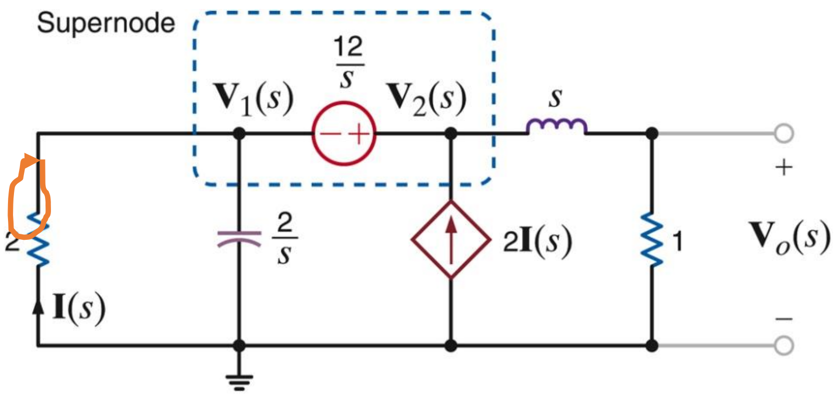
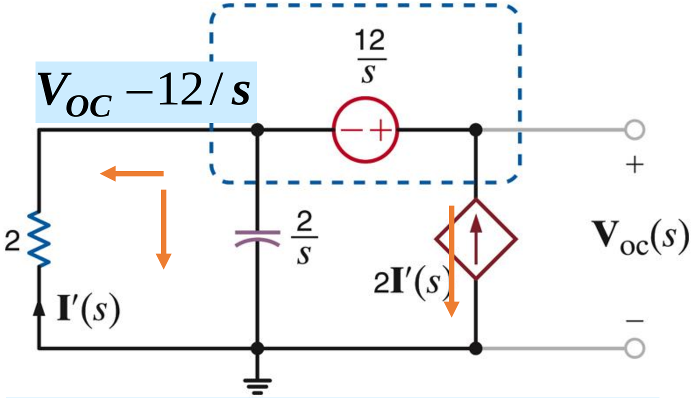
Continued ...

Compute $V_o(s)$ using Thevenin's theorem

- keep dependent source and controlling variable in the same sub-circuit
- Make sub-circuit to be reduced as simple as possible
- Try to leave a simple voltage divider after reduction to Thevenin equivalent



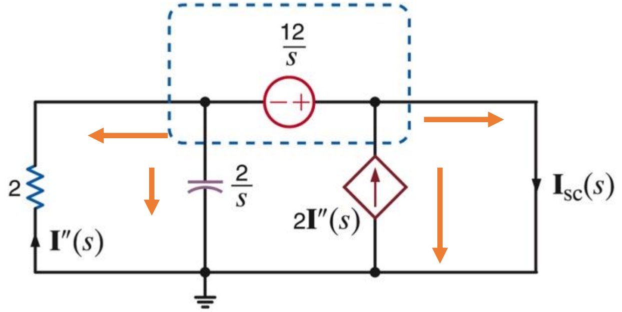


$V_{OC} - 12/s$

$$\frac{V_{OC}-12/s}{2}+\frac{V_{OC}-12/s}{2/s}-2I'=0$$

$$I'=-\frac{V_{OC}-12/s}{2} \qquad V_{OC}(s)=\frac{12}{s}$$

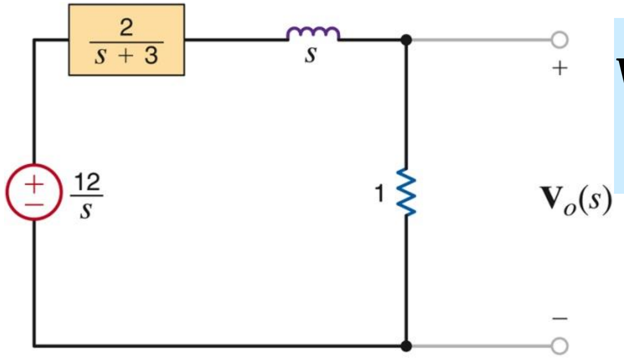
$$I'=0$$



– $12/s$ +

$$I_{SC}-2I''-I''-2I''/(2/s)=0 \qquad I''=6/s$$

$$I_{SC}=\frac{6(s+3)}{s} \qquad Z_{TH}=\frac{V_{OC}(s)}{I_{SC}(s)}=\frac{2}{s+3}$$



$$V_o(s)=\frac{1}{1+s+\dfrac{2}{s+3}}\times\frac{12}{s}$$

GEAUX

Continued … Computing the inverse Laplace transform

Analysis in the s-domain has established that the Laplace transform of the output voltage is

$$V_o(s) = \frac{12(s+3)}{s(s^2+4s+5)}$$

$$s^2+4s+5 = (s+2-j1)(s+2+j1) = (s+2)^2+1$$

$$V_o(s) = \frac{12(s+3)}{s(s+2-j1)(s+2+j1)} = \frac{K_o}{s} + \frac{K_1}{(s+2-j1)} + \frac{K_1^*}{(s+2+j1)}$$

$$K_o = sV_o(s)|_{s=0} = 36/5$$

$$\frac{K_1}{(s+\alpha-j\beta)} + \frac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t + \angle K_1)u(t)$$

$$K_1 = (s+2-j1)V_o(s)|_{s=-2+j1} = \frac{12(1+j1)}{(-2+j1)(j2)} = \frac{12\sqrt{2}\angle 45°}{\sqrt{5}\angle 153.43°(2\angle 90°)}$$

$$= 3.79\angle -198.43° = 3.79\angle 161.57°$$

$$v_o(t) = \left(\frac{36}{5} + 7.59e^{-2t}\cos(t+161.57°)\right)u(t)$$

One can also use quadratic factors...

$$V_o(s) = \frac{12(s+3)}{s\left[(s+2)^2+1\right]} = \frac{C_o}{s} + \frac{C_1(s+2)}{(s+2)^2+1} + \frac{C_2}{(s+2)^2+1}$$

$$C_o = sV_o(s)|_{s=0} = 36/5$$

$$\frac{C_1(s+\alpha)}{(s+\alpha)^2+\beta^2} + \frac{C_2\beta}{(s+\alpha)^2+\beta^2} \leftrightarrow e^{-\alpha t}[C_1\cos\beta t + C_2\sin\beta t]u(t)$$

$$12(s+3) = C_o((s+2)^2+1) + s[C_1(s+2)+C_2]$$

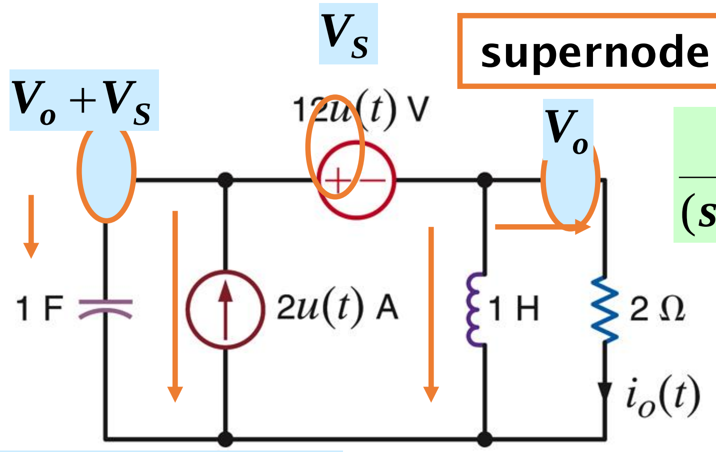
$$s=-2 \Rightarrow 12 = C_o - 2C_2 \Rightarrow C_2 = 36/10 - 6 = -12/5$$

Equating coefficients of s^2 : $0 = C_o + C_1 \Rightarrow C_1 = -36/5$

$$v_o(t) = \left[\frac{36}{5}(1-e^{-2t}\cos t) - \frac{12}{5}e^{-2t}\sin t\right]u(t)$$

LEARNING EXTENSION Find $i_o(t)$ using node equations

Assume zero initial conditions
Implicit circuit transformation to s-domain



$$\frac{K_1}{(s+\alpha-j\beta)} + \frac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t + \angle K_1)u(t)$$

KCL at supernode

$$Cs(V_o(s)+V_S(s)) - \frac{2}{s} + \frac{V_o(s)}{s} + \frac{V_o(s)}{2} = 0$$

$$V_S(s) = \frac{12}{s}, \quad I_o(s) = \frac{V_o(s)}{2}$$

Doing the algebra

$$I_o(s) = \frac{1-6s}{s^2+0.5s+1} = \frac{1-6s}{\left(s+\frac{1}{4}\right)^2 + \frac{15}{16}}$$

$$I_o(s) = \frac{1-6s}{\left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)\left(s+\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)} = \frac{K_1}{\left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)} + \frac{K_1^*}{\left(s+\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)}$$

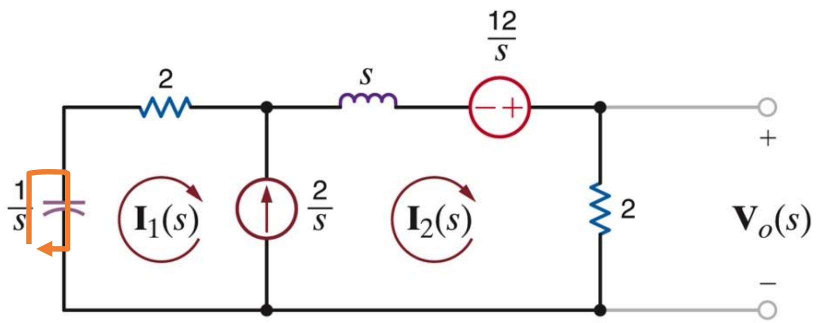
$$K_1 = \left(s+\frac{1}{4}-j\frac{\sqrt{15}}{4}\right)I_o(s)\Big|_{s=-\frac{1}{4}+j\frac{\sqrt{15}}{4}} = \frac{1-6\left(-\frac{1}{4}+j\frac{\sqrt{15}}{4}\right)}{2j\frac{\sqrt{15}}{4}}$$

$$K_1 = \frac{6.33\angle -66.72°}{0.97\angle 90°} = 6.53\angle -156.72°$$

$$i_o(t) = 13.06e^{-\frac{t}{4}}\cos\left(\frac{\sqrt{15}}{4}t - 156.72°\right)$$

GEAUX

LEARNING EXTENSION Find $v_o(t)$ using loop equations



supermesh

constraint due to source

$$\frac{2}{s} = I_2 - I_1$$

KVL on supermesh

$$\frac{1}{s}I_1 + 2I_1 + sI_2 - \frac{12}{s} + 2I_2 = 0$$

$$I_2(s) = \frac{16s+2}{s(s^2+4s+1)} = \frac{16s+2}{s(s+0.27)(s+3.73)}$$

Determine inverse transform

$$I_2(s) = \frac{K_0}{s} + \frac{K_1}{s+0.27} + \frac{K_2}{s+3.73}$$

$$K_0 = sI_2(s)|_{s=0} = 2$$

$$K_1 = (s+0.27)I_2(s)|_{s=-0.27} = \frac{16(-0.27)+2}{(-0.27)(-0.27+3.73)} = 2.48$$

$$K_2 = (s+3.73)I_2(s)|_{s=-3.73} = \frac{16(-3.73)+2}{(-3.73)(-3.73+0.27)} = -4.47$$

$$i_2(t) = \left(2 + 2.48e^{-0.27t} - 4.47e^{-3.73t}\right)u(t)$$

$$v_o(t) = 2i_2(t)$$

GEAUX

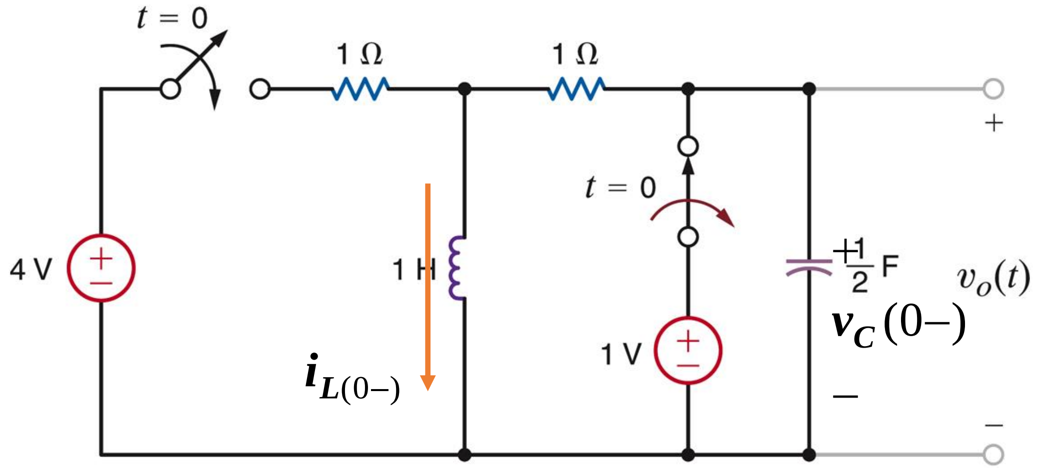
TRANSIENT CIRCUIT ANALYSIS USING LAPLACE TRANSFORM

For the study of transients, especially transients due to switching, it is important to determine initial conditions. For this determination, one relies on the properties:

1. Voltage across capacitors cannot change discontinuously
2. Current through inductors cannot change discontinuously

LEARNING EXAMPLE

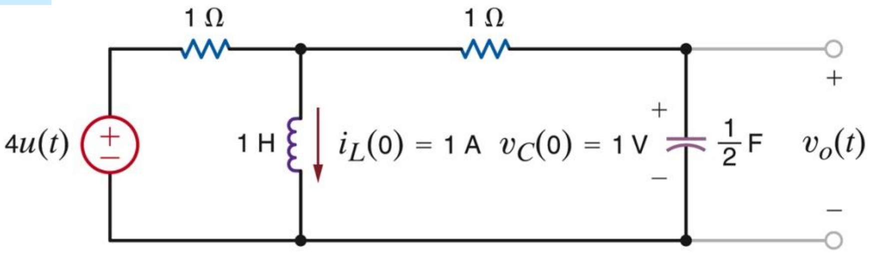
Determine $v_o(t), t > 0$



Assume steady state for t<0 and determine voltage across capacitors and currents through inductors

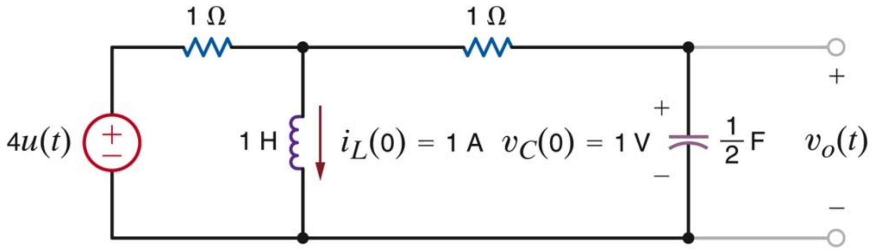
For DC case capacitors are open circuit inductors are shortcircuit

$v_C(0-) = 1V,\ i_L(0-) = 1A$



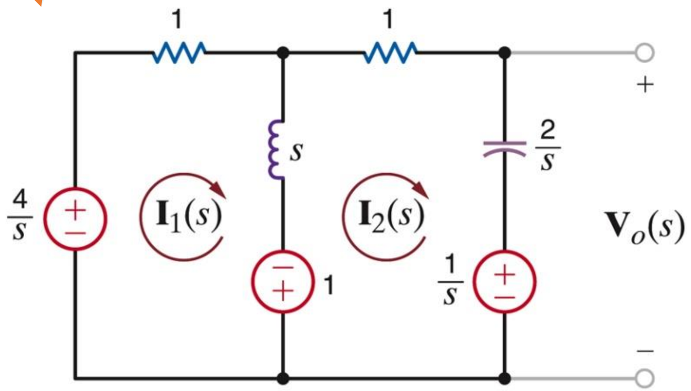
Circuit for t>0

GEAUX



Laplace

Circuit for t>0



Use mesh analysis

$$(s+1)I_1 - sI_2 = \frac{4}{s} + 1$$

$$-sI_1 + (s+1+\frac{2}{s})I_2 = -\frac{1}{s} - 1$$

$$I_2(s) = \frac{2s-1}{2s^2+3s+2}$$

$$V_o(s) = \frac{2}{s}I_2(s) + \frac{1}{s}$$

$$V_o(s) = \frac{2s+7}{2s^2+3s+2}$$

Now determine the inverse transform

$$b^2 - 4ac < 0 \Rightarrow \text{complex conjugate roots}$$

$$V_o(s) = \frac{K_1}{s + \frac{3}{4} - j\frac{\sqrt{7}}{4}} + \frac{K_1^*}{s + \frac{3}{4} + j\frac{\sqrt{7}}{4}}$$

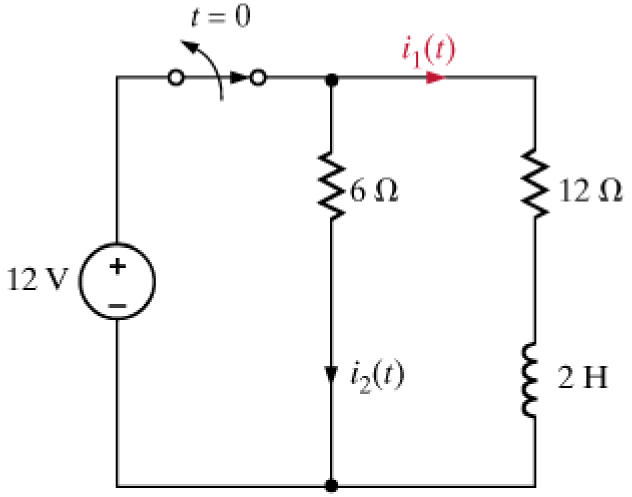
$$K_1 = \left(s + \frac{3}{4} - j\frac{\sqrt{7}}{4}\right)V_o(s)\Big|_{s=-\frac{3}{4}+j\frac{\sqrt{7}}{4}} = 2.14\angle -76.5°$$

$$\frac{K_1}{(s+\alpha-j\beta)} + \frac{K_1^*}{(s+\alpha+j\beta)} \leftrightarrow 2|K_1|e^{-\alpha t}\cos(\beta t + \angle K_1)u(t)$$

$$v_o(t) = 4.28e^{-\frac{3}{4}t}\cos(\frac{\sqrt{7}}{4}t - 76.5°)$$

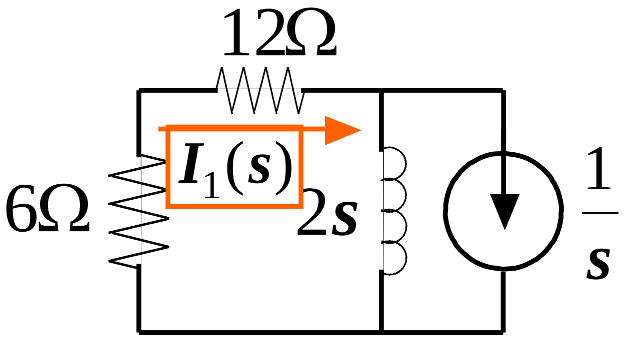
GEAUX

LEARNING EXTENSION Determine $i_1(t),\ t > 0$



Initial current through inductor

$$i_L(0-) = i_L(0+) = 1A$$



$$I_1(s) = \frac{2s}{2s+18} \times \frac{1}{s}$$

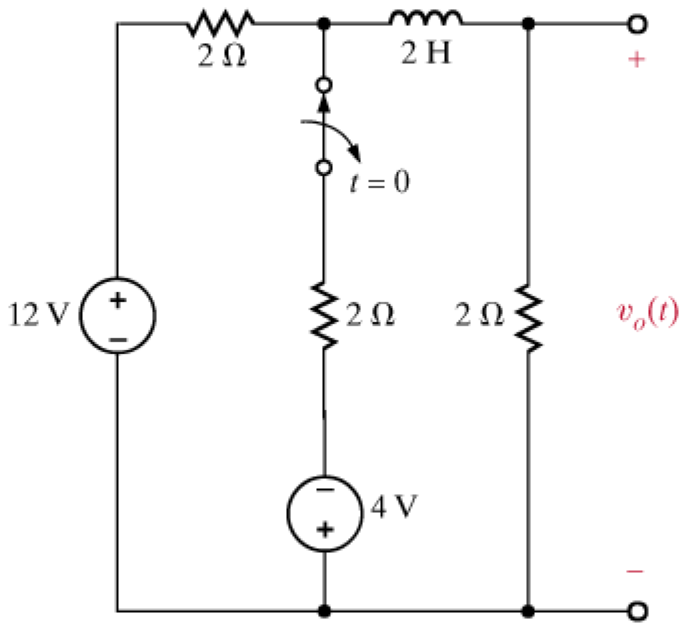
Current divider

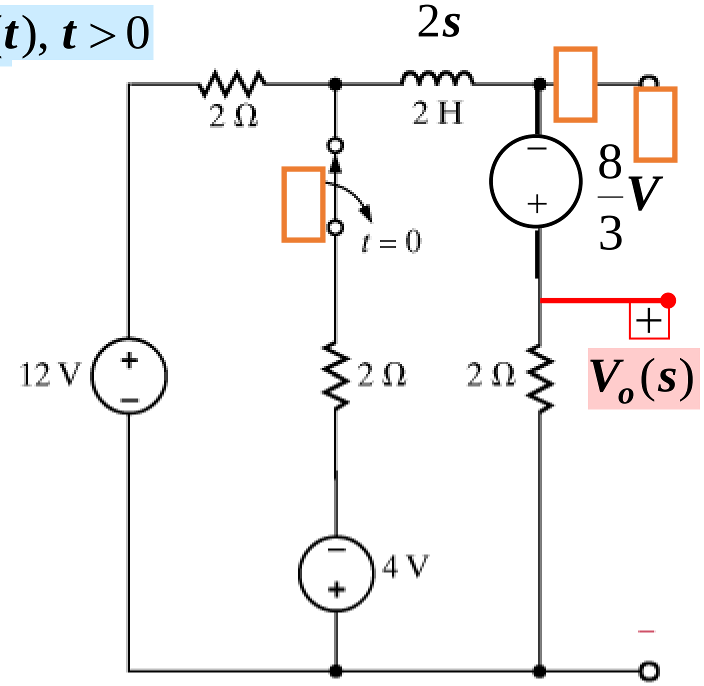
$$I_1(s) = \frac{1}{s+9} \rightarrow i_1(t) = e^{-9t}u(t)$$

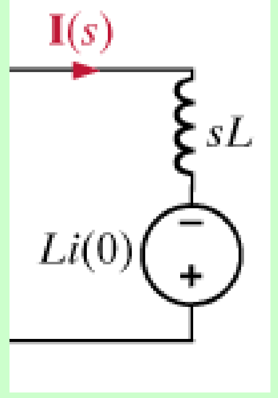
GEAUX

LEARNING EXTENSION

Determine $v_o(t),\ t > 0$







$$V_o(s) = \frac{2}{4+2s} \times \left(\frac{12}{s} + \frac{8}{3}\right)$$ (voltage divider)

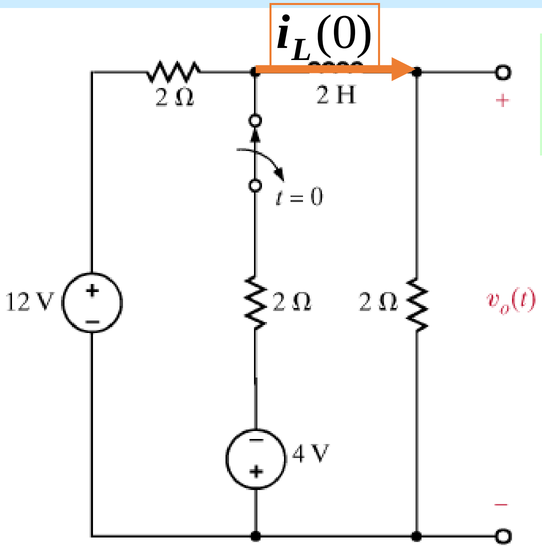
$$V_o(s) = \frac{(8s+36)}{3s(s+2)} = \frac{K_1}{s} + \frac{K_2}{s+2}$$

$$K_1 = sV_o(s)|_{s=0} = 6$$

$$K_2 = (s+2)V_o(s)|_{s=-2} = -\frac{10}{3}$$

$$v_o(t) = \left(6 - \frac{8}{3}e^{-2t}\right)u(t)$$

Determine initial current through inductor



Use source superposition

$$i_{12V} = 2A$$

$$i_{4V} = -\frac{2}{3}A$$

$$i_L(0) = \frac{4}{3}A$$

GEAUX

TRANSFER FUNCTION

$$H(s) = \frac{Y(s)}{X(s)}$$

If the model for the system is a differential equation

$$b_n \frac{d^n y}{dt^n} + b_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots + b_1 \frac{dy}{dt} + b_o y = a_m \frac{d^m x}{dt^m} + a_{m-1} \frac{d^{m-1} x}{dt^{m-1}} + \ldots + a_1 \frac{dx}{dt} + a_o x$$

If all initial conditions are zero

$$\mathcal{L}\left[\frac{d^k y}{dt^k}\right] = s^k Y(s)$$

$$b_n s^n Y(s) + \ldots + b_1 s Y(s) + b_0 Y(s) = a_m s^m X(s) + \ldots + a_1 s X(s) + a_0 X(s)$$

$$Y(s) = \frac{b_n s^n + \ldots + b_1 s + b_0}{a_m s^m + \ldots + a_1 s + a_0} X(s)$$

$$H(s) = \frac{b_n s^n + \ldots + b_1 s + b_0}{a_m s^m + \ldots + a_1 s + a_0}$$

For the impulse function

$$x(t) = \delta(t) \Rightarrow X(s) = 1$$

H(s) can also be interpreted as the Laplace transform of the output when the input is an impulse and all initial conditions are zero

The inverse transform of H(s) is also called the impulse response of the system

If the impulse response is known then one can determine the response of the system to ANY other input

LEARNING EXAMPLE A network has impulse response $h(t) = e^{-t}u(t)$

Determine the response, $v_o(t)$, for the input $v_i(t) = 10e^{-2t}u(t)$

In the Laplace domain, $Y(s)=H(s)X(s)$

$$\therefore V_o(s) = H(s)V_i(s)$$

$$h(t) = e^{-t}u(t) \Rightarrow H(s) = \frac{1}{s+1}$$

$$v_i(t) = 10e^{-2t}u(t) \Rightarrow V_i(s) = \frac{10}{s+2}$$

$$V_o(s) = \frac{10}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$K_1 = (s+1)V_o(s)|_{s=-1} = 10$$

$$K_2 = (s+2)V_o(s)|_{s=-2} = -10$$

$$v_o(t) = 10\left(e^{-t} - e^{-2t}\right)u(t)$$

GEAUX