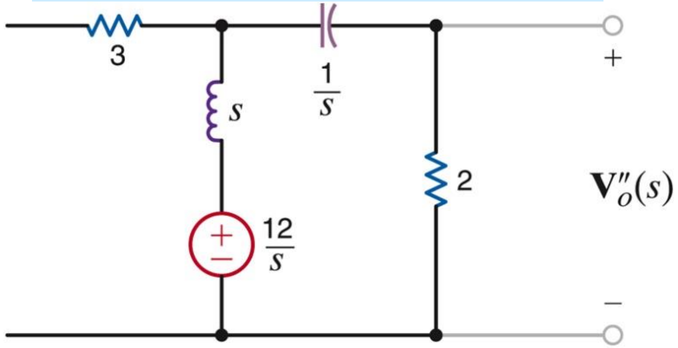
Applying current source



Current divider

$$V_o'(s) = 2 \times \frac{s}{2 + \frac{1}{s} + s} \times \frac{4}{s}$$

Applying voltage source



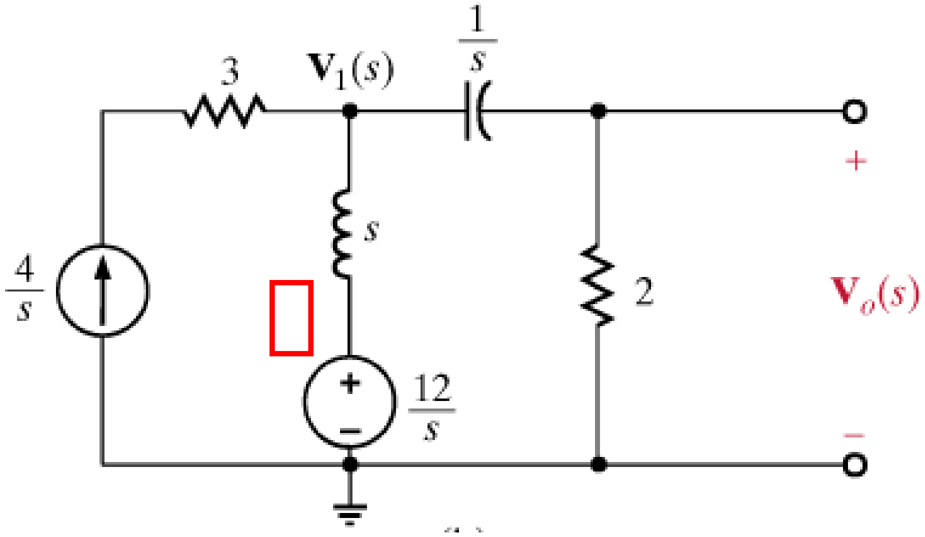
Voltage divider

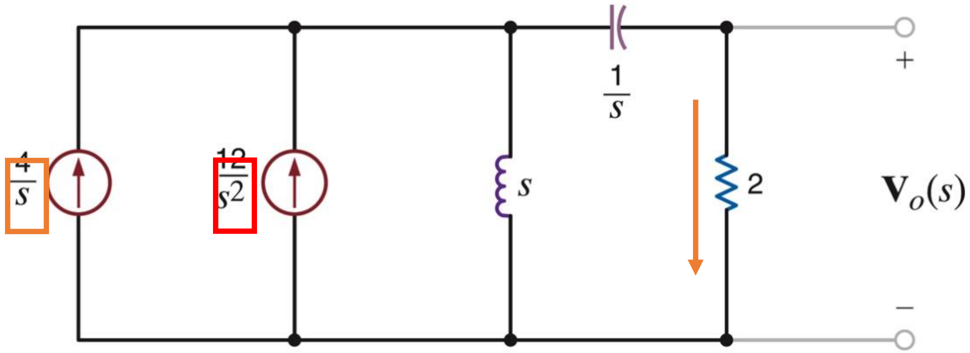
$$V_o''(s) = \frac{2}{2 + \frac{1}{s} + s} \times \frac{12}{s}$$

$$V_o(s) = V_o'(s) + V_o''(s) = \frac{8(s+3)}{(s+1)^2}$$

GEAUX

Source Transformation





The resistance is redundant

Combine the sources and use current divider

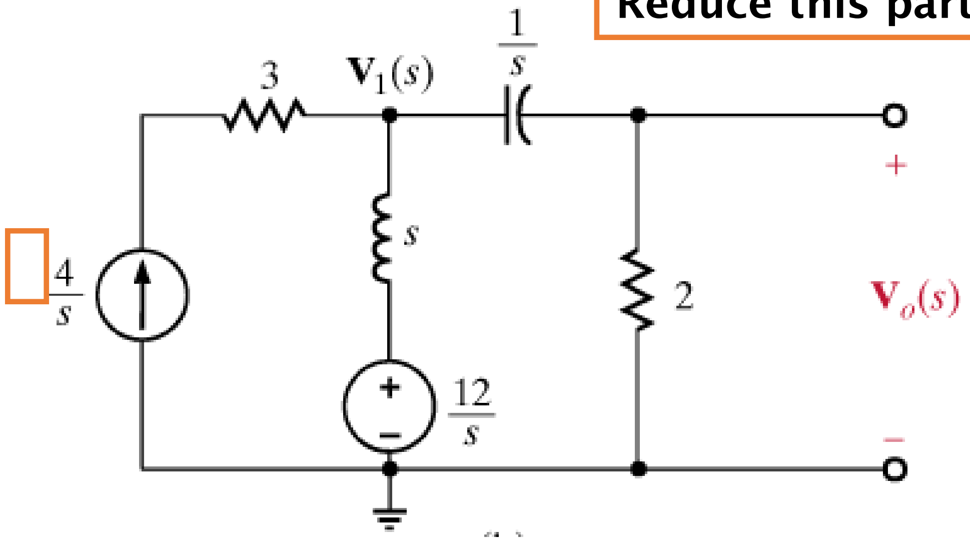
$$V_o(s) = 2 \times \frac{s}{s + \frac{1}{s} + 2}\left(\frac{4}{s} + \frac{12}{s^2}\right)$$

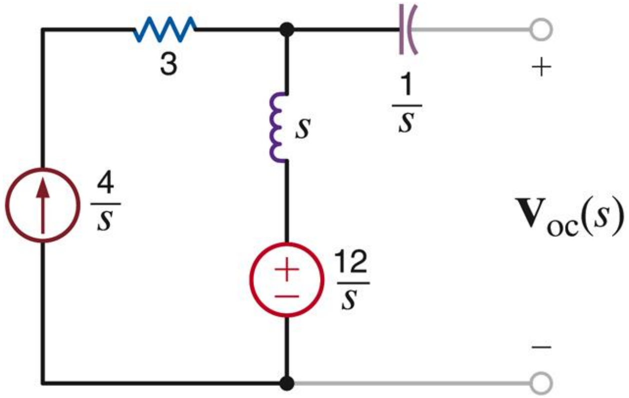
$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

GEAUX

Using Thevenin's Theorem

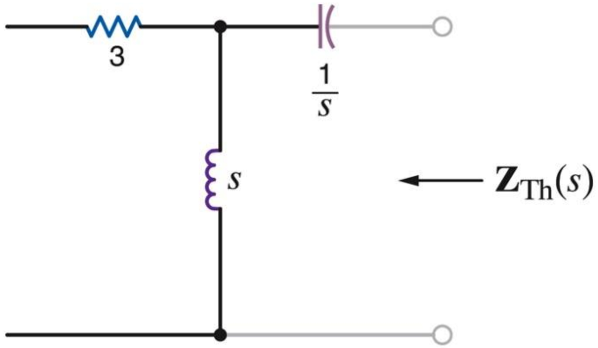
Reduce this part



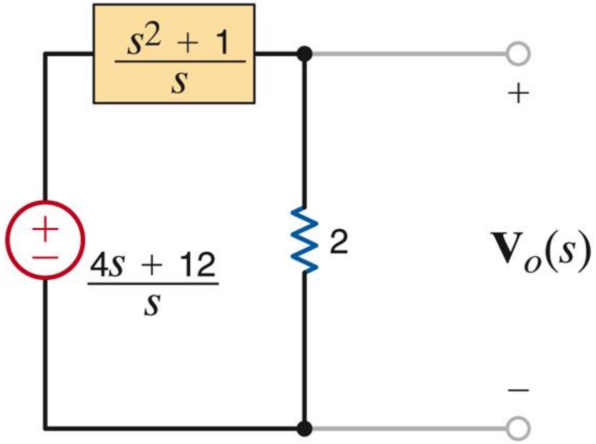


$$V_{OC}(s) = \frac{12}{s} + s\frac{4}{s} = \frac{4s+12}{s}$$

Only independent sources



$$Z_{Th} = \frac{1}{s} + s = \frac{s^2+1}{s}$$



Voltage divider

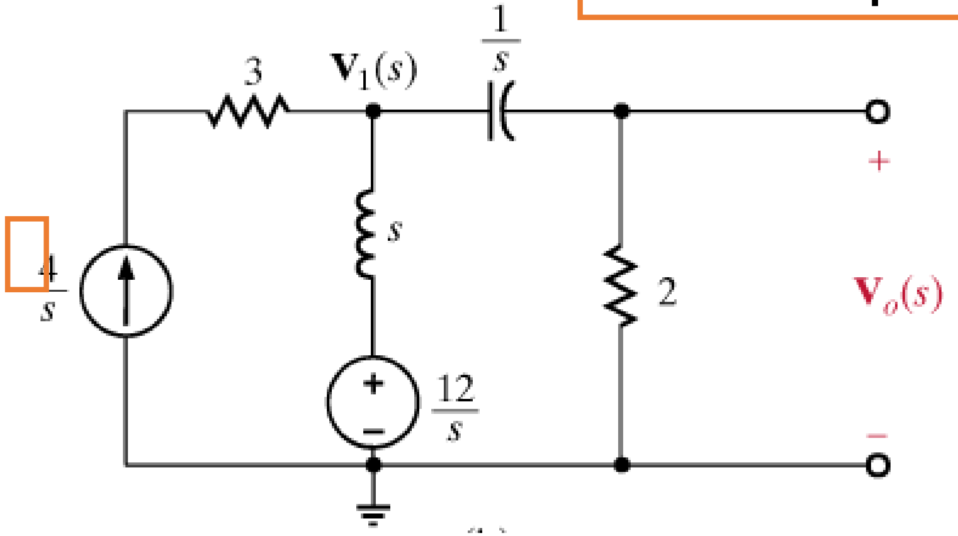
$$V_o(s) = \frac{2}{2 + \frac{s^2+1}{s}} \frac{4s+12}{s}$$

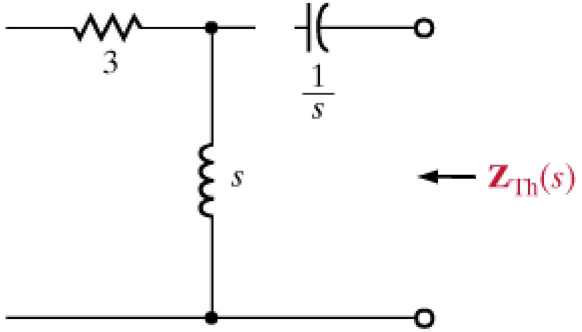
$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

GEAUX

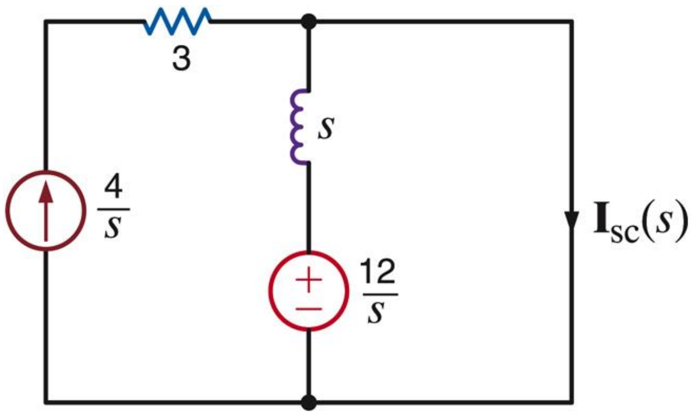
Using Norton's Theorem

Reduce this part

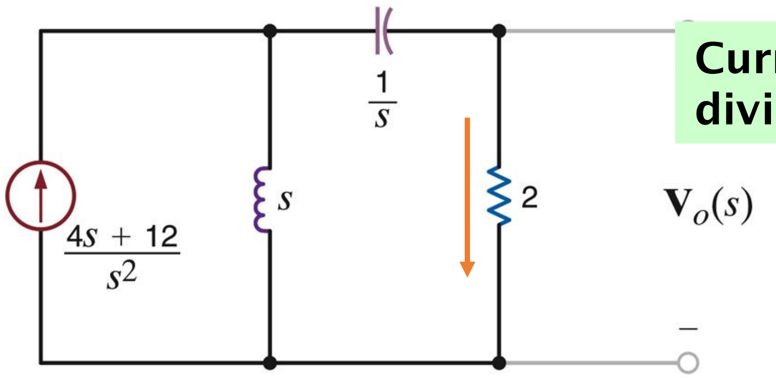




$$Z_{Th} = s$$



$$I_{SC}(s) = \frac{4}{s} + \frac{12/s}{s} = \frac{4s+12}{s^2}$$



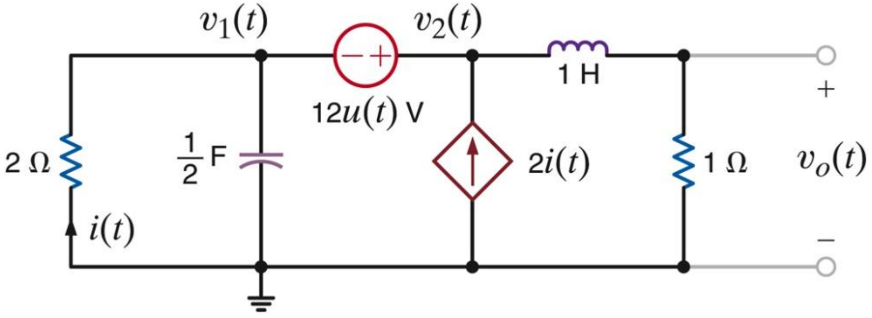
Current division

$$V_o(s) = 2 \times \frac{s}{s + \frac{1}{s} + 2} \frac{4s+12}{s^2}$$

$$V_o(s) = \frac{8(s+3)}{(s+1)^2}$$

GEAUX

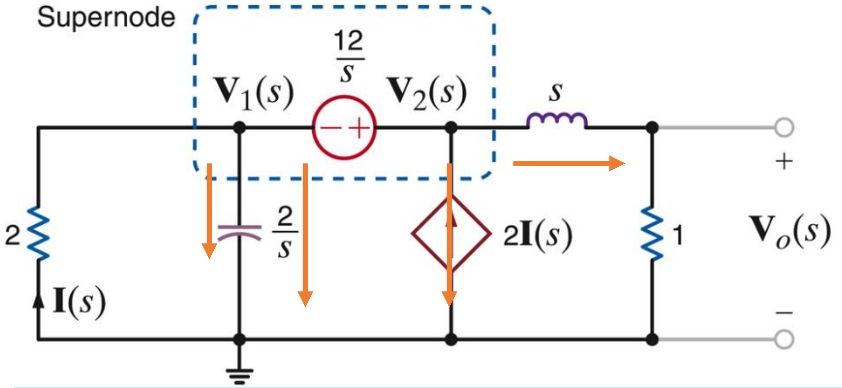
LEARNING EXAMPLE Determine the voltage $v_o(t)$. Assume all initial conditions to be zero



Selecting the analysis technique:

- Three loops, three non-reference nodes
- One voltage source between non-reference nodes - supernode
- One current source. One loop current known or supermesh
- If v_2 is known, v_o can be obtained with a voltage divider

Transforming the circuit to s-domain



Supernode constraint: $V_2(s) - V_1(s) = \dfrac{12}{s}$

KCL@ supernode: $\dfrac{V_1(s)}{2} + \dfrac{V_1(s)}{2/s} - 2I(s) + \dfrac{V_2(s)}{s+1} = 0$

Controlling variable: $I(s) = -\dfrac{V_1(s)}{2}$

Voltage divider: $V_0(s) = \dfrac{1}{s+1} V_2(s)$

Doing the algebra: $V_1(s) = V_2(s) - 12/s$

$$I(s) = -V_2(s)/2 + 6/s$$

$$(1/2)(s+1)\left(V_2(s) - 12/s\right) - 2(-V_2(s)/2 + 6/s) + V_2(s)/(s+1) = 0$$

$$V_2(s) = \frac{12(s+1)(s+3)}{s(s^2+4s+5)}$$

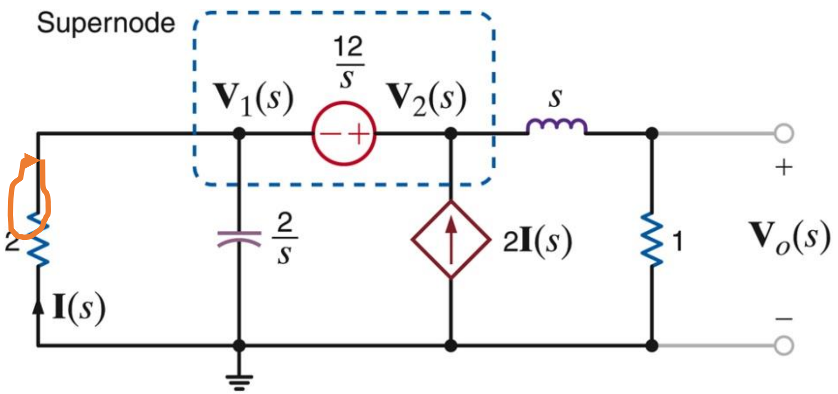
$$V_o(s) = \frac{12(s+3)}{s(s^2+4s+5)}$$

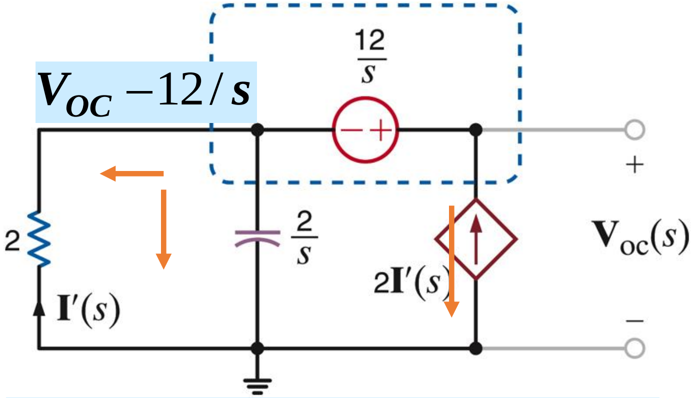
GEAUX

Continued ...

Compute $V_o(s)$ using Thevenin's theorem

- keep dependent source and controlling variable in the same sub-circuit
- Make sub-circuit to be reduced as simple as possible
- Try to leave a simple voltage divider after reduction to Thevenin equivalent



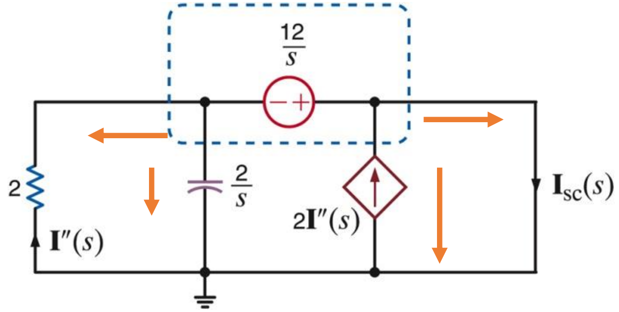


$V_{OC} - 12/s$

$$\frac{V_{OC} - 12/s}{2} + \frac{V_{OC} - 12/s}{2/s} - 2I' = 0$$

$$I' = -\frac{V_{OC} - 12/s}{2} \qquad V_{OC}(s) = \frac{12}{s}$$

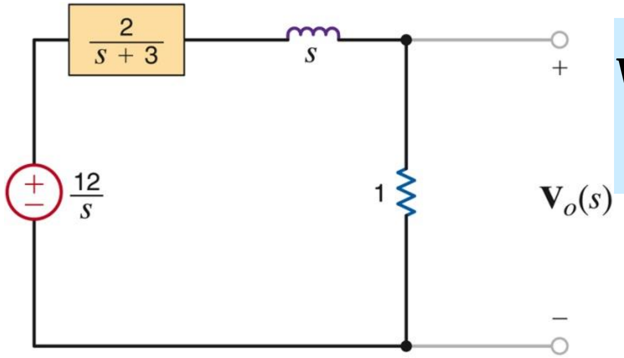
$$I' = 0$$



$-\ 12/s\ +$

$$I_{SC} - 2I'' - I'' - 2I''/(2/s) = 0 \qquad I'' = 6/s$$

$$I_{SC} = \frac{6(s+3)}{s} \qquad Z_{TH} = \frac{V_{OC}(s)}{I_{SC}(s)} = \frac{2}{s+3}$$



$$V_o(s) = \frac{1}{1 + s + \frac{2}{s+3}} \times \frac{12}{s}$$

GEAUX