## MILLMAN'S THEOREM

- Through the application of Millman's theorem, any number of parallel voltage sources can be reduced to one.
- Showing in Fig, for example, the three voltage sources can be reduced to one. This would permit finding the current through or voltage across RL without having to apply a method such as mesh analysis, nodal analysis, superposition, and so on.



# Basically, three steps are included in its application 

- Convert all voltage sources to current sources
- Combine parallel current sources
- Convert the resulting current source to a voltage source, and the desired single-source network is obtained


$$
\begin{gathered}
E_{e q}=\frac{\frac{E_{1}}{R_{1}}+\frac{E_{2}}{R_{2}}+\cdots+\frac{E_{n}}{R_{n}}}{\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots+\frac{1}{R_{n}}} \\
R_{e q}=\frac{1}{\frac{1}{R_{1}+R_{2}+R_{3}+\cdots+R_{n}}}
\end{gathered}
$$

## MILLMAN'S THEOREM FOR NETWORKS

For AC networks Millman's theorem states that " if ' $n$ ' number of voltage sources $\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}, \ldots \ldots \ldots, \mathrm{~V}_{\mathrm{n}}$ having internal impedances $\mathrm{Z}_{1}, \mathrm{Z}_{2}, \mathrm{Z}_{3}, \ldots \ldots ., \mathrm{Z}_{\mathrm{n}}$ are connected in parallel across the load $\mathrm{Z}_{\mathrm{L}}$ than this arrangement may be replaced by a single voltage source $\mathrm{V}_{\mathrm{eq}}$ in series with equivalent impedance $\mathrm{Z}_{\mathrm{eq}}$. Millman's equivalent circuit is shown in fig.1.



Fig. 1b

Fig. 1a

## MILLMAN'S THEOREM FOR AC NETWORKS

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{eq}}=\frac{\mathrm{V}_{1} \mathrm{Y}_{1}+\mathrm{V}_{2} \mathrm{Y}_{2}+\mathrm{V}_{3} \mathrm{Y}_{3}-----+\mathrm{V}_{\mathrm{n}} \mathrm{Y}_{\mathrm{n}}}{\mathrm{Y}_{1}+\mathrm{Y}_{2}+\mathrm{Y}_{3}---------+\mathrm{Y}_{\mathrm{n}}} \\
& \mathrm{Z}_{\mathrm{eq}}=\frac{1}{\mathrm{Y}_{1}+\mathrm{Y}_{2}+\mathrm{Y}_{3}--------+\mathrm{Y}_{\mathrm{n}}}-------(2)
\end{aligned}
$$

Where $\mathrm{Z}_{1}, \mathrm{Z}_{2}, \mathrm{Z}_{3} \ldots \ldots . . \mathrm{Z}_{\mathrm{n}}$ are theimpedances and $\mathrm{Y}_{1}, \mathrm{Y}_{2}, \mathrm{Y}_{3}, \ldots \ldots . . \mathrm{Y}_{\mathrm{n}}$ are theadmitatances.
$\mathrm{Y}=\frac{1}{\mathrm{Z}}$
This theorem is applicable only to solve the parallel branch with one impedance or resistance connected to voltage or current source. The voltage sources can be converted into current sources by transformation of sources.

## MILLMAN'S THEOREM FOR AC NETWORKS

EXAMPLE: Using Millman's theorem, find the current through 'ab'


Fig. 2

## MILLMAN'S THEOREM FOR AC NETWORKS

 SOLUTION:$$
\begin{aligned}
& V_{1}=10 \angle 0^{\circ} \mathrm{V} \\
& \mathrm{~V}_{2}=20 \angle 45^{0} \mathrm{~V}
\end{aligned}
$$

$$
\mathrm{Y}_{1}=\frac{1}{\mathrm{Z}_{1}}=\frac{1}{5}=0.2 \mathrm{~S}
$$

$$
Y_{1}=\frac{1}{Z_{2}}=\frac{1}{-j 3}=0.33 \angle 90^{\circ} S
$$

$$
V_{e q}=\frac{V_{1} Y_{1}+V_{2} Y_{2}}{Y_{1}+Y_{2}}
$$

$$
\mathrm{V}_{\mathrm{eq}}=\frac{\left(10 \angle 0^{0}\right)(0.2)+\left(20 \angle 45^{0}\right)\left(0.33 \angle 90^{0}\right.}{0.2+0.33 \angle 90^{0}}
$$

$$
\mathrm{V}_{\mathrm{eq}}=13.94 \angle 60.97^{\circ} \mathrm{V}
$$

$$
\mathrm{Z}_{\text {eq }}=\frac{1}{\mathrm{Y}_{\text {eq }}}=\frac{1}{0.2+0.33 \angle 90^{0}}=2.59 \angle-58.78^{0} \Omega
$$

## MILLMAN'S THEOREM FOR AC NETWORKS

Millman's Equivalent Circuit:


Fig. 3
b

$$
\begin{aligned}
& I_{a b}=\frac{V_{e q}}{Z_{e q}+(2+j 4)}=\frac{13.94 \angle 60.97^{0}}{(2.59 \angle-58.78)+(2+j 4)} \\
& I_{a b}=3.68 \angle 32.79_{0} A
\end{aligned}
$$

Current through branch $\mathrm{ab}=3.68 \angle 32.79^{\circ} \mathrm{A}$


