Unit-1
Introduction

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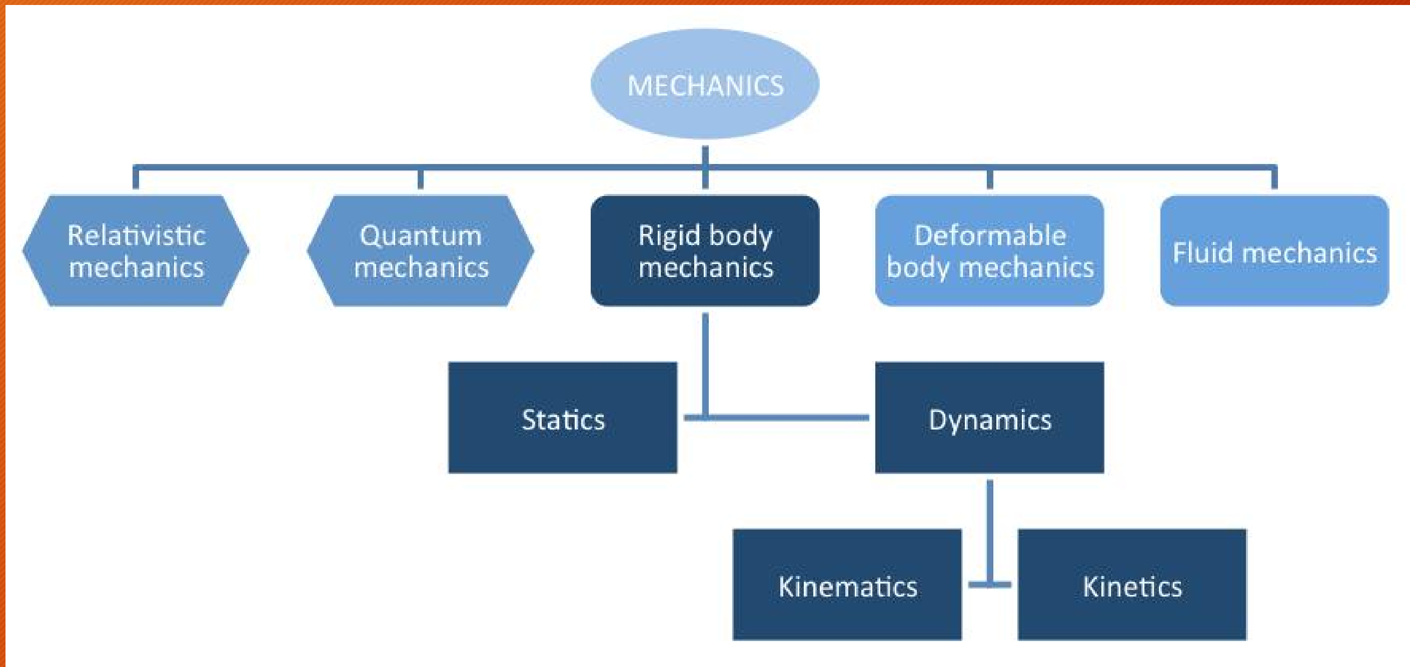
Introduction to Fluids and Continuum

- Fluid mechanics deals with the study of all fluids under static and dynamic situations.
- Fluid mechanics is a branch of continuous mechanics which deals with a relationship between forces, motions, and static conditions in a continuous material.
- This study area deals with many and diversified problems such as surface tension, fluid statics, and flow in enclose bodies, or flow round bodies (solid or otherwise), flow stability, etc. In fact, almost any action a person is doing involves some kind of a fluid mechanics problem.

- There are two main approaches of presenting an introduction of fluid mechanics materials.
- The first approach introduces the fluid kinematic and then the basic governing equations, to be followed by stability, turbulence, boundary layer, and internal and external flow.
- The second approach deals with the Mathematical Analysis to be followed with Differential Analysis, and continue with Empirical Analysis

Fluid mechanics is a sub discipline of continuum mechanics



Solids, liquids and gases

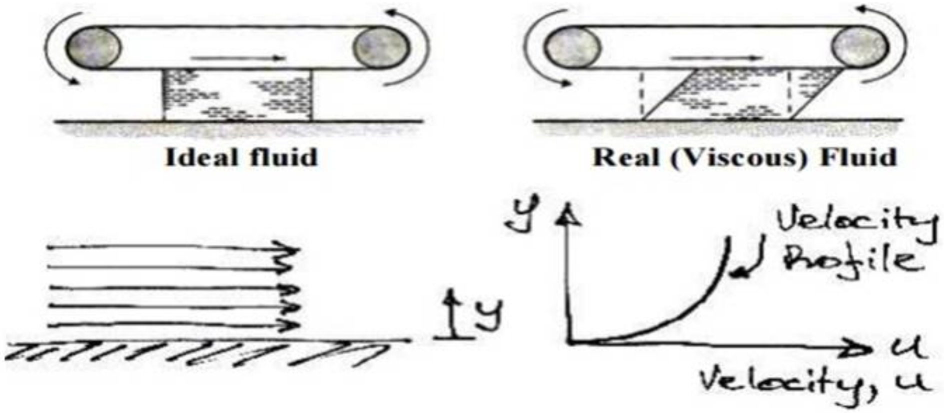
- In general matter can be distinguished by the physical forms known as solid, liquid, and gas.
- The liquid and gaseous phases are usually combined and given a common name of fluid.
- Solids differ from fluids on account of their molecular structure (spacing of molecules and ease with which they can move).
- The intermolecular forces are large in a solid, smaller in a liquid and extremely small in gas.

Fluids

- A fluid is a substance that continually deforms (flows) under an applied shear stress, no matter how small.
- Fluids are a subset of the phases of matter and include liquids, gases, plasmas and, to some an extent, plastic solids.
- In common usage, ‘fluid’ is often used as a synonym for ‘liquid‘, with no implication that gas could also be present.
- **For static fluids:**According to this definition, if we apply a shear force to a fluid it will deform and take up a state in which no shear force exists. Therefore, we can say:
- *If a fluid is at rest there can be no shearing forces acting and therefore all forces in the fluid must be perpendicular to the planes in which they act.*

- Consider the fluid shown flowing along a fixed surface. At the surface there will be little movement of the fluid (it will ‘stick’ to the surface), whilst further away from the surface the fluid flows faster (has greater velocity)
- If one layer of is moving faster than another layer of fluid, there must be shear forces acting between them. For example, if we have fluid in contact with a conveyor belt that is moving we will get the behaviour shown:

For fluids in motion:



Density or Mass Density

- Density or Mass density of a fluid is defined as the ratio of mass and volume. Thus mass per unit volume of a fluid is called density and is denoted by ρ . The SI unit of density is kg /m3
- The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature. Mathematically, mass density if written as
- ρ =Mass of fluid/ Volume of fluid

DIMENSIONS AND UNITS

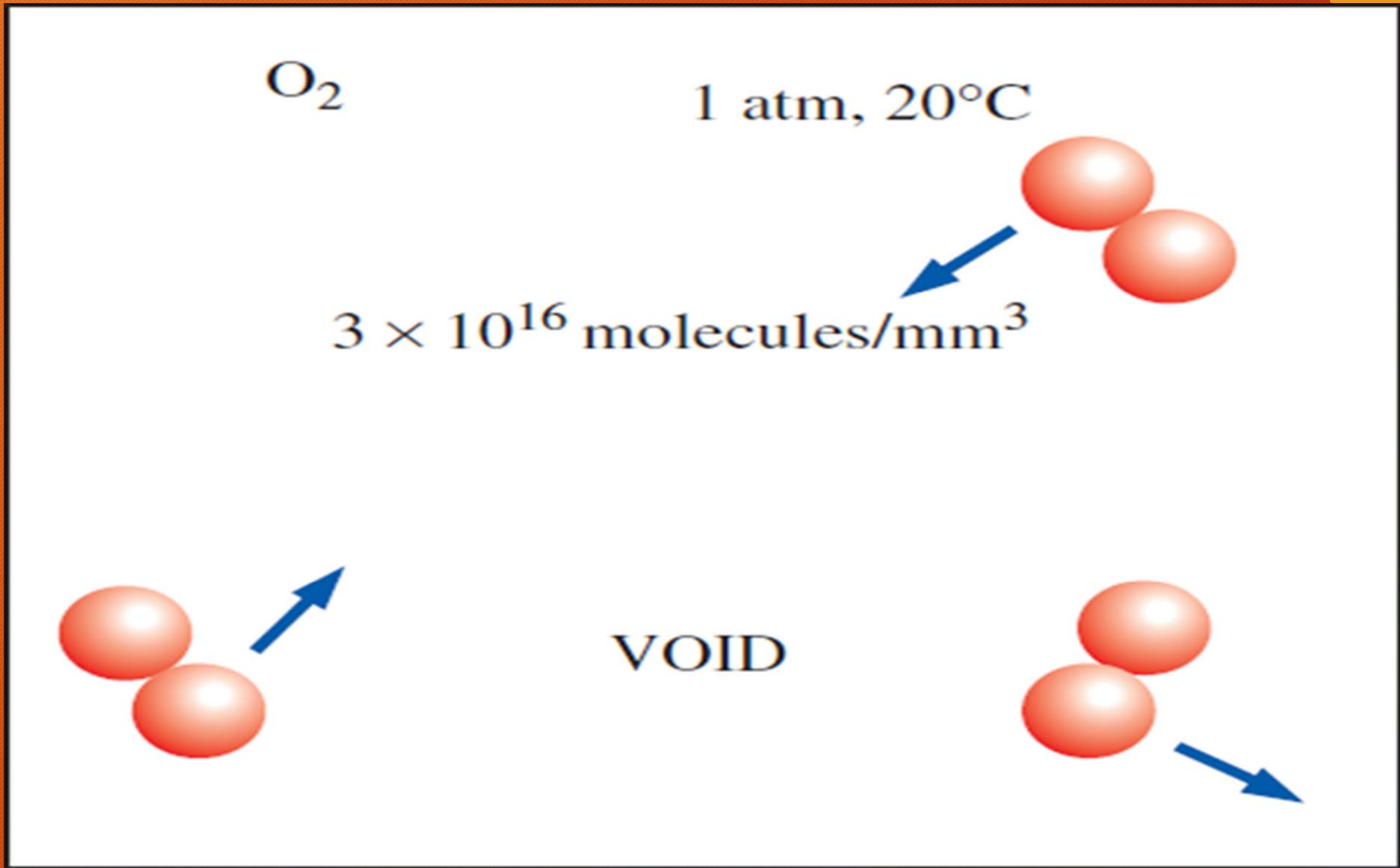
- Dimension = A dimension is the measure by which a physical variable is expressed quantitatively.
- Unit = A unit is a particular way of attaching a number to the quantitative dimension.
- Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters or meters are both numerical units for expressing length

Fluid as a continuum

- Fluid mechanics is supposed to describe motion of fluids and related phenomena at macroscopic scales, which assumes that a fluid can be regarded as a continuous medium. This means that any small volume element in the fluid is always supposed so large that it still contains a very great number of molecules
- For the continuum model to be valid, the smallest sample of matter of practical interest must contain a large number of molecules so that meaningful averages can be calculated. In the case of air at sea-level conditions, a volume of 10-9 mm3 contains 3×107 molecules. In engineering sense, this volume is quite small, so the continuum hypothesis is valid.

- Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**.
- The continuum idealization allows us to treat properties as point functions and to assume the properties vary continually in space with no jump discontinuities.
- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- This is the case in practically all problems.
- In this text we will limit our consideration to substances that can be modeled as a continuum.

Despite the relatively large gaps between molecules, a substance can be treated as a continuum because of the very large number of molecules even in an extremely small volume.


O_2
1 atm, 20°C
3×10^{16} molecules/mm^3
VOID

Density of Ideal Gases

- Equation of state: Any equation that relates the pressure, temperature, and density (or specific volume) of a substance.
- Ideal-gas equation of state: The simplest and best-known equation of state for substances in the gas phase.

$$Pv = RT \quad \text{or} \quad P = \rho RT \qquad R = R_u / M$$

$$R_u = 8.314 \text{ kJ/kmol} \cdot \text{K} \qquad PV = mRT \text{ or } PV = NR_uT$$

R_u: The universal gas constant

$$T(\text{K}) = T(°\text{C}) + 273.15 = T(R)/1.8$$

$$T(\text{R}) = T(°\text{F}) + 459.67 = 1.8\, T(\text{K})$$

Properties of Fluid

- Any characteristic of a system is called property.
- It may either be intensive (mass independent) or extensive (that depends on size of system).
- The state of a system is described by its properties.
- The number of properties required to fix the state of the system is given by state postulates.

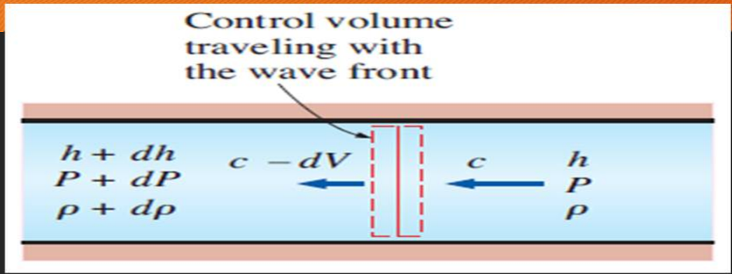
- 1. Pressure (p) : It is the normal force exerted by a fluid per unit area. More details will be available in the subsequent section
- 2. Density: The density of a substance is the quantity of matter contained in unit volume of the substance.
- 3.Temperature (T): It is the measure of hotness and coldness of a system. In thermodynamic sense, it is the measure of internal energy of a system. Many a times, the temperature is expressed in centigrade scale (°C) where the freezing and boiling point of water is taken as 0°C and 100°C, respectively. In SI system, the temperature is expressed in terms of absolute value in Kelvin scale (K = °C+ 273)

- 4.Viscosity (μ): When two solid bodies in contact, move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. The property that represents the internal resistance of a fluid to motion (i.e. fluidity) is called as viscosity
- 5.Thermal Conductivity(k): Thermal conductivity varies with temperature for liquids as well as gases in the same manner as that of viscosity. The reference value of thermal conductivity (k0) for water and air at reference temperature is taken as, 0.6 W/m.K and 0.025 W/m.K, respectively

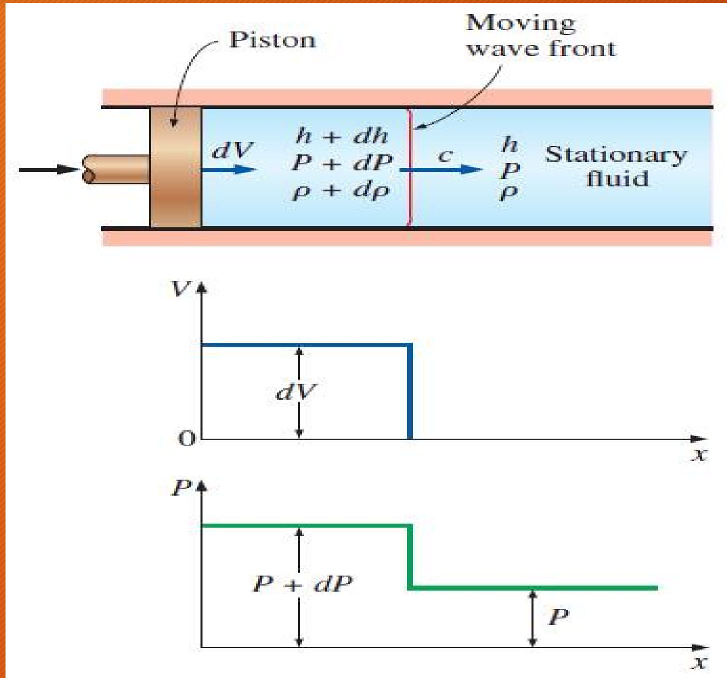
- . 6.Coefficient of compressibility/Bulk modulus(Ev): It is the property of that fluid that represents the variation of density with pressure at constant temperature.
- 7.Coefficient of volume expansion(β): It is the property of that fluid that represents the variation of density with temperature at constant pressure. It can be shown easily that Ev for an ideal gas at a temperature T is equivalent to inverse of the absolute temperature.

- 8.Specific heats: It is the amount of energy required for a unit mass of a fluid for unit rise in temperature. Since the pressure, temperature and density of a gas are interrelated, the amount of heat required to raise the temperature depends on whether the gas is allowed to expand during the process so that the energy supplied is used in doing the work instead of raising the temperature.
- 9.Speed of sound (c): An important consequence of compressibility of the fluid is that the disturbances introduced at some point in the fluid propagate at finite velocity. The velocity at which these disturbances propagate is known as “acoustic velocity/speed of sound”

- The speed of sound in air increases with temperature. At typical outside temperatures, *c is about 340 m/s. In* round numbers, therefore, the sound of thunder from a lightning strike travels about 1 km in 3 seconds. If you see the lightning and then hear the thunder less than 3 seconds later, you know that the lightning is close, and it is time to go indoors!



$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T \qquad c = \sqrt{kRT}$$



- 10. Vapor pressure (pv) : It is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. For a pure substance, it is same as the saturation pressure. In a fluid motion, if the pressure at some location is lower than the vapor pressure, bubbles start forming. This phenomenon is called as cavitation because they form cavities in the liquid.

VAPOR PRESSURE AND CAVITATION

- Saturation temperature T_{sat}: The temperature at which a pure substance changes phase at a given pressure.
- Saturation pressure P_{sat}: The pressure at which a pure substance changes phase at a given temperature.
- Vapor pressure (P_v): The pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. It is identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$).
- Partial pressure: The pressure of a gas or vapor in a mixture with other gases. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.

- There is a possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization.
- The vapor bubbles (called cavitation bubbles since they form “cavities” in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called cavitation, and it is an important consideration in the design of hydraulic turbines and pumps.

ENERGY AND SPECIFIC HEATS

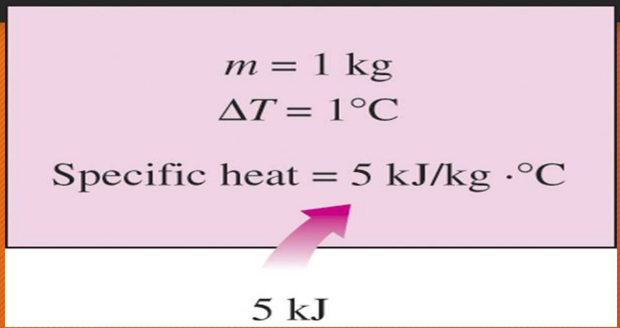
- Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the **total energy, *E*** of a system.
- Thermodynamics deals only with the ***change*** of the total energy.
- **Macroscopic forms of energy**: Those a system possesses as a whole with respect to some outside reference frame, such as kinetic and potential energies.
- **Microscopic forms of energy**: Those related to the molecular structure of a system and the degree of the molecular activity.
- **Internal energy, *U***: The sum of all the microscopic forms of energy.

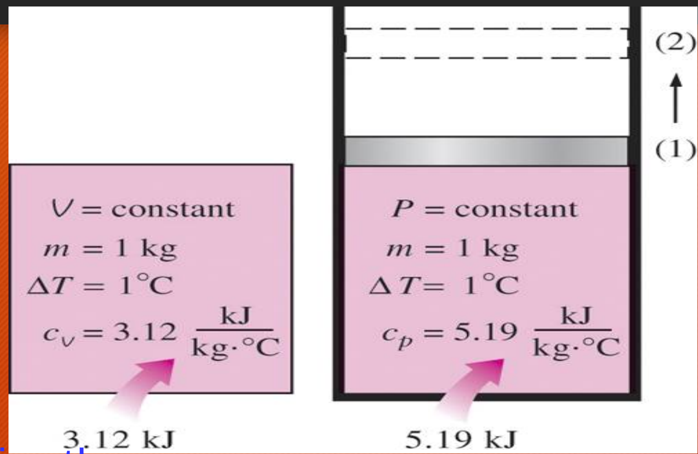
- Kinetic energy, KE: The energy that a system possesses as a result of its motion relative to some reference frame.
- Potential energy, PE: The energy that a system possesses as a result of its elevation in a gravitational field.

Specific Heats:

-Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.





Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

COMPRESSIBILITY AND SPEED OF SOUND

- Coefficient of Compressibility

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure.

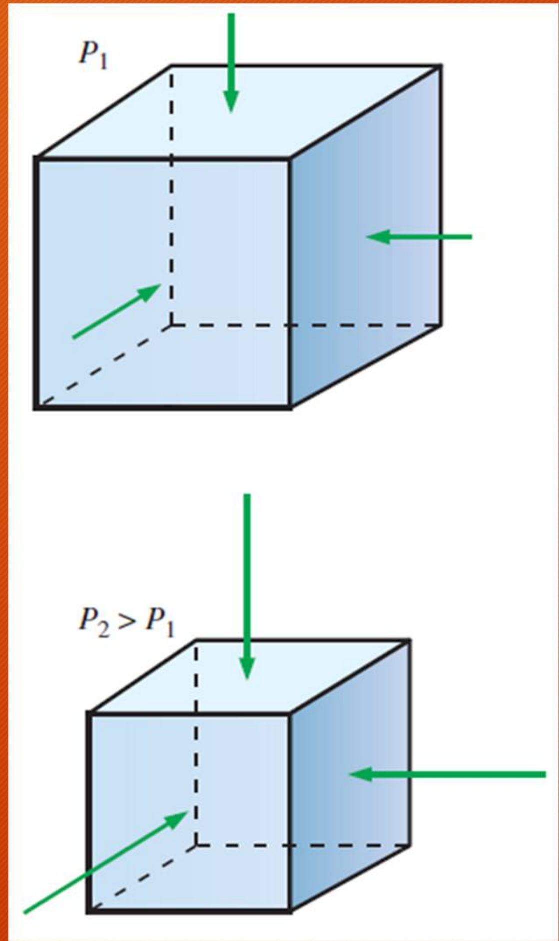
Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized.

But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature.

Two such properties are:

the bulk modulus of elasticity κ

the coefficient of volume expansion β.



Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2.

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{(Pa)}$$

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \qquad (T = \text{constant})$$

Coefficient of compressibility

(also called the bulk modulus of compressibility or bulk modulus of elasticity) for fluids

The coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.

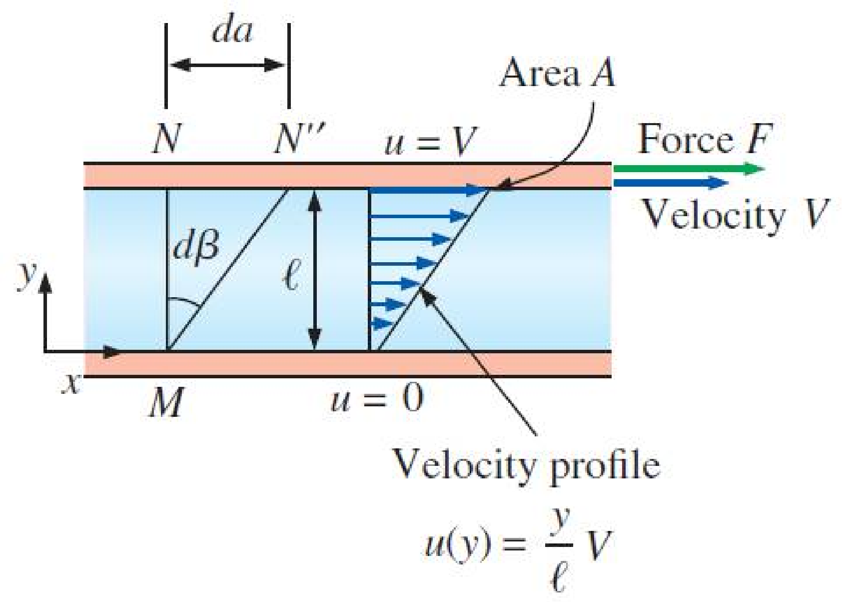
What is the coefficient of compressibility of a trusubstance (v = constant)?

A large value of κ indicates that a large change in pressure is needed to cause a small fractional change in volume, and thus a fluid with a large κ is essentially incompressible.

This is typical for liquids, and explains why liquids are usually considered to be *incompressible*.

VISCOSITY

- Viscosity: A property that represents the internal resistance of a fluid to motion or the "fluidity".
- Drag force: The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity
- The viscosity of a fluid is a measure of its "*resistance to deformation*."
- Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.



The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

$$\tau = \frac{F}{A}$$

$$u(y) = \frac{y}{\ell} V \quad \text{and} \quad \frac{du}{dy} = \frac{V}{\ell}$$

$$d\beta \approx \tan d\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt$$

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

Newtonian fluids: Fluids for which the rate of deformation is proportional to the shear stress.

$$\tau \propto \frac{d(d\beta)}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \quad (\text{N/m}^2)$$

Shear stress

Shear force

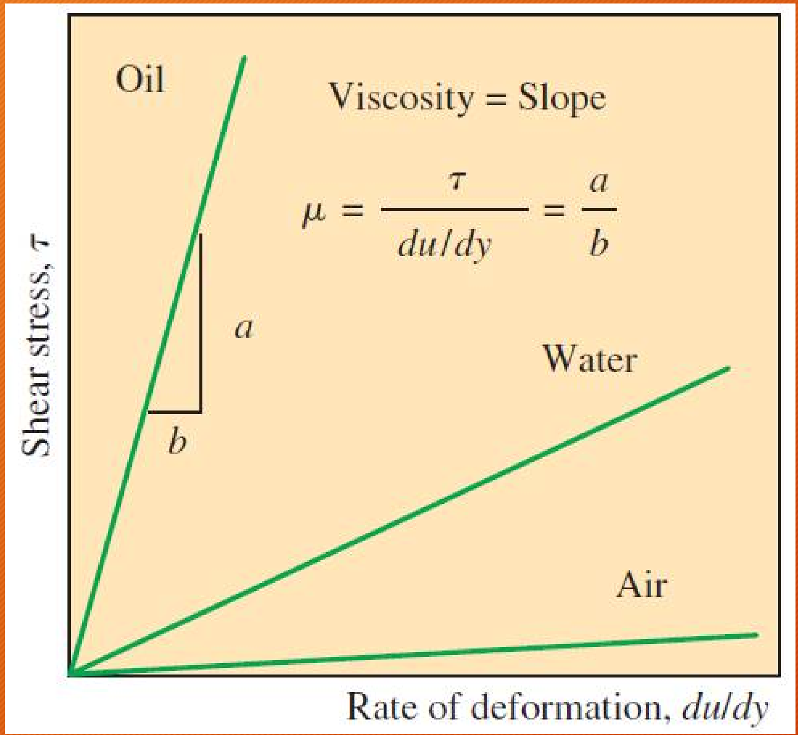
$$F = \tau A = \mu A \frac{du}{dy} \quad (\text{N})$$

μ coefficient of viscosity

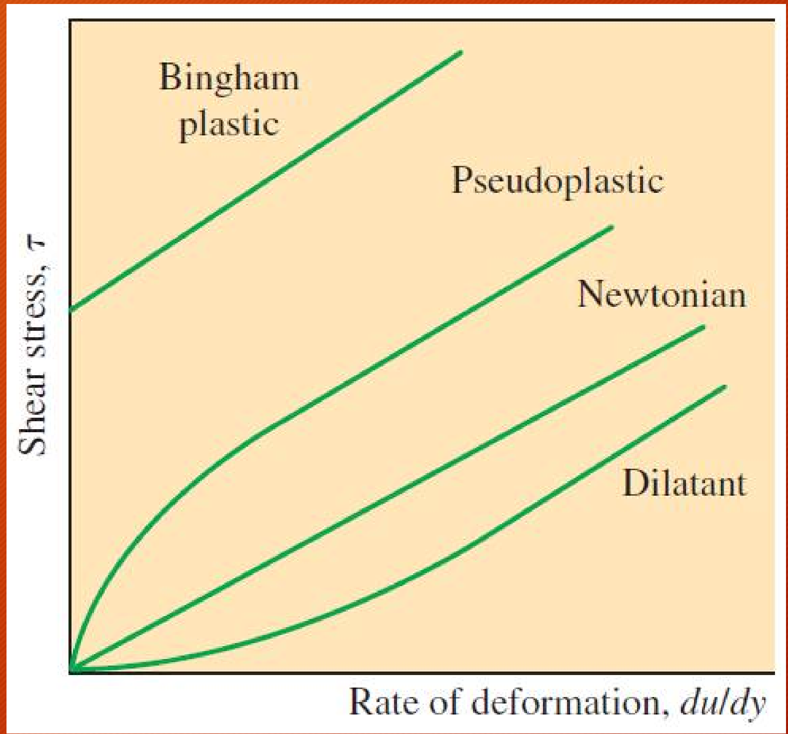
Dynamic (absolute) viscosity

kg/m · s or N · s/m^2 or Pa · s

1 poise = 0.1 Pa · s



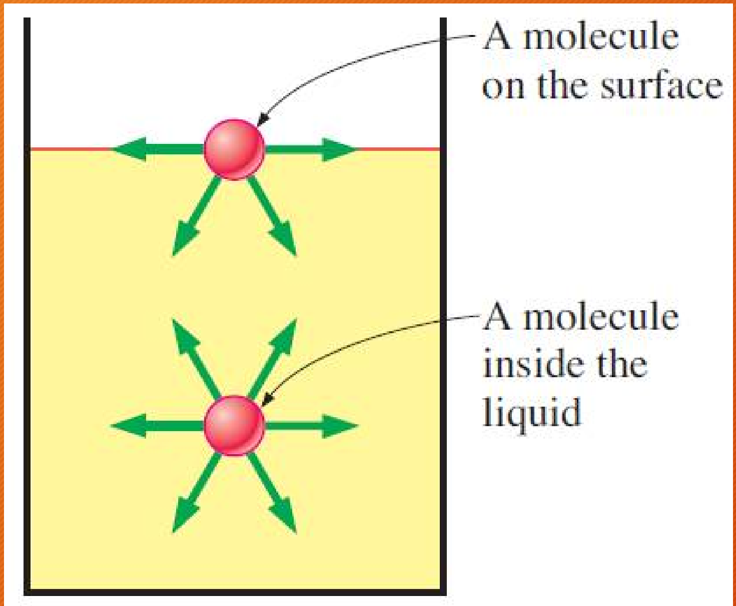
The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

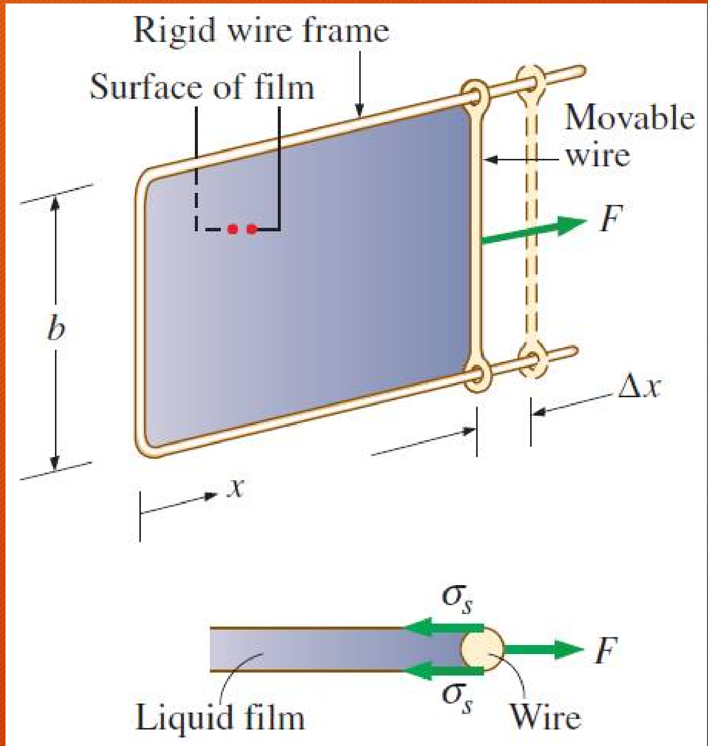


Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

SURFACE TENSION AND CAPILLARY EFFECT

- Liquid droplets behave like small balloons filled with the liquid on a solid surface, and the surface of the liquid acts like a stretched elastic membrane under tension.
- The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.
- The magnitude of this force per unit length is called surface tension (or *coefficient of surface tension*) and is usually expressed in the unit N/m.
- This effect is also called *surface energy* [per unit area] and is expressed in the equivalent unit of $N \cdot m/m^2$.





$$\sigma_s = \frac{F}{2b}$$

Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

Surface tension: The work done per unit increase in the surface area of the liquid.

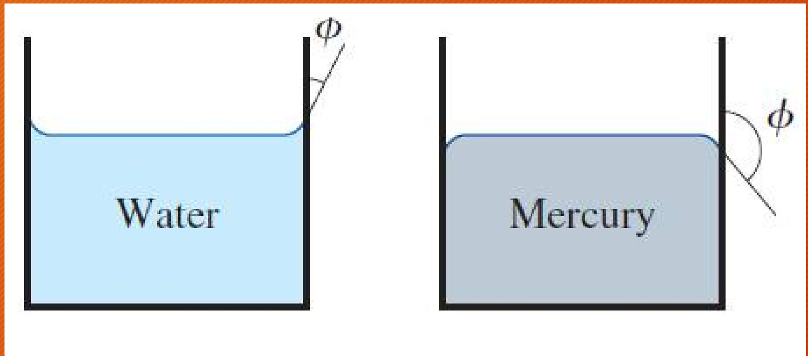
Capillary Effect

Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

Capillaries: Such narrow tubes or confined flow channels.

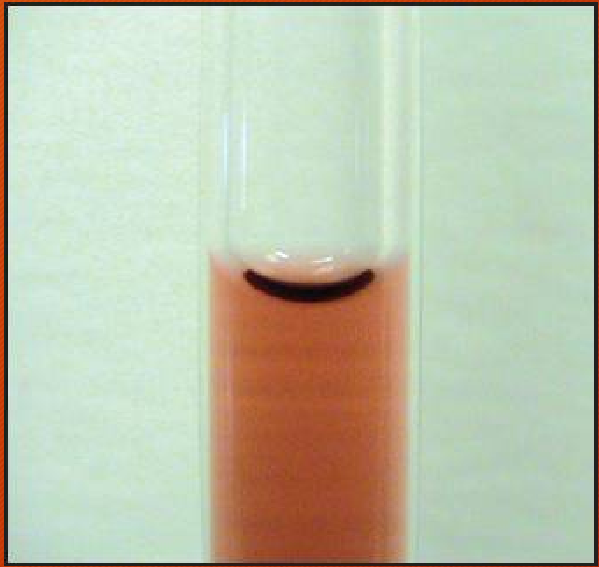
The capillary effect is partially responsible for the rise of water to the top of tall trees.

Meniscus: The curved free surface of a liquid in a capillary tube



(a) Wetting fluid

(b) Nonwetting fluid



The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.

Introduction to Rheology

- Introduction to the Rheology of Fluids
- Rheology
- Study of deformation and flow of matter

- A **fluid** is a substance that deforms continuously under the action of a shearing force.
  - Intuitively, a fluid flows!

- Inquiry into the flow behavior of complex fluids

- Complex fluids do not follows Newton's Law or Hooke's Law (of elasticity)

Newton and Simple Fluids

- Reflected upon the resistance of liquids to a cylinder rotating in a vessel.

- **Newton (-Stokes) Law**
  - Deformation rate is expected to be proportional to stress and the constant coefficient of proportionality is called viscosity.

$$\tau = \eta\dot{\gamma}$$

- The study of simpler fluids have their own well-defined field, called *fluid mechanics*.

- Purely viscous fluid.

What is Rheology Anyway?
An answer for your baffled family and friends. *

- "Rheology is the study of the flow of materials that behave in an interesting or unusual manner. Oil and water flow in familiar, normal ways, whereas mayonnaise, peanut butter, chocolate, bread dough, and silly putty flow in complex and unusual ways. In rheology, we study the flows of unusual materials."

- "… all normal or Newtonian fluids (air, water, oil, honey) follow the same scientific laws. On the other hand, there are also fluids that do not follow the Newtonian flow laws. These non-Newtonian fluids, for example mayo, paint, molten plastics, foams, clays, and many other fluids, behave in a wide variety of ways. The science of studying these types of unusual materials is called rheology

Examples of Complex Fluids

- Foods
  - Emulsions (mayonaisse, ice cream)
  - Foams (ice cream, whipped cream)
  - Suspensions (mustard, chocolate)
  - Gels (cheese)
- Biofluids
  - Suspension (blood)
  - Gel (mucin)
  - Solutions (spittle)
- Personal Care Products
  - Suspensions (nail polish, face scrubs)
  - Solutions/Gels (shampoos, conditioners)
  - Foams (shaving cream)
- Electronic and Optical Materials
  - Liquid Crystals (Monitor displays)
  - Melts (soldering paste)
- Pharmaceuticals
  - Gels (creams, particle precursors)
  - Emulsions (creams)
  - Aerosols (nasal sprays)
- Polymers

Rheology’s Goals

1. Establishing the relationship between applied forces and geometrical effects induced by these forces at a point (in a fluid).

- The mathematical form of this relationship is called the rheological equation of state, or **the constitutive equation.**
- The constitutive equations are used to solve macroscopic problems related to continuum mechanics of these materials.
- Any equation is just a model of physical reality.

Rheology's Goals

1. Establishing the relationship between rheological properties of material and its molecular structure (composition).

- Related to:
  - Estimating quality of materials
  - Understanding laws of molecular movements
  - Intermolecular interactions
- Interested in what happens inside a point during deformation of the medium.

What happens inside a point?

Material Structure

- More or less well-organized and regularly spaced shapes
- Arrangements, organization or intermolecular interactions
- Structured Materials – properties change due to the influence of applied of applied forces on the structure of matter
- Rheology sometimes is referred to as mechanical spectroscopy.
- “Structure Mechanisms” are usually proposed, analogous to reaction mechanisms in reaction kinetics
- Structural probes are used to support rheological studies and proposed mechanisms.

Rheological analysis is based on the use of continuum theories

meaning that:

- There is no discontinuity in transition from one geometrical point to another, and the mathematical analysis of infinitesimal quantities can be used; discontinuities appear only at boundaries
- Properties of materials may change in space (due to gradients) but such changes occur gradually
  - changes are reflected in space dependencies of material properties entering equations of continuum theories
- Continuity theories may include an idea of anisotropy of properties of material along different directions.

Rheology as an Interdisciplinary Science

Physics

Explanation and prediction of rheological properties

- molecular physics
- statistical physics
- thermodynamics, etc…

Rheology (of Liquids)

Chemistry

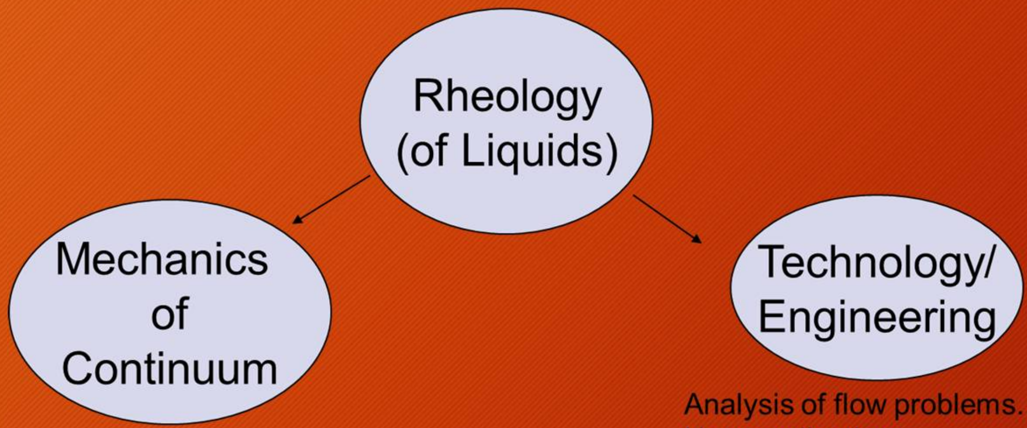
Direct correlation between chemical parameters and rheological properties

- molecular mass
- MWD
- chemical structures
- intermolecular interactions

Material Design

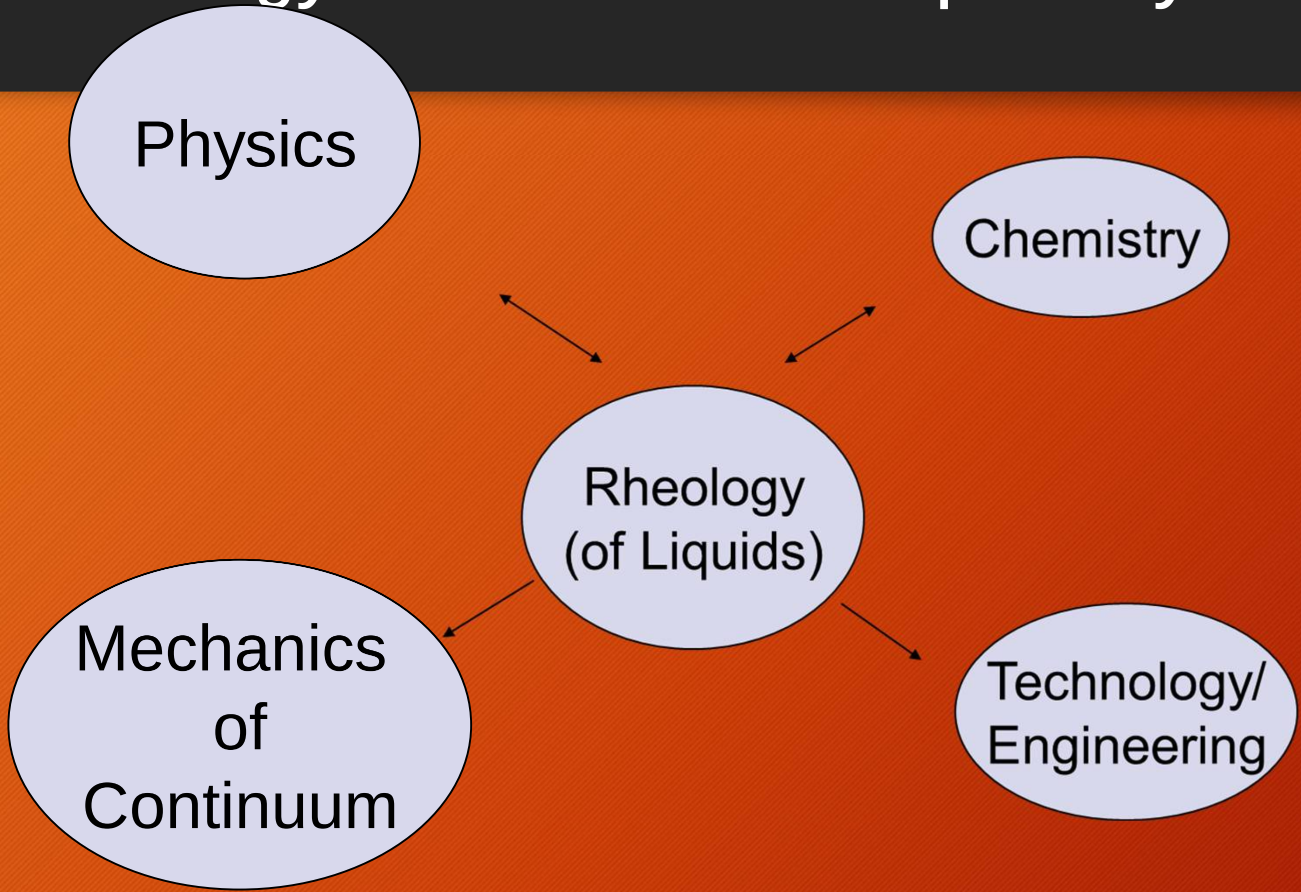
Rheology as an Interdisciplinary Science

Rheological studies give background for formulation of boundary problems in dynamics of liquids (governing equations and their solutions) to find numerical values of macro properties.



Analysis of flow problems.
New applications

Rheology as an Interdisciplinary Science



Rheological Properties

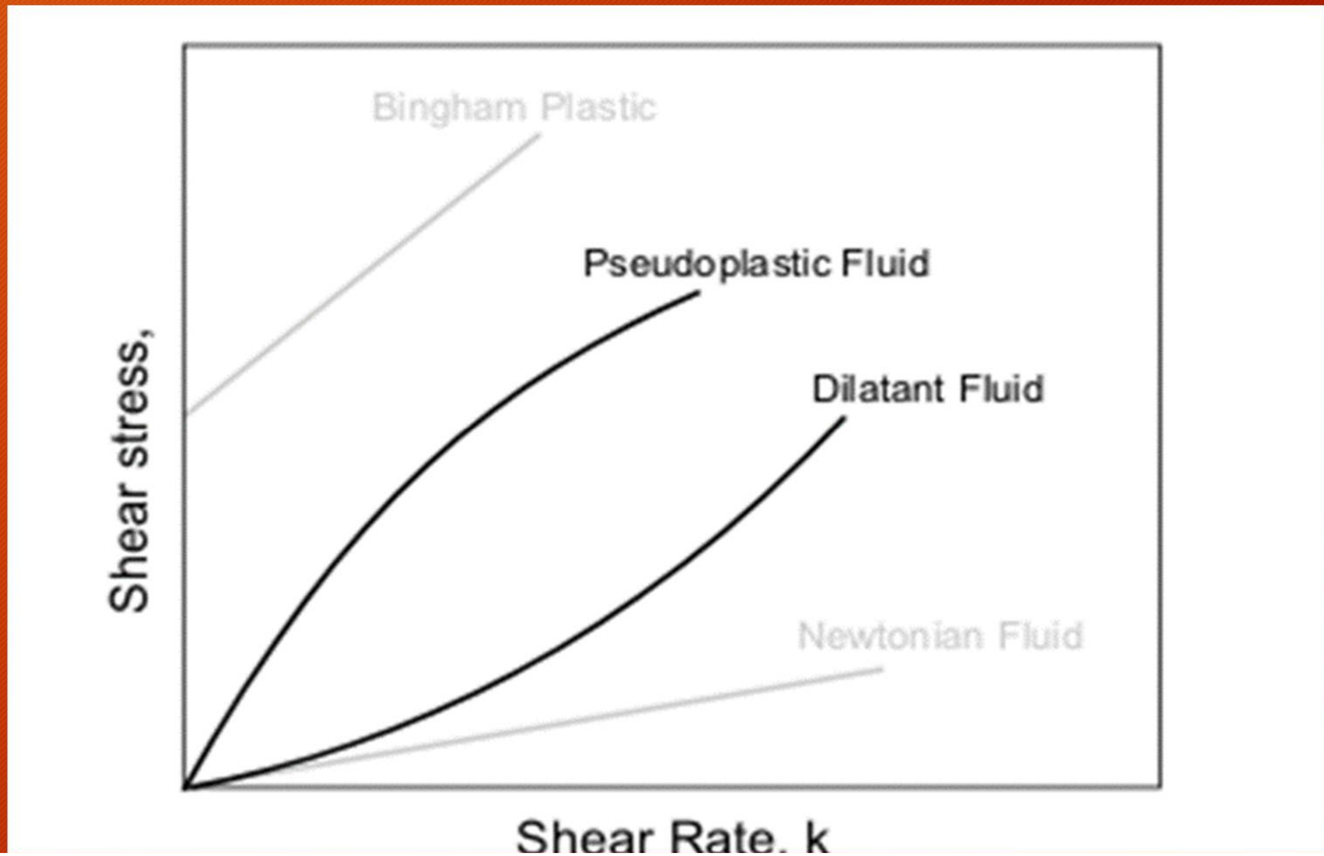
- Stress
  - Shear stress
  - Normal stress
  - Normal Stress differences
- Viscosity

  ← most commonly sought rheological quantity

  - Steady-state (i.e. shear)
  - Extensional
  - Complex
- Viscoelastic Modulus
  - G' – storage modulus
  - G" – loss modulus
- Creep, Compliance, Decay
- Relaxation times
- and many more …

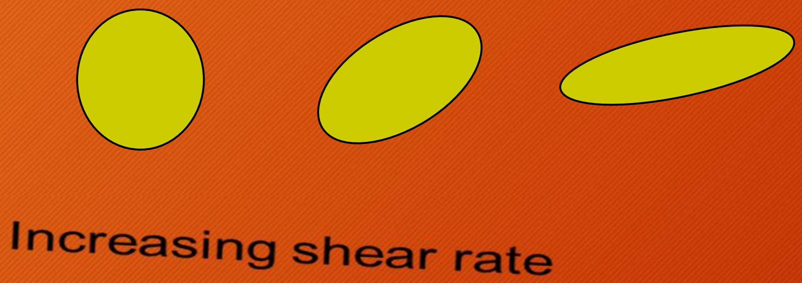
Common Non-Newtonian Behavior

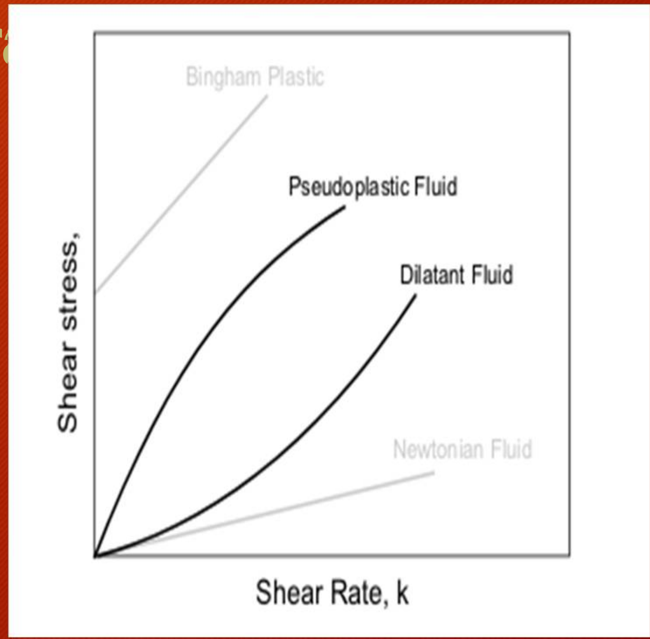
- shear thinning
- shear thickening
- yield stress
- viscoelastic effects
  - Weissenberg effect
  - Fluid memory
  - Die Swell



Shear Thinning and Shear Thickening

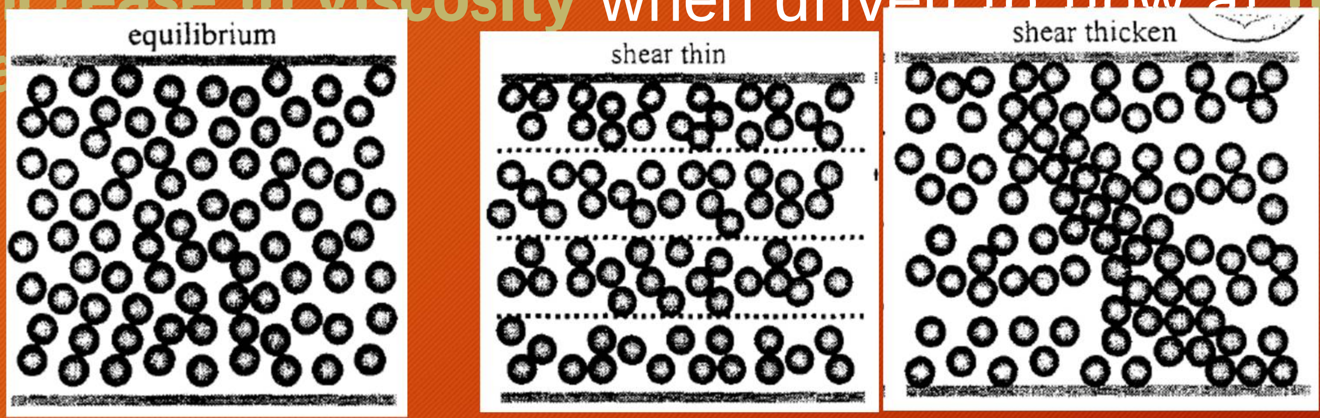
- shear thinning – tendency of some materials to **decrease in viscosity** when driven to flow at **high shear ra** such as by higher pressure drops





Shear Thickening

- shear thickening – tendency of some materials to **increase in viscosity** when driven to flow at **high shear ra**



Quicksand – A Non-Newtonian Fluid

- Quicksand is a colloid hydrogel (sand, clay and salt water).
- When undisturbed behaves as a solid gel, but minor changes in the stress will cause a sudden decrease in its viscosity
- After the initial perturbation, water and sand separate and dense regions of sand sediment
  - High volume fraction regions -> viscosity increases
- Sufficient pressure must be applied to reintroduced water into the compacted sand.
- The forces required to remove a foot from quicksand at a speed of 1 cm/s are about the same as "that needed to lift a medium-sized car."

Phenomenological Modeling of Shear Thinning and Thickening

- Generalized Newtonian Equation: $\tau = \eta(\dot{\gamma})\dot{\gamma}$
-
- Power Law Model: $\eta = m\dot{\gamma}^{n-1}$

  - m = μ n = 1 Newtonian
  - m n > 1 Shear Thickening, Dilatant
  - m n < 1 Shear Thinning

- Slope of log η vs log γ is constant
- Advantages: simple, success at predicting Q vs ΔP
- Disadvantages: does not describe Newtonian Plateau at small shear rates

Modeling of Shear Thinning and Thickening

- Carreau-Yasuda Model

$$\frac{\eta(\dot{\gamma})-\eta_\infty}{\eta_0-\eta_\infty}=[1+(\dot{\gamma}\lambda)^a]^{\frac{n-1}{a}}$$

a – affects the shape of the transition region
λ – time constant determines where it changes from constant to power law
n – describes the slope of the power law
η_0, η_∞ - describe plateau viscosities

- Advantages: fits most data
- Disadvantages: contains 5 parameters, do not give molecular insight into polymer behavior

Yield Stress

- Tendency of a material to flow only when stresses are above a threshold stress

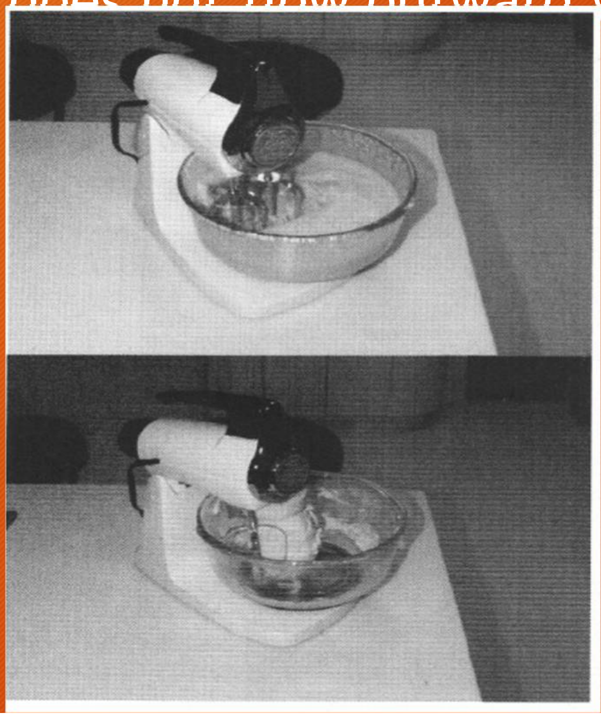
- Bingham Model:

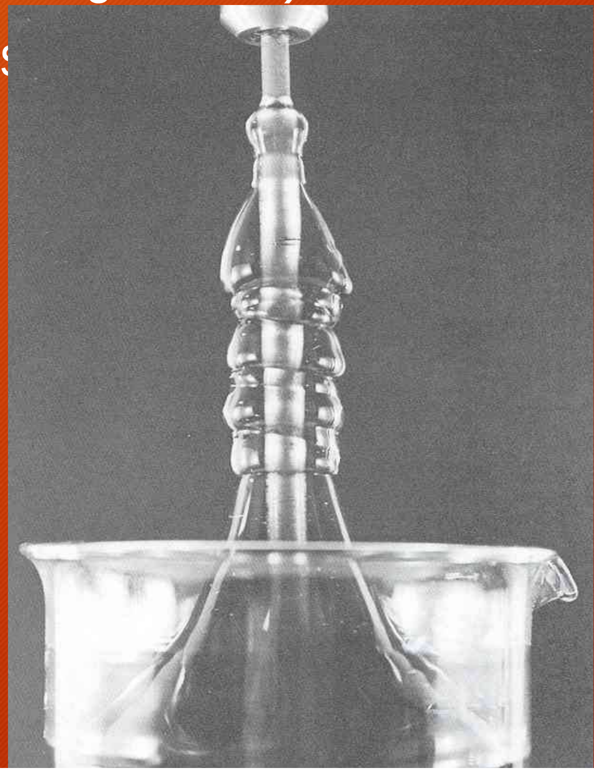
$$\eta(\dot{\gamma}) = \begin{cases} \infty & \tau \leq \tau_y \\ \mu_0 + \dfrac{\tau_y}{\dot{\gamma}} & \tau \geq \tau_y \end{cases}$$

τ_y = yield stress, always positive
μ_0 = viscosity at higher shear rates

Elastic and Viscoelastic Effects

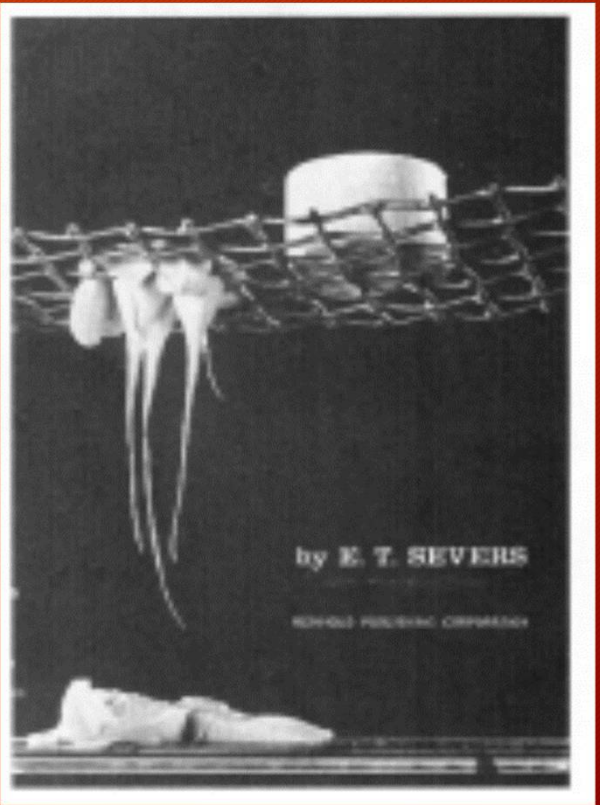
- Weissenberg Effect (Rod Climbing Effect)
- does not flow outward when s





Elastic and Viscoelastic Effects

- Fluid Memory
  - Conserve their shape over time periods or seconds or
  - Elastic like rubber
  - Can bounce or partially retract
  - Example: clay (plasticina)

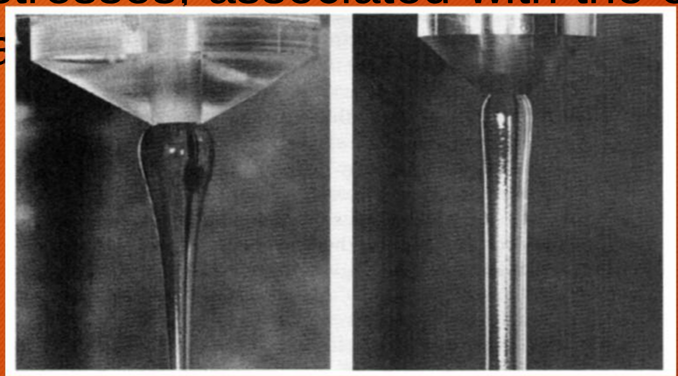


Elastic and Viscoelastic Effects

- Viscoelastic fluids subjected to a stress deform
  - when the stress is removed, it does not instantly vanish
  - internal structure of material can sustain stress for some time
  - this time is known as the relaxation time, varies with materials
  - due to the internal stress, the fluid will deform on its own, even when external stresses are removed
  - important for processing of polymer melts, casting, etc..

Elastic and Viscoelastic Effects – Die Swell

- as a polymer exits a die, the diameter of liquid stream increases by up to an order of magnitude
- caused by relaxation of extended polymer coils, as stress is reduced from high flow producing stresses present within the die to low stresses, associated with the extruded stream moving through a



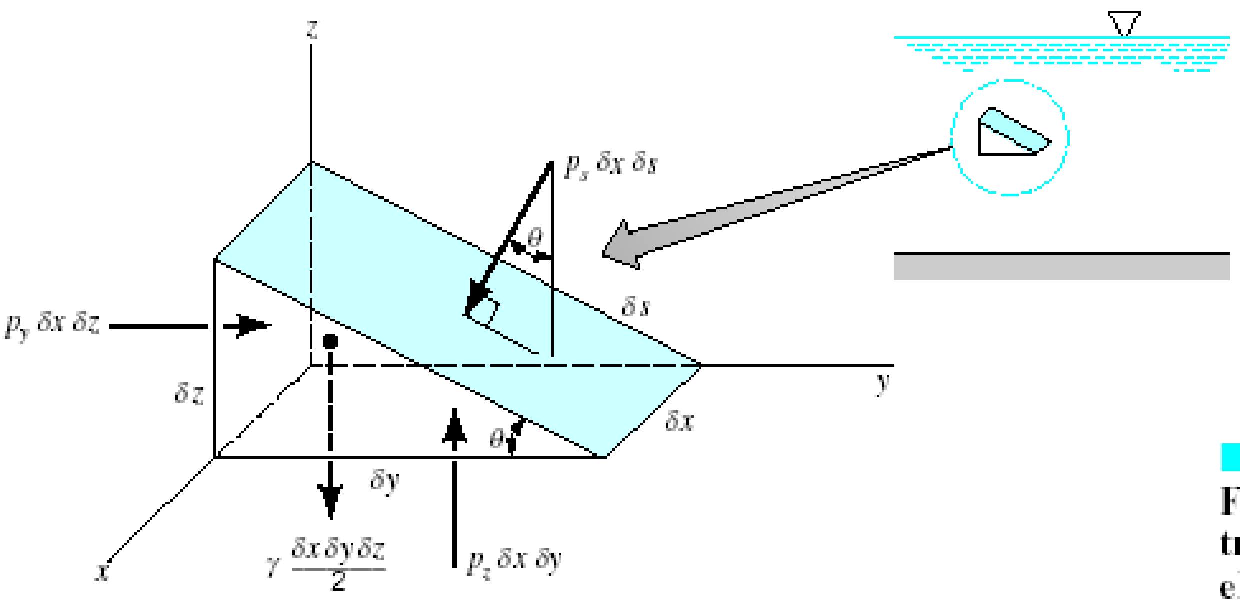
Fluid Statics

- When a surface is submerged in a fluid at rest, hydrostatic forces develop on the surface due to the fluid pressure. These forces must be perpendicular to the surface since there is no shear action present. These forces can be determined by integrating the static pressure distribution over the area it is acting on.

Pressure distribution in a static fluid and its effects on solid surfaces and on floating and submerged bodies.

- Fluid either at rest or moving in a manner that there is no relative motion between adjacent particles.
- No shearing stress in the fluid
- Only pressure (force that develop on the surfaces of the particles)

2.1 Pressure at a point N/m^2 (Force/Area)



■ FIGURE 2.1 Forces on an arbitrary wedge-shaped element of fluid.

$$\vec{F} = m\vec{a}$$

$$\sum F_y = p_y \delta x \delta z - P_s \delta x \delta s \sin\theta$$

Y: $$= \rho \frac{\delta x \delta y \delta z}{2} a_y$$

$$\sum F_z = p_z \delta x \delta y - p_z \delta x \delta s \cos\theta - \gamma \frac{\delta x \delta y \delta z}{2} a_z$$

Z: $$= \rho \frac{\delta x \delta y \delta z}{2} a_z$$

$$\delta y = \delta s \cos\theta \ ; \ \delta z = \delta s \sin\theta$$

$$y : p_y - p_s = \rho a_y \frac{\delta y}{2}$$

$$z : p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$

What happen at a pt. ?

$$\delta x, \delta y, \delta z \rightarrow 0$$

$$p_y = p_s$$

$$p_z = p_s$$

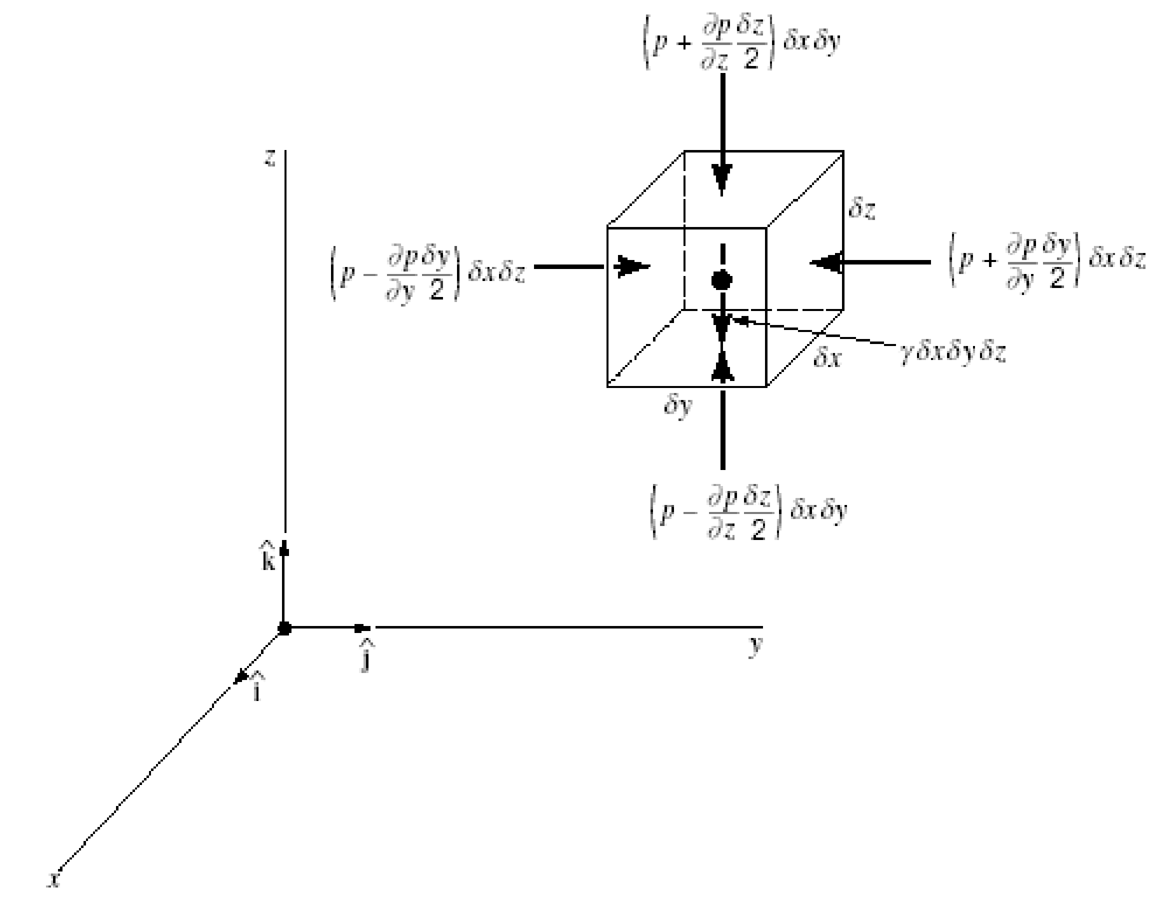
$$\Longrightarrow p_y = p_z = p_s$$

θ is arbitrarily chosen

Pressure at a pt. in a fluid at rest, or in motion, is independent of direction as long as there are no shearing stresses present. (Pascal's law)

2.2 Basic equation for Pressure Field

How does the pressure in a fluid which there are no shearing stresses



■ FIGURE 2.2
Surface and body forces acting on small fluid element.

Surface & body forces acting on small fluid element

↘ pressure ↘ weight

Surface forces:

$$y : \delta F_y = (p - \frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x \delta z - (p + \frac{\partial p}{\partial y}\frac{\delta y}{2})\delta x \delta z$$

$$\delta F_y = -\frac{\partial p}{\partial y}\delta x \delta y \delta z$$

Similarly, in *z* and *x* directions:

$$\delta F_x = -\frac{\partial p}{\partial x}\delta x \delta y \delta z \qquad \delta F_z = -\frac{\partial p}{\partial z}\delta x \delta y \delta z$$

$$\delta F_s = \delta F_x \vec{i} + \delta F_y \vec{j} + \delta F_z \vec{k} = -(\frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k})\delta x \delta y \delta z$$

$$= -(\nabla p)\delta x \delta y \delta z \qquad \nabla \equiv \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Newton's second law

$$\sum \delta\vec{F} = \delta m\vec{a} = \delta\vec{F}_s + \delta\vec{W} = -\nabla p\,\delta x\delta y\delta z - \gamma\delta x\delta y\delta z$$

$$= \rho\left(\delta x\delta y\delta z\right)\vec{a}$$

$$\therefore \underline{-\nabla p - \gamma\vec{k} = \rho\vec{a}}$$

General equation of motion for a fluid in which there are no shearing stresses.

2.3 Pressure variation in a fluid at rest

$$\vec{a} = 0 \quad \rightarrow \nabla p + \gamma\vec{k} = 0$$

$$\frac{\partial p}{\partial z} = -\gamma$$

$$\underline{\frac{dp}{dz} = -\gamma} \qquad\qquad (\text{Eq.}2.4)$$

2.3.1 Incompressible

$$\gamma = \rho g = const$$

$$\int_{p_1}^{p_2} dp = -\gamma \int_{z_2}^{z_1} dz \rightarrow p_1 - p_2 = \gamma (z_2 - z_1) = \gamma h$$

Hydrostatic Distribution

$$p_1 = \gamma h + p_2$$

*see Fig. 2.2

$$h = \frac{p_1 - p_2}{\gamma}$$

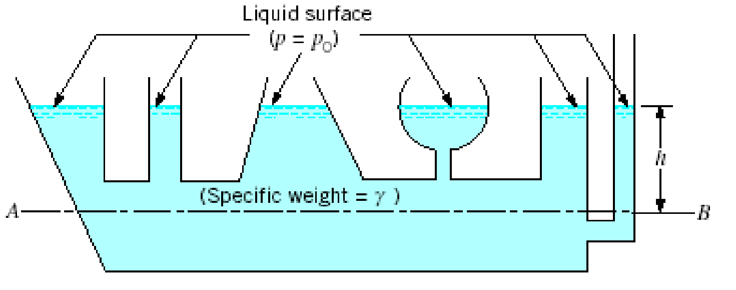
pressure head

Ex: $10\,psi = p_1 - p_2 \rightarrow h = 23.1\,ft \text{ or } 518\,mmHg$

$(\gamma = 62.4\,lb/ft^2)\ (\gamma = 133\,KN/m^3)$

$$p = \gamma h + p_0$$

Pressure in a homogeneous, incompressible fluid at rest: ~ reference level, indep. of size or shape of the container.



■ FIGURE 2.4 Fluid equilibrium in a container of arbitrary shape.

The required equality of pressures at equal elevations
Throughout a system.

$$\rightarrow F_2 = \frac{A_2}{A_1} F_1 \qquad (Fig.2.5)$$

Transmission of fluid pressure

2.3.2 Compressible Fluid perfect gas: $p = \rho RT$

$$\frac{dp}{dz} = -\gamma = -\rho g = -\frac{gp}{RT}$$

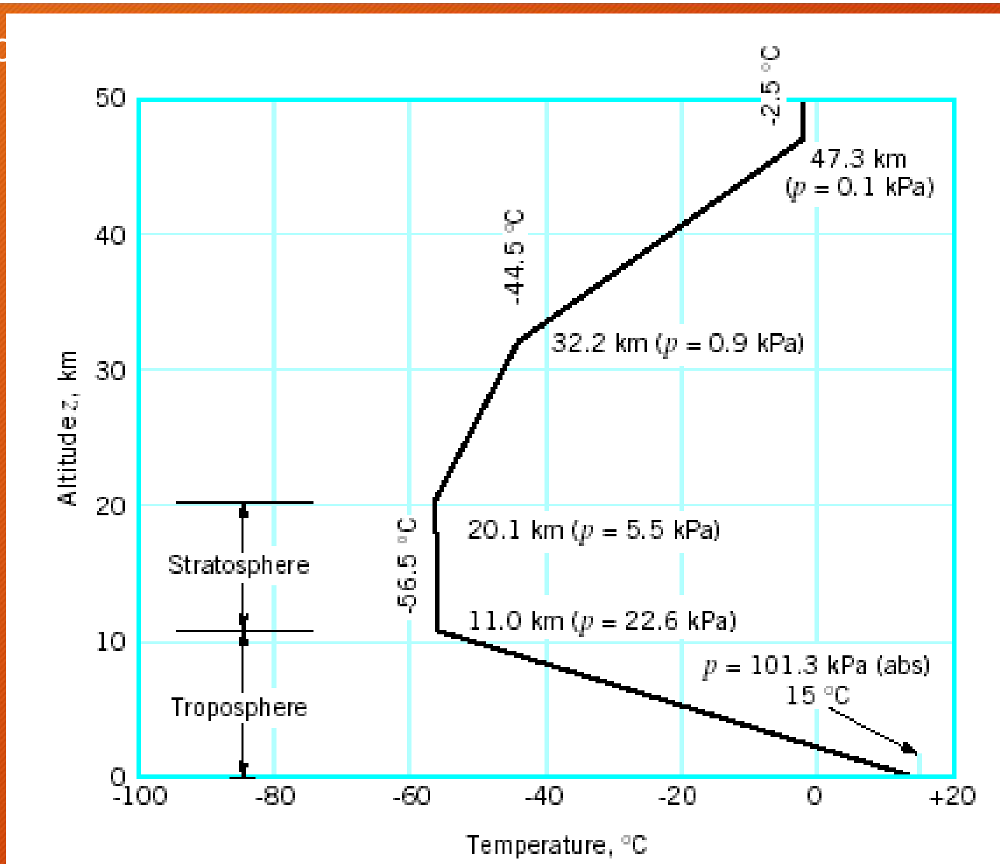
$$\int_{p_1}^{p_2} \frac{dp}{p} = \ln\frac{p_2}{p_1} = -\frac{g}{R}\int_{Z_1}^{Z_2} \frac{dz}{T} \qquad g, R \text{ const.} (z_1 \rightarrow z_2)$$

Assume

$$T = T_0 \quad over[z_1, z_2] \text{ isothermal conditions}$$

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

2.4 Standard Atmo



Troposphere:

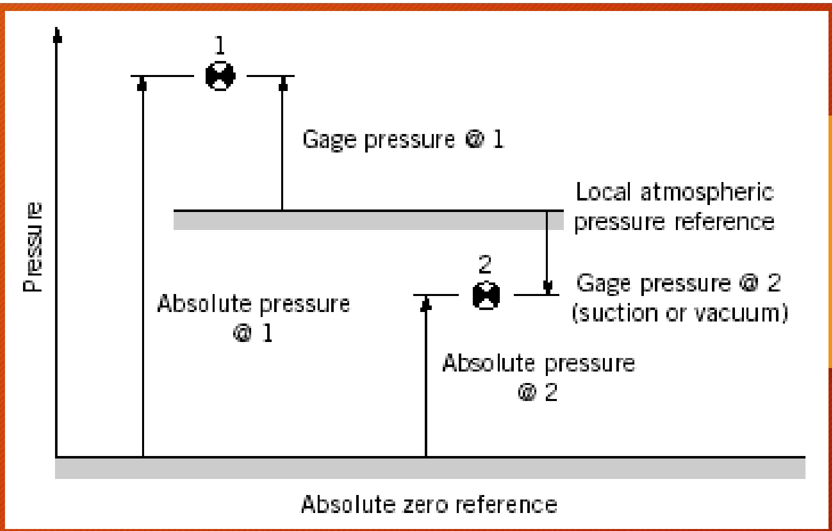
$$T = T_a - \beta z \qquad T_a \; @ \; z = 0$$

$$0.0065 \, {}^\circ K / m \approx \beta \; lapose \; rate$$

$$0.00357 \, {}^\circ R / ft$$

$$p = p_a \left(1 - \frac{\beta z}{T_a}\right)^{g / R\beta}$$

2.5 Measurement of Pressure
See Fig. 2.7
Absolute &
Gage pressure



$$p_{atm} = \gamma\ h + p_{vapor}$$

(Mercury barometer)
Example 2.3

$$pa = \frac{N}{m^2}(\ pascal\ )$$

2.6 Manometry
1. Piezometer Tube:
2. U-Tube Manometer:
3. Inclined-tube manometer

1. $p > p_a$ 2. h_1 is reasonable $p - p_a$不大 3. liquid, not a gas

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

see examples

*explain Fig. 2.11 Differential U-tube manometer

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Example 2.5

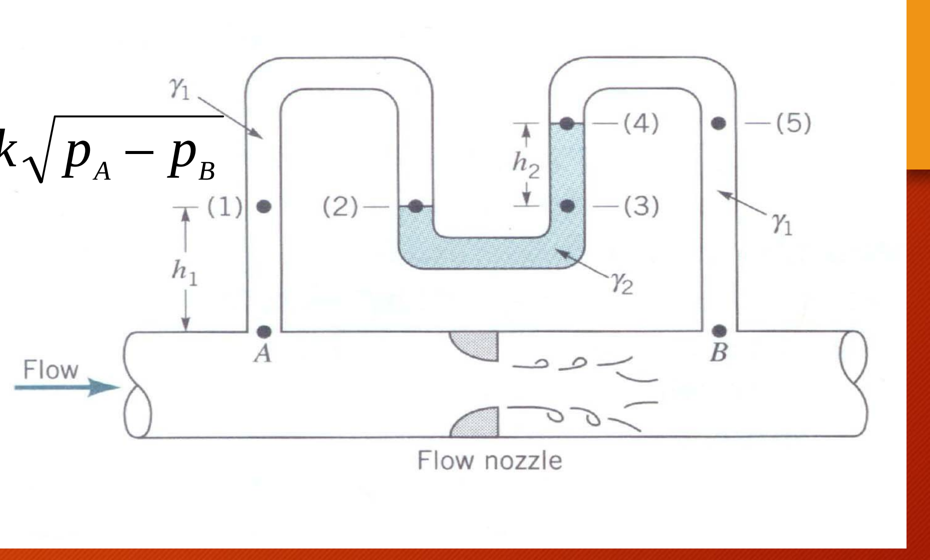
Ex. 2.5

$$\Delta u \uparrow, \Delta p \downarrow, \Delta p = p_A - p_B$$

$$Q(\text{the volume rate of the flow}) = k\sqrt{p_A - p_B}$$

$$p_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1 (h_1 + h_2) = p_B$$

$$p_A - p_B = h_2(\gamma_2 - \gamma_1)$$



2.6.3 Fig. 2.12 Inclined tube manometer

$$p_A - p_B = \gamma_2 l_2 \sin\theta$$

$$l_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$

Small difference in gas pressure
If pipes A & B contain a gas

2.7 Mechanical and Electronic Pressure Measuring Device

- Bourdon pressure gage (elastic structure)

  Bourdon Tube

  $p\uparrow,$ curved tube $\rightarrow$ straight

  deformation $\rightarrow$ dial

- A zero reading on the gage indicates that the measured pressure

- Aneroid barometer—measure atmospheric pressure (absolute pressure)

- Pressure transducer—pressure V.S. time

  Bourdon tube is connected to a linear variable differential transformer(LVDT), Fig. 2.14

  coil; voltage

This voltage is linear function of the pressure, and could be recorded on an oscillograph, or digitized for storage or processing on computer.

Disadvantage-elastic sensing element

meas. pressure are static or only changing slowly(quasistatic).

relatively mass of Bourdon tube

<diaphragm>

1Hz

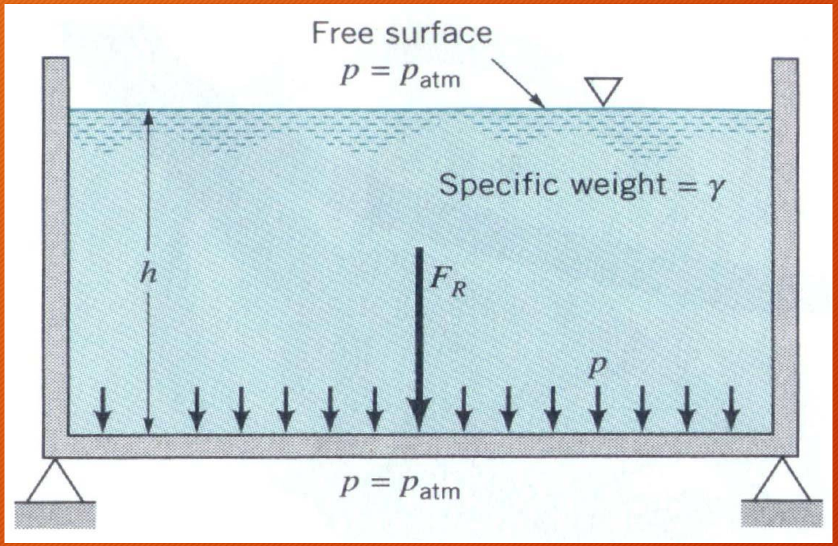
*strain-gage pressure transducer *

Fig. 2.15 (arterial blood pressure)

piezo-electric crystal. (Refs. 3, 4, 5)

2.8 Hydrostatic Force on a Plane Surface

Fig. 2.16 Pressure and resultants hydrostatic force developed on the bottom of an open tank.



$$F_R = pA$$

Storage tanks, ships

. For fluid at rest we know that the force must be perpendicular to the surface, since there are no shearing stress present.

. Pressure varies linearly with depth if incompressible

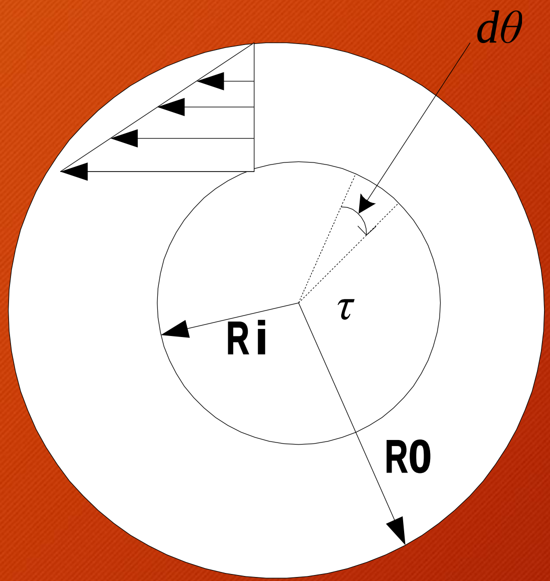
$$\frac{dp}{dz} = -\gamma = -\rho g \qquad p = \gamma h$$ for open tank, Fig. 2.16

The resultant force acts through the centroid of the area

* Exercise 1.66

$$dT = R_i \tau dA$$

torque   shearing stress



$$dA = (R_i d\theta) l$$

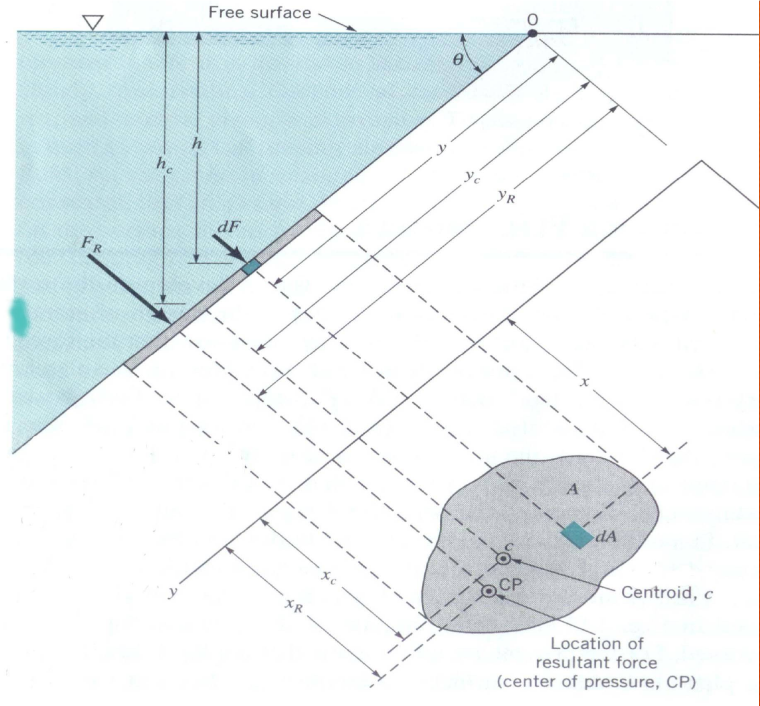
$$dT = R_i^2 l \tau d\theta$$

$$T = R_i^2 l \tau \int_0^{2\pi} d\theta = 2\pi R_i l \tau$$

Assume velocity distribution in the gap is linear

$$\tau = \mu \frac{R_i w}{R_0 - R_i}$$

$$\therefore T = \frac{2\pi R_i^3 l \mu w}{R_0 - R_i}$$



$$dF = \gamma h dA$$

$$F_R = \int_A \gamma h dA = \int_A \gamma y \sin\theta dA$$

if θ, γ are constants.

$$F_R = \gamma \sin\theta \underline{\int_A y dA}$$

first moment of the area

$\int_A ydA = y_c A$

$$\therefore F_R = \gamma A y_C \sin\theta = \underline{\gamma h_c A}$$

Indep. Of θ

The moment of the resultant force must equal the moment of the Distributed pressure force

$$F_R y_R = \int_A y dF = \int_A \gamma \sin\theta y^2 dA$$

$$\because F_R = \gamma A y_C \sin\theta \qquad \therefore y_R = \frac{\int_A y^2 dA}{y_c A}$$

$I_x = \int_A y^2 dA$ – second moment of the area (moment of inertia)

$$y_R = \frac{I_x}{y_c A} \quad ; \quad I_x = I_{xc} + Ay_c^2$$

$$y_R = \frac{I_{xc}}{y_{cA}} + y_c \qquad y_R > y_c$$

$$x_R = \frac{I_{xyc}}{y_{cA}} + x_c \qquad I_{xc}, I_{xyc} \text{ ect see Fig. 2.18}$$

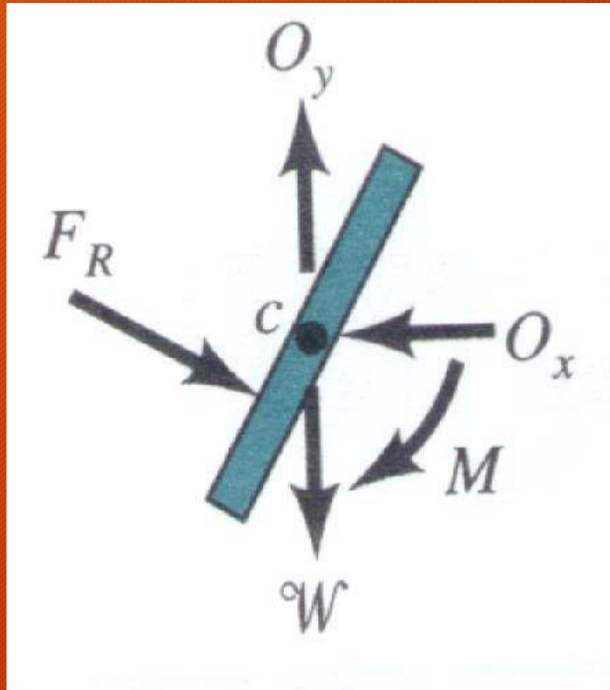
Note: I_{xy}-the product of inertia wrt the x& y area.
I_{xyc}-the product of inertia wrt to an orthogonal coord. system passing through the centroid of the area.

If the submerged area is symmetrical wrt an axes passing through the centroid and parallel to either the x or y axes, the resultant force must lie along the line $x=x_c$, since $I_{xyc}= 0$.

Center of pressure (Resultant force acts points)

Example 2.6 求a. $F_R;(x_R,y_R)$

b. $M\,(moment)$



(c)

a. $F_R \rightarrow \text{Eq. } 2.18 \qquad F_R = 1.23\times10^6\,N$

$x_R \rightarrow \text{Eq. } 2.19, 2.20 \qquad x_R = 0$

$y_R \qquad\qquad y_R = 11.6\text{m}$

b. $\sum M_c = 0 \quad \text{(shaft ; water)}$

$$M = F_R(y_R - y_c) = 1.01\times10^5\,N\cdot m$$

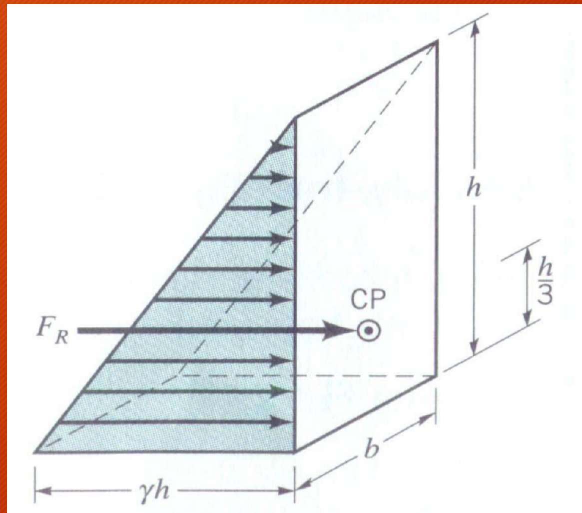
2.9 Pressure Prism

the pressure varies linearly with depth. See Fig. 2.19

$$F_R = P_{Ave}A = \gamma(\frac{h}{2})A$$

$$F_R = volume\ of\ pressure\ prism$$

$$= \frac{1}{2}(\gamma h)(bh) = \frac{h}{2}\gamma A$$

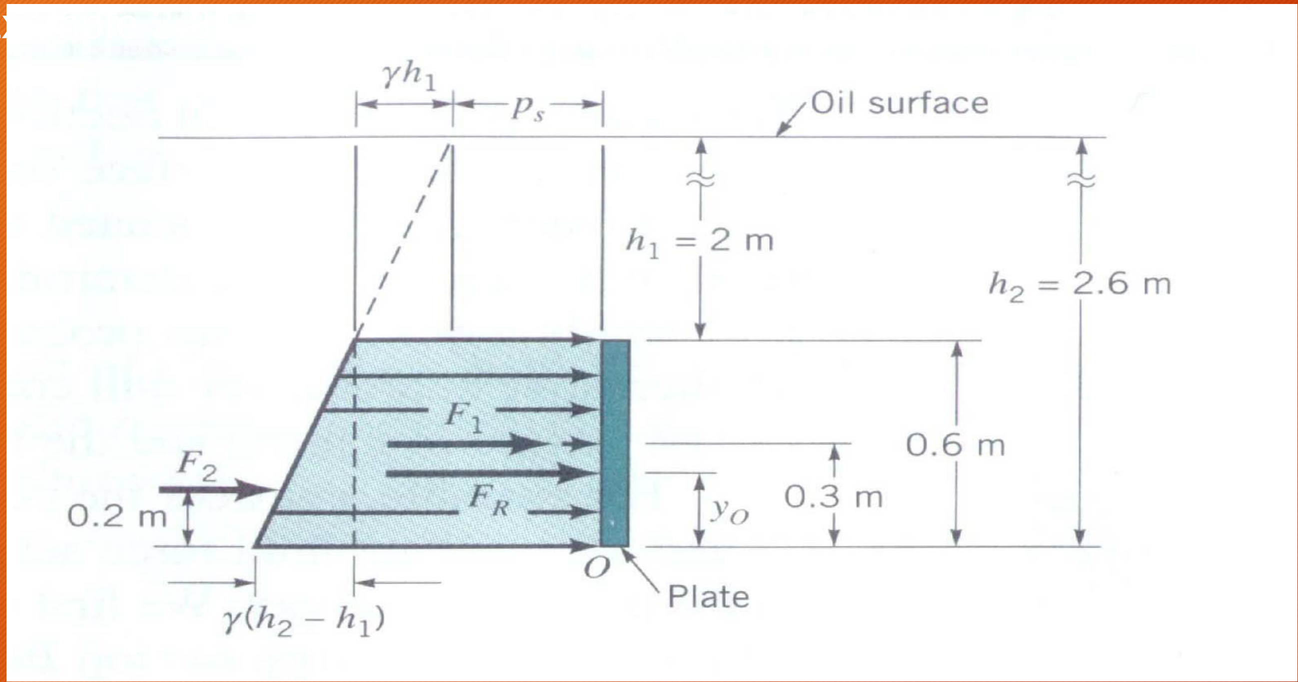


No matter what the shape of the pressure prism is, the resultant force is still equal in magnitude to the volume of the pressure Prism, and it passes through the centroid of the volume.

First, draw the pressure prism out. $\frac{dp}{dz} = -\gamma$

$$p = -\gamma z + p_0$$

Ex



$$F_1 = (\gamma h_1 + p_s)A = 2.44\times10^4\,N$$

$$F_2 = \gamma\left(\frac{h_2 - h_1}{2}\right)A = 0.954\times10^3\,N$$

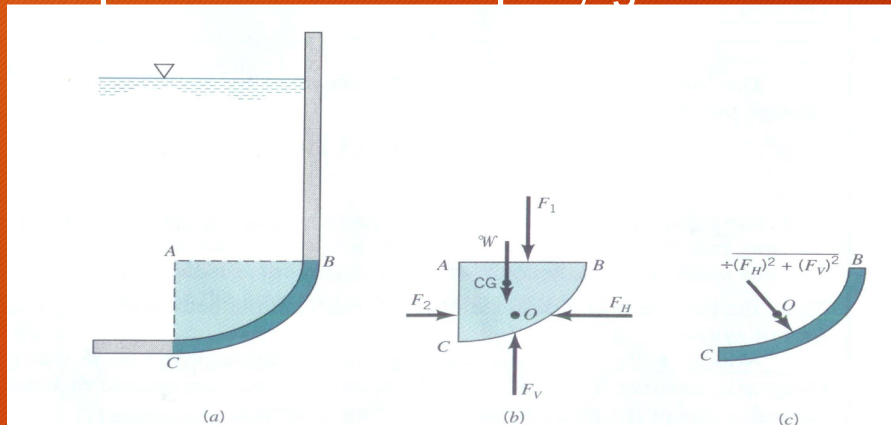
$$F_R = F_1 + F_2 = 25.4KN$$

$$F_R y_0 = F_1(0.3m) + F_2(0.2m)$$

$$\underline{y_0 = 0.296m}$$

2.10 Hydrostatic Force on a Curved Surface

- Eqs. Developed before only apply to the plane surfaces
  - ⟼ magnitude and location of F_R
- Integration: tedious process/ no simple, general formulas can be developed.
- Fig. 2.23



$$F_1; F_2 \rightarrow \text{plane surface}$$

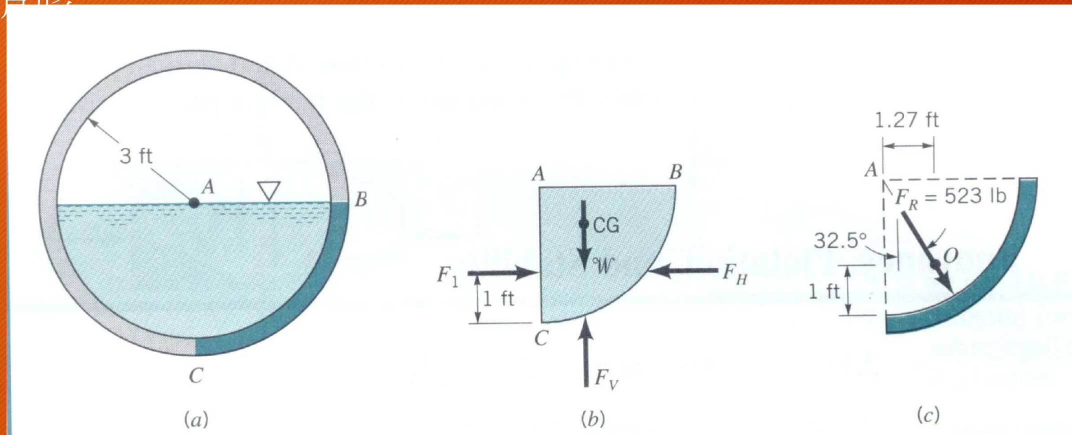
$$\overline{W} \rightarrow \gamma x V; \text{through C.G(center of gravity)}$$

$$F_H, F_V \rightarrow \text{The compoments of force that the tank exerts on the fluid.}$$

For equilibrium, $F_H = F_2;$ collinear. through pt

$$F_V = F_1 + \overline{W}$$

Example 2.9 排水管受力情形



$$F_1 = \gamma h_c A$$
$$= 62.4\frac{\text{lb}}{\text{ft}^3}\times\frac{3}{2}\text{ft}\times\left(3\times1\text{ft}^2\right)$$
$$= 281\text{lb}$$

$$\bar{\omega} = \gamma\forall = \rho g\forall = 62.4\frac{\text{lb}}{\text{ft}^3}\times\frac{\pi\times3^2}{4}\text{ft}^2\times1\text{ft}$$

See Fig. 2.18

$$= 441\text{b}$$ at C.G

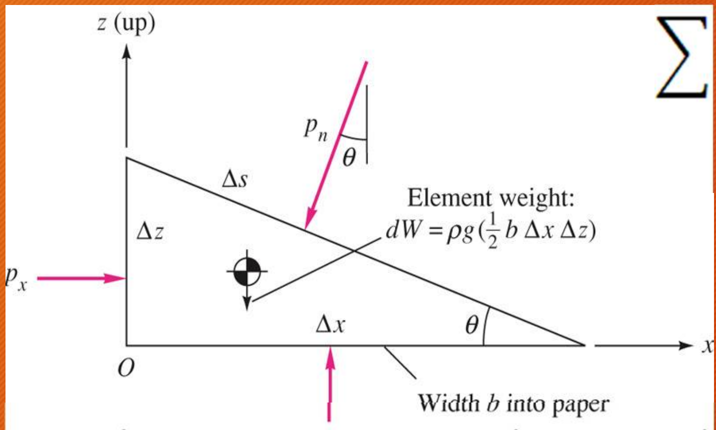
(Centroid; center of pressure, CP; center of gravity)

$$y_R = y_C + \frac{I \times C}{y_c A} = \frac{3}{2}\text{ft} + \frac{\frac{1}{12}\times3^4}{\frac{3}{2}\times3^2}\text{ft}$$

$$= 2\text{ ft}$$

Fluid at rest

- hydrostatic condition: when a fluid velocity is zero, the pressure variation is due only to the weight of the fluid.



$$\sum F_z = 0 = p_z\, b\, \Delta x - p_n b\, \Delta s\, cos\theta - \frac{1}{2}\rho g\, b\, \Delta x\, \Delta z$$

$$\sum F_x = 0 = p_x\, b\, \Delta z - p_n b\, \Delta s\, sin\theta$$

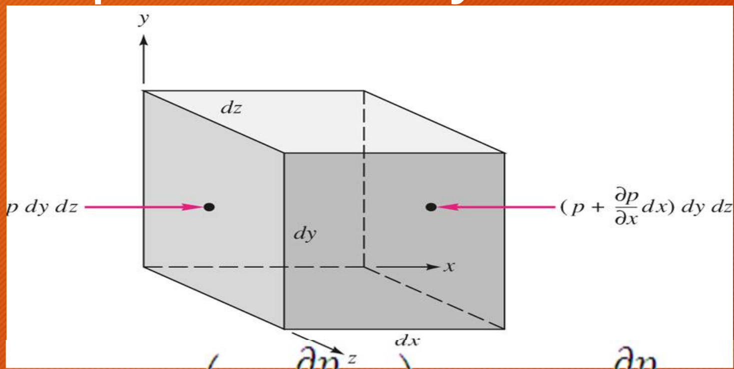
$$p_x = p_n \qquad p_z = p_n + \frac{1}{2}\rho g\, \Delta z$$

- There is no pressure change in the horizontal direction.
- There is a pressure change in the vertical direction proportional to the density, gravity, and depth change.

$$p_x = p_z = p_n = p$$

Pressure forces (pressure gradient)

- Assume the pressure vary arbitrarily in a fluid, $p=p(x,y,z,t)$.



$$dF_x = p dy dz - \left(p + \frac{\partial p}{\partial x} dx\right) dy dz = -\frac{\partial p}{\partial x} dx\, dy\, dz$$

$$dF_{press} = -\left(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} i\right) dx\, dy\, dz$$

$$f_{press} = -\nabla p$$

- The pressure gradient is a <u>surface force</u> that acts on the sides of the element.
- Note that the pressure gradient (not pressure) causes a net force that must be balanced by gravity or acceleration.

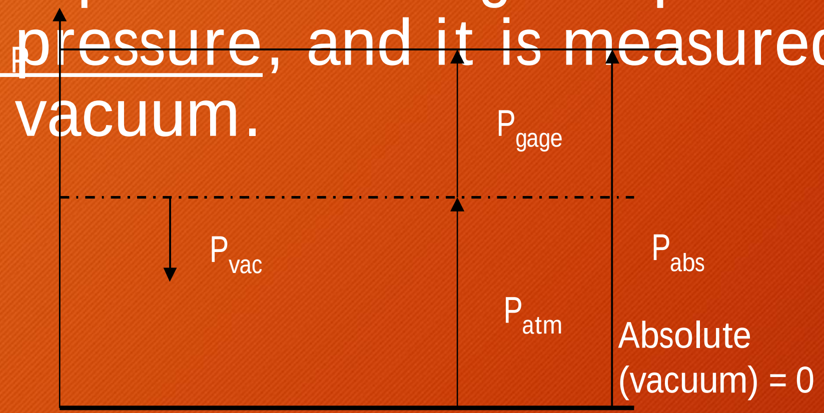
Equilibrium

- The pressure gradient must be balanced by gravity force, or weight of the element, for a fluid at rest.
- $dF_{gravity} = \rho g dx\, dy\, dz$ $\quad f_{gravity} = \rho g$

- The gravity force is a body force, acting on the entire mass of the element. Magnetic force is another example of body force.

Gage pressure and vacuum

- The actual pressure at a given position is called the <u>absolute pressure</u>, and it is measured relative to absolute vacuum.



$p > p_a$ *Gage pressure* $p_{gage} = p - p_a$

$p < p_a$ *Vacuum pressure* $p_{vacuum} = p_a - p$

Hydrostatic pressure distribution

- Recall: ∇p is perpendicular everywhere to surface of constant pressure *p*.

$$\nabla p = \rho g$$

- In our customary coordinate z is "upward" and the gravity vector is:

$$g = -gk$$

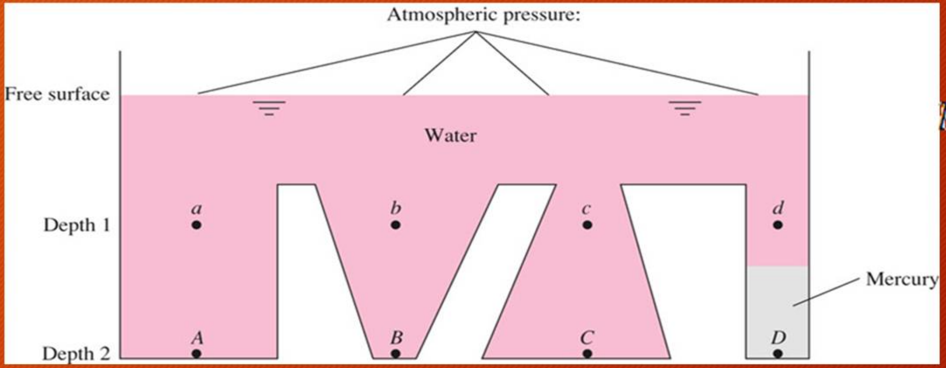
where g = 9.807 m/s^2. The pressure gradient vector becomes:

$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

$$\frac{dp}{dz} = -\gamma \qquad p_2 - p_1 = -\int_1^2 \gamma dz$$

Hydrostatic pressure distribution

- Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container.
- The pressure is the same at all points on a given horizontal plane in a fluid

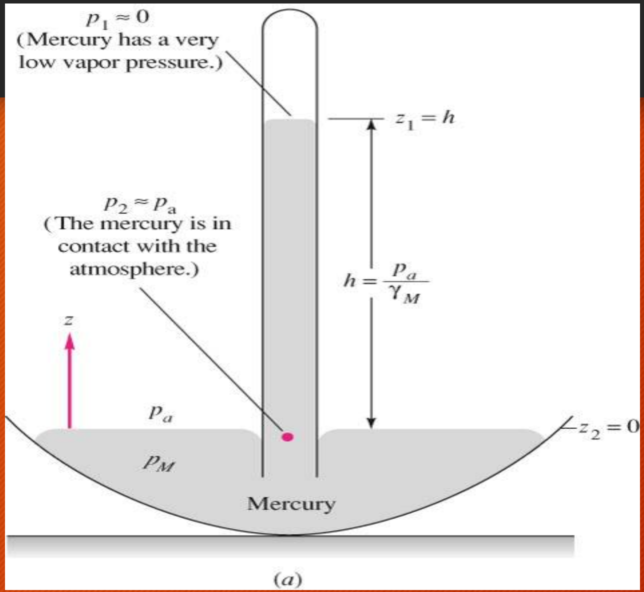


$$p_a = p_b = p_c$$

$$p_A = p_B = p_C \neq p_D$$

$$p_2 - p_1 = -\gamma(z_2 - z_1) \quad or \quad z_1 - z_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma}$$

The mercury barometer



P_{atm} = 761 mmHg

Mercury has an extremely small vapor pressure at room temperature (almost vacuum), thus $p_1 = 0$. One can write:

$$p_a - 0 = -\gamma_{mercury}(0 - h) \quad or \quad h = \frac{p_a}{\gamma_{mercury}}$$

Hydrostatic pressure in gases

- Gases are compressible, using the ideal gas equation of state, $p=\rho RT$:

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT}g$$

- For small variations in elevation, "isothermal atmosphere" can be assumed:

$$p_2 = p_1 exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

- In general (for higher altitudes) the atmospheric temperature drops off linearly with z

$$T \approx T_0 - Bz$$

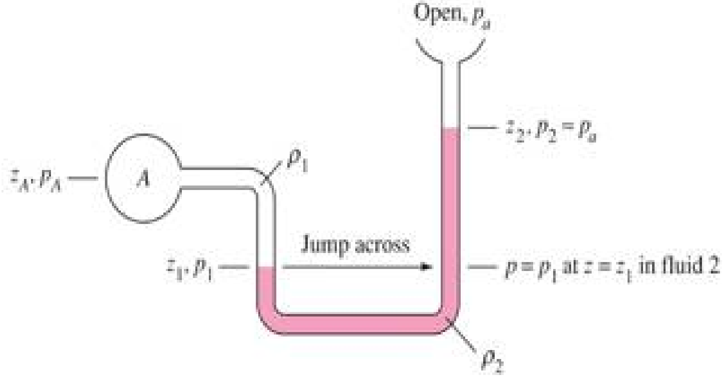
where T_0 is the sea-level temperature (in Kelvin) and B=0.00650 K/m.

$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/RB} \quad \text{for air } \frac{g}{RB} = 5.26$$

- Note that the P_{atm} is nearly zero (vacuum condition) at z = 30 km.

Manometry

- A static column of one or multiple fluids can be used to measure pressure difference between 2 points. Such a device is called manometer.



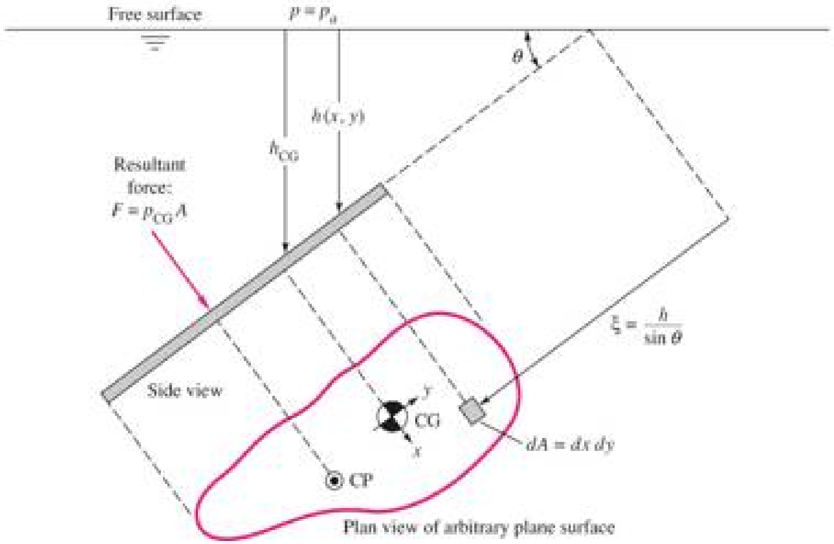
$$p_A + \gamma_1|z_A - z_1| - \gamma_2|z_1 - z_2| = p_2 = p_{atm}$$

- Adding/ subtracting γΔz as moving down/up in a fluid column.

- Jumping across U-tubes: any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

Hydrostatic forces on surfaces

- Consider a plane panel of arbitrary shape completely submerged in a liquid.



$$p = p_a + \gamma h$$

- The total hydrostatic force on one side of the plane is given by:

$$F = \int p dA = \int (p_a + \gamma h)\, dA = p_a A + \gamma \int h dA$$

Hydrostatic forces on surfaces

- After integration and simplifications, we find:

$$F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG})A = p_{CG} A$$

- The force on one side of any plane submerged surface in a uniform fluid equals the pressure at the plate centroid times the plate area, independent of the shape of the plate or angle θ.

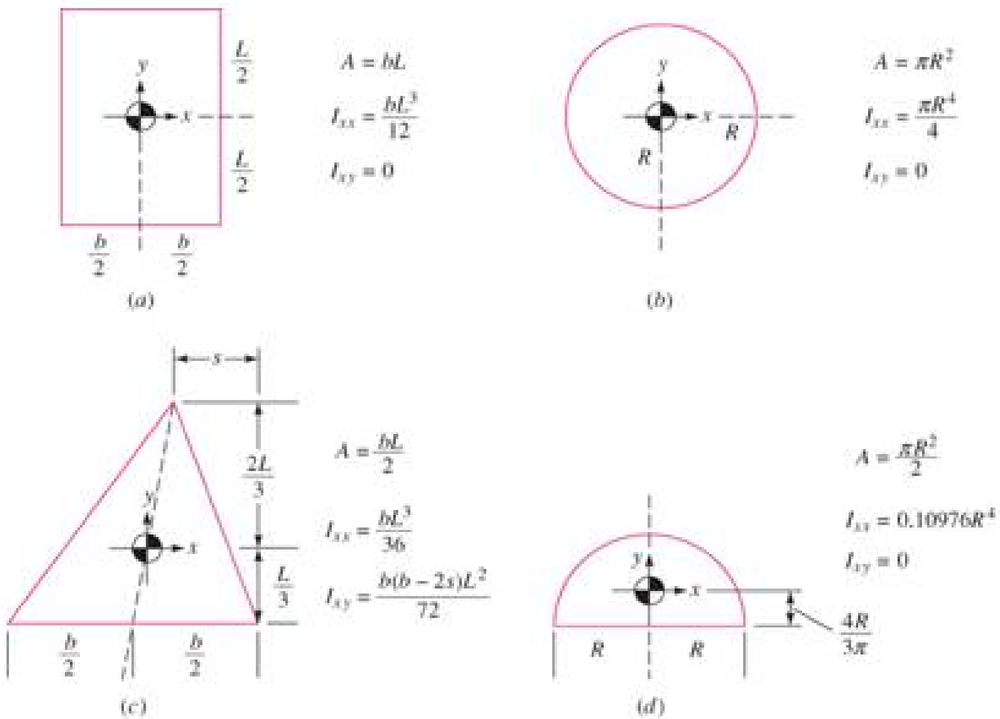
- The resultant force acts not through the centroid but below it toward the high pressure side. Its line of action passes through the centre of pressure CP of the plate (x_{CP}, y_{CP}).

$$F y_{CP} = \int y p dA = \int y(p_a + \gamma \xi sin\theta)\, dA = \gamma sin\theta \int y\xi dA$$

$$y_{CP} = -\gamma sin\theta \frac{I_{xx}}{p_{CG} A} \qquad x_{CP} = -\gamma sin\theta \frac{I_{xy}}{p_{CG} A}$$

Hydrostatic forces on surfaces

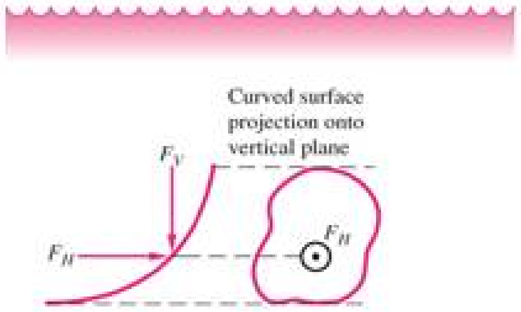
- Centroidal moments of inertia for various cross-sections.

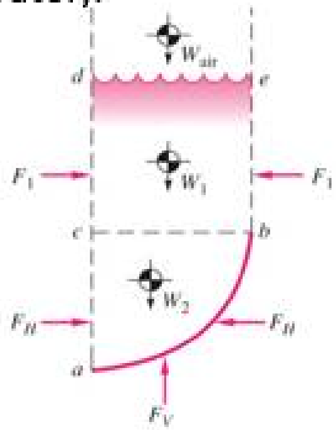


- Note: for symmetrical plates, $I_{xy} = 0$ and thus $x_{CP} = 0$. As a result, the center of pressure lies directly below the centroid on the y axis.

Hydrostatic forces: curved surfaces

- The easiest way to calculate the pressure forces on a curved surface is to compute the horizontal and vertical forces separately.



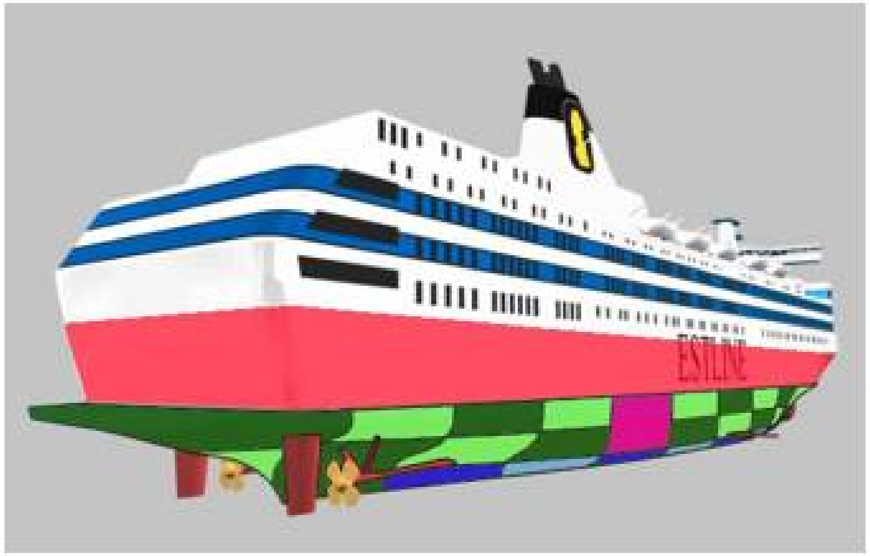


- The horizontal force equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.
- The vertical component equals to the weight of the entire column of fluid, both liquid and atmospheric above the curved surface.

$$F_V = W_2 + W_1 + W_{air}$$

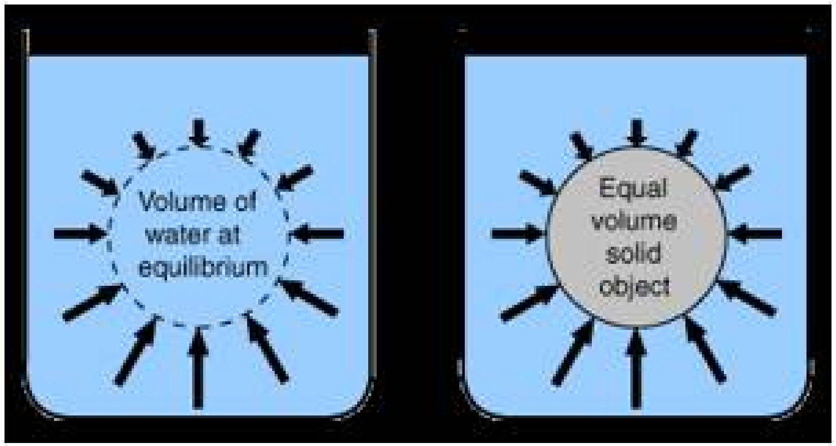
Buoyancy

- From Buoyancy principle, we can see whether an object floats or sinks. It is based on not only its weight, but also the amount of water it displaces. That is why a very heavy ocean liner can float. It displaces a large amount of water.

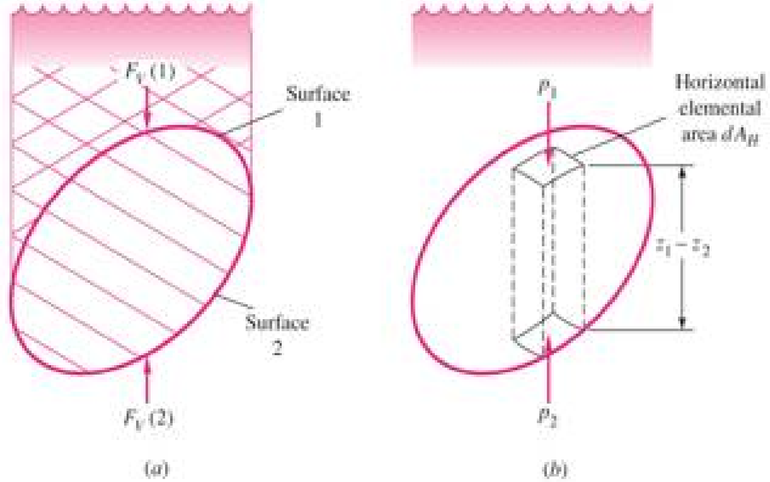


Archimedes 1st law

- A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces



Buoyancy force

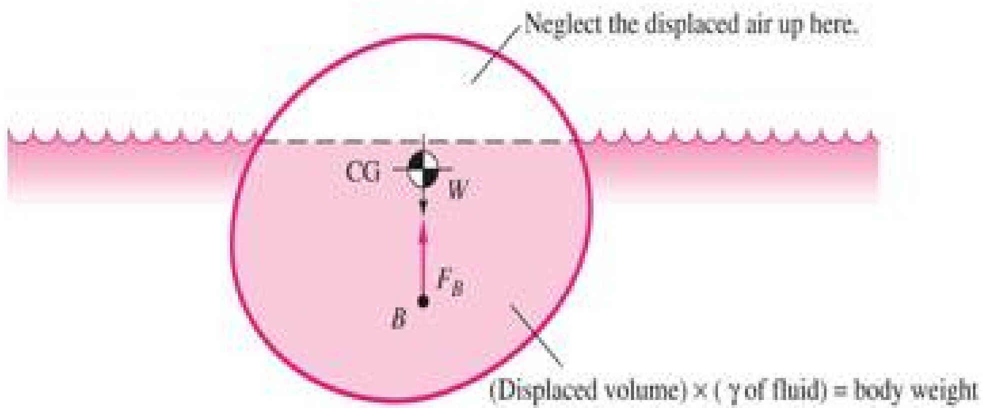


$$F_B = \int_{body} (p_2 - p_1)\, dA_H = -\gamma \int (z_2 - z_1)\, dA_H = \gamma(body\ volume)$$

- The line of action of the buoyant force passes through the center of volume of the displaced body; i.e., the center of mass is computed as if it had uniform density. The point which F_B acts is called the *center of buoyancy*.

Archimedes 2nd law

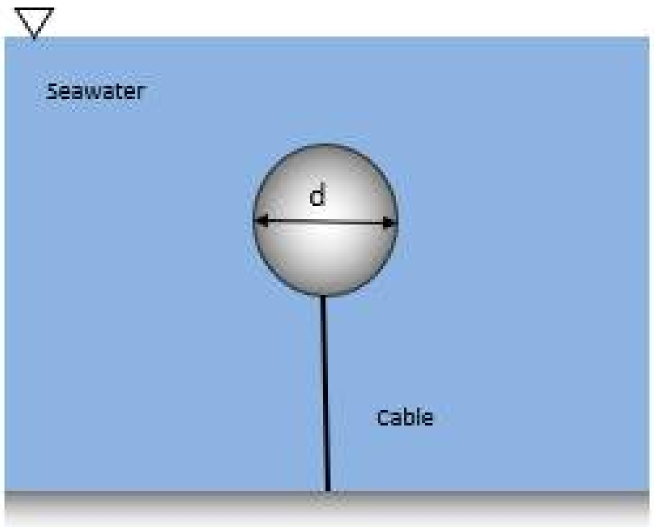
- A floating body displaces its own weight in the fluid in which it floats. In the case of a floating body, only a portion of the body is submerged.

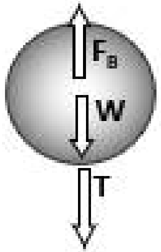


$$F_B = \gamma(displaced\ volume) = \text{weight of the floating body}$$

Example

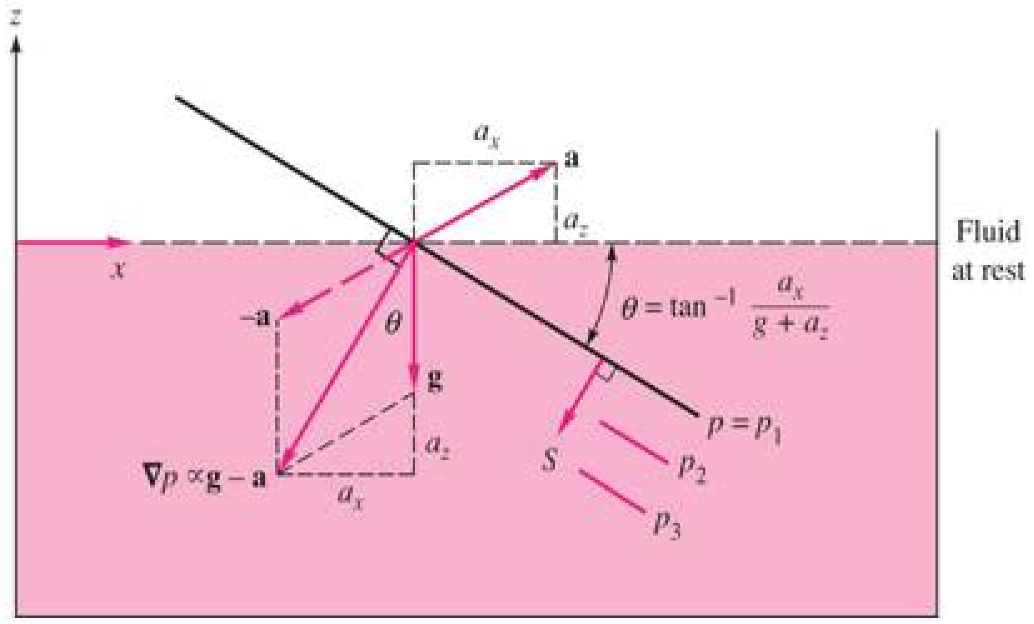
- A spherical body has a diameter of 1.5 *m*, weighs 8.5 *kN*, and is anchored to the sea floor with a cable as is shown in the figure. Calculate the tension of the cable when the body is completely immersed, assume $\gamma_{sea\text{-}water}$ =10.1 kN/m³.





Pressure in rigid-body motion

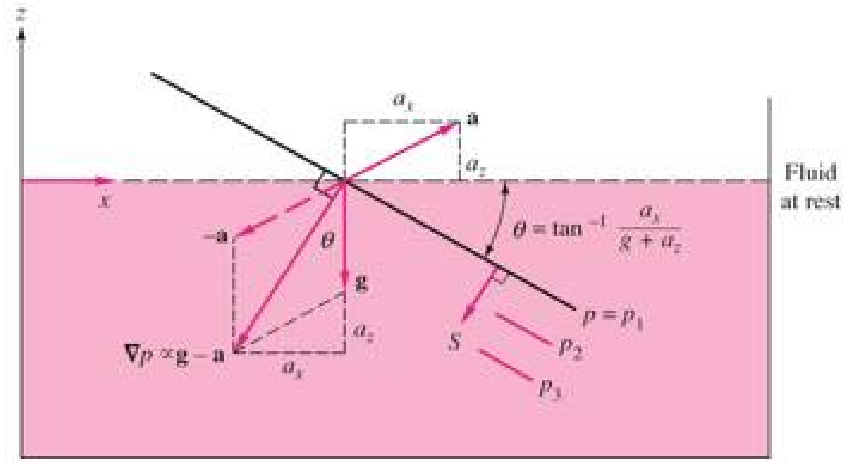
- Fluids move in rigid-body motion only when restrained by confining walls. In rigid-body motion, all particles are in combined translation and rotation, and there is no relative motion between particles.



$$\nabla p = \rho(\vec{g} - \vec{a})$$

Rigid-body motion cont'd

- The pressure gradient acts in the direction of g – a and lines of constant pressure are perpendicular to this direction and thus tilted at angle θ



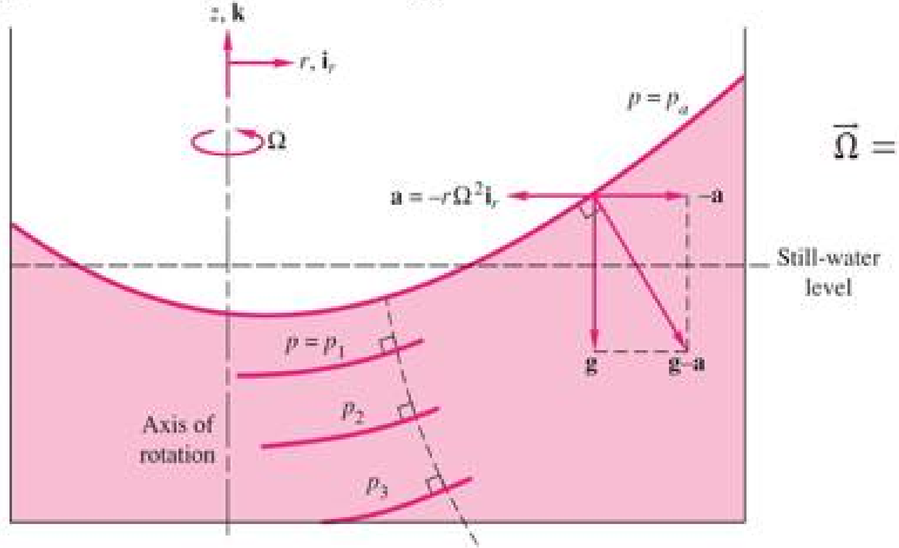
$$\theta = tan^{-1} \frac{a_x}{g + a_z}$$

- The rate of increase of pressure in the direction g – a is greater than in ordinary hydrostatics

$$\frac{dp}{ds} = \rho G \;\; where \;\; G = [a_x^2 + (g + a_z)^2]^{1/2}$$

Rigid-body rotation

- Consider a fluid rotating about the z-axis without any translation at a constant angular velocity Ω for a long time.



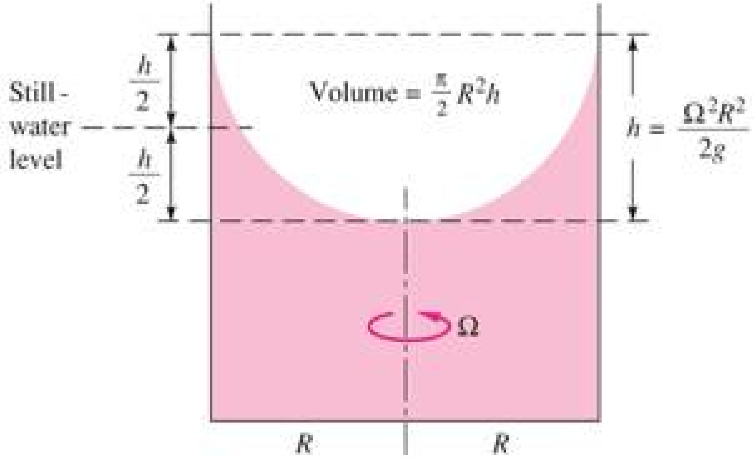
$\vec{\Omega} = \Omega k \quad \vec{r} = r i_r$

The acceleration id given by: $\Omega \times (\Omega \times \vec{r}) = -r\Omega^2 i_r$

The forced balance becomes: $\nabla p = \frac{\partial p}{\partial r} i_r + \frac{\partial p}{\partial z} k = \rho(g - a) = \rho(-gk + r\Omega^2 i_r)$

Rigid-body motion cont'd

The pressure field can be found by equating like components



$$\frac{\partial p}{\partial r} = \rho r\Omega^2 \quad \frac{\partial p}{\partial z} = -\gamma$$

- After integration with respect to *r* and *z* with $p=p_0$ at $(r,z) = (0,0)$:

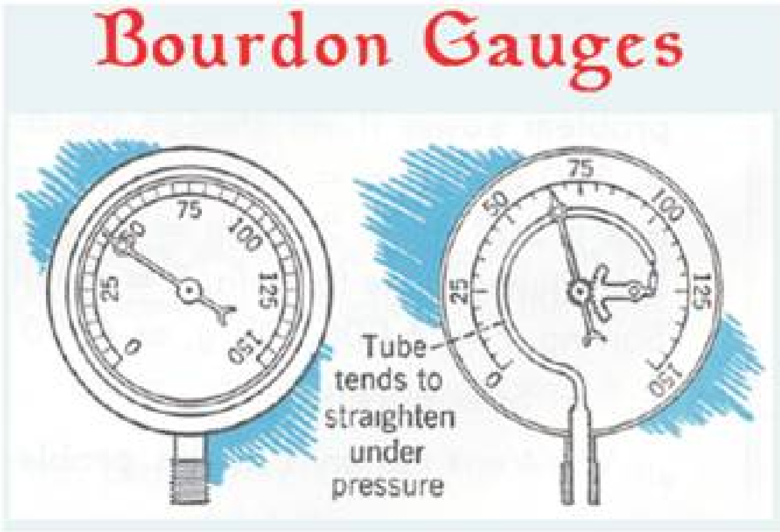
$$p = p_0 - \gamma z + \frac{1}{2}\rho r^2\Omega^2$$

- The pressure is linear in z and parabolic in r. The constant pressure surfaces can be calculated using

$$z = \frac{p_0 - p_1}{\gamma} + \frac{r^2\Omega^2}{2g} = a + br^2$$

Pressure measurement

- Pressure is the force per unit area and can be imagined as the effects related to fluid molecular bombardment of a surface.
- There are many devices for both a static fluid and moving fluid pressure measurements. Manometer, barometer, Bourdon gage, McLeod gage, Knudsen gage are only a few examples.



Dimensional Analysis

Dimensional Analysis

It is a pure mathematical technique to establish a relationship between physical quantities involved in a fluid phenomenon by considering their dimensions.

In dimensional analysis, from a general understanding of fluid phenomena, we first predict the physical parameters that will influence the flow, and then we group these parameters into dimensionless combinations which enable a better understanding of the flow phenomena. Dimensional analysis is particularly helpful in experimental work because it provides a guide to those things that significantly influence the phenomena; thus it indicates the direction in which experimental work should go.

Dimensional Analysis

Dimensional Analysis refers to the physical nature of the quantity (**Dimension**) and the type of unit used to specify it.

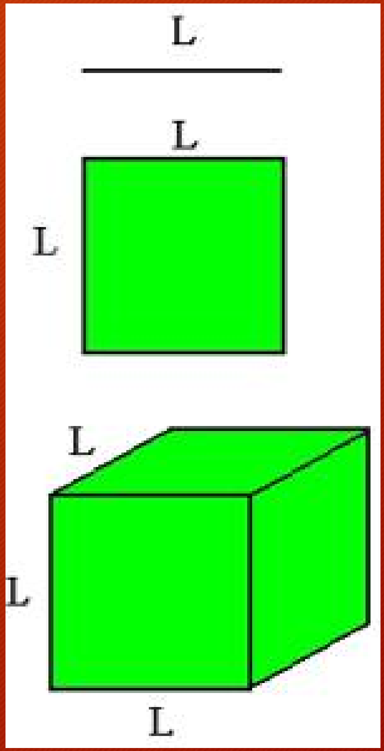
Distance has dimension L.

Area has dimension L^2.

Volume has dimension L^3.

Time has dimension T.

Speed has dimension L/T



Application of Dimensional Analysis

- Development of an equation for fluid phenomenon
- Conversion of one system of units to another
- Reducing the number of variables required in an experimental program
- Develop principles of hydraulic similitude for model study

Dimensional Reasoning & Homogeneity

- Principle of Dimensional Homogeneity

  The fundamental dimensions and their respective powers should be identical on either side of the sign of equality.

- Dimensional reasoning is predicated on the proposition that, for an equation to be true, then both sides of the equation must be numerically and dimensionally identical.

- To take a simple example, the expression x + y = z when x =1, y =2 and z =3 is clearly numerically true but only if the dimensions of x, y and z are identical. Thus

  1 elephant +2 aeroplanes =3 days is clearly nonsense but

  1 metre +2 metre =3 metre is wholly accurate.

  An equation is only dimensionally homogeneous if all the terms have the same dimensions.

Fundamental Dimensions

- We may express physical quantities in either mass-length-time (MLT) system or force-length-time (FLT) system.

  This is because these two systems are interrelated through Newton's second law, which states that force equals mass times acceleration,

  $F = ma$ 2nd Law of motion

  $F = ML/T^2$

  $F = MLT^{-2}$

- Through this relation, we can covert from one system to the other. Other than convenience, it makes no difference which system we use, since the results are the same.

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Expansion coefficient	β	Θ^{-1}	Θ^{-1}

Dimensions of Some Common Physical Quantities

[x], Length – L

[m], Mass – M

[t], Time – T

[v], Velocity – LT^{-1}

[a], Acceleration – LT^{-2}

[F], Force – MLT^{-2}

[Q], Discharge – L^3T^{-1}

[ρ], Mass Density – ML^{-3}

[P], Pressure – $ML^{-1}T^{-2}$

[*E*], Energy – ML^2T^{-2}

Basic Concepts

- All theoretical equations that relate physical quantities must be dimensionally homogeneous. That is, all the terms in an equation must have the same dimensions. For example

  Q = A.V (homogeneous)

  $L^3T^{-1} = L^3T^{-1}$

- We do, however sometimes use no homogeneous equation, the best known example in fluid mechanics being the Manning equation.

$$Q = VA = \left(\frac{1.49}{n}\right) AR^{\frac{2}{3}} \sqrt{S} \quad [U.S.]$$

$$Q = VA = \left(\frac{1.00}{n}\right) AR^{\frac{2}{3}} \sqrt{S} \quad [SI]$$

Mannings e[...]enerally the use of such equations is limited to specialized areas.

- To illustrate the basic principles of dimensional analysis, let us explore the equation for the speed V with which a pressure wave travels through a fluid. We must visualize the physical problem to consider physical factors probably influence the speed. Certainly the compressibility E_v must be factor; also the density and the kinematic viscosity of the fluid might be factors. The dimensions of these quantities, written in square brackets are

$$V=[LT^{-1}],\ E_v=[FL^{-2}]=[ML^{-1}T^{-2}],\ \rho=[ML^{-3}],\ v=[L^2T^{-1}]$$

Here we converted the dimensions of E_v into the MLT system using $F=[MLT^{-2}]$. Clearly, adding or subtracting such quantities will not produce dimensionally homogenous equations. We must therefore multiply them in such a way that their dimensions balance. So let us write

$$V=C\ E_v^a\ \rho^b\ v^d$$

Where C is a dimensionless constant, and let solve for the exponents a, b, and d substituting the dimensions, we get

$$(LT^{-1})=(ML^{-1}T^{-2})^a (ML^{-3})^b (L^2T^{-1})^d$$

To satisfy dimensional homogeneity, the exponents of each dimension must be identical on both sides of this equation. Thus

For M: $0 = a + b$

For L: $1 = -a -3b +2d$

For T: $-1 = -2a - d$

Solving these three equations, we get

$$a=1/2,\ b=-1/2,\ d=0$$

So that $V = C\sqrt{(E_v/\rho)}$

This identifies basic form of the relationship, and it also determines that the wave speed is not effected by the fluid's kinematic viscosity, v.

Dimensional analysis along such lines was developed by Lord Rayleigh.

Methods for Dimensional Analysis

- Rayleigh's Method
- Buckingham's ∏-method

Rayleigh's Method

Functional relationship between variables is expressed in the form of an exponential relation which must be dimensionally homogeneous

if "y" is a function of independent variables $x_1, x_2, x_3, \ldots.x_n$, then

In exponential form as $y = f(x_1, x_2, x_3, \ldots\ldots.x_n)$

$$y = \phi[(x_1)^a, (x_2)^b, (x_3)^c, \ldots\ldots.(x_n)^z]$$

Rayleigh's Method

Procedure

- Write fundamental relationship of the given data
- Write the same equation in exponential form
- Select suitable system of fundamental dimensions
- Substitute dimensions of the physical quantities
- Apply dimensional homogeneity
- Equate the powers and compute the values of the exponents
- Substitute the values of exponents
- Simplify the expression
- Ideal up to three independent variables, can be used for four.

Buckingham's ∏ method

- A more generalized method of dimension analysis developed by E. Buckingham and others and is most popular now. This arranges the variables into a lesser number of dimensionless groups of variables. Because Buckingham used ∏ (pi) to represent the product of variables in each groups, we call this method Buckingham pi theorem.
- "If 'n' is the total number of variables in a dimensionally homogenous equation containing 'm' fundamental dimensions, then they may be grouped into (n-m) ∏ terms.

$$f(X_1, X_2, \ldots\ldots X_n) = 0$$

then the functional relationship will be written as

$$\Phi\ (\Pi_1\ ,\ \Pi_2\ , \ldots\ldots\ldots\ldots \Pi_{n-m}) = 0$$

The final equation obtained is in the form of:

$$\Pi_1 = f\ (\Pi_2, \Pi_3, \ldots\ldots\ldots\ldots \Pi_{n-m}) = 0$$

- Suitable where $n \geq 4$
- Not applicable if $(n-m) = 0$

Buckingham's ∏ method

Procedure

- List all physical variables and note 'n' and 'm'.
  n = Total no. of variables
  m = No. of fundamental dimensions (That is, [M], [L], [T])
- Compute number of ∏-terms by (n-m)
- Write the equation in functional form
- Write equation in general form
- Select repeating variables. Must have all of the 'm' fundamental dimensions and should not form a ∏ among themselves
- Solve each ∏-term for the unknown exponents by dimensional homogeneity.

Buckingham's ∏ method

Example:

- Let us apply Buckingham's ∏ method to an example problem that of the drag forces F_D exerted on a submerged sphere as it moves through a viscous fluid. We need to follow a series of following steps when applying Buckingham's ∏ theorem.
- **Step 1:** Visualize the physical problem, consider the factors that are of influence and list and count the n variables.

We must first consider which physical factors influence the drag force. Certainly, the size of the sphere and the velocity of the sphere must be important. The fluid properties involved are the density ρ and the viscosity μ. Thus we can write

$$f\,(F_D, D, V, \rho, \mu) = 0$$

Here we used D, the sphere diameter, to represent sphere size, and f stands for "some function". We see that n = 5. Note that the procedure cannot work if any relevant variables are omitted. Experimentation with the procedure and experience will help determine which variables are relevant.

Buckingham's ∏ method

- **Step 2:** Choose a dimensional system (MLT or FLT) and list the dimensions of each variables. Find m, the number of fundamental dimensions involved in all the variables.

  Choosing the MLT system, the dimensions are respectively

  MLT^{-2} , L , LT^{-1} , ML^{-3} , $ML^{-1}T^{-1}$

  We see that M, L and T are involved in this example. So m = 3.

- **Step 3:** Determine n-m, the number of dimensionless ∏ groups needed. In our example this is 5 – 3 = 2, so we can write $\Phi(\Pi_1. \Pi_2) = 0$

- **Step 4:** Form the ∏ groups by multiplying the product of the primary (repeating) variables, with unknown exponents, by each of the remaining variables, one at a time. We choose ρ, D, and V as the primary variables. Then the ∏ terms are

$$\Pi_1 = D^a V^b \rho^c F_D$$

$$\Pi_2 = D^a V^b \rho^c \mu^{-1}$$

Buckingham's ∏ method

- **Step 5:** To satisfy dimensional homogeneity, equate the exponents of each dimension on both sides on each pi equation and so solve for the exponents

$\Pi_1 = D^a V^b \rho^c F_D = (L)^a (LT^{-1})^b (ML^{-3})^c (MLT^{-2}) = M^0L^0T^0$

Equate exponents:

L: $a + b - 3c + 1 = 0$

M: $c + 1 = 0$

T: $-b - 2 = 0$

We can solve explicitly for

$b = -2, \quad c = -1, \quad a = -2$

Therefore

$$\Pi_1 = D^{-2} V^{-2} \rho^{-1} F_D = F_D/(\rho V^2 D^2)$$

Buckingham's ∏ method

Finally, add viscosity to D, V, and ρ to find Π_2 . Select any power you like for viscosity. By hindsight and custom, we select the power -1 to place it in the denominator

$\Pi_1 = D^a V^b \rho^c \mu^{-1} = (L)^a (LT^{-1})^b (ML^{-3})^c (ML^{-1}T^{-1})^{-1} = M^0L^0T^0$

Equate exponents:

L: $a + b - 3c + 1 = 0$

M: $c - 1 = 0$

T: $-b + 1 = 0$

We can solve explicitly for

$b = 1, \quad c = 1, \quad a = 1$

Therefore,

$\Pi_2 = D^1 V^1 \rho^1 \mu^{-1} = (D V \rho)/(\mu) = R$ = Reynolds Number

R = Reynolds Number= Ratio of inertia forces to viscous forces

Check that all Π_s are in fact dimensionless

Buckingham's ∏ method

Rearrange the pi groups as desired. The pi theorem states that the $\prod_s$ are related. In this example hence

$$F_D/(\rho V^2 D^2) = \Phi (R)$$

So that $F_D = \rho V^2 D^2 \ \Phi (R)$

We must emphasize that dimensional analysis does not provide a complete solution to fluid problems. It provides a partial solution only. The success of dimensional analysis depends entirely on the ability of the individual using it to define the parameters that are applicable. If we omit an important variable. The results are incomplete, and this may lead to incorrect conclusions. Thus, to use dimensional analysis successfully, one must be familiar with the fluid phenomena involved.

Modeling and Similitude

- To develop the procedures for designing models so that the model and prototype will behave in a similar fashion…….

Model vs Prototype Model Prototype

Model ? A model is a representation of a physical system that may be used to predict the behavior of the system in some desired respect. Mathematical or computer models may also conform to this definition, our interest will be in physical model.

Prototype? The physical system for which the prediction are to be made.

Models that resemble the prototype but are generally of a different size, may involve different fluid, and often operate under different conditions.

Usually a model is smaller than the prototype.

Occasionally, if the prototype is very small, it may be advantageous to have a model that is larger than the prototype so that it can be more easily studied. For example, large models have been used to study the motion of red blood cells.

Model vs Prototype

- With the successful development of a valid model, it is possible to predict the behavior of the prototype under a certain set of conditions.
- There is an inherent There is an inherent danger in the use of models danger in the use of models in that predictions can be made that are in error and the error not detected until the can be made that are in error and the error not detected until the prototype is found not to perform as predicted.
- It is imperative that It is imperative that the model be properly designed and tested and the model be properly designed and tested and that the results be interpreted correctly.

Similarity of Model and Prototype

- What conditions must be met to ensure the similarity of model and prototype?
- **Geometric Similarity**
  - ⇨Model and prototype have same shape.
  - ⇨Linear dimensions on model and prototype correspond within constant scale factor.
- **Kinematic Similarity**
  - ⇨Velocities at corresponding points on model and prototype differ only by a constant scale factor.
- **Dynamic Similarity**
  - ⇨Forces on model and prototype differ only by a constant scale factor.

Theory of Models

The theory of models can be readily developed by using the principles of dimensional analysis.

For given problem which can be described in terms of a set of pi terms as

$$\Pi_1 = \phi(\Pi_2, \Pi_3, ,, \Pi_n)$$

This relationship can be formulated with a knowledge of the general nature of the physical phenomenon and the variables involved.

This equation applies to any system that is governed by the same variables.

Theory of Models

A similar relationship can be written for a model of this prototype; that is,

$$\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, , , , \Pi_{nm})$$

where <u>the form of the function will be the same</u> as long as the same phenomenon is involved in both the prototype and the model.

The prototype and the model must have the same phenomenon.

Theory of Models

Model design (the model is designed and operated) conditions, also called similarity requirements or modeling laws.

$$\Pi_2 = \Pi_{2m} \qquad \Pi_3 = \Pi_{3m} \ldots\ldots \Pi_n = \Pi_{nm}$$

The form of Φ is the same for model and prototype, it follows that

$$\Pi_1 = \Pi_{1m}$$

This is the **desired prediction equation** and indicates that the measured of Π_{1m} obtained with the model will be equal to the corresponding Π_1 for the prototype as long as the other Π parameters are equal.

Theory of Models

The prototype and the model must have the same phenomenon.

For prototype $\Pi_1 = \phi(\Pi_2, \Pi_3, ,, \Pi_n)$

For model $\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, ,,, \Pi_{nm})$

Theory of Models

- The model is designed and operated under the following conditions (called design conditions, also called similarity requirements or modeling laws)

$$\Pi_2 = \Pi_{2m} \qquad \Pi_3 = \Pi_{3m} \ldots\ldots \Pi_n = \Pi_{nm}$$

- The measured of Π_{1m} obtained with the model will be equal to the corresponding Π_1 for the prototype.

$$\Pi_1 = \Pi_{1m}$$ Called prediction equation